

Risk-limiting, Market-based Power Dispatch and Pricing

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Abstract—The purpose of this work is to enable risk-limiting electricity dispatch through a market mechanism. There has been a solid body of work on centralized risk-limiting dispatch, which guarantees that the risk of energy shortage is within a user-specified bound. The current trading mechanism of a day-ahead electricity market can be viewed as a market that deals with the expectation (i.e., the first moment) of future supply and load. We show in this paper that distributed and risk-limiting dispatch is enabled by also trading the *standard deviation* (i.e., the second moment). In the proposed mechanism, a dispatchable power provider, such as spinning reserve and battery, can “sell” the standard deviation in a market by contributing to absorbing the uncertainty of energy demand and supply. The market-clearing prices of the mean and the standard deviation of electricity are found through the Walrasian auction. This approach allows each power provider to specify the probability density function (pdf) of the amount of energy that it has to generate in the future. As a result, a power provider can quantitatively limit the risk of power shortage by imposing chance constraints in a decentralized manner. The decentralized risk-limiting dispatch and pricing problem are solved at each time step with a receding time horizon. We demonstrate the capabilities of the proposed approach by simulations using real data.

I. INTRODUCTION

A. Overview of Contingent Power Dispatch and Pricing

Among various types of electricity market, futures markets, such as a day-ahead market, play a central role in power dispatch. An existing day-ahead electricity market typically concerns only the prediction of the uncertain future energy supply and demand. In other words, although the probability distribution of future energy supply and demand is often available through a statistical method, existing market mechanisms consider only the mean of the probability distribution.

We observe that an electricity market becomes more capable in dealing with uncertainty by fully exploiting the information of the probability distribution. This observation leads to our novel market-based power dispatch and pricing approach, namely *contingent power dispatch and pricing*, which trades the *standard deviation* of future power demand and supply in a market. A market finds the market-clearing prices of the mean and the standard deviation of the future electricity by a price-adjusting mechanism called Walrasian auction, which is also known as *tâtonnement* [1]. In Walrasian auction, a market adjusts prices based on the current

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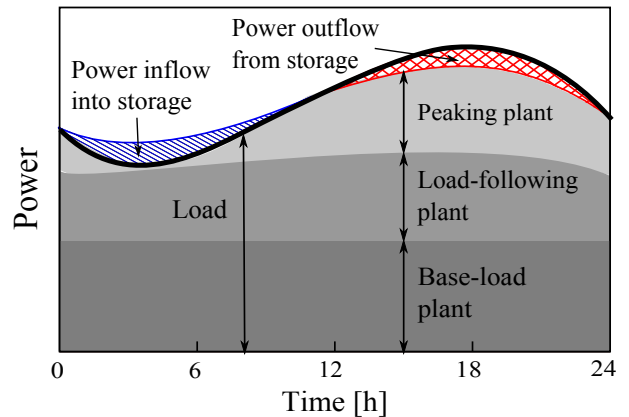


Fig. 1. An example of absorbing uncertainty of power demand by using peaking plant.

supply-demand imbalance, while each self-interested player optimizes the quantities of supply and demand to maximize its own profit at the given prices.

The primary purpose of employing this market mechanism is to enable decentralized and risk-limiting power dispatch. More specifically, unlike existing centralized risk-limiting approaches such as [2]–[4], our approach manages the risk of power shortage within a user-specified bound by imposing chance constraints on *each* dispatchable power provider. This is made possible since the proposed approach allows each dispatchable power provider to specify the probability density function (pdf) of the amount of energy that it must provide in the future. As a result, risk-limiting dispatch is realized through a market mechanism.

There are two additional contributions of the proposed approach. First, each dispatchable power provider can optimize its output by considering the expected cost, $\mathbb{E}[J(G)]$, instead of the nominal cost, $J(\mathbb{E}[G])$, where J is a cost function and G is the amount of future energy production. Second, each dispatchable power plant can optimally choose the amount of uncertainty that it commits to absorb by considering the cost of deviating its output. As illustrated in Fig. 1, peaking and load-following plants, such as gas and hydro power plants, typically absorb the majority of fluctuation in energy demand, while the output of base-load plants, such as nuclear stations, are kept almost constant since it is difficult and/or costly to change their outputs. The proposed approach achieves such an optimal allocation of power generation over time by a purely market-based mechanism.

We also provide a capability of optimally operating storage facilities, such as a battery and a pumped-storage hydroelec-

tricity, within the framework of contingent power dispatch and pricing. The existence of storage capacities in a grid introduces couplings between different points of time. Therefore, instead of considering an optimization problem at a single point of time, we optimize the nominal outputs of dispatchable power plants over multiple time steps in the future. The resulting algorithm is a decentralized stochastic model predictive control, which solves at each time step a contingent power dispatch and pricing problem over a receding finite prediction horizon.

B. Related Work

There is a solid body of literature in risk-limiting power dispatch, as well as in decentralized power dispatch and pricing. The main contribution of this work is to achieve both in a unified framework.

A need for risk-limiting power dispatch arises from rapidly growing penetration of intermittent wind and solar generations, which significantly increases the uncertainty in energy supply. Moreover, several recent incidents, most notably the Northeast blackout of 2003, showed that the traditional $N-1$ security criterion is vulnerable to “hidden” failures that occur probabilistically [5]. Recently, an alternative security criterion for power dispatch, called risk-based security or stochastic security, has emerged [2], [5], [6]. Although its definition varies, it seems to be a consensus that the risk of a blackout needs to be managed quantitatively based on a stochastic model of the power grid. For example, [7] developed a power dispatch method that guarantees market clearing under uncertainty. Their approach is “scenario based” in a sense that the continuous probability distribution of wind generation is approximated by a discrete one with a finite number of “slices”, on each of which a market-clearing constraint is imposed at all time steps. An issue of this approach is that the computational cost increases exponentially with the number of slices and the number of time steps. A similar approach is presented by [8], which employs scenario-based stochastic model predictive control (SMPC) that randomly samples from the distribution. This approach is “market-based” in a sense that a market specifies the dual variable (i.e., a shadow price) of the optimal dispatch problem, but the optimization is conducted in a centralized manner. Recently, [2] has proposed a risk-limiting dispatch approach that explicitly imposes a chance constraint on the overall supply-demand balance. [3] proposed a probabilistic extension to the $N-1$ security criterion as well as a power dispatch algorithm that satisfies the probabilistic criterion. However, these power dispatch methods are centralized.

In general, a decentralized power dispatch has two practical advantages over centralized ones. First, it can be readily applied to existing deregulated electricity markets, where the quantity of generation and consumption of each self-interested player is determined through a competitive market mechanism. Second, it can scale to a real-world power grid system, which consists of thousands of components. Various decentralized dispatch approaches have been proposed. For example, [9] developed a distributed power dispatch algo-

rithm called DYMOND, which is similar to our approach in that it is built upon dual decomposition and uses a Walrasian auction-based pricing mechanism. Optimal electricity pricing, which can be viewed as a decentralized power dispatch, is also a well studied area. For example, [10] considered a Walrasian auction-based pricing mechanism with demand response, while [11] proposed a double auction mechanism. However, these decentralized approaches assume a deterministic model, which does not allow to consider stochastic security criterion such as chance constraint.

Overall, to the best of our knowledge, ours is the first decentralized dispatch algorithm with chance constraints. Decentralized chance-constrained dispatch is difficult because, although the probability distribution of the future *net* load is typically available, the probability distribution of the amount power that *each* electricity provider must supply in the future cannot be specified in a conventional market mechanism since it only deals with the mean (i.e., expectation) of the probability distributions. The proposed market mechanism overcomes this challenge by trading the mean *and* the standard deviation of the probability distributions.

The rest of this paper is organized as follows. In Section II, we intuitively explain the concept of contingent power dispatch. In Section III, we formulate the contingent power dispatch problem. Section IV presents a decentralized reformulation of the contingent power dispatch problem, and proposes a decentralized stochastic MPC algorithm that solves the problem at every time step. Finally, Section IV demonstrates the proposed method by simulations.

II. WALK-THROUGH EXAMPLE

In this section, we intuitively explain the concept of contingent power dispatch and pricing using an example. For the sake of simplicity, we only consider a dispatching problem at a single time step in this section.

In our proposed approach, two types of power are traded in a market: nominal power and *contingent power*. Nominal power represents the expected demand and supply of power, while contingent power represents the deviation from the nominal. The key idea is to determine the *percentage* of the contingent power that each power provider covers through a market mechanism.

For example, consider a power grid with three dispatchable generators, as shown in Fig. 2. Plant 1 is a base-load plant, which produces nominal power with the lowest cost, but requires the largest cost to deviate the output (i.e., to produce contingent power) from the constant level. Plant 2 is a load-following plant, which has moderate cost to produce both nominal and contingent power. Plant 3 is a peaking plant, which has the highest cost of nominal power production, but the output can be easily adjusted with the least cost.

We consider a day-ahead market that deals with the net load, which is the total demand minus the total renewable production. In Fig. 2 we assume that the predicted net load at 8:00 p.m., January 2nd, is 300 MWh. Hence, on the preceding day, January 1st, at 8:00 p.m., 300 MWh nominal power is sold to the three plants in the day-ahead market.

Our proposed contingent power dispatch mechanism is different from regular electricity market in that the percentage of the contingent power covered by each plant is also allocated by a market mechanism. For example, in Fig. 2, plants 1, 2, and 3 commit to provide 20%, 30%, and 50% of contingent power, respectively. Each plant sells such a commitment at a price specified by the market. The market adjusts the price so that the total percentage of commitment is equal to 100%.

Then, 24 hours later, the actual net load turns out to be 320 MWh, resulting in 20 MWh excess load. We call the excess load as *contingent load*. The contingent load is allocated to each plant according to the levels of commitment agreed in the day-ahead market. As a result, plants 1, 2, and 3 produce 4 MWh, 6 MWh, and 10 MWh of contingent power, respectively. Hence, the total power produced by each plant is 154 MWh, 106 MWh, and 60 MWh. These sum up to 320 MWh, which matches the actual load.

The advantage of this contingent power dispatch approach is that each plant can know *a priori* the probability distribution of the amount of power that it has to generate, given the probability distribution of the future net load. As a result, each plant can quantitatively bound the risk of exceeding capacity, and statistically evaluate the expected cost of power generation.

In the example in Fig. 2, consider the future net load has a known probability distribution with a standard deviation of 10 MW. Then, the probability distributions of the power generation of the three plants have the same shape with the standard deviation: 2 MW, 3 MW, and 5 MW, respectively. Therefore, an allocation of the percentage of contingent power can be viewed as an allocation of standard deviation. In the following section, we formulate the contingent power dispatch problem as an optimal allocation problem of the mean *and* standard deviation of future net load.

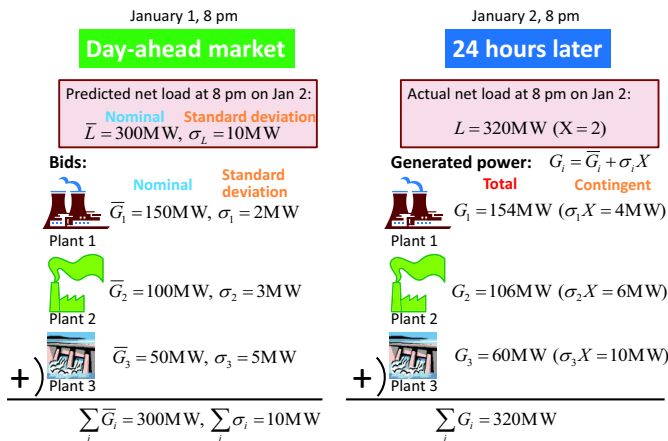


Fig. 2. Walk-through example of contingent power dispatch for a grid with three dispatchable power plants.

III. PROBLEM FORMULATION

In this section we formulate the contingent power dispatch and pricing problem introduced above into a decentralized chance-constrained programming. After presenting the problem setups in Sections III-A, III-B, and III-C, we first present a centralized formulation of the contingent power dispatch problem in III-D. Then, we reformulate it into a decentralized optimization problem using dual decomposition in III-E.

A. Definitions of Load, Generation, and Storage

We consider a power grid system consisting of non-dispatchable renewable generators, dispatchable power plants, batteries, and consumers. In our formulation, wind and solar generations, which are non-dispatchable and intermittent, are considered as negative load. We use the term *net load* to mean the total energy demand minus the wind and solar generations. The net load at time k is represented by a random variable $L(k)$, which is assumed to have a known probability distribution. We decompose $L(k)$ into its mean and the deviation from the mean as follows:

$$L(k) = \bar{L}(k) + \sigma_L(k)X(k), \quad (1)$$

where $X(k)$ is a zero-mean random variable with its standard deviation being one. Note that \bar{L} and σ_L in (1) are deterministic parameters representing the mean and the standard deviation of the predicted load. We call the first term in (1), $\bar{L}(k)$, as nominal load at time k , while referring to the second term, $\sigma_L(k)X(k)$, as contingent load.

We assume that there are N_g dispatchable generators and N_b batteries in the grid. In the electricity market, the i th generator commits to produce the following amount:

$$G_i(k) = \bar{G}_i(k) + \sigma_{g_i}(k)X(k), \quad (2)$$

where X is the same random variable as (1). \bar{G}_i is the nominal power production of the generator. Similarly, the j th battery commits to discharge the following amount:

$$R_j(k) = \bar{R}_j(k) + \sigma_{r_j}(k)X(k), \quad (3)$$

where \bar{R}_j is the nominal power outflow from the battery, which is positive when discharging and negative when charging. We assume a lossless battery dynamics, which is commonly assumed in existing literature such as [4]:

$$B_j(k) = B_j(k-1) + R_j(k), \quad (4)$$

where B_j is the storage level of the j th battery. We note that the loss of energy storage can also be considered in the proposed contingent power dispatch and pricing framework by replacing (4). Such an extension is beyond the scope of this paper. The second term in (2) and (3) represents *contingent power production*. Since they share X with (1), the i th generator and j th battery are committed to provide the fixed portion of the contingent load represented by σ_{g_i}/σ_L and σ_{r_j}/σ_L .

An electricity market must balance the load and generation. Hence,

$$L(k) = \sum_{i=1}^{N_g} G_i(k) + \sum_{j=1}^{N_b} R_j(k). \quad (5)$$

This can be achieved by ensuring that the nominal and contingent power productions from all dispatchable generators and batteries are equal to the net nominal and contingent load, respectively:

$$\bar{L}(k) = \sum_{i=1}^{N_g} \bar{G}_i(k) + \sum_{j=1}^{N_b} \bar{R}_j(k) \quad (6)$$

$$\sigma_L(k) = \sum_{i=1}^{N_g} \sigma_{g_i}(k) + \sum_{j=1}^{N_b} \sigma_{r_j}(k). \quad (7)$$

Hence, instead of directly balancing a random quantity as in (5), the market seeks to balance two quantities: nominal power and standard deviation, as in (6) and (7).

The proposed mechanism is similar to the conventional balancing market in that each participating generator holds regulating reserve on top of the expected output, $\bar{G}_i(k)$. The difference is that the proposed approach also gives the probability distribution of the amount of energy that needs to be provided by each generator. As a result, each generator can quantitatively limit the risk of the shortage of reserve capacity by imposing a chance constraint, as we explain in detail in Section III-C.

B. Cost Function

In our problem formulation, the step cost function of each plant and battery is a function of the supply of nominal power and standard deviation, denoted by $J_{g_i}(\bar{G}_i(k), \sigma_{g_i}(k))$ and $J_{r_j}(\bar{R}_j(k), \sigma_{r_j}(k))$. In this subsection, we omit the argument k for simplicity of notation. This formulation is justified by the following two observations.

a) Expected Cost: The first observation is that this formulation allows us to explicitly represent the expected cost of future energy generation and storage. Note that the expectation of any function of G_i is a function of \bar{G}_i and σ_{g_i} . Likewise, the expectation of any function of R_i is a function of \bar{R}_i and σ_{r_i} . Recall that the probability density function of X is assumed to be known. Therefore,

$$\mathbb{E}[J'(G_i)] = \int_{-\infty}^{+\infty} f(x) J'(\bar{G}_i + \sigma_{g_i} x) dx := J(\bar{G}_i, \sigma_{g_i}), \quad (8)$$

where f is a probability density function of X .

The integral in (8) is obtained in a closed-form in many cases. For example, when $J'(G_i)$ is a quadratic function, $J'(G_i) = a_1 G_i + a_2 G_i^2$, we have:

$$J(\bar{G}_i, \sigma_{g_i}) := \mathbb{E}[J'(G_i)] = a_1 \bar{G}_i + a_2 (\bar{G}_i^2 + \sigma_{g_i}^2). \quad (9)$$

More generally, consider a polynomial function given by:

$$J'(G_i) = \sum_{n=0}^N a_n G_i^n.$$

We assume that the N th moment of X exists. For $n \geq 1$, we denote by μ_n the n th raw moment of X . By assumption,

$\mu_0 = 1$, $\mu_1 = 0$, and $\mu_2 = 1$. Then,

$$J(\bar{G}_i, \sigma_{g_i}) := \mathbb{E}[J'(G_i)] = \sum_{n=0}^N a_n \sum_{l=0}^n \binom{n}{l} \mu_l \bar{G}_i^{n-l} \sigma_{g_i}^l,$$

where $\binom{n}{l}$ is a binomial coefficient. Note that the expected cost of energy production of each power plant cannot be evaluated in an existing market mechanism that only concerns \bar{G}_i .

b) Cost of Output Deviation: The second observation is that our objective function formulation allows us to consider the cost of deviating the output of a generator in real time. Recall that the contingent power generation, represented by $\sigma_{g_i}(k)X(k)$ in (2), means the deviation from the expected power output, $\bar{G}_i(k)$. The amount of contingent power that a power plant must supply is unknown until the moment of dispatch. Therefore, its standard deviation, $\sigma_{g_i}(k)$, can be interpreted as the expected deviation of power output.

A certain type of power plant can change its output easily while others cannot. For example, gas and hydro power plants are typically operated as peaking and load-following plants since their output can be varied easily. On the other hand, it is typically hard to quickly change the output of a nuclear power plant, which is operated as a base-load plant. With the contingent power dispatch and pricing framework, we can explicitly consider the degree of difficulty in changing the output of each power plant since the cost is a function of $\sigma_{g_i}(k)$. In other words, each plant can optimize the degree of the variation in its output by setting the cost of deviation appropriately.

C. Constraints

We assume that each generator and battery has a generation and storage capacity, G_i^m and B_j^m , which is a positive real constant:

$$0 \leq \bar{G}_i(k) + \sigma_{g_i}(k)X(k) \leq G_i^m \quad (10)$$

$$0 \leq B_j(k) \leq B_j^m. \quad (11)$$

Moreover, we assume that each battery has bounds on its charging and discharging rate:

$$R_j^d \leq \bar{R}_j(k) + \sigma_{r_j}(k)X(k) \leq R_j^c. \quad (12)$$

Note that, when X has an unbounded probability distribution, it is impossible in general to guarantee the satisfaction of constraint (10) and (12). This means that there is a risk of power shortage when unexpectedly large net electricity demand exceeds generation and discharging capacity of plants and batteries. In order to manage such a risk, we impose following chance constraints:

$$\Pr[\bar{G}_i(k) + \sigma_{g_i}(k)X(k) \leq G_i^m] \geq 1 - \epsilon_{g_i} \quad (13)$$

$$\Pr[\bar{R}_j(k) + \sigma_{r_j}(k)X(k) \leq R_j^c] \geq 1 - \epsilon_{r_j}, \quad (14)$$

where ϵ are *risk bounds* that are specified by users. Note that we do not bound the probability of violating the lower bounds of (10) and (12). This is because we assume that excessive production of energy does not pose a risk since the output of renewable generation can be arbitrarily reduced, by

employing the pitch control of the blades of wind turbines, for example.

We next convert the chance-constraints in (13) and (14) into deterministic constraints. Let $F_X(\cdot)$ be the cumulative distribution function of X . We also denote by $F_X^{-1}(\zeta)$ the inverse function of the cumulative distribution function:

$$F_X(y) = \zeta \iff F_X^{-1}(\zeta) = y.$$

Using these notations, the chance constraint (13) is transformed into an equivalent deterministic constraint as follows:

$$\begin{aligned} & \Pr [\bar{G}_i(k) + \sigma_{g_i}(k)X(k) \leq G_i^m] \geq 1 - \epsilon_{g_i} \\ \iff & F_X \left(\frac{G_i^m - \bar{G}_i(k)}{\sigma_{g_i}(k)} \right) \geq 1 - \epsilon_{g_i} \\ \iff & G_i^m - \bar{G}_i(k) \geq \sigma_{g_i}(k)F_X^{-1}(1 - \epsilon_{g_i}). \end{aligned}$$

The last equivalence is derived from the fact that a cumulative distribution function is always a non-decreasing function. (14) can also be transformed into an equivalent deterministic constraint in the same manner.

D. Centralized Contingent Power Dispatch

We now present the overall formulation of the contingent power dispatch problem. Present in this subsection is a centralized formulation, which does not involve pricing since the allocation of electricity generation is optimized by a centralized process instead of a market. The pricing problem is introduced in the decentralized formulation presented in Section III-E.

The objective of contingent power dispatch is to find the optimal allocation of nominal and contingent powers to dispatchable power providers so that the overall cost over a finite time horizon is minimized. After applying the deterministic transformation described above to chance-constraints (13) and (14), we now formulate the contingent power dispatch problem as a finite-horizon chance-constrained optimal control problem, as below:

Problem 1: Centralized Contingent Power Dispatch

$$\begin{aligned} \min_{\substack{\bar{G}_{1:N_g}, \sigma_{g_{1:N_g}}, \\ \bar{R}_{1:N_g}, \sigma_{r_{1:N_g}}}} & \sum_{k=\tau}^{\tau+H-1} \left(\sum_{i=1}^{N_g} J_{g_i}(\bar{G}_i(k), \sigma_{g_i}(k)) \right. \\ & \left. + \sum_{j=1}^{N_b} J_{r_j}(\bar{R}_j(k), \sigma_{r_j}(k)) \right) \end{aligned} \quad (15)$$

$$\text{s.t.} \quad \bar{L}(k) = \sum_{i=1}^{N_g} \bar{G}_i(k) + \sum_{j=1}^{N_b} \bar{R}_j(k) \quad (16)$$

$$\sigma_L(k) = \sum_{i=1}^{N_g} \sigma_{g_i}(k) + \sum_{j=1}^{N_b} \sigma_{r_j}(k) \quad (17)$$

$$\bar{G}_i(k) \leq G_i^m - \sigma_{g_i}(k)F_X^{-1}(1 - \epsilon_{g_i}) \quad (18)$$

$$\bar{R}_j(k) \leq R_j^c - \sigma_{r_j}(k)F_X^{-1}(1 - \epsilon_{r_j}) \quad (19)$$

$$0 \leq B_j(k) \leq B_j^m \quad (20)$$

where H is a prediction horizon. The decision variable $\bar{G}_{1:N_g}$ consists of the nominal power output of *all* power plants at all time steps in the horizon, defined as follows:

$$\begin{aligned} \bar{G}_{1:N_g} & := \{\bar{G}_1 \cdots \bar{G}_{N_g}\} \\ \bar{G}_i & := \{\bar{G}_i(\tau) \cdots \bar{G}_i(\tau + H - 1)\}. \end{aligned}$$

The other decision variables, $\sigma_{g_{1:N_g}}$, $\bar{R}_{1:N_g}$, and $\sigma_{r_{1:N_g}}$, are likewise defined.

E. Decentralized Formulation of Contingent Power Dispatch and Pricing

The advantage of our problem formulation, Problem 1, is that it can be reformulated into a decentralized optimization problem by using dual decomposition [12]. The resulting decentralized formulation consists of two parts. In the first part (Problem 2), each self-interested power provider maximizes its own profit by optimizing the supply quantity of nominal and contingent power, given their prices. Chance constraints are imposed on each individual power provider in order to limit the risk of power shortage. In the second part of the decentralized optimization (Problem 3), the prices of the mean and the standard deviation of electricity are optimized by a market. It turns out that the market-clearing prices achieve the minimum overall cost.

Let $p_N(k)$ and $p_\sigma(k)$ be the dual variables for (16) and (17), respectively. The dual variables correspond to the prices of nominal power and standard deviation. Given the prices, the following problem is solved by each power provider:

Problem 2: Decentralized Contingent Power Dispatch

For i th generator

$$\begin{aligned} \min_{\bar{G}_i, \sigma_{g_i}} & \sum_{k=\tau}^{\tau+H-1} \{ J_{g_i}(\bar{G}_i(k), \sigma_{g_i}(k)) \\ & - (p_N(k)\bar{G}_i(k) + p_\sigma(k)\sigma_{g_i}(k)) \} \end{aligned} \quad (21)$$

$$\text{s.t.} \quad \bar{G}_i(k) \leq G_i^m - \sigma_{g_i}(k)F_X^{-1}(1 - \epsilon_{g_i}) \quad (22)$$

For j th battery

$$\begin{aligned} \min_{\bar{R}_j, \sigma_{r_j}} & \sum_{k=\tau}^{\tau+H-1} \{ J_{r_j}(\bar{R}_j(k), \sigma_{r_j}(k)) \\ & - (p_N(k)\bar{R}_j(k) + p_\sigma(k)\sigma_{r_j}(k)) \} \end{aligned} \quad (23)$$

$$\text{s.t.} \quad \bar{R}_j(k) \leq R_j^c - \sigma_{r_j}(k)F_X^{-1}(1 - \epsilon_{r_j}) \quad (24)$$

$$0 \leq B_j(k) \leq B_j^m. \quad (25)$$

Note that this optimization problem only involve variables of a single power plant or a battery. Hence, it can be solved in a decentralized manner. Also note that, in (21), $p_N\bar{G}_i + p_\sigma\sigma_{g_i}$ corresponds to the revenue of the i th generator obtained by selling \bar{G}_i of nominal power and σ_{g_i} of standard deviation in the market. Hence, minimizing the objective function in (21) and (23) means maximizing the benefit of the i th generator and the j th battery.

Let $\bar{G}_i^*(k; p_N, p_\sigma)$, $\sigma_{g_i}^*(k; p_N, p_\sigma)$, $\bar{R}_j^*(k; p_N, p_\sigma)$, and $\sigma_{r_j}^*(k; p_N, p_\sigma)$ be the optimal solutions to (21)-(25) given

the prices $p_N := \{p_N(\tau) \cdots p_N(\tau + H - 1)\}$ and $p_\sigma := \{p_\sigma(\tau) \cdots p_\sigma(\tau + H - 1)\}$. The market finds market-clearing prices by solving the following root-finding problem:

Problem 3: Contingent Power Pricing

For $k = \tau \cdots \tau + H - 1$, find $[p_N(k), p_\sigma(k)]$ such that:

$$\bar{L}(k) = \sum_{i=1}^{N_g} \bar{G}_i^*(k; p_N, p_\sigma) + \sum_{j=1}^{N_b} \bar{R}_j^*(k; p_N, p_\sigma) \quad (26)$$

$$\sigma_L(k) = \sum_{i=1}^{N_g} \sigma_{g_i}^*(k; p_N, p_\sigma) + \sum_{j=1}^{N_b} \sigma_{r_j}^*(k; p_N, p_\sigma). \quad (27)$$

The above equalities correspond to the stationary condition for the dual of Problem 1. Since the dual objective function is guaranteed to be concave, it is a sufficient condition for optimality. Therefore, with our formulation, the market-clearing prices achieve the dual optimality. If J_{g_i} and J_{r_j} are convex functions, the dual solution has no duality gap. Therefore, an optimal solution to Problems 2 and 3 is guaranteed to be an optimal solution to Problem 1. Although the decentralized optimization with nonconvex cost functions may result in a suboptimal solution, an upper bound on the duality gap can be evaluated posteriori.

IV. MARKET-BASED SOLUTION METHOD TO CONTINGENT POWER DISPATCH AND PRICING

The goal of this section is to develop a decentralized solution algorithm to Problems 2 and 3 that can be readily used in a market. To this end, we build our algorithm upon Walrasian auction, where the prices are iteratively updated by a market while each power provider responds to the price signals by adjusting the supply of nominal power and standard deviation. At each time step, Problems 2 and 3 are solved with a finite prediction horizon. Hence, the resulting algorithm can be viewed as a decentralized stochastic model predictive control.

A. Finite Horizon Contingent Power Dispatch and Pricing

In this subsection we develop a finite-horizon optimization algorithm that finds optimal solutions to Problems 2 and 3. Given the prices p_N, p_σ , in each iteration, each self-interested power providers solves Problem 2 to obtain the optimal supply of nominal power and standard deviation, $\bar{G}_i^*(k; p_N, p_\sigma)$, $\sigma_{g_i}^*(k; p_N, p_\sigma)$, $\bar{R}_j^*(k; p_N, p_\sigma)$, and $\sigma_{r_j}^*(k; p_N, p_\sigma)$, for $k = \tau \cdots \tau + H - 1$. Problem 3 is solved by a market to find the prices that balance the supply and demand of nominal power and standard deviation. It is known that an optimal solution to Problem 3 can be found by a subgradient method with a diminishing step size [13]. At each iteration, the subgradient method updates the prices with an increment that is proportional to the difference between the supply and the demand, i.e., the right hand sides of (26) and (27). In economics, such a price adjusting algorithm is referred to as Walrasian auction, which is frequently used as a model of the price dynamics in a competitive market [1].

The complete description of the algorithm is given in Algorithm 1. The algorithm is initialized with initial prices,

Algorithm 1 Finite-Horizon Contingent Power Dispatch and Pricing

- 1: **function** FiniteHorizonContingentPowerDispatch(k, p_N, p_σ)
- 2: $\alpha \leftarrow \alpha_0$ and $\lambda \in (0, 1)$
- 3: **while** $|\varepsilon_N(k : k + H - 1, k)| > \gamma_N \vee |\varepsilon_\sigma(k : k + H - 1, k)| > \gamma_\sigma$ **do**
- 4: The market announces the prices $p_N(k : k + H - 1, k)$, $p_\sigma(k : k + H - 1, k)$ to plants and batteries.
- 5: Each power provider computes $\bar{G}_i^*(k : k + H - 1, k)$, $\sigma_{g_i}^*(k : k + H - 1, k)$, $\bar{R}_j^*(k : k + H - 1, k)$, $\sigma_{r_j}^*(k : k + H - 1, k)$ by solving Problem 2.
- 6: $\varepsilon_N(k : k + H - 1, k) \leftarrow \bar{L}(k : k + H - 1) - \sum_{i=1}^{N_g} \bar{G}_i^*(k : k + H - 1, k) + \sum_{j=1}^{N_b} \bar{R}_j^*(k : k + H - 1, k)$
- 7: $\varepsilon_\sigma(k : k + H - 1, k) \leftarrow \sigma_L(k : k + H - 1) - \sum_{i=1}^{N_g} \sigma_{g_i}^*(k : k + H - 1, k) + \sum_{j=1}^{N_b} \sigma_{r_j}^*(k : k + H - 1, k)$
- 8: $p_N(k : k + H - 1, k) \leftarrow p_N(k : k + H - 1, k) + \alpha \varepsilon_N(k : k + H - 1, k)$
- 9: $p_\sigma(k : k + H - 1, k) \leftarrow p_\sigma(k : k + H - 1, k) + \alpha \varepsilon_\sigma(k : k + H - 1, k)$
- 10: $\alpha \leftarrow \lambda \alpha$
- 11: **end while**

Algorithm 2 Receding Horizon Algorithm

- 1: Initialize $p_N(1 : H, 1)$ and $p_\sigma(1 : H, 1)$.
- 2: **for** $k = 1, 2, \dots$ **do**
- 3: FiniteHorizonContingentPowerDispatch($k, p_N(k, k + H - 1, k), p_\sigma(k, k + H - 1, k)$)
- 4: $p_N(k + 1 : k + H, k + 1) \leftarrow [p_N^*(k + 1 : k + H - 1, k), p_{N, \text{init}}]$
- 5: $p_\sigma(k + 1 : k + H, k + 1) \leftarrow [p_\sigma^*(k + 1 : k + H - 1, k), p_{\sigma, \text{init}}]$
- 6: **end for**

$p_N(0 : H - 1, 0)$ and $p_\sigma(0 : H - 1, 0)$ (Line 1). The step size of the subgradient method, α , diminishes throughout iteration with a discount factor $\lambda \in (0, 1)$ (Line 10). The initial step size and the discount factor λ are also initialized appropriately (Line 2). In each iteration, each power provider solves Problem 2 to find the optimal supply levels of nominal power and standard deviation at the given prices (Line 5). Then, the market adjusts the prices by the subgradient method (Line 8 and 9). This sequence is repeated until the differences between the demands and the aggregate supplies are within specified tolerance levels (Line 3).

B. Receding Horizon Contingent Power Dispatch and Pricing

Algorithm 1 is solved repeatedly at each time step with a receding prediction horizon, as described in Algorithm 2. Here, Algorithm 1 is called at each time step as a subroutine (Line 3). In order to enhance the computation efficiency, we use the last $H - 1$ optimal prices at time k , denoted by $p_N^*(k + 1 : k + H - 1, k)$ and $p_\sigma^*(k + 1 : k + H - 1, k)$, as the initial prices of Algorithm 1 at time $k + 1$ (Lines 4 and 5).

TABLE I
COST FUNCTION PARAMETERS [USD/MWh]

	a	b	a_σ	b_σ	c	c_σ
Plant 1	10	0.1	0	1000		
Plant 2	30	0.3	0	10		
Plant 3	50	0.5	0	0.5		
Battery 1					0.1	0.1
Battery 2					0.2	0.2

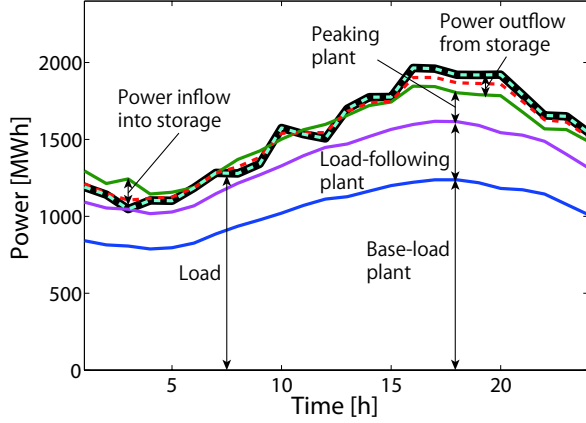


Fig. 3. A stacked graph showing the outputs from of power plants and batteries. Legends: blue solid line-Plant 1, purple solid line-Plant 2, green solid line-Plant 3, red dashed-line-Battery 1, cyan dashed line-Battery 2, thick black solid line-net load.

V. SIMULATION RESULTS

A. Simulation Settings

We consider three power plants and two batteries, each of which has a quadratic step cost function as follows:

$$J_{g_i}(\bar{G}_i(k), \sigma_{g_i}(k)) = \sum_{k=\tau}^{\tau+H-1} (a_i \bar{G}_i(k) + b_i \bar{G}_i(k)^2 + a_{\sigma_i} \sigma_{g_i}(k) + b_{\sigma_i} \sigma_{g_i}(k)^2)$$

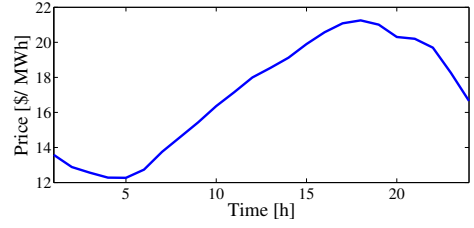
$$J_{r_j}(\bar{R}_j(k), \sigma_{r_j}(k)) = \sum_{k=\tau}^{\tau+H-1} (c_j \bar{R}_j(k)^2 + c_{\sigma_j} \sigma_{r_j}(k)^2),$$

where a_i , b_i , a_{σ_i} , b_{σ_i} , c_j , and c_{σ_j} are constant parameters given in Table I. Plants 1, 2, and 3 model a baseload plant, a load-following plant, and a peaking plant, respectively. The cost functions of the peaking plant (Plant 3) and the batteries are the expected cost derived from (9). The baseload and load-following plants (Plants 1 and 2) has greater a_σ in order to represent the degree of difficulty in deviating their outputs. Also note that the baseload plant has the lowest cost of nominal power while the peaking plant has the highest cost of nominal power.

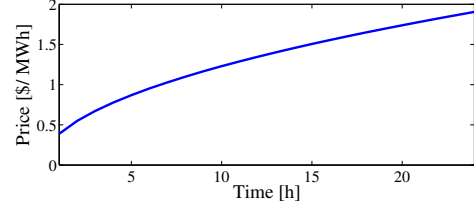
The capacities of the power plants and the batteries are set as follows:

$$G_i^m = 1500 \text{ MW} \quad (i = 1, 2, 3)$$

$$B_j^m = 1500 \text{ MWh}, \quad R_j^d = R_j^c = 100 \text{ MW} \quad (j = 1, 2).$$



(a) The price of nominal power



(b) The price of standard deviation

Fig. 4. The market-clearing prices of nominal power and standard deviation over 24 hours.

The risk bounds, ϵ_{g_i} and ϵ_{r_i} , are set to 0.01 for all plants and batteries.

The profile of the nominal net load, \bar{L} , We use a 24 hour-long data of electricity demand in California on August 23, 2012¹ as the nominal net demand, \bar{L} . We scaled the data to fit into our simulation with three plants and two batteries. The probability distribution of the net load at time k is approximated by a Gaussian distribution. The standard deviation of the net load, $\sigma_L(k)$, is monotonically increasing over time since the prediction of the far future involves greater uncertainty than that of the near future. By assuming that an independent, Gaussian-distributed uncertainty with a constant standard deviation is added at each time step, the accumulated uncertainty has a standard deviation as follows:

$$\sigma_L(k) = \eta \sqrt{k}, \quad (28)$$

where η is a constant parameter, which is set to 30. At each time step in the simulation, the profile of the nominal net load is shifted by a random increment drawn from a zero-mean Gaussian distribution with the standard deviation being η . The initial step size of the subgradient method and the discount factor are given as $\alpha = 0.05$ and $\lambda = 0.995$, respectively. Simulations are conducted on a machine with Intel Core i5-2520M CPU clocked at 2.50 GHz and a 4.00 GB RAM.

B. Results

Fig. 3 shows the amount of energy that is actually dispatched from each plant and battery. Note that the figure shows a similar tendency as Fig. 1 in that the baseload plant (Plant 1) the high-frequency fluctuation of the net load is mostly absorbed by the peaking plant (Plant 3) and the two batteries, while the baseload plant has relatively small rate of change in its output. This is because the

¹The data is available at the California ISO's webpage: <http://www.caiso.com/Pages/Today's-Outlook-Details.aspx>

TABLE II

PERFORMANCE COMPARISON OF THE PROPOSED ALGORITHM AND THE DETERMINISTIC ALGORITHM

	Probability of failure	Average cost
Proposed algorithm	0.0%	4.7865×10^6
Deterministic algorithm	2.1%	4.8148×10^6

peaking plant and the batteries have smaller cost of providing contingent power (i.e., standard deviation) than the load-following and baseload plants. The most significant portion of the energy is provided by the baseload plant since its cost of nominal power is the cheapest. Also observe that the two batteries store electricity when the demand is low, while discharging it when the demand is high. We emphasize that these seemingly cooperative behaviors of power plants and batteries result from a purely market-based process where each power provider simply maximizes its own profit by solving Problem 2.

Figs. 4(a) and 4(b) shows the market-clearing prices of the nominal power and the standard deviation. As expected, the profile of the price of nominal power is similar to that of the net load, represented by the thick solid line in Fig. 3. On the other hand, the price of standard deviation has a similar tendency as the standard deviation of the net load, given by (28). Intuitively, this is because the market encourages power plants and batteries to “sell” greater amount of standard deviation (i.e., to absorb greater uncertainty) by raising the price when the prediction of the net load has significant uncertainty.

Finally, we conducted a Monte Carlo simulation to demonstrate that the proposed approach can limit the risk of power shortage. We compare the proposed algorithm with a deterministic one, which omits the second term of the right hand side of (22) and (24). This deterministic approach is roughly corresponds to the decentralized algorithm presented in [9], which does not consider chance constraints and contingent power. We run both simulations 1,000 times to evaluate the probability of failure (i.e., risk) as well as the average total cost. Average computation time of the contingent power dispatch and pricing algorithm per time step is 2.09 minutes. A simulation is regarded as a failure if the capacity constraints in (11) by at least one of the power plants or batteries. The results are summarized in Table II. In order to make a fair comparison, we excluded the cost of contingent power generation from the first row of the table. The proposed algorithm results in a slight reduction in the average cost. This is probably because the proposed algorithm explicitly minimizes the expected (i.e., average) cost of Plant 3 and the batteries. Also note that none of the simulations resulted in a failure when using the proposed contingent power dispatch and pricing algorithm. This result indicate that the risk is limited within the given bound, 1%. On the other hand, without chance constraints, the power dispatch algorithm results in the 2.1% probability of failure.

VI. CONCLUSION

In this paper, we proposed a novel market-based contingent power dispatch algorithm that enables a grid with intermittent energy sources to dispatch energy within a user-specified risk bound in a decentralized manner.

We first presented our concept of contingent power dispatch, and formulated a centralized contingent power dispatch problem (Problem 1). This problem can be solved by one optimization process, but it is far from the real power market mechanism. We next reformulated the problem using dual decomposition method to introduce a market-based decentralized contingent power dispatch problem (Problem 2) and a optimal power pricing problem (Problem 3). Then we proposed a decentralized optimization algorithm to solve Problem 2, 3 by using a subgradient method. Finally, we demonstrated the performance of our proposed algorithm in simulation. The result showed that the risk of power imbalance can be bounded quantitatively and the total cost of power generation can be minimized by the proposed algorithm.

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