Massachusetts Institute of Technology
6.888: Wireless Communications Systems — September 6, 2013
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Recitation 1 Notes

The recitation focused on the discussion of transmission over a single link i.e. single input, single output (SISO) wireless systems.

1 Key Concepts

- **Tx Power Budget:** The power consumption of the transmitter is an important parameter and impacts the range of the transmitted signal and the performance of the transmitter. We can't keep increasing the power budget for the following reasons:
  - More power requirements make the devices bulky.
  - You would want to save battery in case of devices like cellphones.
  - FCC regulations govern the limits on the power that you can transmit.
  - **Broadcast Medium:** Unlike wired connections, wireless is a broadcast medium.
  - **Spatial Reuse:** As long as two transmitters are out of each other's range, they can transmit at the same frequency.
- **Tx Bandwidth:** The bandwidth in which you can transmit is also governed by FCC regulations and the circuit used in the transmitter.
- **Signal to Noise Ratio (SNR):** SNR is defined as $SNR = \frac{Received-Signal}{Total-noise-power}$.
- **Bit Error Rate (BER):** BER is defined as the probability that a bit transmitted by the transmitter will arrive flipped at the receiver. Higher SNR reduces BER. Better coding schemes reduce BER too. Lower BER leads to increased throughput.
- **Throughput:** The actual number of correct data bits transferred per second per Hertz is the throughput.
- **Capacity:** The maximum throughput achievable given some bandwidth and SNR. The capacity is defined as $C = B.log(1 + SNR)$ where B is the bandwidth. Active efforts in research attempt to ensure that throughput close to the capacity is achieved.

2 Transmitter and Receiver

A broad overview of the transmitter and receiver layouts is given in Figure 1. The digital processing and analog RF part are discussed separately in the following subsections.

2.1 Digital Processing

As shown in Figure 2, the digital processing part consists of forward error correction (FEC), modulation, along with some additional processing based on the scheme used (e.g. OFDM, CDMA, ...) and the processing required for estimating and correcting for the wireless channel and for synchronization offsets.

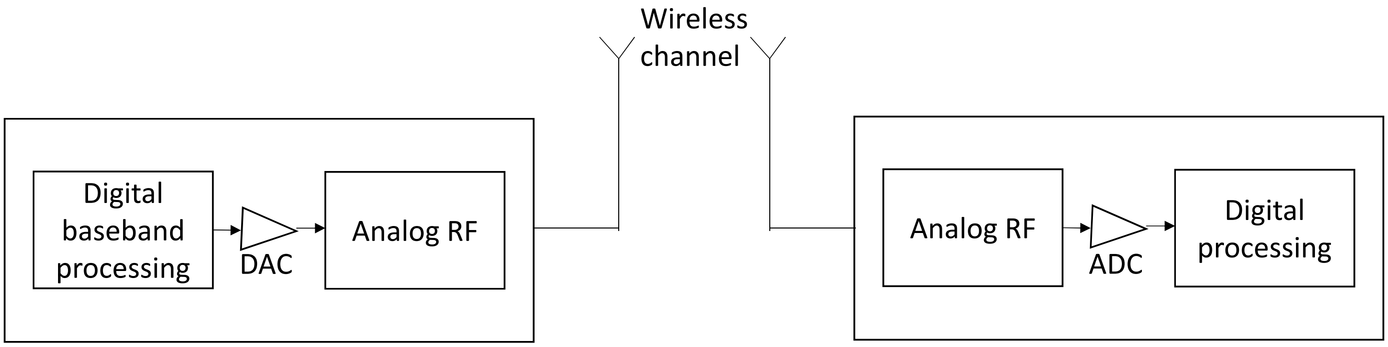


Figure 1: Broad overview of a transmitter and a receiver

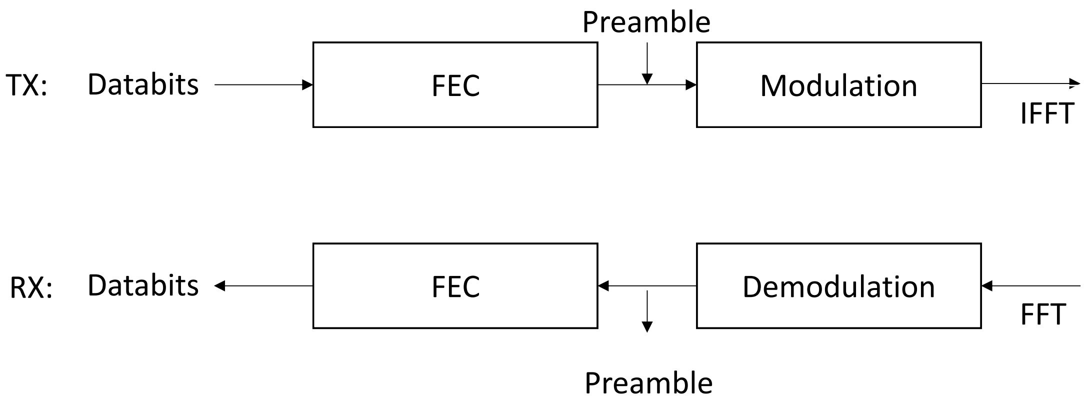


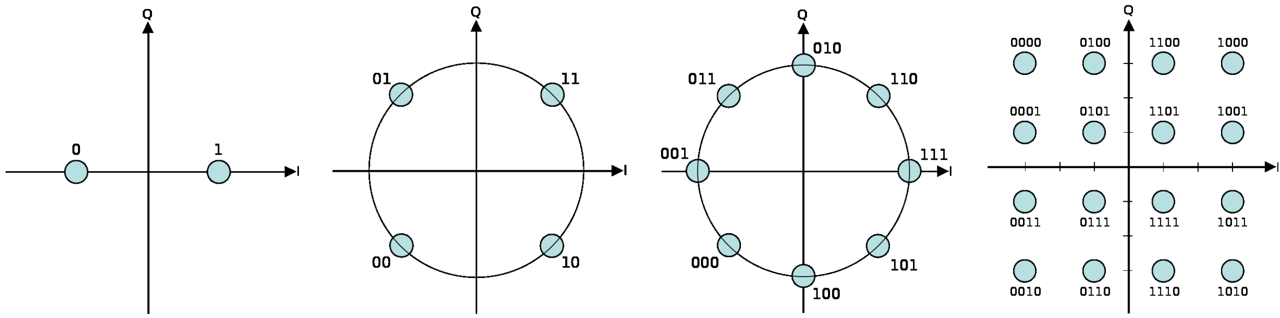
Figure 2: Digital processing in a transmitter and receiver

- **Forward Error Correction:** Forward Error Correction (FEC) is a technique to add redundant bits to the data bits being transmitted to even if they are flipped at the receiver. One naive way of doing this could be repetition code (e.g. sending each bit thrice). However it is not efficient. Other efficient coding schemes have been developed for FEC such as convolutional codes and rateless codes. The code rate is defined as $\frac{\#databits}{\#databits+\#redundantbits}$.

- **Preamble** is inserted to help us to estimate the channel and synchronization offsets and correct for them.

- **Modulation:** Modulation means modifying the properties of the waveforms to encode information. Difference types of modulation techniques include Frequency modulation, Amplitude modulation and Phase modulation. Sequences of bits have to be converted to complex symbols before transmission. The symbols in the complex plane are called constellation points. The real and imaginary parts of the constellation points are referred to as Is and Qs (In-Phase and Quadrature-Phase). Several techniques can be used for it as shown in Figure 3:

  - **BPSK:** Binary phase shift keying maps the bit sequence to a sequence of $\{+1, -1\}$. Each bit is then received at the receiver as sum of the symbol transmitted added with noise (generally Additive White Gaussian Noise).
  - **4PSK/4QAM:** QAM stands for quadrature amplitude modulation where the bits are mapped onto a grid constellation. PSK stands for phase shift keying where the bits are mapped to constellation points on a circle with uniform phase separation. For 4 points, 4PSK and 4QAM are the same and 2 bits are mapped to each constellation point.
  - **8PSK:** 3 bits are mapped to 1 constellation point.



(a) BPSK (b) 4PSK/4QAM (c) 8PSK (d) 16QAM

Figure 3: Constellation Points of some amplitude and phase modulations schemes.

   - **16QAM:** 4 bits are mapped to 1 constellation point.

  A few points are worth noting:

   - Higher order modulation → More bits per symbols → Higher bit rate.
   - Higher order modulations typically have better spectral efficiency (except for OFDM based transceivers).
   - For a given SNR, the minimum distance between the nearest neighbors in the above constellation graphs will determine the bit error rate.
   - For higher order modulations (8,16,...), use gray codes to map bits to symbols. Gray codes ensure that the nearest neighbors vary by only 1 bit and hence one symbol error will result only in one bit error.
   - Given the same power budget, the minimum distance of BPSK modulation will be larger than the minimum distance of 16QAM. Hence, for the same SNR, the BER of BPSK will be smaller than the BER of 16QAM.
   - Higher SNR → Can use higher order modulation → Higher bit rate.

- **Rate Adaptation:** Choose FEC code rate and modulation to achieve the maximum bit rate without exceeding capacity. Since we have to pick the rate before transmission:
   - Rate too high → High packet loss.
   - Rate too low → High inefficiency.
   - Ideas: adjust bit rate based on feedback from receiver : SNR, BER, packet loss ...
   - Rateless code: do not pick a rate in advance, keep transmitting until all bits received correctly. Automatically adjust the rate.

2.2 Analog RF

An overview of the analog processing in the transmitter and receiver is shown in Figure 4.

- **Transmitter:** The digital Is and Qs are converted to analog signals by the DAC (Digital-to-Analog Converter). They are then passed through a low pass filter to remove the high frequencies caused by sharp edges at the output of the DAC. The resulting signal is called a baseband signal i.e. it's center frequency is 0Hz. The signal is then moved to passband using the mixer after which it will be centered around a carrier frequency f_c. It is then passed

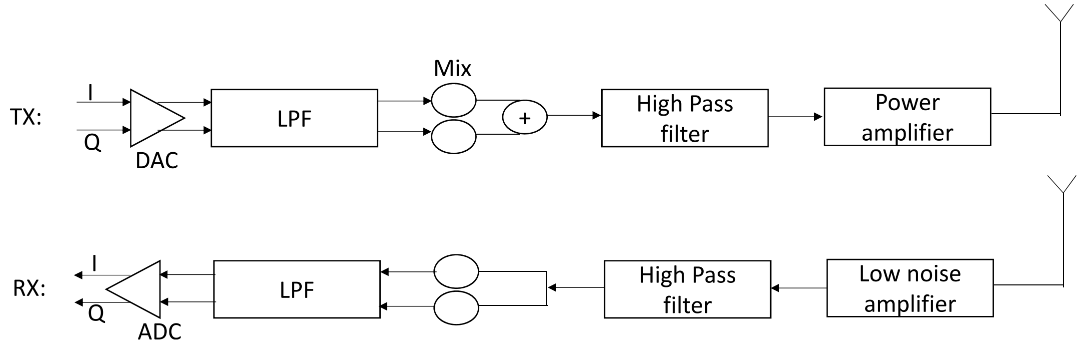


Figure 4: Analog processing in a transmitter and receiver

through a high pass filter and then amplified using a power amplifier before it goes to the TX antenna.

- **Receiver:** The analog signal coming from the RX antenna is amplified using a low noise amplifier. It is then passed through a high pass filter, then mixed down to baseband. It is then passed through a low pass filter and feed to the ADC (Analog-to-Digital Converter) which samples and quantizes the Is and Qs.

- **Up Conversion:** The signal $s(t) = I + jQ$ is converted to passband by the mixers. The Is are multiplied by $cos(2\pi f_c t)$ and the Qs by $cos(2\pi f_c t + \frac{\pi}{2})$, where f_c is the carrier frequency. The two signals are then summed together to create the transmitted signal $x(t)$.

$$
\begin{aligned}
x(t) &= Icos(2\pi f_c t) + Qcos(2\pi f_c t + \frac{\pi}{2}) \\
&= Icos(2\pi f_c t) - Qsin(2\pi f_c t) \\
&= Re\{s(t)e^{2\pi j f_c t}\}
\end{aligned}
$$

- **Down Conversion:** The passband signal is converted to baseband by multiplying it by $cos(2\pi f_c t)$ and $cos(2\pi f_c t + \frac{\pi}{2})$ and then low pass filtered.

$$
\begin{aligned}
x(t)cos(2\pi f_c t) &= Icos^2(2\pi f_c t) - Qsin(2\pi f_c t)cos(2\pi f_c t) \\
&= \frac{I}{2} + \frac{I}{2}cos(2\pi(2f_c)t) - \frac{Q}{2}sin(2\pi(2f_c)t) \\
&= \frac{I}{2} \text{ (after low pass filtering)}
\end{aligned}
$$

$$
\begin{aligned}
x(t)cos(2\pi f_c t + \frac{\pi}{2}) &= -Icos(2\pi f_c t)sin(2\pi f_c t) + Qsin^2(2\pi f_c t) \\
&= -\frac{I}{2}sin(2\pi(2f_c)t) + \frac{Q}{2} - \frac{Q}{2}cos(2\pi(2f_c)t) \\
&= \frac{Q}{2} \text{ (after low pass filtering)}
\end{aligned}
$$

$$
\begin{aligned}
s(t) &= I + jQ = 2 \times LPF\left\{x(t)cos(2\pi f_c t) + jx(t)cos(2\pi f_c t + \frac{\pi}{2})\right\} \\
&= 2 \times LPF\left\{x(t)e^{-2\pi j f_c t}\right\} \\
&= 2 \times LPF\left\{Re\{s(t)e^{2\pi j f_c t}\}e^{-2\pi j f_c t}\right\} \\
&= s(t)e^{2\pi j f_c t}e^{-2\pi j f_c t} = s(t)
\end{aligned}
$$

- **Synchronization Issues:**
  - **Carrier Frequency Offset (CFO):** The oscillators of the transmitter and receiver are not synchronized and as a result the carrier frequency at the transmitter f_c will be slightly different from the carrier frequency at the receiver $f_c + \Delta f_c$. This offset Δf_c is called CFO. As a result, the received signal $s(t)e^{2\pi j f_c t} e^{-2\pi j (f_c + \Delta f_c)t} = s(t)e^{-2\pi j \Delta f_c t}$. The phase of the signal accumulates across time and hence the constellation points will rotate in the complex (I, Q) plane. Thus if we do not correct for CFO, the constellation points will be decoded incorrectly.
  - **Sampling Frequency Offset (SFO):** The DAC on the transmitter and the ADC on the receiver are driven by different clocks and hence have a sampling frequency offset. This sampling frequency offset needs to be compensated so that receiver and transmitter do not drift so far apart.

  We will better model these synchronization issues and how to deal with them in the following lecture.

- **Automatic Gain Controller (AGC):** The ADC quantizes the signal which adds quantization noise. In this case, the SNR is given by $SNR = \frac{P}{N_{th}+N_Q}$, where N_Q is the quantization noise and N_{th} is the thermal noise. To minimize the quantization noise, we want the signal received at the receiver to span the entire range of the ADC to make sure that the maximum number of bits is uses to quantize the signal. The quantization noise will decrease by 6dB/bit. At the same time, we want to make sure that the signal at the input of the ADC is not too large that it saturates the ADC and gets clipped. The AGC at the output of the ADC gives feedback to amplify or attenuate the signal to make sure it spans the full range of the ADC.

- **Half-Duplex vs Full Duplex Radios:** Most radios today are half-duplex which means that they cannot transmit and receive at the same time. This because the signal from the transmit chain of the radio is so large at the receiver chain that it saturates the ADC. Signals received from other transmitters will be lost since they are drowned in the signal from the radios own transmission. Recently, there has been a some research on building full-duplex radios. We will cover full-duplex radios in a later lecture of the course.

2.3 Wireless Channel

In this section, we will mathematically model the wireless channel.

- **Multi-Path:** Between the transmitter and receiver the signal traverses different paths which have different delays. Thus, the receiver sees summation of delayed versions $s(t-\tau)$ of the signal weighted by different attenuation $h(\tau)$. Thus, for a transmitted signal $x(t)$ the received signal $y(t)$ is

$$y(t) = \int h(\tau)s(t-\tau) + n(t) = h(t) * s(t) + n(t)$$

  where $(*)$ is convolution an $n(t)$ is additive white Gaussian noise.

- **Narrow Band Channel:** For narrow band we can approximate the wireless channel $h(t)$ by a impulse function $h \cdot \delta(t)$. This is because sampling at low rate does not allow us to distinguish all the delayed versions of the signal and hence they all appear as a single path.

Thus, for narrow band, convolving with the wireless channel reduces to multiplying by a single complex number h and we can now write the received signal $y(t)$ as:

$$y(t) = hx(t) + n(t).$$

- **Wide Band Channel:** For wide band we can approximate the wireless channel $h(t)$ by a multi-tap channel i.e. multiple delayed impulses as shown in Figure 5. For a k tap channel the received signal $y(t)$ can be written as:

$$y(t) = \sum_{i=0}^{i=k} h(i)s(t - i\tau)$$

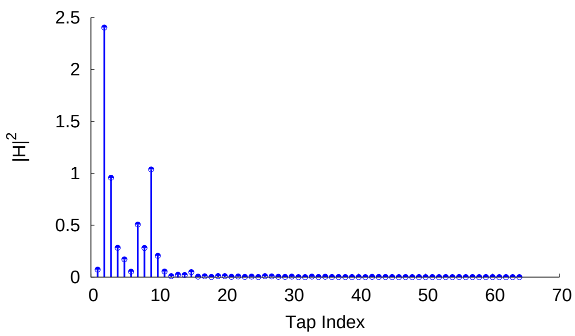


Figure 5: Time Domain Wide Band Channel $h(t)$ (≈11 taps)

- **Frequency Selective Fading:** Convolution with $h(t)$ in the time domain results multiplication with $H(f)$ in the frequency domain. For narrow band, $h(t)$ is an impulse and $H(f)$ is flat. For wide band, $H(f)$ results in different attenuation for different frequencies as shown in Figure 6. The figure also shows that for narrow bands the channel can be approximated as flat.

$$y(t) = h(t) * s(t) + n(t) \Leftrightarrow Y(f) = H(f)S(f) + N$$

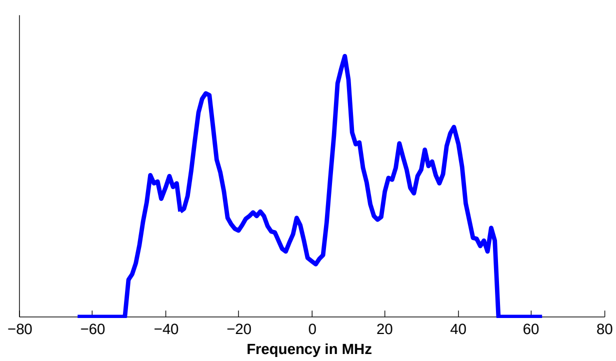


Figure 6: Frequency Selective Fading for 100 MHz channel

- **Inter-Symbol-Interference:** Multi-path results in inter-symbol-interference i.e. delayed symbols interfere with the symbol being decoding. The effect is sever and results in decoding errors for wide band since the symbol length is short and of the order of the delayed taps. The next lecture will discuss how we deal with this problem using OFDM (Orthogonal Frequency Division Multiplexing).