

# Handling Overtraced Strokes in Hand-Drawn Sketches

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## Abstract

Overtracing is the phenomenon in sketching of repeatedly drawing over previously drawn ink. It is a naturally appearing effect and is especially frequent in domains such as architectural drawings. Existing work in sketch recognition focuses on sketches with non-overtraced strokes. In this paper, we describe a method for generating geometric approximations of overtraced strokes in terms of primitives including lines, non-self-intersecting polylines, ellipses and arcs. Our system generates concise approximations for overtraced strokes at an early stage in sketch processing, making it possible to use these concise descriptions in higher level processing and sketch interpretation systems.

## Introduction

Freehand sketches are inherently informal and messy. Unlike clean, computer generated diagrams, manifestations of abstract shapes are imprecise and highly variable in sketches. One of the challenges in sketch recognition is the ability to support this imprecision and high variability, of which overtracing is an example (Fig. 1).

In a paper studying how architects sketch, Do and Gross describe overtracing as one of the drawing techniques heavily used in practice. They list the functions of overtracing as “selecting or drawing attention to an element; shape emergence, attending to one or another shape interpretation; and shape refinement or adding detail to an abstract or roughed out shape”. We want to make it possible for users to employ this heavily used drawing technique in sketch-based interfaces. We do this by generating geometric descriptions of overtraced strokes in terms of common geometric primitives.

Our strategy for handling overtracing is to deal with it early in the recognition process – in the context of model based recognition – before high level interpretations are built. We focus here on generating geometric approximations for single stroke shapes that are overtraced (e.g., Fig. 1). We attempt to solve the problem at the geometric level by fitting geometric primitives to input strokes.

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Figure 1: Example of an overtraced stroke.

## Shape approximation

We generate fits for lines, arcs and circles by computing model parameters<sup>1</sup> that minimize the least squares fitting error. Polylines usually require a feature point detection step, which is complicated by the overtraced nature of the strokes.

### Line approximation

We generate line approximations by finding a total least squares fit to the data, i.e., assuming noise in both coordinates  $(x, y)$  of the data. Simpler regression techniques assume the noise is in only the  $y$  component and measure only the vertical distance to the fitted line result in unreliable fits especially with lines near vertical, where a small error in the  $x$  coordinate causes a large change in the error contributed by that point.

We compute the equation of the total least squares line fit in the form of  $ax + by + c = 0$  by computing the first eigenvector  $v$  of the covariance matrix of the  $(x, y)$  positions. Then the parameters of the line are  $a = v_1$ ,  $b = v_2$  and  $c = -a\bar{x} - b\bar{y}$ .

### Generating arc and circle approximations

We have previously shown how an arc approximation can be obtained by fitting a circular arc to overtraced strokes (Sezgin & Davis 2004a). This is done by writing the equation for a circle as  $(x_i - c_x)^2 + (y_i - c_y)^2 = r^2$  where  $(c_x, c_y)$  is the center of the circular arc and  $r$  is the radius of the arc. Then we find the least squares solution for:

<sup>1</sup>E.g., slope and position for lines; center, radius and extent for arcs.

$$\begin{bmatrix} 2x_1 & 2y_1 & 1 \\ 2x_2 & 2y_2 & 1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 2x_n & 2y_n & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ r' \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \cdot \\ \cdot \\ x_n^2 + y_n^2 \end{bmatrix}$$

Here  $r'^2 = r^2 - c_x^2 - c_y^2$ . Finally we calculate the starting angle and the extent for the arc. If the extent of the arc computed by accumulating direction changes exceeds  $2\pi$ , we have a full circle instead of an arc.

### Ellipse approximation

Among many methods for elliptical approximation of data (Bookstein 1979; Fitzgibbon *et al.* 1999; Faber & Fisher September 2001), we found the direct ellipse fitting method proposed by (Fitzgibbon *et al.* 1999) to be the most appropriate, because it specifically attempts to fit an ellipse (as opposed to a generic conic section) to the data. It is also robust enough for generating fits for point clouds. Their method solves for the parameters of the generalized conic equation  $ax^2 + bxy + cy^2 + dx + ey + f = 0$  subject to the constraint  $4ac - b^2 = 1$  that results in an elliptical fit.

Alternatively, a simpler method that computes the parameters of an ellipse fit based on the rectangular bounding box of the stroke can be used if the ellipses are known to be full.

### Approximation of polylines

Existing work on polyline approximation uses speed and curvature data as two sources of information to detect feature points,<sup>2</sup> and relies on the data points to be sorted along the path of the stroke (Sezgin *et al.* November 2001; Calhoun *et al.* 2002; Bentsson & Eklundh 1992; Rattarangsi & Chin 1992). In non-overtraced strokes, the time at which points are sampled is implicitly assumed to give this ordering. Clearly, with overtraced strokes the timing information cannot be used to order the points spatially, as points far away temporally may end up close spatially.

Another challenge in dealing with overtraced polylines is that due to the messy nature of sketching, ink in the overtraced parts of a stroke does not necessarily lie all on the same line.

### Feature point detection

We present a two step approach to feature point detection. In the first step, we treat the input stroke as a collection of unordered points and apply a point cloud thinning approach based on moving least squares method (Lee 2000) along with the random sample consensus algorithm to move points closer along a path perpendicular to the stroke. This gives us a thinned version of the stroke. Next, we sort the points along the stroke by an edge linking step that combines all the points on a single path. Conventional feature point detection methods can be applied from that point on. We begin with the stroke thinning and point sorting processes.

<sup>2</sup>Informally, feature points are the corners of a stroke, though more generally they are defined as points in the stroke which separate curved or straight segments of a stroke.

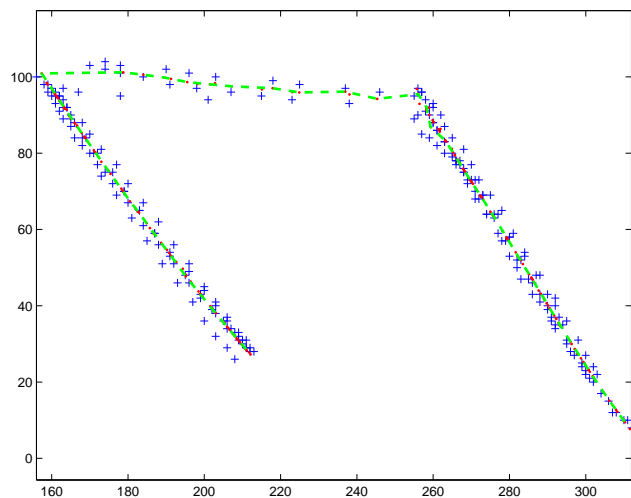


Figure 2: The point cloud obtained by thinning the stroke in Fig. 1. The '+' marks show the original locations of the points in the stroke and the '.' marks show the locations after thinning. The green dashed line shows the result of the edge linking process.

### Stroke thinning

Our approach to stroke thinning is based on the point cloud thinning methods (Lee 2000). As seen in Fig.2, points in an overtraced stroke fall in a band around the intended object. The thinning process aims to bring these points closer to the axis of the point cloud (i.e., to the shape that was possibly intended by the user). In this respect, our method produces shapes unlike the kinds of shapes produced by skeletonization and medial axis transforms.

Thinning is achieved by moving points in a direction perpendicular to the regression line obtained by fitting a line to points in the local neighborhood of the point of interest. In order to obtain reliable regression lines at the corners of a shape, we employ a two step neighborhood selection technique.

### Selecting a neighborhood

For each point  $p_i$  to be moved during thinning, neighborhood selection determines the set of points to be used for obtaining the local regression line towards which  $p_i$  will be moved. The usual approach in cloud thinning is to define neighborhood as all points within a preset radius of  $p_i$ . This approach works well for smooth curves without sharp corners. For polylines or complex objects (i.e., combination of lines and curves) our experiments show that this results in unstable behavior at the corners, where the neighborhood contains points from the edge containing  $p_i$  and points from the next edge. The points from the next edge act as distractors and the resulting linear regression line no longer becomes parallel to either of the edges forming the corner. This is especially problematic for edges intersecting with acute angles.

We solve this problem by employing a neighborhood selection scheme that is robust in the presence of large number of outliers in the data. We use the random sample consen-

algorithm (Fischler & Bolles 1981) to select a subset of points within a radius of  $p_i$  that roughly lie on the same line. We compute the linear regression line using these points. Then  $p_i$  is moved perpendicular to the regression line. After thinning, points less than a pixel apart are merged in a greedy fashion to reduce the number of points.

### Edge linking

After the point cloud is thinned, we connect individual points to obtain a chain of points for the whole stroke. Although there are many solutions to this problem in the literature, we take a two step greedy approach. In the first step, we connect each point to its closest neighbor. This gives us a list of edges. We connect these edges starting with the ones that have their end-points closest, obtaining longer point chains. We continue this process until we are left with a single chain linking all the points together.

The described thinning method also applies to complex objects (i.e., combination of straight and curved segments). Once the stroke is thinned and points are sorted, the corners (and the curved portions where applicable) of the resulting approximation can be detected using any curvature based shape approximation method (Sezgin & Davis 2004b; Sezgin *et al.* November 2001).

### Evaluation

We conducted a preliminary experiment running our algorithms on overtraced stroke data collected from 8 users using an Acer c110 Tablet PC. Although the data was collected out of context, it served to be useful in detecting some of the strengths and weaknesses of our approach.

We collected test data by asking each of the 8 users to draw 5 overtraced examples of lines, circular arcs, circles, ellipses, non-intersecting polylines and complex objects. In each case, we visually compared the generated approximations to the shapes originally intended by the users. For the stroke thinning method, we used a neighborhood radius of 20 pixels.

For primitive types where we could obtain direct least squares fits (i.e., lines, arcs, circles and ellipses), in all cases the computed approximations were in accordance with what each user was asked to draw. This is an indication of the robustness of direct fit methods.

The performance of our approach for polylines and complex shapes was also promising. For the majority of the strokes, the thinning operation yielded the expected result of aligning all the stroke points on a thin path. In a relatively small number of cases (10% of the polylines and 12.5% of the complex objects) the thinning operation failed to align points. This caused the edge linking step to produce incorrect approximations. Fig.3 shows an example where this occurs. After thinning Fig.3-a, we obtain the set of points shown in Fig.3-b. As seen in Fig.3-b the thinned points on the upper (negatively sloped) portion of the stroke don't lie on the same line.

This sort of behavior occurs when ink in a single segment of the stroke lies on two distinct clusters, and the radius constant used for neighborhood computation fails to group points in these clusters together.

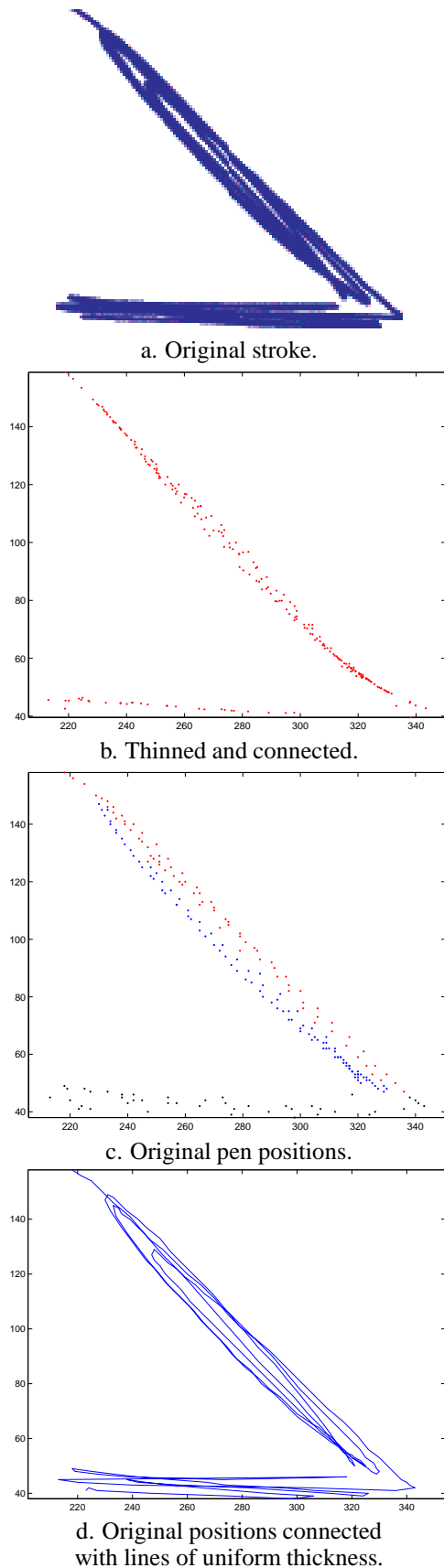


Figure 3:

In Fig.3-c, the two distinct clusters are indicated using blue and red dots. During the thinning process, for a number of points in these clusters, the neighborhood calculation excludes points from the other cluster. As a result points on one cluster get thinned using regression lines that are parallel but are a few pixels apart.

## Related Work

Previous methods on stroke approximation assume that points are sorted spatially along the stroke and use curvature and pen speed information to detect feature points (Sezgin *et al.* November 2001; Calhoun *et al.* 2002; Bentsson & Eklundh 1992; Rattarangsi & Chin 1992). Cloud thinning methods in the literature focus on smooth curves. Smooth curves are generally easier to thin because they don't need the special neighborhood selection strategy similar to the one we employ.

The stroke classification method described in (Shpitalni & Lipson 2002) finds least squares solution of the generalized conic equation. For classification, it relies on classification heuristics such as signs and ratios of certain parameters in the conic equation. Furthermore, it assumes that each stroke is a single entity (i.e., line, arc or a filleted corner). Our approach doesn't have this requirement and it is general enough to be applicable to strokes that are combinations of curved and straight segments. In addition, we obtain *specific* fits for each primitive class and suggest using the errors of each type of fit as the goodness criteria as in (Sezgin *et al.* November 2001) rather than performing the classification based on the relationship between the parameters that solve a generic equation.

## Future Work

As described in the evaluation section, in cases where an individual segment of overtraced stroke forms parallel clusters within itself, the thinning operation fails to group the points on a single line. This problem can be fixed by recursively applying the thinning operation with a stopping criteria based on the stability of points which says if a point cloud becomes stable (i.e., if the points don't get moved significantly by subsequent thinning operations), this may be used as an indication that we converged on a solution. The challenge would be in defining what constitutes as a "significant" change in point positions.

Another direction would be to investigate if factors, other than the pen positions, which affect the way a stroke is rendered to the user can be used during the thinning process. Such factors are especially important, because among other things, they potentially determine how much overtracing a user does (e.g., until the overtraced region is mostly covered with ink). Pen pressure is an example of information other than pen positions that affects rendering. For example, our data collection program renders strokes thicker at points with high pen pressure. This is part of the reason why the clusters in the negatively sloped part of the stroke in Fig.3-a are more apparent in Fig.3-c and d, where we only show the sampled points and connect them with uniform-thickness lines; and less apparent in Fig.3-a. In Fig.3-a, lines connect-

ing the few points between the two clusters is thick enough to close the gap between the two clusters visually. Pressure information can either be used to dynamically adjust neighborhood parameters or it can be used to weigh points with higher pen pressure more heavily using a weighted least squares linear regression method.

The methods we presented worked well for the data collected in an isolated fashion (i.e., outside the context of a specific sketching task or domain). On one hand this is justifiable because our aim has been to do low level processing in a domain and task independent fashion. On the other hand it is conceivable that overtracing behaviors differ across domains. Studying the extent to which this is true and determining what kinds of techniques can be adapted in these situations remains as an interesting area to explore.

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