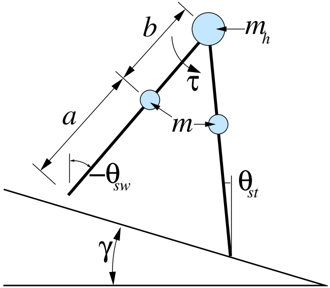
LQR-Trees: Feedback Motion Planning on Sparse Randomized Trees

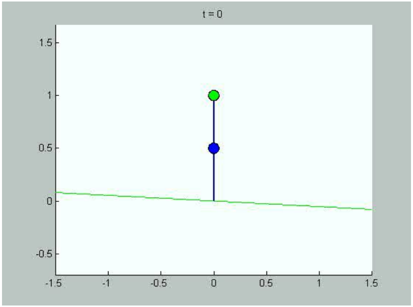
Russ Tedrake

Associate Professor
MIT Computer Science and Artificial Intelligence Lab

RSS, Seattle
June 29, 2009

A Motivating Example: The Compass Gait

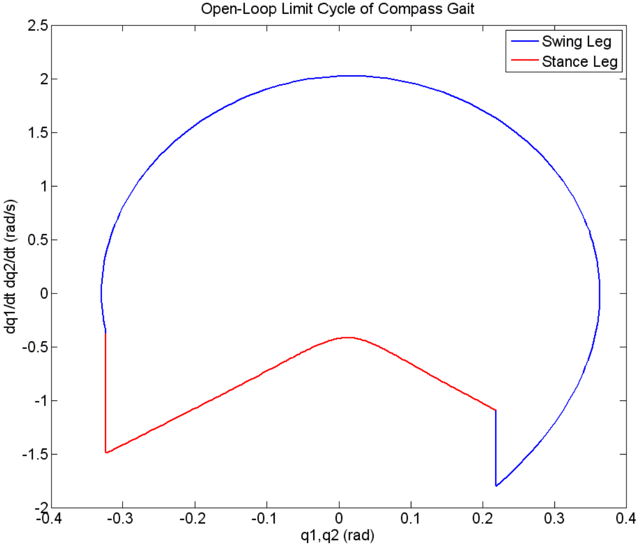




[CLICK IMAGE TO PLAY MOVIE]

- Torque only at the hip.
- No foot scuffing.
- Impulsive, Inelastic Collisions
- Instantaneous transfer of support

Compass Gait: The nominal (passive) limit cycle



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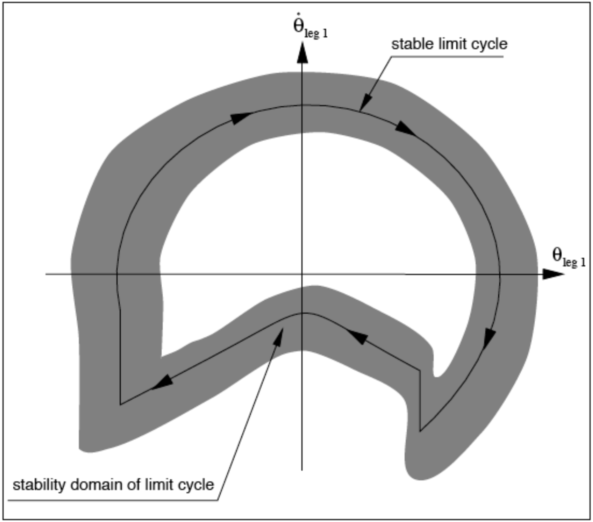

Goswami, 1996

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- but I want more...

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- Observation: Trajectory optimization and trajectory stabilization work very well (locally)
- Possible solution: Trajectory libraries
  - Chris Atkeson has been arguing this for years
  - Can we find a "minimal" set of trajectories that cover the space?

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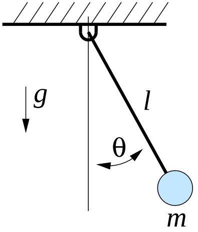
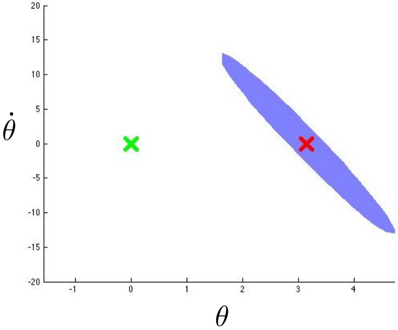
- Pendulum Example:





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- Given polynomial, $p(x)$, with unknown coefficients, c, verify uniform positive definiteness:

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- Can also handle equality constraints, and/or optimize a linear objective

Polynomial Lyapunov functions

- Pablo Parrilo popularized SOS tools for control verification.
- Example: Given a polynomial dynamical system:

$$\dot{x} = \sum_{i=0}^{N} \alpha_i x^i,$$

can search for coefficients of a polynomial Lyapunov function, $V(x)$, such that $\dot{V}(x) \leq 0$.

"Certificates" for LQR Design

- Given $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$
- Linearize around operating point to obtain

$$\dot{\bar{\mathbf{x}}} \approx \mathbf{A}\bar{\mathbf{x}} + \mathbf{B}\bar{\mathbf{u}}.$$

- LQR design gives:

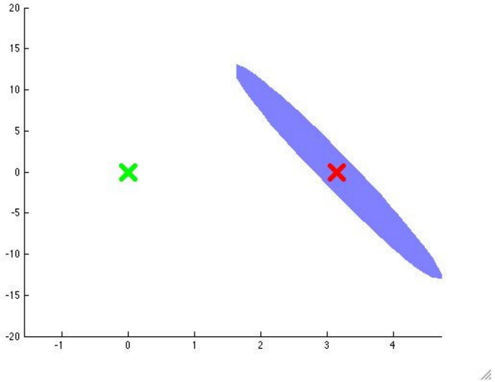
$$\bar{\mathbf{u}} = -\mathbf{K}\bar{\mathbf{x}}, \quad J(\mathbf{x}) \approx \bar{\mathbf{x}}^T\mathbf{S}\bar{\mathbf{x}},$$

  where $J(\mathbf{x})$ is the approximate *cost-to-go*.
- Approximate $\mathbf{f}$ with higher-order Taylor expansion.
- Use SOS tools to find largest scalar ρ for which

$$\forall \mathbf{x} \text{ with } J(\mathbf{x}) \leq \rho, \quad \frac{d}{dt}J(\mathbf{x}) \leq 0.$$

- Also works for LQR trajectory stabilization (time-varying)

Pendulum "Funnels"



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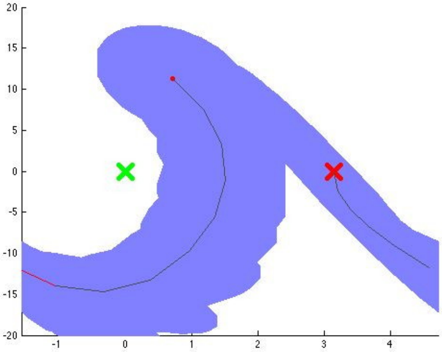

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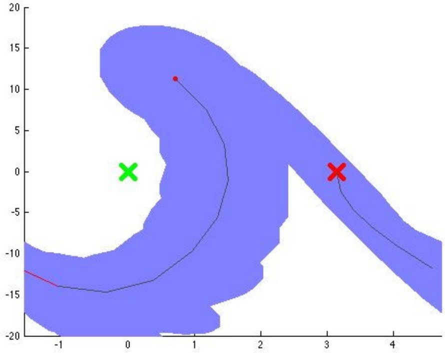

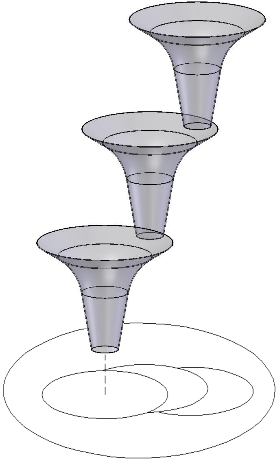

Erdmann,Mason,Koditschek

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  - Conservative in almost every way.
- Combine funnels with randomized motion planning
  - Rapidly-exploring randomized trees (RRTs)
  - Probabilistic Roadmaps (PRMs)

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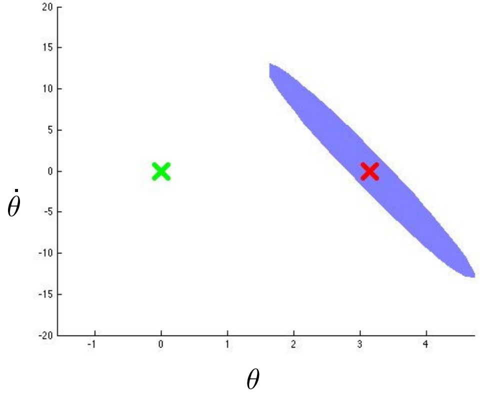
Grow a stabilizing tree backwards from the goal:



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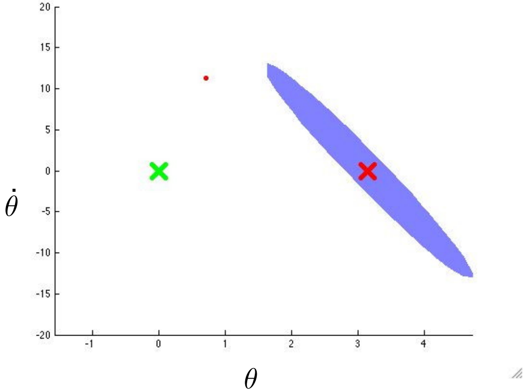

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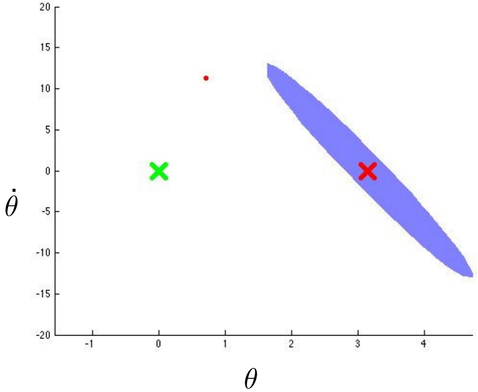

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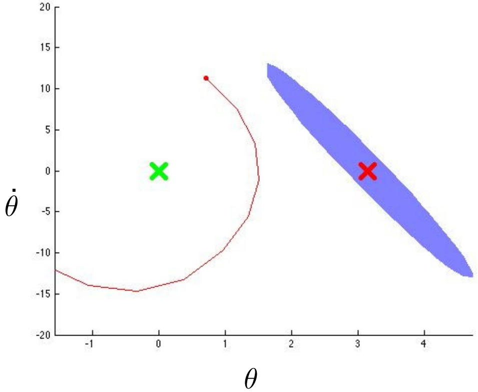

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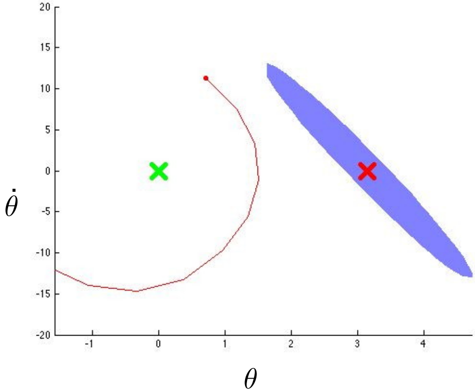

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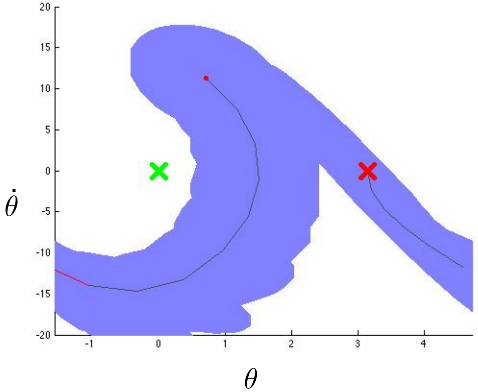

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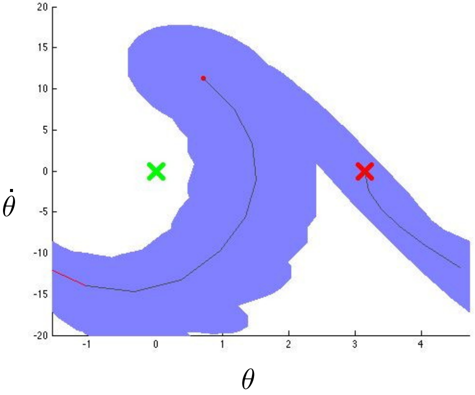
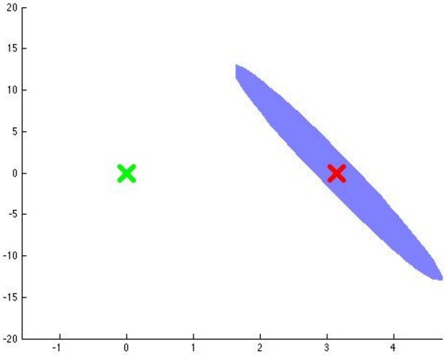
5. Repeat



Simple Pendulum Example



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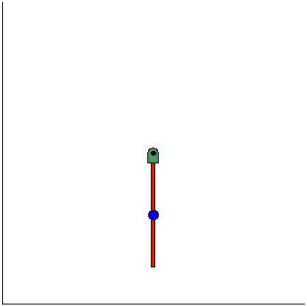

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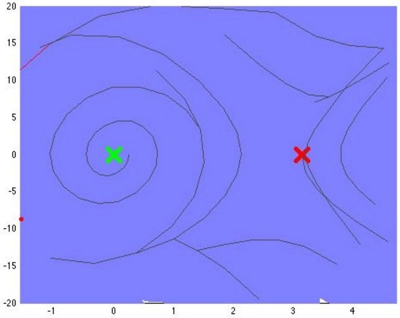
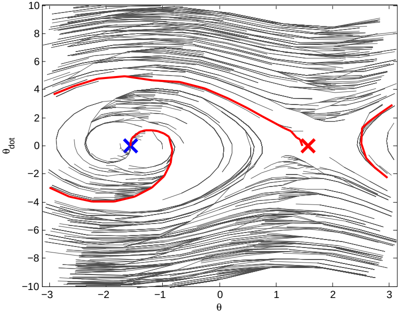
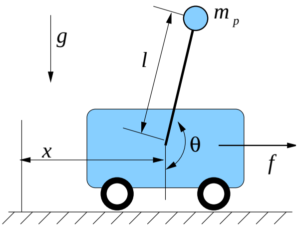


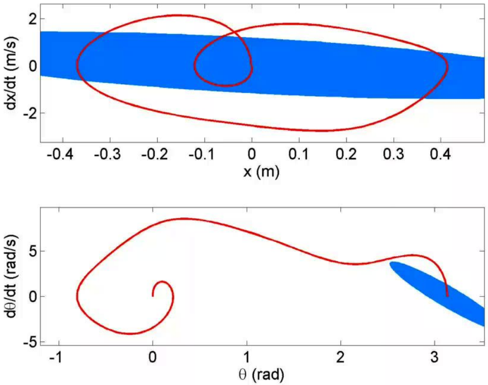
LQR-Trees



RRT w/ Euclidean metric

Certificates for the Cart-Pole system





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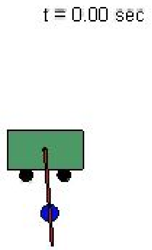
by Philipp Reist

LQR-Trees for the Cart-Pole system



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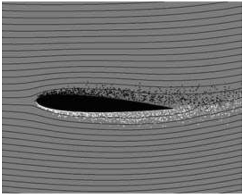
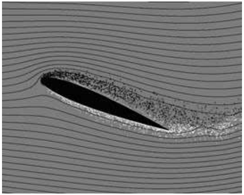
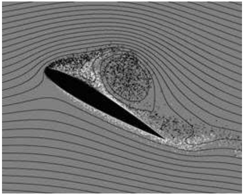
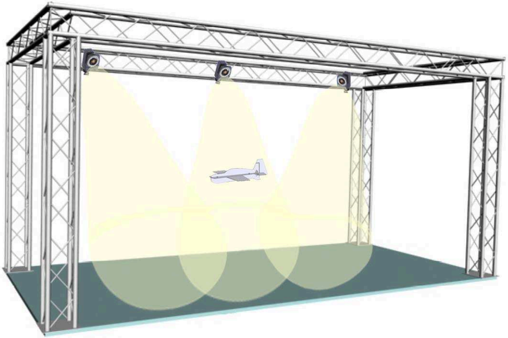
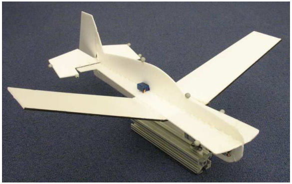
The “Perching” Problem


Experiment Design



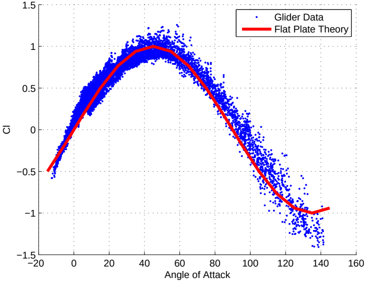
- Glider (no propellor)
- Flat wings
- Dihedral (passive roll stability)
- Offboard sensing and control



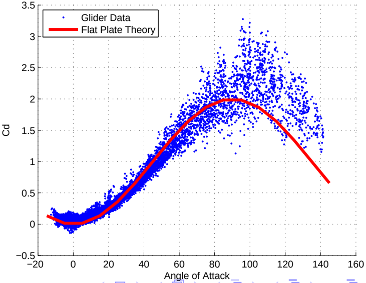
System Identification

- Nonlinear rigid-body vehicle model
- Linear (w/ delay) actuator model
- Real flight data (no wind tunnel)
  - Very high angle-of-attack regimes
  - Relatively small number of physics-based basis functions
  - Vortex shedding

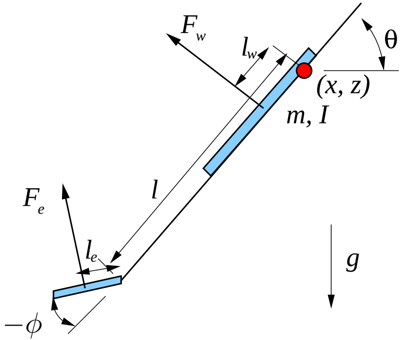
Lift Coefficient



Drag Coefficient



A dynamic model



- Planar dynamics
- Aerodynamics fit from data
- State: $\mathbf{x} = [x, y, \theta, \phi, \dot{x}, \dot{y}, \dot{\theta}]$
- Only actuator is the elevator angle, $\mathbf{u} = \dot{\phi}$

Glider Perching

- Enters motion capture @ 6 m/s.
- Perch is < 3.5 m away.
- Entire trajectory @ 1 second.

Requires Separation!

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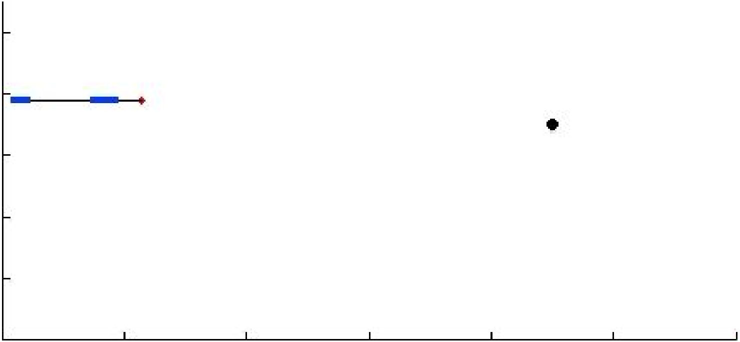
Preliminary results: Trees for Perching



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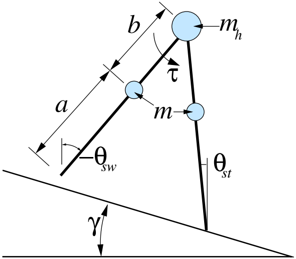
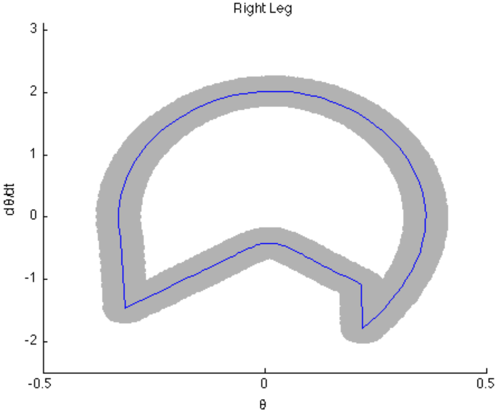
Cartoon: LQR-Trees for bipedal walking





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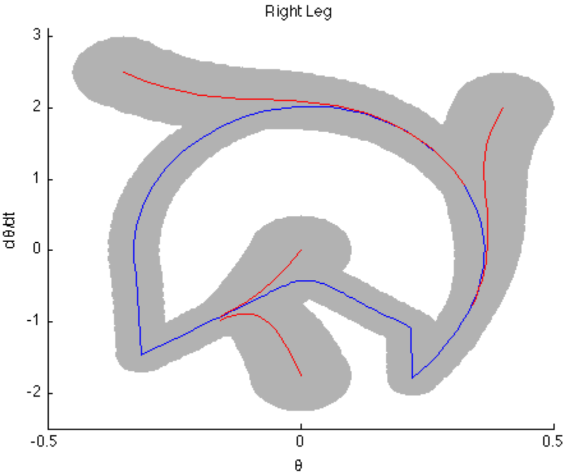

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- Tentative: Combine with policy-gradient methods to adjust to model errors

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- Goal is @ 10 dimensions. Time will tell.

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- Trajectory libraries are a good way to systematically design nonlinear controllers using linear control.
- It pays to reason about the funnels as you plan:
  - Efficient - thanks to new tools from verification
  - Sparseness - relatively few trajectories required
  - Stronger guarantees - "probabilistic feedback coverage"