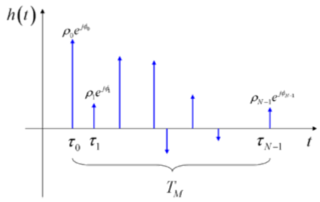


Figure 7: Impulse diagram shows the effect of multipath

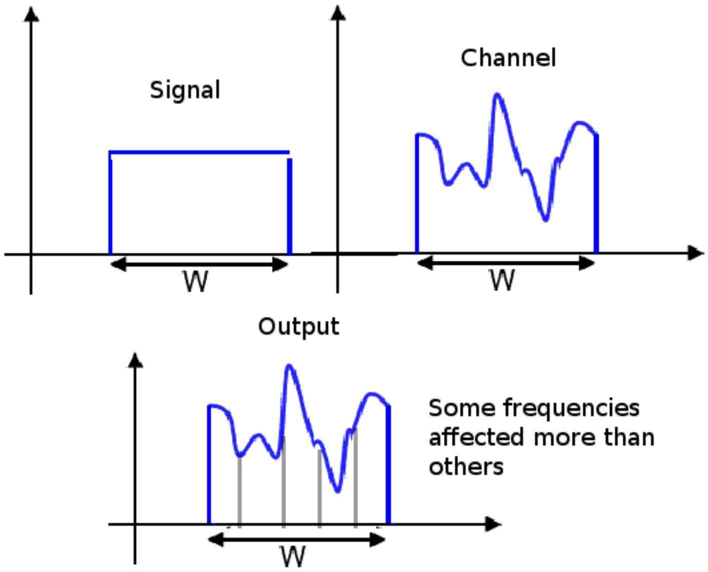


Figure 8: Only some sub-channels are affected at the receiver

During multipath, it is likely that some sub-channels are severely attenuated. If the bits were transmitted using the entire band, all the bits are affected by this attenuation. However, if we transmit some bits on each sub-channel, only some bits are affected by the attenuation. Hence there is know fate sharing among sub channels.

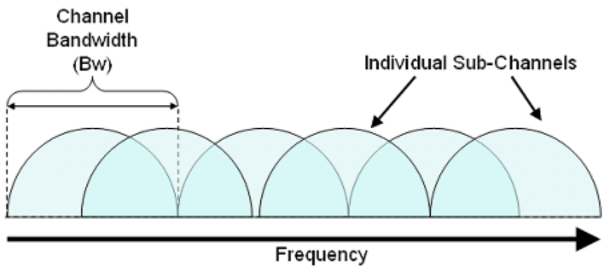
2.2 Achieving Frequency Division

Frequency Division Multiplexing (FDM)

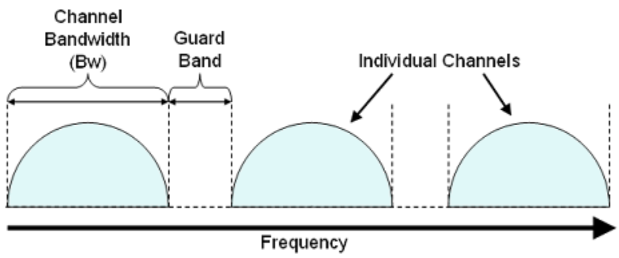
Consider a system where subchannels are equal ranges of frequency. In this case, the frequency spectrum of the signal appears as shown in Figure 9(a) leading to interference of adjacent frequencies. To prevent this, we need to introduce *guard bands* between adjacent frequency bands (see Figure 9(b)). However this is a wastage of bandwidth and leads to low spectral efficiency.

Orthogonal Frequency Division Multiplexing (OFDM)

This is frequency division multiplexing where we choose frequencies that do not interfere. Our sub-carriers are of the form $e^{j2\pi\frac{k}{N}}$ where $k = 0, 1, \cdots, N-1$. (see Figure 10)



(a) Adjacent sub-channels interfere



(b) Guard bands protect leakage from adjacent frequencies

Figure 9: Frequency Division Multiplexing

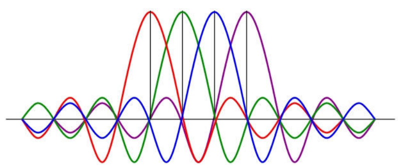


Figure 10: Sub-carriers in OFDM

It is easy to see that these sub-carriers are orthogonal, i.e. they do not interfere with each other.

$$\phi = \sum_{t=-N/2}^{N/2-1} e^{-j2\pi \frac{k}{N} t} e^{j2\pi \frac{p}{N} t} = 0 \quad (p \neq k) \tag{6}$$

Hence we can improve spectral efficiency without causing interference between the sub-carriers.

2.3 OFDM Block Diagram

At the transmitter, we have an input - a stream of D bits. Suppose we have NFFT sub-carriers. Then we must transmit $D/\text{NFFT} = \text{NSYM}$ symbols, where each symbol has NFFT bits. Here we are assuming each signal value in our modulation represents one bit, e.g BPSK. Note that if we use 4-QAM : each symbol will have $2 \times$ NFFT bits, if we use 16-QAM : each symbol will have $4 \times$ NFFT bits, etc. The bits in each is then fed into a serial-to-parallel converter and modulated (BPSK/4-QAM/etc.). Note that it is possible for different sub-carriers to use different modulation schemes. An inverse fast Fourier transform (IFFT) is performed on the NFFT complex numbers. The stream is fed to a parallel to serial converter. Hence the output signal is a sequence of NSYM symbols where each symbol has NFFT samples.

The received signal is fed to a parallel to serial converter. An fast Fourier transform (FFT) is performed on the NFFT complex numbers to produce the symbol. The bits are retrieved through demodulation and the stream is serialized and output. A basic outline of this chain is shown in Figure 11.

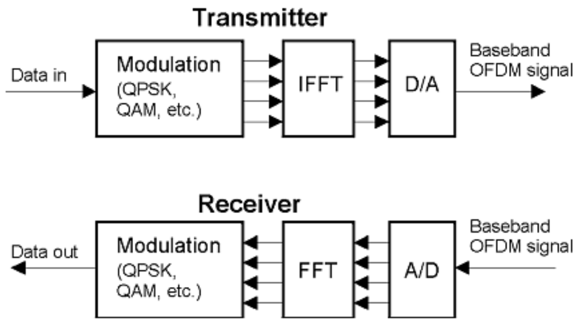


Figure 11: Basic view of the OFDM Transmitter and Receiver

2.4 Inter Symbol Interference (ISI)

Inter-symbol interference is caused when delayed and attenuated versions of the signal (e.g. produced due to multi-path) overlap with the signal. Hence one symbol's delayed version may overlap with an adjacent symbol causing *inter-symbol interference*. One simple technique to avoid this is to introduce a guard-band between adjacent symbols as shown in Figure 12.

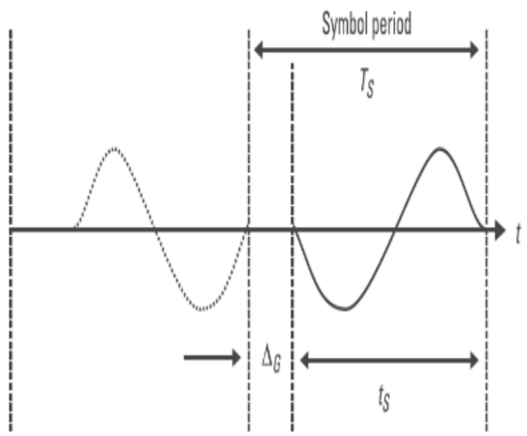


Figure 12: Guard band between symbols in OFDM

However, a better approach is to introduce a *cyclic prefix* where some trailing portion of the symbol is copied in front of it. This gives an illusion to the FFT algorithm that the signal is periodic (the delayed and attenuated signals will be exactly the way it should be if the signal was periodic). Hence equation (5) is valid. This is shown in Figure 13.

The full OFDM transmitter and receiver including the cyclic prefix is shown in Figure 14.

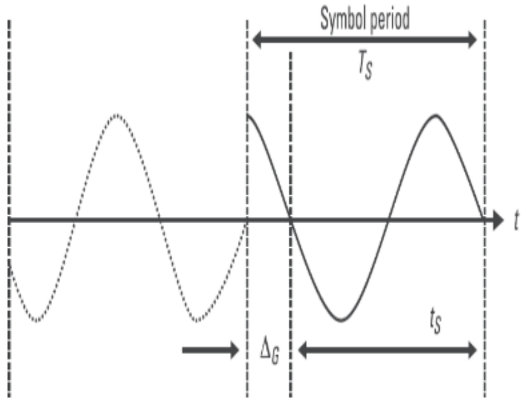


Figure 13: Cyclic prefix between symbols in OFDM

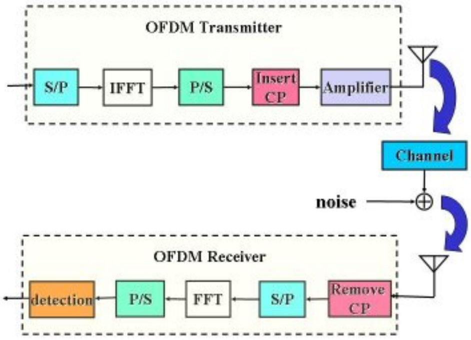


Figure 14: Block diagram of OFDM transmitter and receiver

Cyclic prefix

Overhead: Note that in either case, we have lost some efficiency. However, if each symbol is sufficiently large, the relative size of the interfering region (and hence the cyclic prefix) is small, and hence the loss of efficiency is small.

Advantages: Besides the aforementioned advantage of preventing inter-symbol interference, the CP also allows the signal to be decoded even if the packet was detected after some delay. Delay in the time domain corresponds to rotation in the frequency domain. So we can still obtain the correct signal in the frequency domain by compensating this rotation.

2.5 OFDM Receiver

We discuss some important algorithms in the OFDM receiver.

Packet detection

The packet is detected by having a pair of sliding windows A and B. We say that the packet is detected if the ratio of the energy of A and B crosses a certain threshold. This is illustrated in Figure 15.

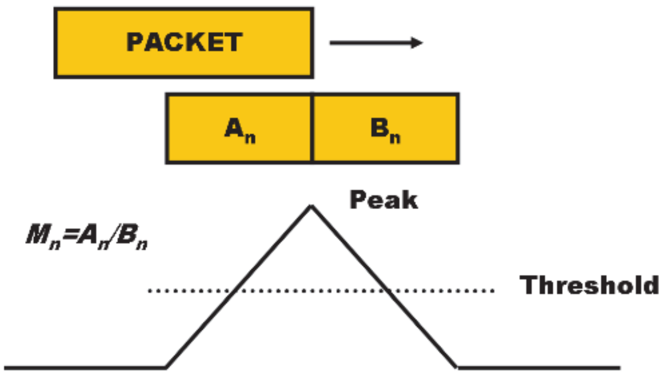


Figure 15: Packet detection

CFO correction

Carrier frequency offset (CFO) is caused because the oscillators of the transmitter and the receiver are not typically tuned to identical frequencies. CFO is the difference in these frequencies. The presence of CFO causes a phase-shift of the signal. Note that:

$$\phi = 2\pi f_{carrier} T_{delay} \tag{7}$$

Suppose we send two identical adjacent symbols. Then the phase difference between any two samples separated by NFFT + NCP may be used to estimate CFO as shown:

$$s^2(i) = s^1(i) e^{j2\pi\delta f T_{symb}} \tag{8}$$

$$CFO = \delta f = \frac{1}{2\pi T_{symb}} \angle s^1(i) s^2(i)^* \tag{9}$$

Channel Estimation

The channel is estimated using the formula:

$$H(f) = \frac{Y(f)}{X(f)} \tag{10}$$

To estimate $H(f)$, we need to know $X(f)$ for some symbols. This is achieved by having a preamble before the packet where known data is sent. One may also transmit multiple symbols in the preamble and average $H(f)$ estimated at each of the symbols. This minimizes the effect of noise.

SFO Estimation

Sampling frequency offset (SFO) is caused because the transmitter and receiver may sample the signal at slightly different offset. Recall that:

$$\phi = 2\pi f_{carrier} T_{delay} \tag{11}$$

Hence, this effectively leads to a difference in phase. All the sub-carriers experience the same sampling delay, but they have different frequencies. Hence, if we plot the change in phase between the transmitted and received signal (after CFO, channel correction) versus frequency, we obtain a straight line (see Figure 16).

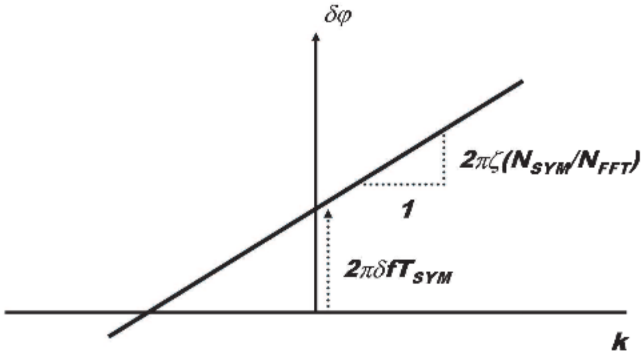


Figure 16: Estimating SFO

The slope of this line relates to the SFO. This line can be computed only if we know what is transmitted. Hence, in every symbol we allot certain sub-carriers to have known values. These sub-carriers are called *pilots*. The SFO is estimated at the receiver by constructing a line based on the phase difference experienced by the pilots (this line is estimated using regression). The y-intercept is caused due to CFO and this can also be corrected.

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