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DYNAMICAL STABILITY IN THE OUTER SOLAR SYSTEM AND THE DELIVERY OF SHORT PERIOD COMETS

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ABSTRACT

Test particle stability in the outer solar system is surveyed. Roughly 7000 test particles are numerically integrated for durations ranging from 20 to 800 million yr. The initial conditions of the test particles fall into two categories: (1) orbits initially near the Lagrange L_4 and L_5 points of the Jovian planets, and (2) circular orbits in the invariable plane with semimajor axes of 5–50 AU. Clusters of test particles near the triangular Lagrange points of Jupiter, Saturn, Uranus, and Neptune survive the full integration. However, the stable regions near Saturn's L_4 and L_5 points exclude the actual Lagrange points. Nearly all particles between the outer planets are removed by close encounters with the planets during the course of these integrations. Numerous test particles between Neptune and 43 AU are removed by close encounters with Neptune, some quite late in the integrations. The particles which have late encounters reach Neptune by a path that roughly preserves semimajor axis while the eccentricity varies irregularly. The distribution of encounter times suggests that the times to first encounter can reach several billion years. The flux of new encounters decays slowly, roughly as the inverse of time. This gives new insight into the dynamics of the delivery of short period comets from the hypothesized Kuiper belt. An estimate of the mass of the Kuiper belt is given.

1. INTRODUCTION

Were there initially asteroids beyond Jupiter? Are there regions between the giant planets in which small bodies are stable against planetary encounters for the age of the solar system? Do Saturn, Uranus, and Neptune have Trojan-like asteroids? Are planetesimals in the hypothesized Kuiper belt stable against close encounters for the age of the solar system or is this region already depleted? On what time scale is material removed from different regions of the solar system? These questions motivate our survey of the long-term stability of small bodies in the outer solar system. In this study we extensively examine the stability of test particles in the regions between the outer planets and beyond Neptune.

In Sec. 2, we review observational searches for slow-moving objects in the outer solar system. Section 3 reviews previous test particle surveys. Section 4 describes the method of this survey. Section 5 presents the details and results of our survey of the regions near the triangular Lagrange points of Jupiter, Saturn, Uranus, and Neptune. Section 6 presents the results of our survey of the invariable plane. Section 7 presents a summary.

2. OBSERVATIONAL SEARCHES

A number of observational surveys to detect slow-moving objects in the outer solar system have been conducted. Tombaugh (1961), Kowal (1989), Luu & Jewitt (1988), and Levison & Duncan (1990) have all conducted such investigations. In reviewing these investigations we consider in which regions each survey could have detected 624 Hektor, a bright Jupiter Trojan asteroid. At $H=7.49$, 624 Hektor would be roughly $V=17$, $V=20$, and $V=22$ if it were at the distance of Saturn, Uranus, or Neptune, respectively (Innanen & Mikkola 1989).

Tombaugh examined the ecliptic region to a limiting magnitude of $m_{pg}=17$ for slow-moving objects. Although he searched the Lagrange points of Saturn, his survey might have just missed the hypothetical bright Trojan at that distance. Other than the discovery of Pluto in 1930, no outer solar system bodies were found.

Kowal photographed about 6400 deg^2 of ecliptic region, to a limit of $V=20$, for slow-moving objects. Only one object was found beyond Saturn, 2060 Chiron. At $V=17$ mag and 17 AU from the Sun at the time of its discovery, Chiron was 3 mag above Kowal's detection limit. Chiron is large (100 km) and cometary (Meech & Belton 1990). Its orbit is planet crossing, highly eccentric, and highly inclined. We have not yet determined if Kowal actually examined the triangular Lagrange points, however, if he did, it would have been possible to detect 624 Hektor at the distance of Saturn and possibly Uranus.

Luu & Jewitt (1988) surveyed 297 deg^2 near the ecliptic to a limiting magnitude of $V\approx 20$ using Schmidt plates and 0.34 deg^2 to a limit of $R\approx 24$ with a CCD camera. Like Kowal's, their plate survey could detect 624 Hektor at distances of Saturn and possibly Uranus. With the CCD survey they could detect the test object even at Neptune distances. At the times of Luu and Jewitt's observations, from February to June, 1987, Saturn, Uranus, and Neptune were quite close together. In fact, the L_5 points of these three planets were near opposition. Consequently, the centers of about two-thirds of the fields observed by Luu

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and Jewitt are within 30°, along the ecliptic, of the L_5 points of Saturn, Uranus, or Neptune. This amounts to observations of about 200 deg^2 near the L_5 points observed to $V \approx 20$. A few of the fields are quite close to the L_5 points. Considering the range of libration observed for stable test particles in our integrations, their choice of fields is appropriate for detecting objects at the Lagrange points. However, they found no slow-moving objects in this search. Recently, Luu & Jewitt (1992) reported the discovery of a slow-moving object beyond Neptune, 1992 QB1. Although its orbit is not well determined at this time, one possible solution places its semimajor axis at 41 AU. However, it is also possible that this object is on a parabolic or near-parabolic orbit.

Finally, Levison & Duncan (1990) examined 4.9 deg^2 of the ecliptic, explicitly limiting their survey to a search for slow-moving objects in the range 25–60 AU. Their detection limit was $V \approx 22.5$. 624 Hektor at the distance of Neptune would be very near the detection limit. Detecting nothing, they report a 99% confidence level result that fewer than one object per deg^2 brighter than $V \approx 22.5$ lies between 25 and 60 AU. Of the 26 fields examined 8 lie within 30° of the L_4 or L_5 points Neptune. This amounts to observations of about 1.5 deg^2 near Neptune's triangular Lagrange points. Several more fields are within 45°, still within range of the widest librators. Even so, this is not a great deal of area considering the range of libration.

In summary, the surveys of Tombaugh, Kowal, and Luu and Jewitt could all have detected bright Saturn Trojans. Some of the observations, including those of Levison and Duncan, could have detected bright objects at the distances as far as Neptune. To date, only one candidate for a small body in a nearly circular orbit in the outer solar system has been found, but its orbit is as yet undetermined.

3. PREVIOUS TEST PARTICLE SURVEYS

There have been test particle surveys of both the Lagrange points of the outer planets and the regions between and beyond the outer planets. The Trojan asteroids occupy the regions near Jupiter's Lagrange points. Similar Trojan-like configurations have been seen in satellite systems, and the asteroid 1990 MB appears to be a Mars Trojan (Holt & Levy 1990; Bowell 1990a,b; Kinoshita 1990). Yet, are the L_4 and L_5 points of other planets stable? Can material placed around these points remain there for the age of the solar system, or will the perturbations of the other planets induce its removal? Analytic methods are inadequate to answer these questions. We must rely on numerical exploration to investigate the long-term stability in realistic models. Levison *et al.* (1991) studied the stability field of the Jupiter Trojans. They integrated 110 test particles in the field of the Sun and four Jovian planets with a fourth-order symplectic integrator. They explored a two-dimensional grid of proper eccentricity and libration amplitude (see Shoemaker *et al.* 1989 and Erdi 1978, 1979). Their integrations extend to 150 000 Jupiter periods or about 1.8 million yr. Zhang & Innanen (1988a,b,c) and Innanen & Mikkola (1989) investigated the stability of the triangular Langrange points of all the Jovian planets. They studied the evolution of a few test particles for 10 million yr, subject to the perturbations of the four Jovian planets. They found that test particles placed at the triangular Lagrange points of Jupiter, Uranus, and Neptune survived without close encounter for 10 million yr. However, test particles placed at Saturn's L_4 and L_5 points approach Saturn on short time scales; whereas those initially placed a small distance away from the Lagrange points librate without close approach. Recently, Mikkola and Innanen (1992) provide a detailed description of the evolution of a number of orbits over 20 million yr. We build upon the work of Innanen & Mikkola (1989) and Mikkola & Innanen (1992) in the context of our model.

There have been a number of previous surveys of test particle stability in the regions between the outer planets, most of which focus on the region between Jupiter and Saturn (Lecar & Franklin 1973; Franklin *et al.* 1989; Soper *et al.* 1990; Weibel *et al.* 1990; Duncan *et al.* 1989, hereafter referred to as DQT89; Gladman & Duncan 1990, hereafter referred to as GD90). Various models have been used to study this region. These models range in complexity from planar models that include Jupiter and Saturn following circular orbits to three-dimensional models that permit Jupiter and Saturn to fully interact. As the models have become more realistic a general consensus on this region has emerged. It is observed that most test particles develop planet crossing orbits or suffer a close encounter with Jupiter or Saturn within 10^4–10^5 yrs; a few test particles survive longer, 10^6 yr. Our study is the first to investigate the Jupiter-Saturn region including all four giant planets in a self-consistent n-body integration.

Fewer studies have examined the test particle stability in the regions between the other outer planets. DQT89 studied the stability of test particles in the region 0.6–34 AU using a simplified two-planet mapping approach. With this method they examined the regions between each pair of adjacent planets, including only those two planets as perturbers and approximating the perturbations on the test particles as impulses at each conjunction. Their integrations extended to 4.5 billion yr. Aside from including only the two adjacent planets as perturbers other approximations were: (1) the planets and test particles were coplanar; (2) the planets were restricted to fixed circular orbits; and, (3) the eccentricities of the test particles were assumed to be small. In the model of DQT89 many nearly circular orbits in the Saturn-Uranus and Uranus-Neptune regions survive for the age of the solar system. GD90 studied the invariable plane from 3 to 40 AU with direct numerical integration of the three-dimensional n-body equations of motion, with a fourth-order symplectic integrator (Candy & Rozmus 1990; Forest & Ruth 1990). For test particles beyond Saturn they included all four giant planets as perturbers. GD90 integrated roughly a thousand test particles for periods up to 22.5 million yr, removing any test particle which encountered a planet or left the system. GD90 reach a very different conclusion from DQT89. GD90 observe that almost all of the test particles on orbits between the planets are unstable against close encounters on a time scale of about 10 million yr. Thus GD90 found that DQT89 overestimated the planet-crossing times by a substantial factor. Our survey extends and refines the study of GD90.

The test particle stability of the region beyond Neptune has received even less attention. Duncan *et al.* (1988) and Quinn *et al.* (1990) find that the distribution of the orbits of short period comets is more consistent with an origin in the hypothesized Kuiper belt of comets in the region beyond Neptune than with an origin in the isotropic Oort cloud. However, the model from which this result emerged has a questionable feature: in order to make their study computationally feasible the masses of the planets were enhanced by a considerable factor. This may be adequate for demonstrating that short period comets more likely come from a low inclination source region beyond Neptune, but is surely inadequate to evaluate the dynamics prior to planetary encounters. DQT89 and GD90 integrate a small number of test particles initially beyond Neptune with models that do not rely on enhancing the planetary masses. However, both studies report that the test particles, except for those quite close to Neptune, retain nearly circular orbits for the duration of their integrations. Torbett & Smoluchowski (1990, hereafter referred to as TS90) more extensively examined the evolution of test particles beyond Neptune using conventional numerical integration. However, in their model the outer planets moved on fixed elliptical orbits. They followed the evolution of about 200 test particles, 40 of which were initially placed on nearly circular orbits, for 10 million yr. TS90 discovered a chaotic zone beyond Neptune which corresponds roughly to orbits with perihelia between 30 and 45 AU. They observed Neptune crossing for only the test particles with initial perihelia quite close to the orbit of Neptune, within 2 AU. They noted that several test particles seemed to random walk through a-e space approximately along lines of constant perihelion. They suggest that the chaotic zone is connected and that orbits diffuse throughout it roughly maintaining constant perihelia. They conjecture that the larger semimajor axis portion of this chaotic zone provides a storage place for short period comets which were initially formed at low eccentricity. The diffusion of comets to Neptune crossing orbits implies an exponential decay in the number of comets stored in the chaotic zone. Clearly, the dynamics of the Kuiper belt deserves much more extensive exploration in more realistic models.

4. METHOD

Our approach is simple and direct. We integrate the motion of test particles in the field of the Sun and massive outer planets, Jupiter through Neptune, with the symplectic mapping method of Wisdom & Holman (1991, hereafter referred to as WH91). The Sun, planets, and test particles interact in the full three-dimensional n-body sense. The test particles have infinitesimal mass; they are perturbed by the massive planets, but do not perturb them in return. The initial positions and velocities of the planets and Sun are taken from Cohen *et al.* (1973, hereafter referred to as CHO73). The accuracy and stability of the symplectic mapping method are analyzed and discussed in Wisdom & Holman (1992).

During the integration the test particles are examined at each time step for close encounters with planets; those which enter the sphere of influence of a planet are terminated. The sphere of influence or activity sphere is a measure of the distance from a planet within which it becomes reasonable to consider a test particle in orbit around the planet perturbed by the Sun, rather than in orbit around the Sun perturbed by the planet (see Danby 1988). The radius of the sphere of influence is

$$r_s = a\mu^{2/5}, \tag{1}$$

where a is the initial semimajor axis of the planet and μ is the ratio of the mass of the planet and that of the Sun. This criterion is slightly different from that used by GD90:

$$r_s = a(2\mu)^{2/5} \approx 1.32 a\mu^{2/5}. \tag{2}$$

We do not believe the exact size of the sphere is important to the qualitative results. Although a planetesimal could survive a close approach without cataclysm the orbital elements of the particle would be radically altered. In addition to close encounters, we examine the test particles for parabolic or hyperbolic orbits. During our integrations, no nonelliptic orbits were detected before a close encounter.

5. SURVEY OF THE LAGRANGE POINTS OF THE OUTER PLANETS

In this segment of our survey we study the evolution, for intervals up to 20 million yr, of about 4000 test particles distributed near the Lagrange points of the outer planets. The test particles are given the same eccentricity, inclination, longitude of ascending node, and mean anomaly as one of the Jovian planets. The argument of pericenter is offset from that of the planet by a wide range of angles. The initial semimajor axes of the test-particles are equal to that of the planet multiplied by a "semimajor axis factor" ranging from 0.96 to 1.04. In the initial survey the argument of pericenter was varied from 0° to 360° with a step of 5°; the semimajor axis factor was varied from 0.96 to 1.04 at steps of 0.01. Additional initial conditions were used to trace detail in the most interesting regions. This choice of initial conditions places the test particles initially in the plane of the corresponding planet. The idea of modifying the initial semimajor axis is due to Innanen & Mikkola (1989) who first used this technique to investigate the stability of test particles at Saturn's triangular Lagrange points. In the case of the particles with the same initial semimajor axis as the planet (semimajor axis factor = 1.0) and the argument of pericenter offset by 60° or −60° the test particle would maintain this configuration if the perturbations of the planets other than the one in question were neglected. Those which enter the sphere of influence of a planet or develop nonelliptic orbits are removed. A time step of 1.0 yr is used. We record the range of the difference in mean longitude of each particle from its corresponding planet, as well as the range of semimajor axis, eccentricity, and inclination. These statistics are updated after each 10 time-steps.

To test that the time-step and interval between updating the range of elements were not too short we repeated one of the runs of Jupiter Trojans with a time-step of 0.5 yr and 5 time-steps per statistics update. The results are qualitatively the same as the run with a 1.0 yr time-step and 10 integration steps before updating the statistics.

In Fig. 1 we plot a point for each test particle that survived the full integration, 20 million yr. We plot the offset of the argument of perihelion from the planet as the initial longitude vs the semimajor axis factor. A stable region surrounds the triangular Lagrange points of each of the outer planets. Innanen & Mikkola (1989) earlier found that test particles near the triangular Lagrange points of Saturn, Uranus, and Neptune could endure integrations of up to 10 million yr, but now we have a two-dimensional view of initial conditions for test particles that endure integrations of 20 million yr without close encounters.

A collection of stable initial conditions surrounds each of the triangular Lagrange points, but this does not imply that the test particles are confined to this region. In fact, over the course of the integration even a test particle initially at the L_4 and L_5 points can explore a large range of longitude with respect to its corresponding planet, 35° for Neptune's L_4 point as an example. Those initially placed further from the Lagrange points explore a larger longitudinal range, in some cases 100°!

Figure 1 immediately raises some questions. What governs the profile of the stable regions? Why do Saturn's L_4 and L_5 stable regions have holes in the center where the others do not? Innanen & Mikkola (1989) suggest the near 5:2 resonance between Jupiter and Saturn as a possible cause for the instability near Saturn's triangular Lagrange points. What is the cause of the apparent asymmetry for the Neptune L_4 and L_5? Although we do not expect the stable regions to be precisely symmetric due to asymmetric planetary phases, the asymmetry is pronounced in the case of Neptune. Can we estimate the phase volume of the stable region in order to predict the likelihood of observing material at these points? Are these regions just seemingly stable and will disappear with further integration? Future effort is clearly needed, but, for now, it has been established that for a non-negligible range of initial conditions test particles near the L_4 and L_5 points of Saturn, Uranus, and Neptune, as well as Jupiter, can endure without close encounter for up to 20 million yr.

6. SURVEY OF THE INVARIABLE PLANE

In this segment of our survey we place 3000 test particles on heliocentric circular orbits in the CHO73 invariable plane. The initial longitude of each test particle is assigned one of six values: 0, $3\pi/10$, $7\pi/10$, $11\pi/10$, $15\pi/10$, and $19\pi/10$ radians, measured from the x axis of the CHO73 coordinate system. Along each of the six longitudes, we uniformly distribute 500 test particles in the range 5–50 AU. Using the symplectic mapping method described above, each test particle is evolved in the field of the Sun and Jovian planets. Interior to Neptune the integrations have been extended to 800 million yr, and exterior to Neptune the integrations have been advanced to 200 million yr. The integration time-step is 1.0 yr. During the integration, values of the minimum and maximum semimajor axis, eccentricity, and inclination explored by each particle are updated after every 100 mapping steps.

Figure 2 and Table 1 present results from our invariable plane integrations. In Fig. 2 the time survived by each test particle is plotted as a function of initial semimajor axis for the full range of initial semimajor axes explored. As mentioned above each bin in semimajor axis contains 6 test particles started at different longitudes in the invariable plane. The vertical bars mark the minimum of the six termination times. The survival times of the remaining 5 test particles are marked by small dots. The points along the top represent test particles that survived the full integration.

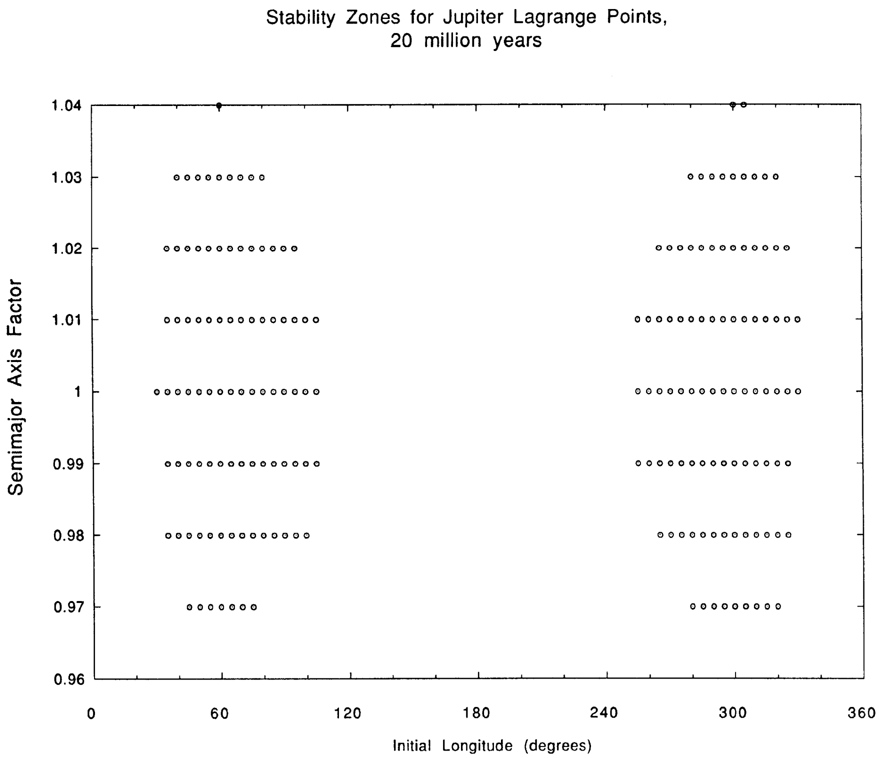
There are several features in the Fig. 2 to point out. The spikes at 5.2, 9.5, 19.2, and 30.1 AU, at the semimajor axes of the planets, correspond to test particles librating about the triangular Lagrange points. Surrounding each of these spikes is a range of semimajor axes in which test particles quickly encounter one of the planets. The width of these regions corresponds well with the range of semimajor axis near a planet within which a test particle is predicted to undergo large-scale chaotic motion due to the overlap of first-order mean motion resonances (Wisdom 1980, DQT89). The half-width is approximately

$$\Delta a \approx 1.5 a \mu^{2/7}, \tag{3}$$

where a is the semimajor axis of the planet and μ is the ratio of the masses of the planet and the Sun. GD90 derive an expression for the range of semimajor axis in the restricted three-body problem for which the Jacobi constant permits initially circular orbits to become crossing orbits. Their expression for the half-width is

$$\Delta a \approx 2.1 a \mu^{1/3}. \tag{4}$$

Both expressions predict the region of rapid removals fairly well, though the estimate based on the chaotic zone is more generally valid for two reasons. First, the Jacobi constant alone does not determine whether particular orbits are quasiperiodic or chaotic. The restricted three-body problem has the usual divided phase space with a mixture of chaotic and quasiperiodic trajectories for each value of the Jacobi constant (Hénon 1966). Orbits do not generally explore the full range of the phase space permitted by the Jacobi constant. In fact, the validity of the crossing zone calculation rests on the fact that the chaotic zone is larger than the crossing zone, implying that the orbits considered in the crossing zone calculation are chaotic and free to explore the phase space which happens to include the crossing orbits. Second, when the perturbations of all the planets are considered the Jacobi constant does not exist, and the calculation has no rigorous generalization. The only generalization is by analogy. On the other hand, the resonance overlap estimate of the extent of the chaotic zone is still valid in the more general case. Further, it can



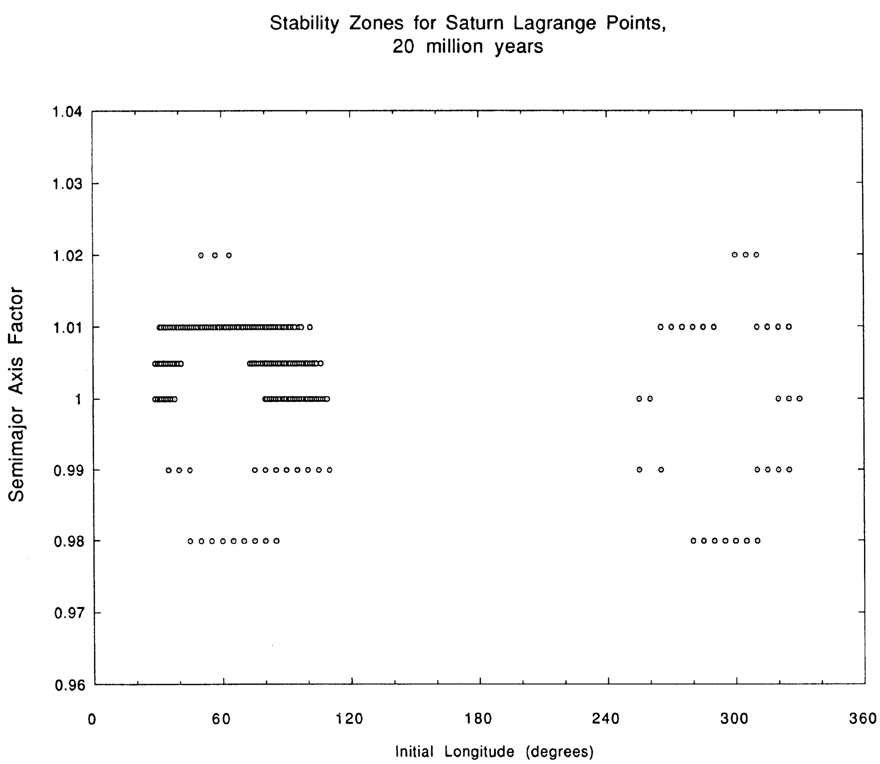
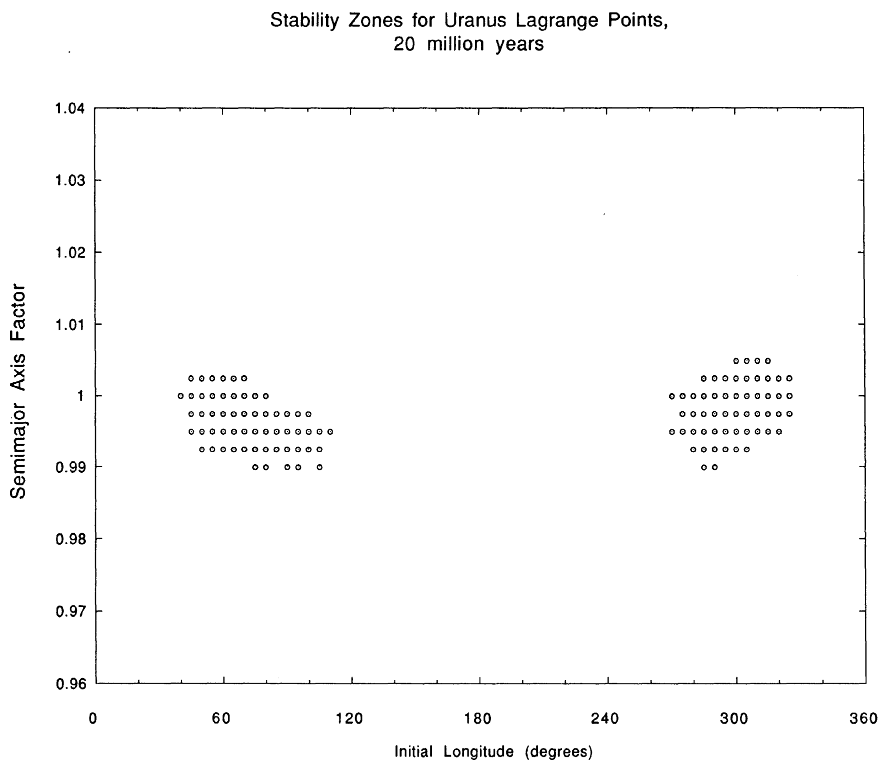


FIG. 1. A point is plotted for each test particle that survived the full 20 million yr integration. The axes show the initial displacement in longitude from the corresponding planet and factor by which that planet's semimajor axis is multiplied to initialize the semimajor axis of the test particle. A two-dimensional stable region lies near the triangular Lagrange points of each of the planets surveyed.



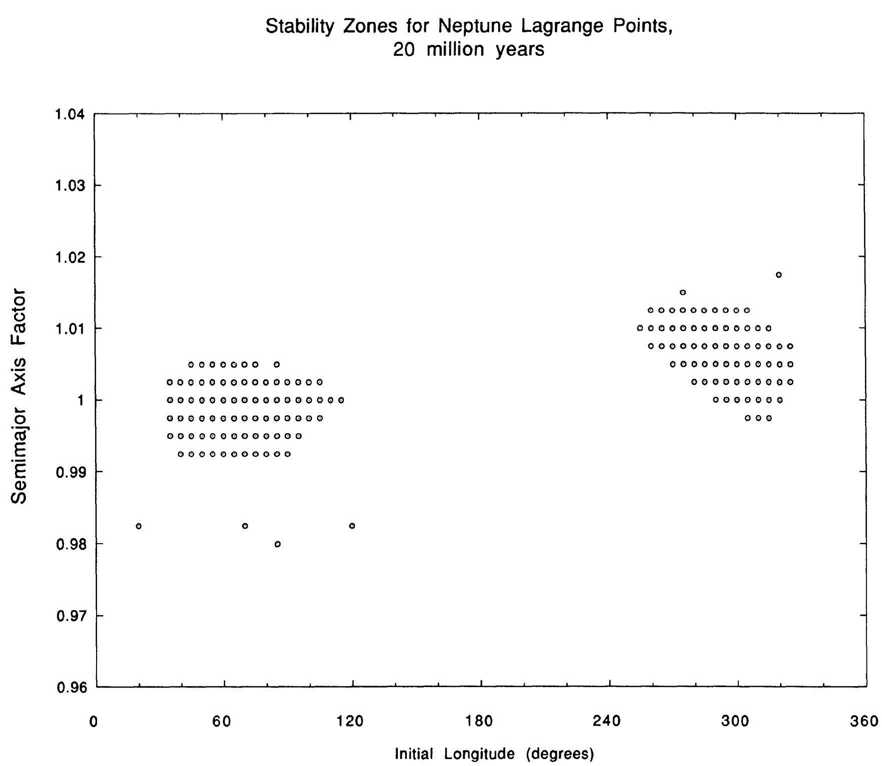


FIG. 1. (continued)

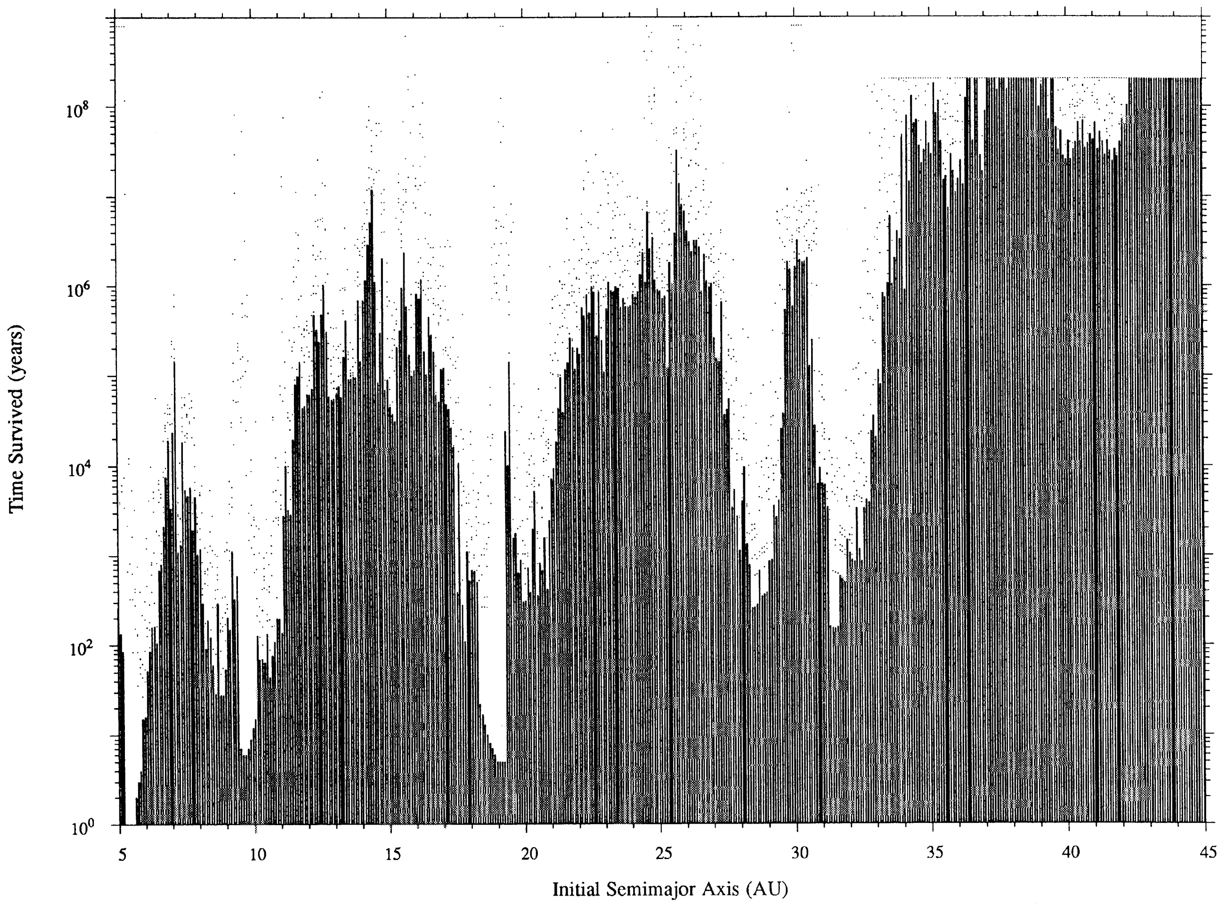


FIG. 2. The time survived by each test particle is plotted as a function of initial semimajor axis. For each semimajor axis bin, six test particles were started at different longitudes. The vertical bars mark the minimum of the six termination times. The points mark the termination time of the other five test particles. The scatter of points gives an idea of the spread of termination times for any given semimajor axis. The envelope at the top is the mark of those test particles surviving the full integration. The spikes at 5.2, 9.5, 19.2, and 30.1 AU, at the semimajor axes of the planets, correspond to test particles librating in Trojan or horseshoelike orbits before close encounter. Interior to Neptune the integration extends to 800 million yr; exterior to Neptune the integration reaches 200 million yr. Beyond about 43 AU all the test particles survive the full integration.

presumably be made more and more accurate by considering higher order resonances with the perturbing planets. GD90 choose not to examine regions within the crossing zones of the planets. Here, we have placed test particles from 5 to 50 AU, leaving no gaps. This is not a costly decision because the test particles in these zones are rapidly removed from the integration. For a small price we get a complete picture.

TABLE 1. The number of outcomes of particles beginning in various ranges of semimajor axis. The table lists the number of encounters with each planet, the number of survivors, and the total number of particles in each semimajor axis range.

	$a_{\min}$–$a_{\max}$	Jup	Sat	Ura	Nep	surv	total
Jupiter	5.00–6.35	75	3	1	1	10	90
Jupiter-Saturn	6.35–8.24	33	92	1	0	0	126
Saturn	8.24–10.94	2	176	2	0	0	180
Saturn-Uranus	10.94–17.60	1	164	272	7	0	444
Uranus	17.60–20.93	0	0	214	5	3	222
Uranus-Neptune	20.93–27.50	0	0	194	238	6	438
Neptune	27.50–32.81	0	0	4	345	5	354
Beyond Neptune	32.81–50.00	0	0	0	266	880	1146

As can be seen in Fig. 2 most of the test particles initially between Jupiter and Saturn have had close encounters within 10^4–10^5 yr. By 10^6 yr all the test particles in the Jupiter-Saturn region have been eliminated; most of the test particles in the region encounter Jupiter or Saturn; a few encounter Uranus and Neptune. This is the first time that the Jupiter-Saturn region has been examined with a n-body calculation that includes all of the Jovian planets as perturbers. Despite the addition of Uranus and Neptune as perturbers, our results qualitatively agree with a number of other studies that employ different models and methods to investigate test particle stability in the Jupiter-Saturn region.

As an aside, GD90 observe that test particles with moderate initial inclinations in the Jupiter-Saturn region begin to be removed by close encounters later than those initially in the plane, but are then removed by close encounters more rapidly. The end result is that by 10^5 yr the number of remaining test particles is roughly the same regardless of initial inclination. This observation indicates that restricting the initial conditions of the test particles to the invariable plane should not unreasonably skew the overall results, although we would have to examine a larger portion of the phase space to verify this assumption.

After 800 million yr no test particles initially in the region between Saturn and Uranus remain; only 6 test particles between Uranus and Neptune survived the full integration. From the figure it can be seen that most test particles in the Saturn-Uranus and Uranus-Neptune regions are eliminated within 10^7 yr. Except for a few test particles librating about Neptune's triangular Lagrange points, the region surrounding Neptune is rapidly depleted, roughly in accord with the extent of the chaotic zone described above. In the region further beyond Neptune, numerous particles have had close encounters within 200 million yr. Thus we are witnessing the depletion of a much larger region than has been seen before in shorter integrations.

TS90 observed Neptune crossing for particles with initial perihelia within 2 AU of Neptune in 10 million yr. GD90 integrated 20 test particles uniformly distributed in 32.8–40 AU for 22.5 million yr. Three test particles between 32.8 and 33.7 AU encountered Neptune, but the other test particles, with semimajor axes between 33.7 and 40 AU, retained nearly circular orbits during their 22.5 million yr integration. For comparison, we observe that 42 of 60 test particles in the range 32.8–33.7 AU had close encounters within 22.5 million yr. Also, we observed that 23 of 420 test particles with initial semimajor axes between 33.7 and 40 AU had close encounters within 22.5 million yr. If we had sampled at the rate of GD90 we would expect to have observed 2–3 close encounters in the range 32.8–33.7 AU and fewer than one close encounter beyond 33.7 AU. Further, we observed no close encounters of test particles beyond 39 AU before 22.5 million yr. Thus, our results are consistent with those of GD90 and TS90; the depletion of the regions beyond 32 AU which we observe begins later.

Note that the regions in which both GD90 and TS90 find Neptune crossing are similar and correspond roughly to the chaotic zone surrounding Neptune predicted by the overlap of first-order mean motion resonances [Eq. (3)]. Beyond this region, those two studies did not observe test particles on initially near circular orbits encountering Neptune. Our study shows that integrations of only 10–20 million yr are inadequate to begin to see the extensive depletion of the region beyond 33 AU. GD90 suggest that there may be a slow outward erosion of the Kuiper belt. We observe that the inner edge of this disk is not simply eroded outward, rather the disk is undergoing a more extensive irregular depletion.

In connection with the removal of test particles in the semimajor axis range 40–42 AU, Figs. 3 and 4 reveals a bump in maximum eccentricity and inclination attained by test particles. Without examining the dynamics more closely we note the 3:2 mean motion resonance with Neptune near 40 AU and secular resonances near 41 AU (Heppenheimer 1979; see also Knezevic *et al.* 1991). We also note the 2:1 mean motion resonance with Neptune near 48 AU.

We observe the phenomenon reported by TS90 that in many cases orbits appear to random walk in the space of semimajor axis and eccentricity roughly along lines of constant perihelion. However, we do not find that motion in the more extensive chaotic zone reported by TS90 has this character. Rather, we find that particles typically retain a semimajor axis near the initial semimajor axis while the eccentricities grow irregularly. Only when the perihelia get in the vicinity of Neptune do the particles begin to random walk along lines of constant perihelia. Typically they then relatively quickly have close encounters with Neptune. More rarely particles then begin again to random walk along another path of near constant semimajor axis with varying eccentricity. There are two classes of pathways: those roughly preserving perihelion distance and those roughly preserving semimajor axis. The constant perihelion paths most often lead to close encounter, but occasionally also serve to connect paths which preserve semimajor axis. The process is quite reminiscent of Arnold diffusion, in which particles successively travel along chaotic zones associated with different resonances (Arnold 1974). The typical evolution is illustrated in Fig. 5. In this figure lines of constant semimajor axis are diagonal; lines of constant perihelion are vertical. The diagonal portion of the trajectory is quite narrow. Figure 6 shows the eccentricity versus time for this trajectory. Thus our simulations suggest a different storage mechanism for the short period comets than suggested by TS90. As of 200 million yr, the particles having late encounters with Neptune come from the region beyond 32 AU and were stored in one of the semimajor axis preserving chaotic zones. We do not observe long storage in the constant perihelion chaotic zone as suggested by TS90. Thus the dynamics of the delivery of short period comets appears to be more analogous to the dynamics of delivery of meteorites from mean motion resonances and secular resonances in the asteroid belt (Wetherill 1968; Wisdom 1985; Froeschlé & Scholl 1986; Wetherill 1987). The eccentricity grows irregularly while the semimajor axis remains relatively unchanged.

As an aside, note that the opposite of the evolution seen in Fig. 5 is possible. A small body in the vicinity of Neptune, possibly an escaped satellite, could evolve along a path with roughly constant perihelion, and subsequently transfer to one of the paths of constant semimajor axis, ultimately evolving to an orbit of low eccentricity with large semimajor axis. The scenario is provocative, although not necessarily of practical importance. Mikkola & Innanen (1992) recently noted that for some initial conditions test particles placed near Neptune's triangular Lagrange points temporarily develop orbits similar to that of Pluto before being ejected.

It is particularly noteworthy that the survival times for particles between adjacent planets can vary by more than two orders of magnitude. The profile is quite jagged. The profile of survival times beyond Neptune is obviously clipped by the limited time of our integrations; the profile suggests that we are just beginning to see the depletion of

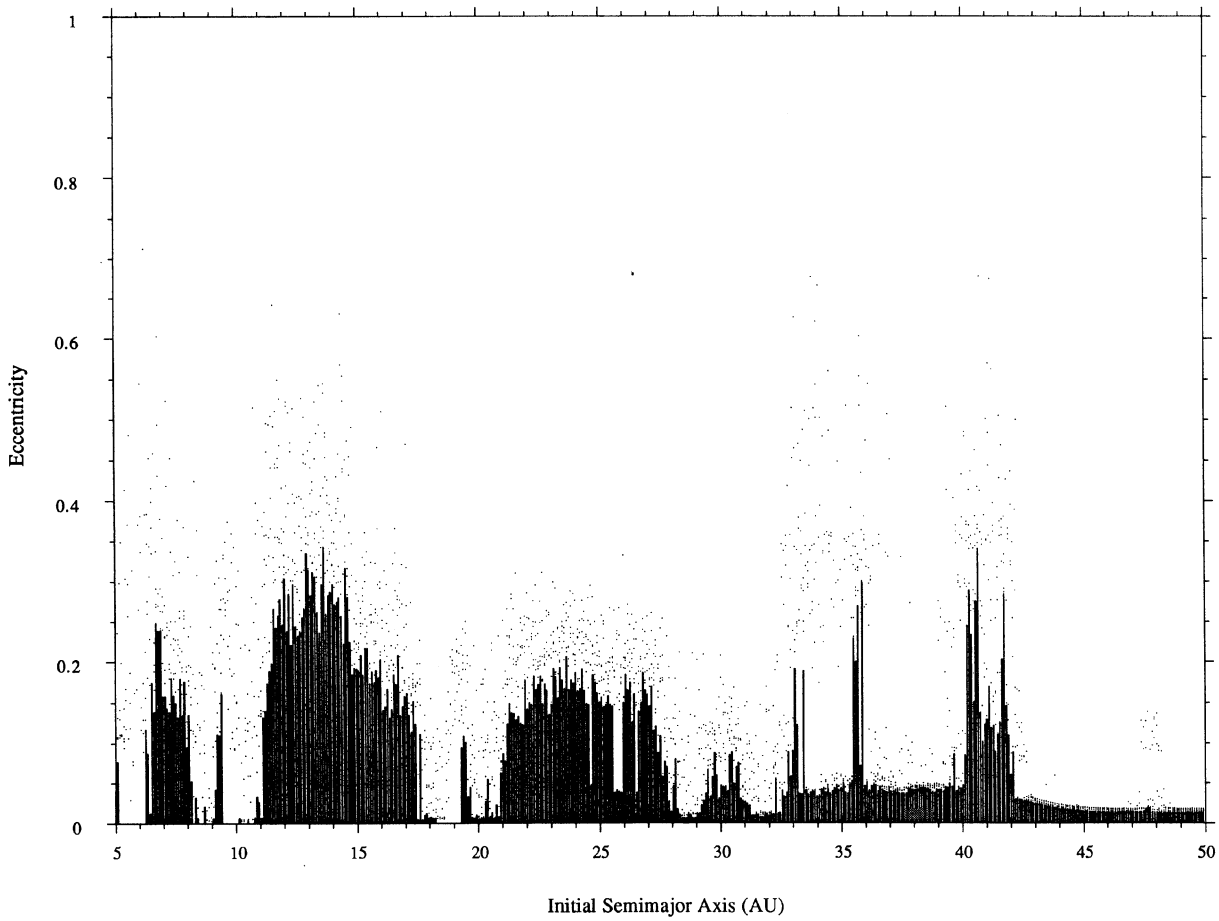


FIG. 3. The maximum eccentricity attained by each test particle during the course of the integration is plotted against initial semimajor axis. The vertical bars mark the minimum of the six values. Notice the features near 41 and 48 AU.

this region, and that the spread of survival times will be equally large. Thus it is likely that particles on initially circular orbits encounter Neptune over a wide range of times from about 10 million yr to, say, 10 billion yr. The range of survival times probably encompasses the age of the solar system.

The plot of the number of particles remaining as a function of time is remarkable (Fig. 7). The population does not decay exponentially as might have been expected. Nor does it decay as a power law. The population is best described as decaying logarithmically! Thus equal numbers of particles encounter Neptune for the first time in equal intervals of the logarithm of time. There was no reason to expect any particular decay law; each particle has its own individual deterministic dynamics. The logarithmic decay is just an average description of the dynamics of a large number of particles. It is interesting to note that a logarithmic decay implies a maximum lifetime; there is a time beyond which all particles which will have encounters have had them. The logarithmic decay is also recognizable in Fig. 2; there are apparently about equal numbers of dots in equal intervals of the logarithm of the survival time. Otherwise stated, the flux of new Neptune encounters is decaying as $1/t$, where t is the time since formation. If this trend continues we can speculate that, for instance, the flux of short period comets 3 billion years ago was only about 4 times the flux today.

We can continue our speculation and estimate how many comets would have had to have been initially in the region between 30 and 50 AU to account for the observed flux of new short period comets. Presuming a flux of new short period comets of about 0.01 per yr (Fernandez 1985; Duncan *et al.* 1988) and presuming that 0.17 of comets which encounter Neptune become visible (Duncan *et al.* 1988) we find that there were initially about 9×10^9 comets in this region and that roughly half of these comets must still be there. Assuming an average mass per comet of $10^{14.5}$ kg (Duncan *et al.* 1988) the current mass of the belt from 30 to 50 AU is comparable to the upper limit of about 0.2 Earth masses placed on the mass of the Kuiper belt from modeling perturbations to the orbit of Halley's comet (Hamid *et al.* 1968; Hogg *et al.* 1991). Our estimate is surely quite crude; for instance the extrapolated flux could easily be off by a factor of several, and our survey only considers initially circular orbits in the invariable plane. The estimate can be easily refined with longer integrations.

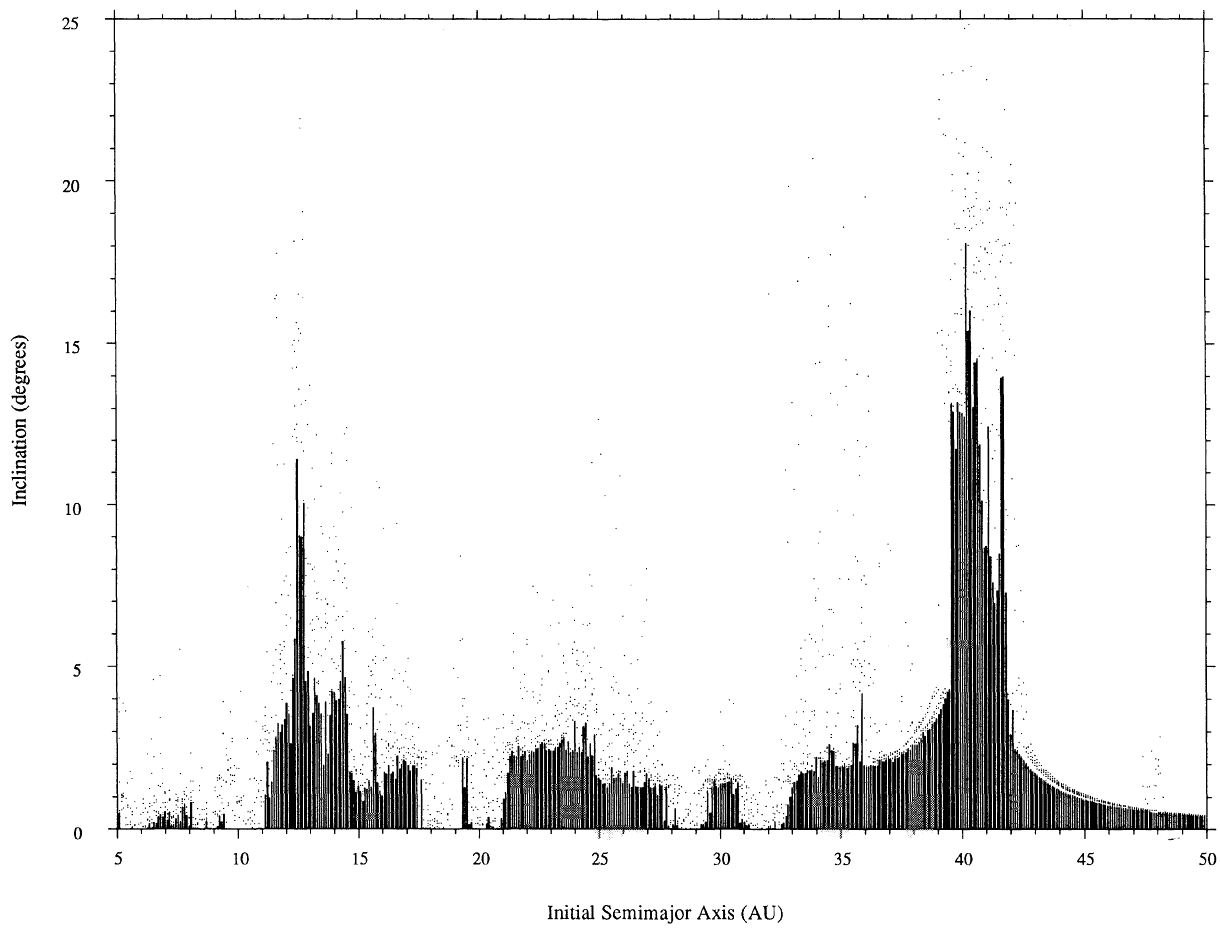


FIG. 4. The maximum inclination attained by each test particle during the course of the integration is plotted against initial semimajor axis. Again, the vertical bars mark the minimum of the six values. The features near 41 and 48 AU can also be seen.

Lecar *et al.* (1992a,b) found an interesting correlation between the maximum Lyapunov exponent and the planet crossing time for test particles in the outer asteroid belt and in the region between Jupiter and Saturn. They report a power law relation between the planet crossing time and the Lyapunov time. In their experiments, the best fit exponents relating the crossing time to the Lyapunov time are roughly 1.8. Figure 8 shows the correlation of the Lyapunov time (the inverse of the maximum Lyapunov exponent) and the time of close encounter for the test particles is our study. We find that the time of close encounter is more nearly directly proportional to the Lyapunov time, perhaps with an exponent as large as 1.4. A strong correlation could be used to estimate the times of close encounter for objects remaining in the region beyond Neptune based on the measured Lyapunov times, leading to another estimate of the flux from the region. However, the termination times range over two orders of magnitude for any given value of the Lyapunov time, limiting predictions of close encounter times to a rough range of values. A more direct approach is to improve our estimate of the flux is to extend the integrations to 4.6 billion yr.

7. SUMMARY

On time scales of 20 million yr we find no evidence that Saturn, Uranus, and Neptune cannot retain Trojan-like asteroids. Test particles in Saturn's orbit plane and placed on orbits near Saturn's L_4 and L_5 points experience close encounters with the planets on short time scales, but test particles further from the Lagrange points remain for the full integration.

In our test particle survey we confirm that test particles on initially circular orbits between Jupiter and Saturn are removed by close encounters with the planets on time scales of 10^4–10^5 yr, even when all four giant planets are included as perturbers. We also find that most test particles in the Saturn-Uranus and Uranus-Neptune regions are removed in 10 million yr, with the exception of small regions between Uranus and Neptune in which a few test particles endure the full 800 million yr integration.

Our results provide essential, new insight concerning the hypothesized Kuiper belt of comets beyond the orbit of Neptune, an expected remnant from the formation of the solar system. These are the first direct integrations that

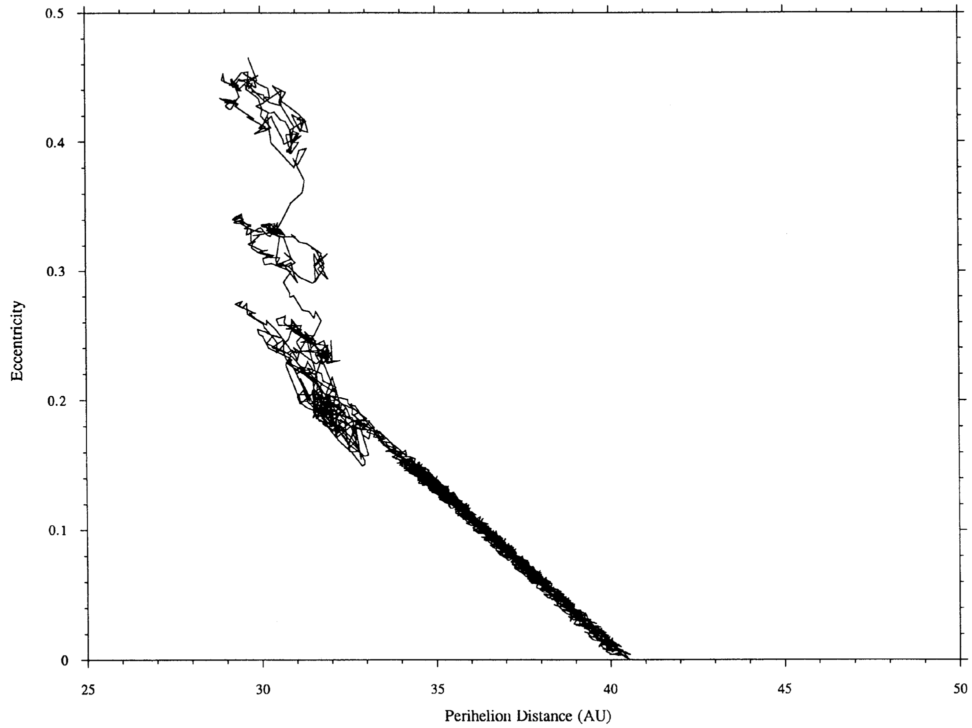


FIG. 5. The eccentricity is plotted vs perihelion distance for a representative trajectory. Lines of constant semimajor axis are diagonal; lines of constant perihelion distance are vertical.

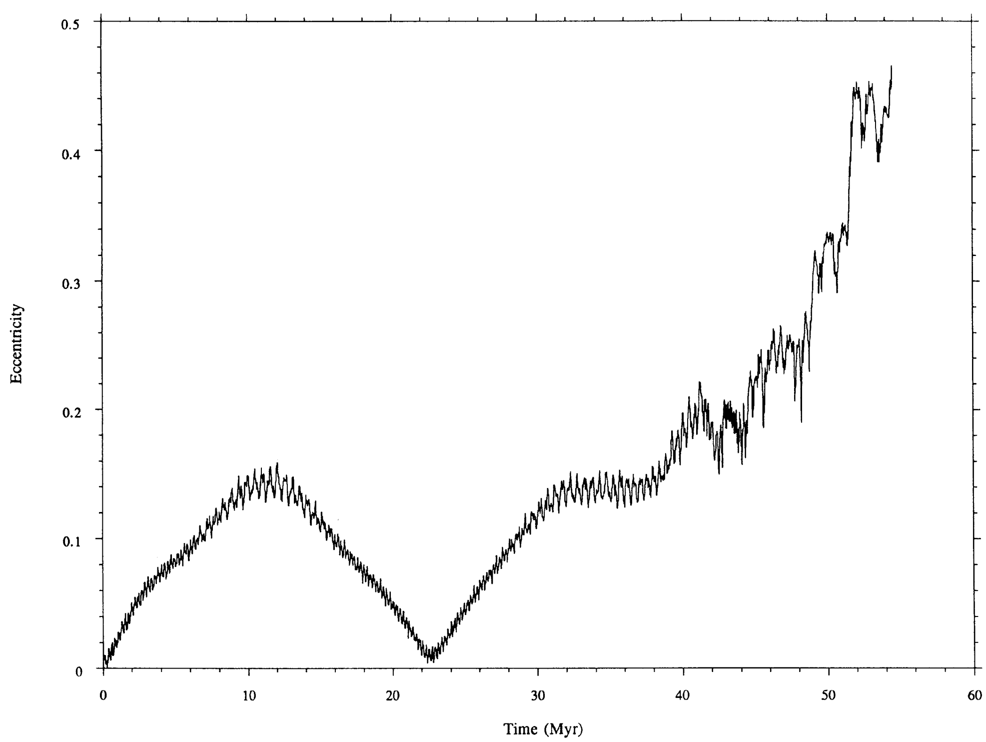


FIG. 6. Eccentricity is plotted vs time for the same trajectory as in Fig. 5.

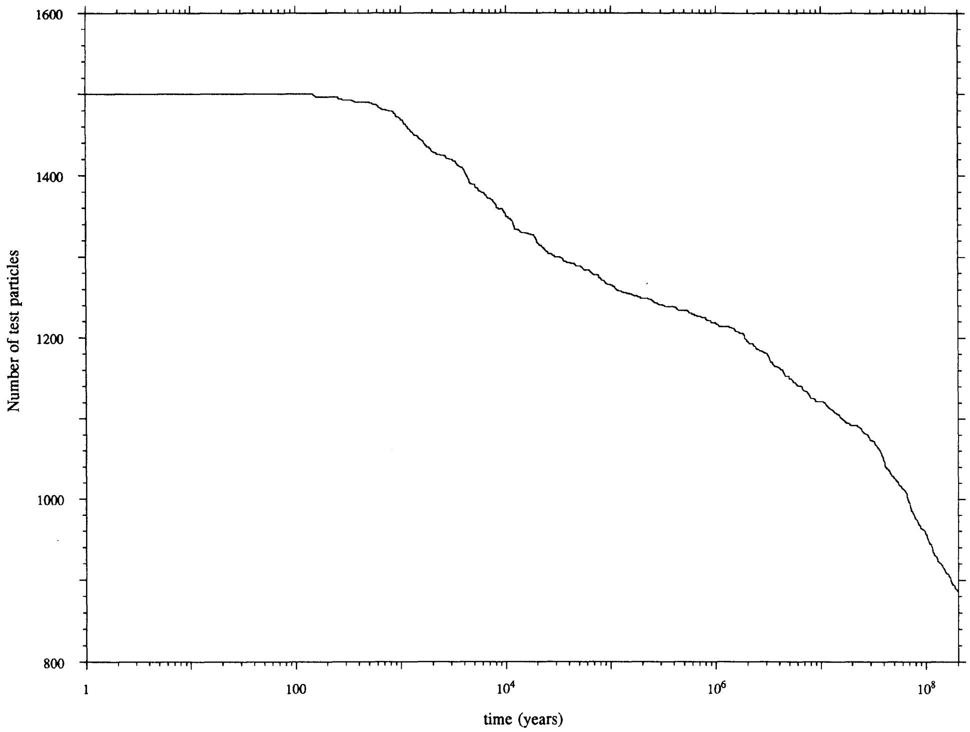


FIG. 7. The number of test particles remaining beyond Neptune is plotted as a function of time. Notice the slow (logarithmic) decay.

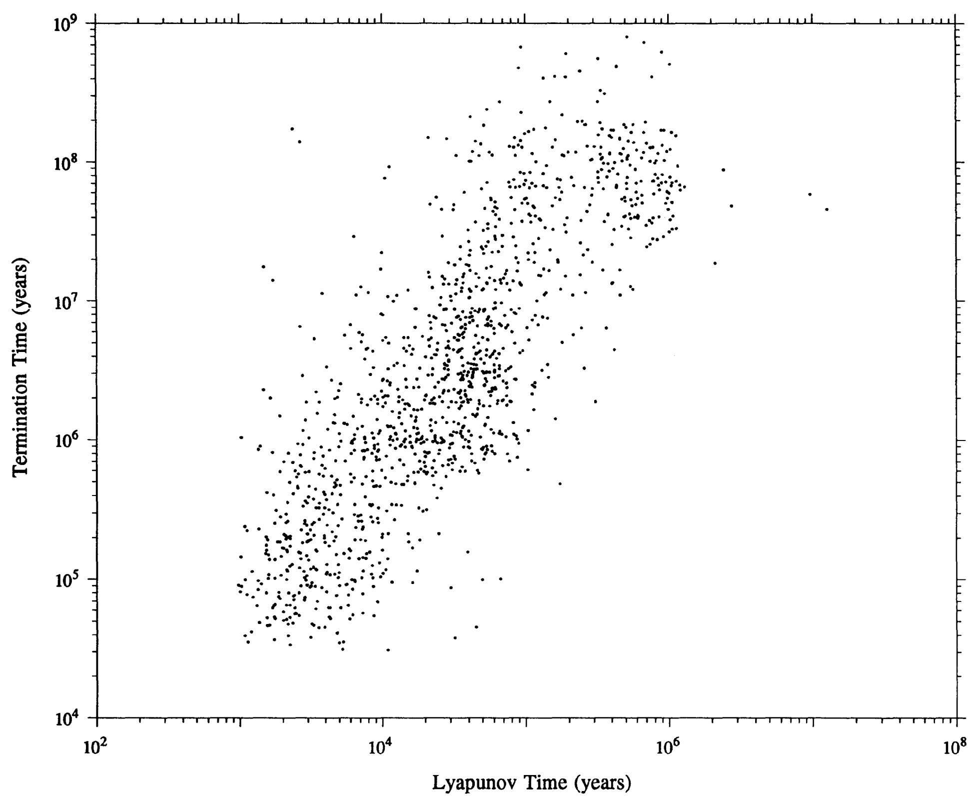


FIG. 8. The time of close encounter is correlated to the Lyapunov time for test particles started on circular orbits in the invariable plane.

demonstrate that small bodies in low inclination, low eccentricity orbits, even as far out as 42 AU, can develop large enough eccentricities to encounter Neptune in 10–100 million yr. As the test particles evolve into Neptune crossing orbits they roughly maintain constant semimajor axis as the eccentricity irregularly increases. Provided small bodies were formed in the Kuiper belt with nearly circular orbits, the distribution of encounter times suggests that they may only now be developing large eccentricities and being scattered by Neptune into the inner solar system.

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