

Swimming in Spacetime: Motion by Cyclic Changes in Body Shape

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Cyclic changes in the shape of a quasi-rigid body on a curved manifold can lead to net translation and/or rotation of the body. The amount of translation depends on the intrinsic curvature of the manifold. Presuming spacetime is a curved manifold as portrayed by general relativity, translation in space can be accomplished simply by cyclic changes in the shape of a body, without any external forces.

The motion of a swimmer at low Reynolds number is determined by the geometry of the sequence of shapes that the swimmer assumes (1). At low Reynolds number, the effects of inertia are negligible and, in the absence of external forces, bodies are at rest. Nevertheless, as a body changes its shape, its location and orientation generally change. A cyclic change in the shape of a body can lead to a net translation or rotation. The net translation or rotation does not depend on the speed with which the shape changes are carried out; it is a consequence of the geometry of the sequence of shapes, a classical example of geometric phase (2). For example, the cilia of a paramecium effectively define and allow changes in its shape, and locomotion is accomplished through cyclic changes in its shape through motion of its cilia.

Consider two concentric spheres in flat space with equal moments of inertia connected at their centers (3–5). If one sphere is rotated with respect to the other, then the orientation of the system adjusts appropriately. The adjustment can be determined using the fact that angular momentum is conserved. If the total angular momentum is zero, the angular velocities of the spheres are equal but opposite. Therefore, if one sphere is rotated about an axis by an angle θ with respect to the other sphere, then the first sphere rotates in space about that axis by $\theta/2$ and the other rotates by $-\theta/2$. If the two spheres undergo a sequence of rotations with respect to each other and are then brought back to their original relative orientation, then their orientation in space can undergo a net rotation. The net rotation of the system does not depend on the speed with which these relative rotations occur; it depends only on the sequence of relative orientations. This is another classical example of geometric phase.

Generalizing the latter example, I show here that it is possible for quasi-rigid bodies to swim on frictionless curved manifolds simply by way

of cyclic changes in their shape. Then, presuming spacetime is a curved manifold as portrayed by general relativity, I show that net translations in space can be accomplished through cyclic, engineered changes in the shape of a body. Motion in space can be accomplished without thrust or external forces.

Swimming with spherical caps. The two-sphere example can be generalized to two rigid bodies of arbitrary shape with a common fixed point (3). Consider two circular spherical caps of angular radius γ , with uniform surface density and mass m , on a sphere of radius R . The relative configuration is specified by two angles (Fig. 1). Let \hat{x} , \hat{y} , and \hat{z} be a right-handed orthonormal basis with origin at the center of the sphere. The caps are symmetrically displaced above and below the x - y plane by the angle ϕ , and they are rotated oppositely about their centers by the angle θ . The angles θ and ϕ are deformation coordinates (6). As the relative configuration changes, the orientation of the system adjusts dynamically. Because of the symmetry of the system, it will move by rotating around the \hat{z} axis. Let ψ be the longitude of the system, the angular displacement of the system around the \hat{z} axis. Assuming zero total angular momentum, the equation of motion of ψ is

$$D\psi(t) = \frac{D\theta(t)C\sin\phi(t)}{A(\cos\phi(t))^2 + C(\sin\phi(t))^2} \quad (1)$$

where D is the derivative operator, C is the moment of inertia of each cap about the radial line through its center, and A is the moment of inertia about any line through the center of the sphere perpendicular to this line. The denominator is half the moment of inertia of the system about the \hat{z} axis; the numerator is half the \hat{z} component of the angular momentum due to the twisting (nonzero $D\theta$) of the cap. Each cap makes the same contribution to the \hat{z} moment of inertia and the \hat{z} component of the angular momentum, so the additional factors of two in the numerator and denominator cancel. Note that the counterro-

tation of the two disks can be accomplished without external torques, for example, by fixing the distance between the centers by a rigid circular arc and then contracting a tension wire symmetrically attached to the outer edge of the two caps. Nevertheless, their contributions to the \hat{z} component of the angular momentum are parallel and add to give a nonzero angular momentum. Other components of the angular momentum are zero. The total angular momentum of the system is zero, so the angular momentum due to twisting must be balanced by the angular momentum of the motion of the system around the sphere.

A net rotation of the system can be accomplished by taking the internal configuration of the system through a cycle. A cycle may be accomplished by increasing θ by $\Delta\theta$ while holding ϕ fixed, then increasing ϕ by $\Delta\phi$ while holding θ fixed, decreasing θ , then decreasing ϕ , which brings the system back to the original relative configuration. In those phases of the cycle in which θ is held fixed, the system does not rotate. The rotation of the system during the two phases in which θ is changed do not balance because ϕ is different. For small γ (small caps), small ϕ (small separation), and small $\Delta\theta$ and $\Delta\phi$ (small deformations), the net motion of the system per cycle of deformation is approximately

$$\Delta\psi = \frac{1}{2} \gamma^2 \Delta\theta \Delta\phi \quad (2)$$

Having made everything small compared with the size of the sphere, it is apparent that the essence of the matter is not that these are rigid bodies, but that the system lives on a curved manifold.

Swimming on curved manifolds. Rigid bodies, defined by a large number of redundant distance constraints between constituents, cannot, in general, move on a manifold of nonconstant curvature because the redundant constraints cannot remain consistent. Consider three masses on the vertices of a triangle, with geodesics of fixed length connecting them. Now place a fourth mass in the middle of the triangle, equidistant from each of the three masses. The distance to those masses will depend on the curvature of the manifold. This system of four masses cannot move to a region of different curvature without some of the constraint distances changing. This is a consequence of the fact that the constraints are redundant—more constraints are specified than are needed to specify the relative location of the masses. If instead of specifying that the fourth mass is equidistant from the three masses it is specified that it is a certain distance from two of the vertices, then all of the constraints can be maintained even as the curvature changes. Such a rigid body with irredundant constraints will be called a quasi-rigid body. A quasi-rigid body has

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a well-defined configuration even though the distances between all the constituents are not fixed. One particular class of quasi-rigid bodies has a tree-like topology. From each vertex extend an arbitrary number of branches or struts. At the end of any strut, more struts may be attached. Masses may be attached to the vertices. The configuration of the system is determined if the lengths of all the struts and the angles they make with the connecting struts are specified. The first example shows that more general quasi-rigid bodies are allowed; they need not be tree-like and the struts need not be geodesics. Struts of quasi-rigid bodies can be constructed from a local lattice or truss of nearly rigid rods. If the rods are small compared with the intrinsic curvature of the manifold, then they can maintain the local constraints of the truss, without much strain, while conforming to the manifold on a larger scale.

For a quasi-rigid body to swim on a curved manifold, it must undergo changes in its shape. For such a change to result in a net rotation or translation of the body on the manifold, the shape changes must go through a nontrivial cycle of deformation. A simple quasi-rigid body that satisfies these requirements is a body consisting of one mass point with mass m_0 connected to two other mass points, each with mass m_1 , by geodesic struts of given length separated by a given angle. The body can be deformed by changing the length of the struts or the angle between them; nontrivial contractible cycles enclose area on the deformation parameter plane.

A Lagrangian for the free geodesic motion of a particle on a manifold is $mg(q)(v, v)/2$, where m is the mass of the particle, g is the

metric, q is a tuple of manifold coordinates, and v is a tuple of the associated generalized velocities (7). A Lagrangian for a quasi-rigid body can be obtained by defining generalized coordinates for the system that incorporate the time-dependent constraints among the constituent particles, deriving the associated transformation of the generalized velocities, and rewriting the free Lagrangians for the constituent masses in terms of these system coordinates (8).

Consider the dynamics of such a quasi-rigid body on two illustrative manifolds: the plane and the sphere (9). Because of the symmetry of the body and the manifold, these three-mass quasi-rigid bodies will move only along the direction bisecting the two struts. The calculation can be simplified by choosing one of the manifold coordinates to coincide with this direction.

For the plane, choose rectangular coordinates, with the x -axis bisecting the struts. The x -coordinate of the vertex at time t is $x(t)$, the length of the strut is $l(t)$, and the angle between the struts and the x -axis is $\alpha(t)$. Momentum conservation, with zero momentum, leads to

$$Dx(t) = \frac{2m_1}{m_0 + 2m_1} (l(t)\sin\alpha(t)D\alpha(t) - \cos\alpha(t)Dl(t)) \tag{3}$$

For a cycle of deformation in the parameter plane (l, α), the net translation, Δx , can be written as an integral of a real-valued 1-form. In this case, the 1-form is closed, so there is no net translation for a cycle of deformation: $\Delta x = 0$.

For the sphere, choose spherical coordinates, with the equator bisecting the struts (Fig. 2). The longitude of the vertex at time t is $\psi(t)$. The length of the geodesic struts, which are spherical arcs, is specified by the angular extent $e(t)$, measured from the center of the sphere. The separation angle between the struts and the line of symmetry is $\alpha(t)$. Conservation of momentum, with zero initial momentum, is enough to determine the motion of the system as it deforms

$$D\psi(t) = \frac{2m_1(D\alpha(t)\sin e(t)\cos e(t)\sin\alpha(t) - \cos\alpha(t)De(t))}{m_0 + 2m_1(\cos\alpha(t))^2 + 2m_1(\sin\alpha(t))^2(\cos e(t))^2} \tag{4}$$

The net translation can be written as a line integral of a real-valued 1-form along the deformation parameter path, or by Stokes's theorem as an integral over the region enclosed by the path of the 2-form that is the exterior derivative of the 1-form. In this case, the real-valued 1-form is not closed and there is a net translation. For small deformations and for bodies with small extent relative to the size of the sphere, the translation per cycle is approximately

$$\Delta\psi = - \frac{4m_0m_1}{(m_0 + 2m_1)^2} (\sin e)^2 \sin\alpha \Delta e \Delta\alpha \tag{5}$$

Swimming on manifolds without symmetry. The examples so far have relied on momentum conservation to deduce the inertial motion of the system from changes in the deformation parameters. If the momentum is not conserved, this does not work. However, if the parameters are varied quickly, then the response of the system is almost the same as if the momenta were conserved. Assume this is the case and consider henceforth the case for which the momentum is zero, whether or not it is exactly conserved or only approximately conserved for fast deformations.

The generalized momentum of the quasi-rigid body is obtained by taking the partial derivative of the system Lagrangian with respect to its generalized velocity argument. If the free Lagrangians are homogeneous quadratic forms in the velocities, then the momenta are sums of terms that are linear in the system generalized velocities and of terms that are linear in the rate of change of the deformation parameters. This is a set of linear equations that can be solved to give the generalized velocities $Dq(t)$ in terms of the momenta and $q(t)$, $c(t)$, and $Dc(t)$, where $q(t)$ are the generalized coordinates of the system, $c(t)$ are the deformation parameters, and $Dc(t)$ are their rates of change, all at time t . For zero momentum, the result is linear in $Dc(t)$

$$Dq(t) = A(c(t), q(t))Dc(t) \tag{6}$$

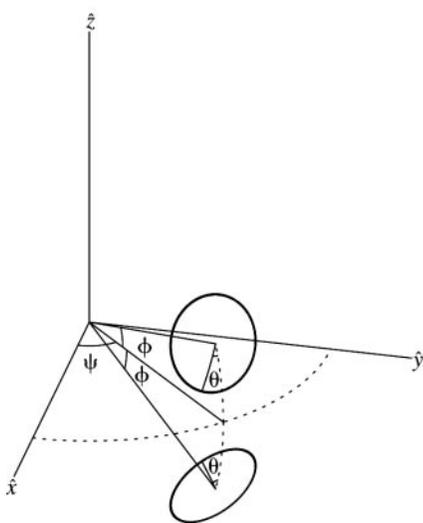


Fig. 1. Two spherical caps on a sphere. The two deformation parameters are θ and ϕ . The caps twist oppositely by the angle θ , and they are symmetrically displaced by the angle ϕ above and below the equator. The angle ψ is the longitude of the system. The deformation parameters follow a specified schedule, and the longitude of the system adjusts dynamically.

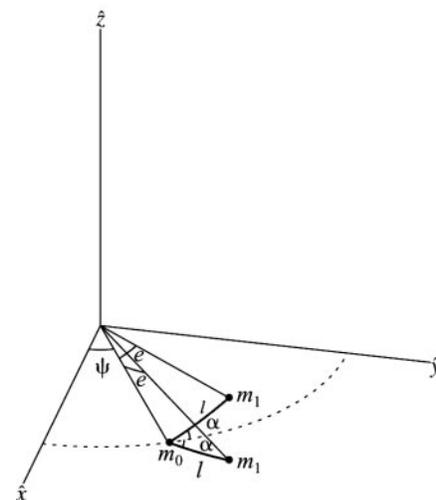


Fig. 2. A quasi-rigid body on the sphere. The two deformation parameters are the length l and separation angle α . The geodesic struts subtend the angle $e = l/R$ at the center of the sphere, where R is the radius of the sphere, and the separation angle α is the angle between the struts and the symmetry line, measured on the plane tangent to the sphere at the vertex. The angle ψ is the longitude of the system. The deformation parameters follow a specified schedule, and the longitude of the system adjusts dynamically.

This may be viewed as defining a vector-valued 1-form that takes tangent vectors along the deformation parameter path, with components $Dc(t)$, to the generalized velocity components. This vector-valued 1-form is a vector gauge potential (3).

Consider piece-wise linear parameter paths. For a segment with tangent $Dc(t) = \xi$, the evolution is governed by the equations

$$\begin{aligned} Dc(t) &= \xi \\ Dq(t) &= A(c(t), q(t))\xi \end{aligned} \quad (7)$$

or, collectively,

$$Ds(t) = G_\xi(s(t)) \quad (8)$$

where $s(t) = (c(t), q(t))$. Let

$$L_G F = DF G \quad (9)$$

define the operator L_G that gives the rate of change of state functions F along solution paths of G . Exponentiating this operator advances state functions (8). Advancing the coordinate selector gives the coordinates

$$q(t_0 + \Delta t) = (e^{\Delta L_G} Q)(c(t_0), q(t_0)) \quad (10)$$

where $Q(c, q) = q$.

Consider the evolution of the system resulting from a small loop in deformation parameter space around a parallelogram specified by two vectors ξ and η . The loop is traversed by moving first along ξ , then η , then $-\xi$, then $-\eta$, which brings the system back to the initial point. The coordinate tuple after evolution around the parallelogram is (10)

$$q(t_0 + \Delta t) = (e^{L_{G_\xi} L_{G_\eta} e^{-L_{G_\xi}} e^{-L_{G_\eta}} Q)(c(t_0), q(t_0)) \quad (11)$$

For small loops, the lowest order change in the coordinates is given by a commutator

$$q(t_0 + \Delta t) = q(t_0) + ([L_{G_\xi}, L_{G_\eta}] Q)(c(t_0), q(t_0)) \quad (12)$$

The commutator defines a vector-valued 2-form of the vectors ξ and η . This 2-form is the field strength associated with the vector gauge potential. Let dc^a and dc^b be dual basis 1-forms on the parameter plane. The components of the vector potential are $A^i(c, q) = A^i_a(c, q) dc^a + A^i_b(c, q) dc^b$ with i running through the component selectors of \dot{q} . The components of the field strength are (11)

$$\begin{aligned} F^i(\xi, \eta)(c, q) &= ([L_{G_\xi}, L_{G_\eta}] Q)(c, q) \\ &= ((\partial_1 A^i_b(c, q)) A_a(c, q) \\ &\quad - (\partial_1 A^i_a(c, q)) A_b(c, q)) (\xi^a \eta^b - \xi^b \eta^a) \\ &\quad + (\partial_0(B^i(\eta)))(c, q) \xi \\ &\quad - (\partial_0(B^i(\xi)))(c, q) \eta \end{aligned} \quad (13)$$

where

$$B^i(\xi)(c, q) = A^i_a(c, q) \xi^a + A^i_b(c, q) \xi^b \quad (15)$$

The field strength can be used to determine whether a contractible cyclic deformation results in translation for more complicated geometries than considered thus far, whether or not symmetries are present. A nonzero field strength F^i implies there is a change in coordinate q^i as the parameters traverse the loop specified by the two deformation parameter vectors ξ and η . In the cases considered thus far, where the translation occurred along a single coordinate, the corresponding component of the field strength reduces to the real-valued 2-form specified earlier.

Swimming with intrinsic curvature.

Swimming on a frictionless plane cannot be accomplished by cyclic deformation of shape, but it is possible to swim on a frictionless sphere. The plane has no intrinsic or extrinsic curvature. The sphere has both. Therefore, it is interesting to consider whether swimming on a cylinder is possible. The cylinder has extrinsic curvature but no intrinsic curvature. The example illustrates the use of the field strength to determine the net translation.

Construct a body from three point masses as follows. The cylindrical coordinates of the vertex of the body, with mass m_0 , are (θ, z) , where θ measures the angle around the cylinder and z the distance along the axis. The radius of the cylinder is R . Two other point particles, with mass m_1 , are connected to the vertex by geodesic struts of length $l(t)$. The angle between each strut and the bisector is $\alpha(t)$. The orientation of the body is specified by ϕ , the angle from the horizontal (which is perpendicular to the axis of the cylinder) to the bisector of the struts.

The components of the gauge potential are

$$\begin{aligned} A_l(l, \alpha; \theta, z, \phi) &= \left(\frac{-2m_1 \cos\phi \cos\alpha}{R(m_0 + 2m_1)}, \frac{-2m_1 \sin\phi \cos\alpha}{R(m_0 + 2m_1)}, 0 \right) \\ A_\alpha(l, \alpha; \theta, z, \phi) &= \left(\frac{2lm_1 \cos\phi \sin\alpha}{R(m_0 + 2m_1)}, \frac{2lm_1 \sin\phi \sin\alpha}{(m_0 + 2m_1)}, 0 \right) \end{aligned} \quad (16)$$

The fact that the ϕ component of the gauge potentials is zero means the tilt of the body does not change as the body deforms. The other components are nonzero, so there is some motion on the cylinder. However, the field strength is identically zero

$$F(\xi, \eta)(l, \alpha; \theta, z, \phi) = (0, 0, 0) \quad (17)$$

for arbitrary deformation vectors ξ and η . Therefore, small cyclic deformations of this body on a cylinder do not give a net translation. Apparently, intrinsic curvature is required for swimming on curved manifolds, and extrinsic curvature does not help.

Swimming in curved spacetime. A quasi-rigid body can swim on a manifold with intrinsic curvature through cyclic deformations of shape. General relativity portrays spacetime as a curved four-dimensional manifold. Is it possible to swim in spacetime through cyclic deformations?

In relativity, forces of constraint move with finite velocity, so if one part of the system receives an impulse then there will be a delay in the response of other parts of the system. Naturally occurring bodies are not described by pure positional constraints. However, there is no obstacle to engineering a quasi-rigid body that does maintain positional constraints, as long as the schedule of deformations of the body is known sufficiently in advance. In this case, the internal stresses that are required to maintain the positional constraints can be precomputed and prespecified, and then executed simultaneously. The engineered quasi-rigid body is choreographed for a particular frame, which defines simultaneity. Ballet is not Lorentz invariant. It is choreographed so that dancers make simultaneous movements in the frame of the audience. In other frames, the dancers would be out of sync, but those observers are invited to slow down and enjoy the performance. This idea of engineered quasi-rigid bodies is introduced so that the analysis follows the previous examples; it seems likely that it would not be strictly required to be able to swim in spacetime.

Of most interest is Schwarzschild geometry, the curved spacetime around a nonrotating mass (12). The Schwarzschild metric is

$$\begin{aligned} g(q)(\xi_0, \xi_1) &= -\left(1 - \frac{2GM}{c^2 r}\right) c^2 \xi_0^t \xi_1^t \\ &\quad + \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)} \xi_0^r \xi_1^r \\ &\quad + r^2 (\xi_0^\theta \xi_1^\theta + (\sin\theta)^2 \xi_0^\phi \xi_1^\phi) \end{aligned} \quad (18)$$

where $q = (t, r, \theta, \phi)$ are Schwarzschild coordinates, $\xi_i = (\xi_i^t, \xi_i^r, \xi_i^\theta, \xi_i^\phi)$ are components of tangent vectors with respect to the Schwarzschild coordinate basis vectors, M is the Schwarzschild mass, G is the gravitational constant, and c is the speed of light.

According to general relativity, test particles follow geodesics on this curved spacetime manifold. Geodesics are solutions of the Lagrange equations for the Lagrangian $g(q)(u, u)/2$, where g is the Schwarzschild metric, q the tuple of Schwarzschild coordinates, and u the tuple of the associated generalized velocities. The independent variable is proper time. The magnitude of the generalized velocity is conserved and equal to c . This conserved quantity can be used to eliminate the proper time in favor of Schwarzschild time t as the independent variable. This allows the “dance” of the swimmer to be choreographed in a frame of constant t .

The reduced Lagrangian for a particle of mass m is (13)

$$\begin{aligned}
 & L_r(t; r, \theta, \phi; \dot{r}, \dot{\theta}, \dot{\phi}) \\
 &= mc \left(c^2 \left(1 - \frac{2GM}{c^2 r} \right) - \left(\frac{\dot{r}^2}{\left(1 - \frac{2GM}{c^2 r} \right)} + r^2 (\dot{\theta}^2 + (\sin\theta)^2 \dot{\phi}^2) \right)^{1/2} \right) \quad (19)
 \end{aligned}$$

The independent variable is Schwarzschild time t . The coordinates are the Schwarzschild spatial coordinates, and the generalized velocities are the rates of change of the Schwarzschild coordinates with respect to t .

The active struts are constructed at fixed Schwarzschild time. They are taken to be geodesics of specified proper length on the submanifold of constant t . The struts must be constantly monitored to make sure that the constraints are maintained. This monitoring may be done locally along the strut by surveying neighboring points on the strut (14). The monitoring may be done sufficiently quickly by giving each surveyor responsibility for an arbitrarily small segment. Each surveyor must know in advance

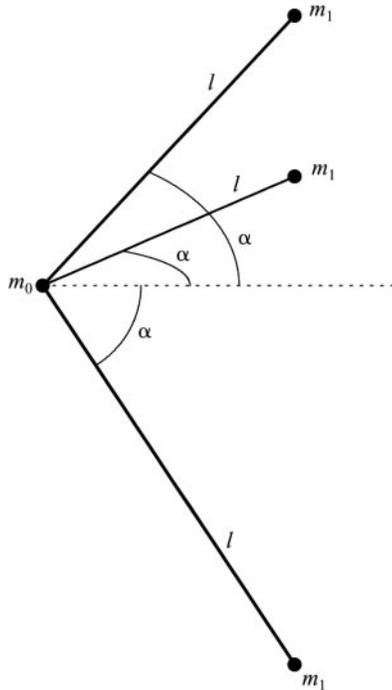


Fig. 3. A quasi-rigid spacetime swimmer. From the vertex, with mass m_0 , extend three geodesic struts of proper length l . In a local Lorentz frame at the vertex, each strut is tilted by the angle α from the axis of the swimmer. The three struts are equally spaced around this axis. The axis is pointing radially away from the central mass, which is to the left. The spatial Schwarzschild coordinates of the vertex are (r, θ, ϕ) .

what stresses to apply so that the system maintains the required positional constraints.

Consider the following spacetime swimmer (Fig. 3). Place one point mass, with mass m_0 , at the vertex. Then extend three equal length struts, with proper length $l(t)$ in the Schwarzschild frame. In a local stationary Lorentz frame, let $\alpha(t)$ be the angle each of these struts makes with an axis defining the orientation of the body, and distribute the struts equally in angle about the axis. For three struts, the angle between them is $2\pi/3$ radians (15). At the end of each of these three struts, place a point mass of mass m_1 . The deformation parameters are $l(t)$ and $\alpha(t)$. The system Lagrangian is obtained from the individual free Lagrangians, as before.

The calculation is simplified by introducing one additional assumption. The free Lagrangian L_r is not quadratic in the velocities, so the momenta are not linear in the generalized velocities. In this case, the solution for the generalized velocities in terms of the rates of change of the deformation parameters would involve the solution of nonlinear equations. But in the limit, where the velocities are small compared with the velocity of light, the mass matrix is constant. For simplicity, this assumption is made here.

The goal is to show that it is possible to swim in spacetime. So it is enough to consider a special orientation of the body. If the axis of the body is oriented radially away from the central mass, then the symmetry of the Schwarzschild geometry and the three-fold symmetry of the swimmer guarantee that any translation due to cyclic deformation will occur only in the radial direction. The problem is, then, to compute the radial component of the deformation field strength, which is a real-valued 1-form, as in the example of the plane and sphere.

The calculation follows the same outline as the simpler examples presented above. The general form of the displacement can be anticipated, given Eq. 5. The displacement will be proportional to the square of the ratio of the size of the object to the radius of curvature of the manifold. For Schwarzschild geometry, the components of the Riemann curvature tensor are proportional to $GM/(c^2 r^3)$, which may be thought of as the inverse of the square of a characteristic radius of curvature. Therefore, the displacement should be proportional to $l^2 GM/(c^2 r^3)$. In addition, the displacement should be proportional to the change in length, to the change in separation angle, and to a factor that is homogeneous of degree zero in the masses. Detailed calculation confirms these expectations. For large r , where the body is small compared to the radius of curvature of spacetime, the displacement is found to be

$$\begin{aligned}
 \Delta r = & \\
 & - \frac{3m_0 m_1}{(m_0 + 3m_1)^2} l^2 \frac{GM}{c^2 r^3} \sin\alpha \Delta l \Delta\alpha \quad (20)
 \end{aligned}$$

Therefore, it is indeed possible to swim in spacetime (16). Translation in space can be accomplished merely by cyclic changes in shape, without thrust or external forces.

The curvature of spacetime is very slight, so the ability to swim in spacetime is unlikely to lead to new propulsion devices. For a meter-sized object performing meter-sized deformations at the surface of the Earth, the displacement is of order 10^{-23} m (17). Nevertheless, the effect is interesting as a matter of principle. You cannot lift yourself by pulling on your bootstraps, but you can lift yourself by kicking your heels.

References and Notes

1. A. Shapere, F. Wilczek, *J. Fluid. Mech.* **198**, 557 (1989).
2. Geometric phase is also called anholonomy. Some restrict the term geometric phase to quantum phases and use the term anholonomy for classical processes.
3. A. Shapere, F. Wilczek, Eds., in *Geometric Phases in Physics* (World Scientific, London, 1989) pp. 449–460.
4. A. Guichardet, *Ann. Inst. Henri Poincaré* **40**, 329 (1984).
5. R. Montgomery, in *The Geometry of Hamiltonian Systems*, T. Ratiu, Ed. (Springer, New York, 1991).
6. Contrast the fact that here the deformation is specified by coordinates (the associated basis vector fields commute) with the fact that the successive angles of rotation used in the two-sphere example are not coordinates (the associated vector fields do not commute).
7. A tuple is an ordered list.
8. G. J. Sussman, J. Wisdom, with M. E. Meyer, *Structure and Interpretation of Classical Mechanics* (MIT Press, Cambridge, MA, 2001).
9. It is informative to consider these manifolds even though they have constant curvature, so the assumption of quasi-rigidity is not required.
10. Note that functions compose in the reverse order to the operators that generate them: $(e^{L_{C_2}} I) \circ (e^{L_{C_1}} I) = e^{L_{C_1}} e^{L_{C_2}} I$, where I is the identity function.
11. In this expression, the structure produced by the ∂_1 contracts with the vector components of the following A . Similarly, the structure generated by the ∂_0 contracts with the components of the following ξ or η (8). Equation 14 is fully expanded for clarity. The first term of Eq. 14 is the product of two factors. The first factor can be written in terms of a commutator of vector fields on the configuration manifold with coefficients $A_a(c, q)$ and $A_b(c, q)$; the second factor is the area of the parallelogram formed by ξ and η on the deformation parameter plane.
12. C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1970).
13. E. Bertschinger, notes for MIT class 8.962, "General Relativity" (1999).
14. A passive fixed-length strut moving uniformly with respect to the Schwarzschild frame would appear Lorentz contracted in the Schwarzschild frame. A surveyor moving with the strut would have to account for this and add to the schedule of deformations a change in the length of the strut so that the length in the Schwarzschild frame, the frame of choreography, is as expected.
15. If the axis of the body were aligned with the North

pole, then α would be the co-latitude of the struts and the longitudinal angle between the struts would be 120° .

16. The presence of the factor of c^2 confirms that this is a relativistic effect. There is no swimming effect in the analogous Newtonian problem.
17. The fact that the swimming displacement per stroke is so small means that, strictly speaking, one should consider the swimming effect relative to the ordinary nonswimming geodesic motion of the

swimmer. However, the calculation that is presented is enough to show the existence of the effect that is surely also present in more complicated situations. Perhaps the most interesting case to consider would be a swimmer in a circular orbit, where the swimming effect could be used to gradually increase the radius of the orbit.

18. I thank J. Touma for infecting me with his interest in geometric phase and for bringing the articles of A. Shapere and F. Wilczek to my atten-

tion. I thank H. Abelson, E. Bertschinger, D. Finkelstein, R. Hermann, P. Kumar, G. J. Sussman, J. Touma, and F. Wilczek for helpful and pleasant conversations.

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Coherent Quantum Dynamics of a Superconducting Flux Qubit

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We have observed coherent time evolution between two quantum states of a superconducting flux qubit comprising three Josephson junctions in a loop. The superposition of the two states carrying opposite macroscopic persistent currents is manipulated by resonant microwave pulses. Readout by means of switching-event measurement with an attached superconducting quantum interference device revealed quantum-state oscillations with high fidelity. Under strong microwave driving, it was possible to induce hundreds of coherent oscillations. Pulsed operations on this first sample yielded a relaxation time of 900 nanoseconds and a free-induction dephasing time of 20 nanoseconds. These results are promising for future solid-state quantum computing.

It is becoming clear that artificially fabricated solid-state devices of macroscopic size may, under certain conditions, behave as single quantum particles. We report on the controlled time-dependent quantum dynamics between two states of a micron-size superconducting ring containing billions of Cooper pairs (I). From a ground state in which all the Cooper pairs circulate in one direction, application of resonant microwave pulses can excite the system to a state where all pairs move oppositely, and make it oscillate coherently between these two states. Moreover, multiple pulses can be used to create quantum operation sequences. This is of strong fundamental interest because it allows experimental studies on decoherence mechanisms of the quantum behavior of a macroscopic-sized object. In addition, it is of great importance in the context of quantum computing (2) because these fabricated structures are attractive for a design that can be scaled up to large numbers of quantum bits or qubits (3).

Superconducting circuits with mesoscopic Josephson junctions are expected to behave according to the laws of quantum mechanics if they are separated sufficiently from external degrees of

freedom, thereby reducing the decoherence. Quantum oscillations of a superconducting two-level system have been observed in the Cooper pair box qubit using the charge degree of freedom (4). An improved version of the Cooper pair box qubit showed that quantum oscillations with a high quality factor could be achieved (5). In addition, a qubit based on the phase degree of freedom in a Josephson junction was presented, consisting of a single, relatively large Josephson junction current-biased close to its critical current (6, 7).

Our flux qubit consists of three Josephson junctions arranged in a superconducting loop threaded by an externally applied magnetic flux near half a superconducting flux quantum $\Phi_0 = h/2e$ [(8); a one-junction flux qubit is described in (9)]. Varying the flux bias controls the energy level separation of this effectively two-level system. At half a flux quantum, the two lowest states are symmetric and antisymmetric superpositions of two classical states with clockwise and anticlockwise circulating currents. As shown by previous microwave spectroscopy studies, the qubit can be engineered such that the two lowest eigenstates are energetically well separated from the higher ones (10). Because the qubit is primarily biased by magnetic flux, it is relatively insensitive to the charge noise that is abundantly present in circuits of this kind.

The central part of the circuit, fabricated by electron beam lithography and shadow

evaporation of Al, shows the three in-line Josephson junctions together with the small loop defining the qubit in which the persistent current can flow in two directions, as shown by arrows (Fig. 1A). The area of the middle junction of the qubit is $\alpha = 0.8$ times the area of the two outer ones. This ratio, together with the charging energy $E_C = e^2/2C$ and the Josephson energy $E_J = hI_C/4\pi e$ of the outer junctions (where I_C and C are their critical current and capacitance, respectively), determines the qubit energy levels (Fig. 2A) as a function of the superconductor phase γ_q across the junctions (Fig. 1B). Close to $\gamma_q =$

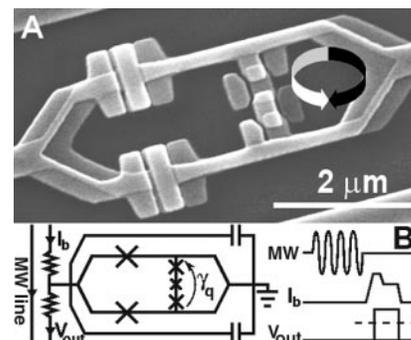


Fig. 1. (A) Scanning electron micrograph of a flux qubit (small loop with three Josephson junctions of critical current $\sim 0.5 \mu\text{A}$) and the attached SQUID (large loop with two big Josephson junctions of critical current $\sim 2.2 \mu\text{A}$). Evaporating Al from two different angles with an oxidation process between them gives the small overlapping regions (the Josephson junctions). The middle junction of the qubit is 0.8 times the area of the other two, and the ratio of qubit/SQUID areas is about 1:3. Arrows indicate the two directions of the persistent current in the qubit. The mutual qubit/SQUID inductance is $M \approx 9 \text{ pH}$. (B) Schematic of the on-chip circuit; crosses represent the Josephson junctions. The SQUID is shunted by two capacitors ($\sim 5 \text{ pF}$ each) to reduce the SQUID plasma frequency and biased through a resistor ($\sim 150 \text{ ohms}$) to avoid parasitic resonances in the leads. Symmetry of the circuit is introduced to suppress excitation of the SQUID from the qubit-control pulses. The MW line provides microwave current bursts inducing oscillating magnetic fields in the qubit loop. The current line provides the measuring pulse I_b and the voltage line allows the readout of the switching pulse V_{out} . The V_{out} signal is amplified, and a threshold discriminator (dashed line) detects the switching event at room temperature.

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