Randomized Wait-Free Consensus using An Atomicity Assumption

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1. Introduction
    - Problem Statement
    - Assumptions
2. Proposed Algorithm
    - Main Ideas
    - Example: Binary Consensus
    - Correctness
3. Model Checking with PRISM
4. Conclusions

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Randomization: processes can toss coins.

- (*Probabilistic Termination*) With probability 1, every live process eventually decides on some value.

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- Adversary model:
  - atomic random-write operation.

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Adversaries may have complete or partial access to dynamic information, thus different complexity results.

Adversary Models and Expected Total Work

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\begin{tikzcd}
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\text{Atomic RandWrite} \arrow[dr] & & \text{Write-Oblivious} \arrow[dl] & \\
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Strong: $\tilde{O}(N^2)$

Atomic RandWrite, Write-Oblivious, Oblivious: $\left.\right\} \tilde{O}(N)$

Complete information over execution history.

- Bracha and Rachman, 1991: $O(N^2 \log N)$
- Aspnes, 1998: $\Omega(N^2 / \log^2 N)$

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Consensus gets easier when adversaries "know" less.

Example: $O(N \log N)$ against *write-oblivious* adversaries in MWMR model [Aumann, 1997].

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Based on [Chor, Isreali and Li, 1994]: $O(N^2)$.

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Different from consensus from shared-coin (often based on voting) e.g. [Bracha and Rachman, 1991] and [Aumann, 1997].

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$K \times R$ one-bit registers

$K = 2$

$R = 2 \log N + 2 = 6$

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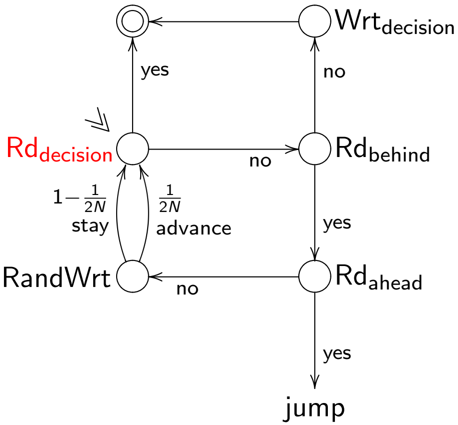

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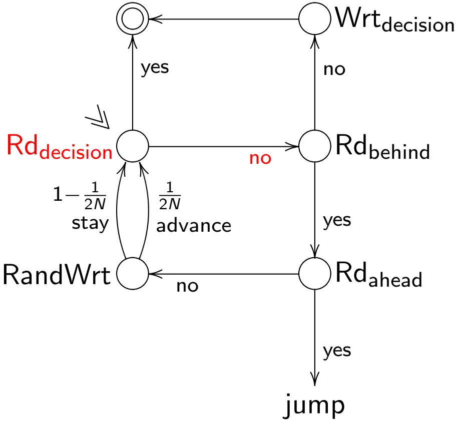

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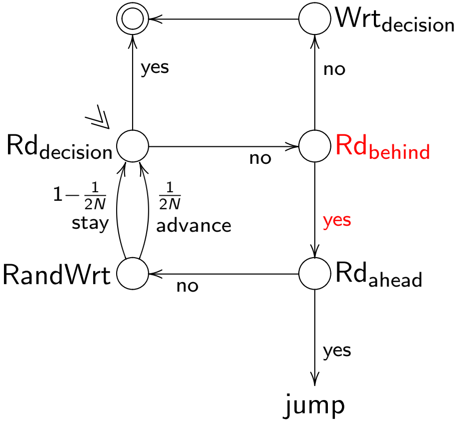

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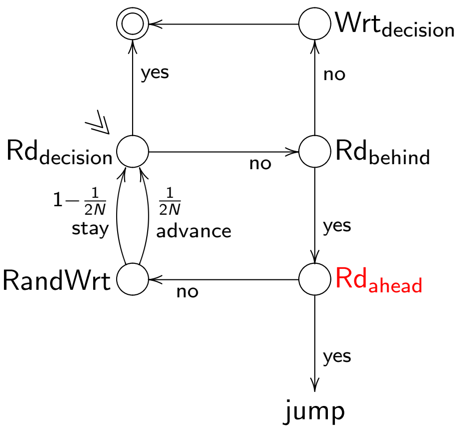

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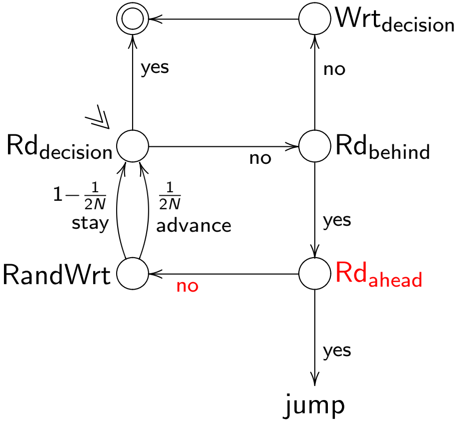

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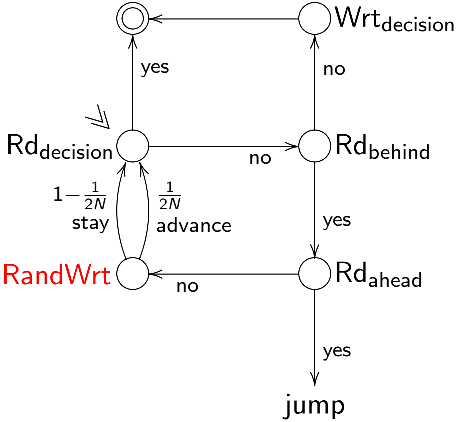

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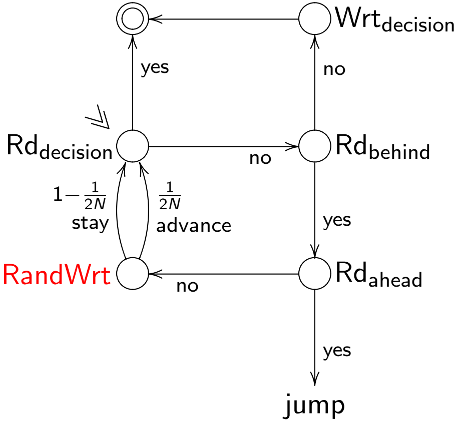

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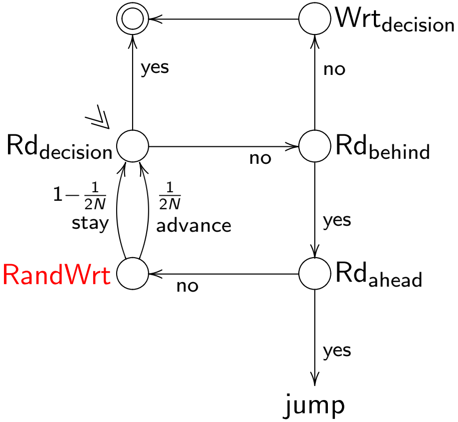

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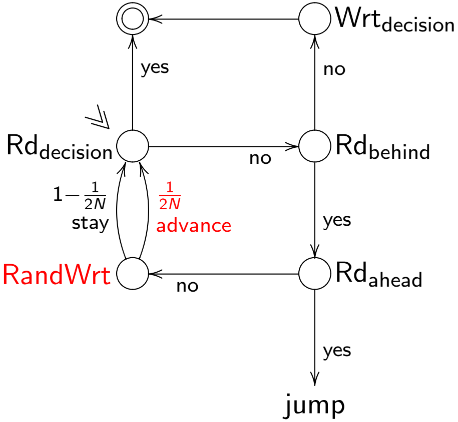

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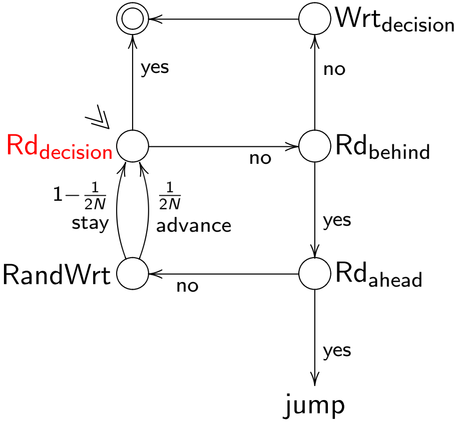

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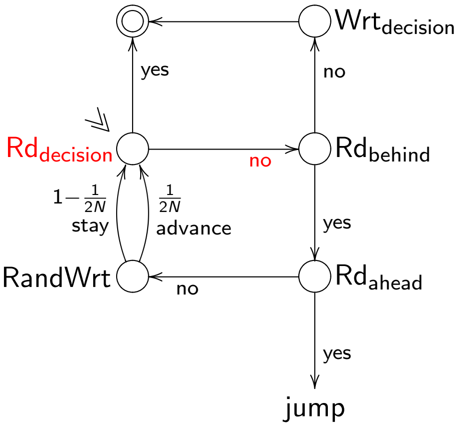

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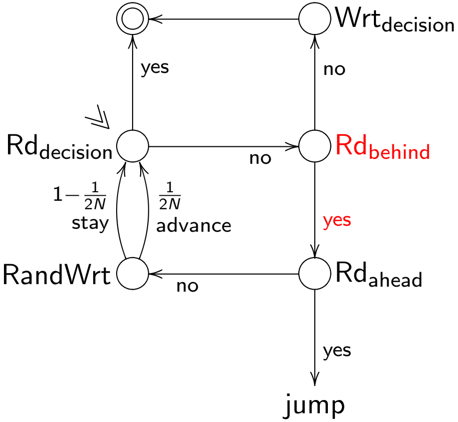

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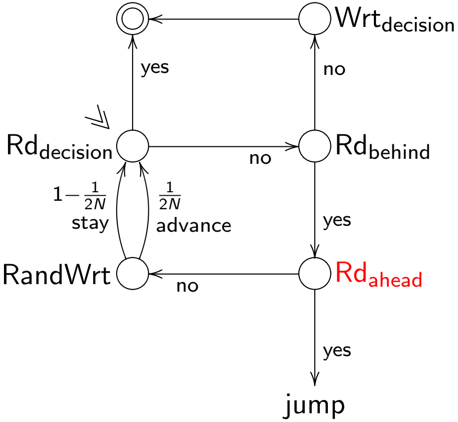

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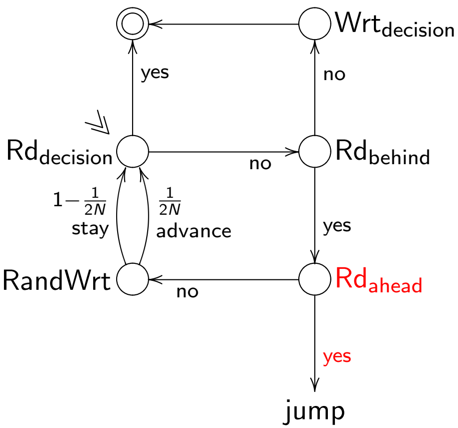

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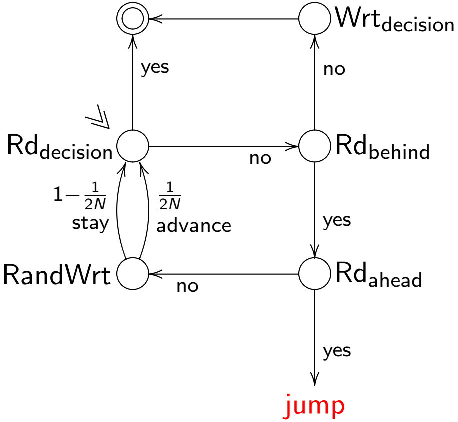

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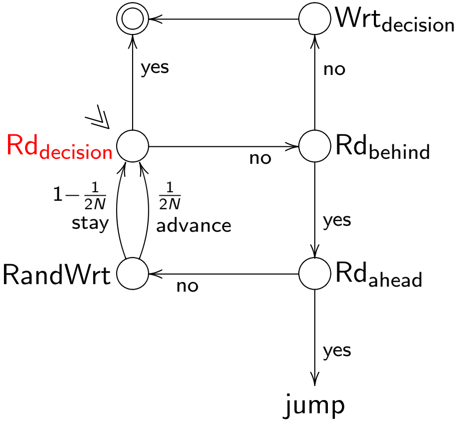

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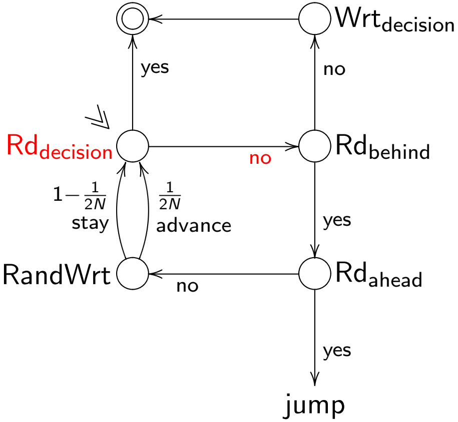

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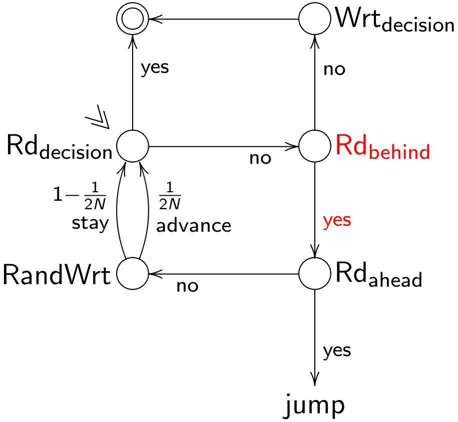

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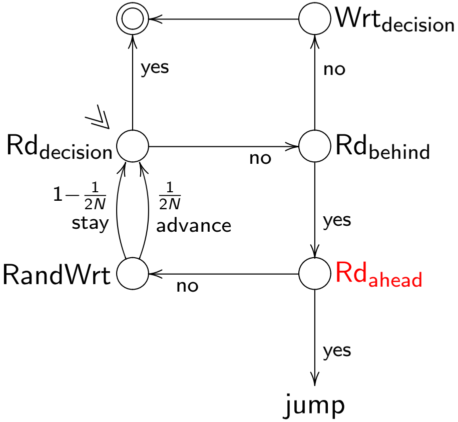

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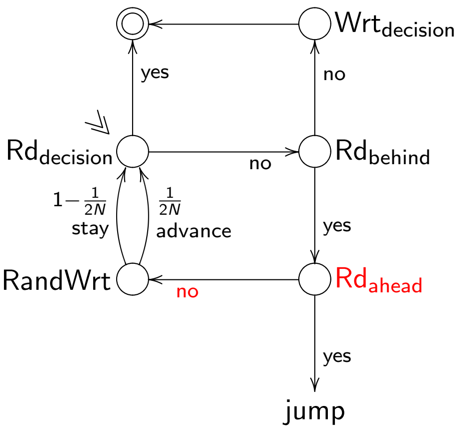

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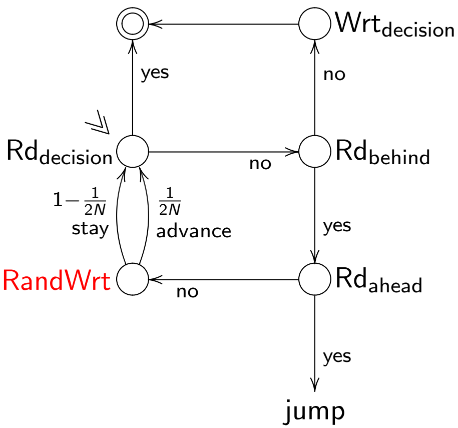

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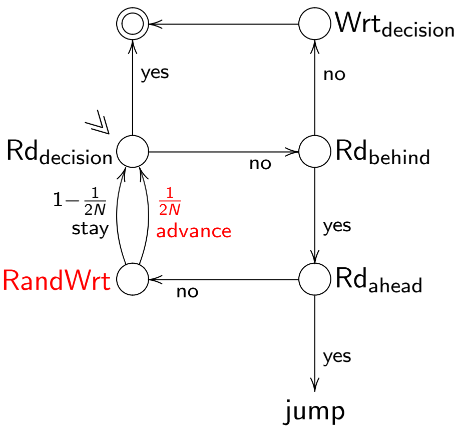

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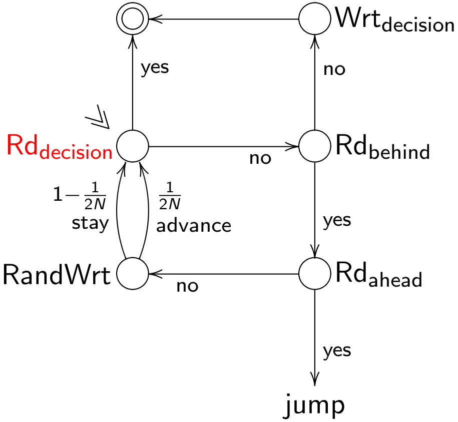

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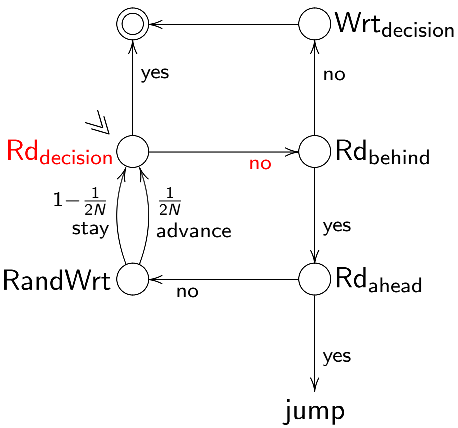

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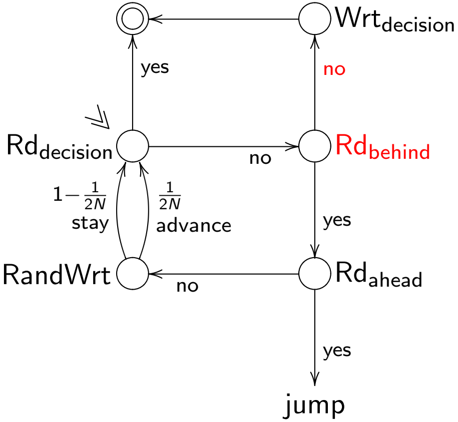

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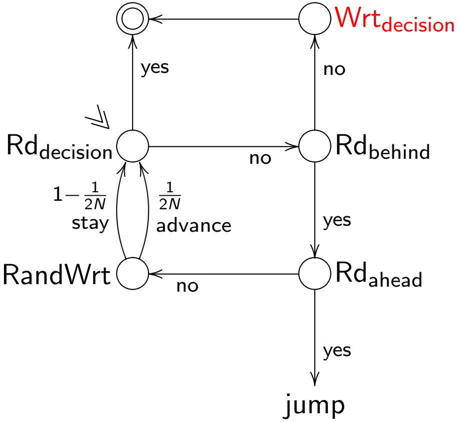
0	<u>1</u>
0	0
0	1
0	1
1	1
1	1



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Proof by contradiction: disagreement implies two distinct values eliminate each other.

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- Wait-free; $O(N \log(\log N))$.

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Caution: non-determinism resolved under perfect information.

Model Checking Results

N	R	#Phases	Model Construction		Agreement
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2	2	30	42,320	4	0.025
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			Time(sec)	MinProb	AnalyticBd
2	2	30	6	0.745	0.511
3	4	90	2,662	0.971	0.667
4	2	60	602	0.755	0.511
4	4	40	55,795	0.765	0.750

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- Partial information model checking?

– End –

Other Weak-Adversary Algorithms

Write-oblivious: unread register content hidden from adversary.

- Chandra, 1996: $O(N \log^2 N)$, MWMR
- Aumann, 1997: $O(N \log^4 N)$, SWMR; $O(N \log N)$, MWMR

Value-oblivious: all parameter values hidden from adversary.

- Aumann and Kapah-Levy, 1999: $O(N \log N \cdot e^{\sqrt{\log N}})$, SWSR
- Aumann and Bender, 2004: $O(N \log^2 N)$, MWMR

Oblivious: predetermined list of process names, independent of dynamic random choices.

- Aumann, Bender and Zhang, 1997: $O(N \log N \log(\log N))$ for N processes *and* N words, MWMR