COMPARISON BETWEEN SUBSONIC FLOW SIMULATION AND PHYSICAL MEASUREMENTS OF FLUE PIPES

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ABSTRACT

Direct simulations of wind musical instruments using the compressible Navier Stokes equations have recently become possible through the use of parallel computing and through developments in numerical methods. As a first demonstration, the flow of air and the generation of musical tones inside a soprano recorder are simulated numerically. In addition, physical measurements are made of the acoustic signal generated by the recorder at different blowing speeds. The comparison between simulated and physically measured behavior is encouraging and points towards ways of improving the simulations.

1. INTRODUCTION

A flue pipe consists of the flue channel (where the jet of air is formed), the labium (sharp edge), and the pipe (resonant cavity). The jet of air emerges from the flue channel, impinges the labium, and oscillates around the labium generating acoustic waves which travel into the pipe and return to interact with the jet of air in a complex feedback cycle. Predicting the acoustic frequencies which are generated by a flue pipe is generally difficult, and is possible only in simple cases and well-studied geometries. The study of flue pipes dates back to Helmholtz and Rayleigh and perhaps earlier, and remains very active today because the problem is very difficult. Approximate physical models of flue pipes have been developed over the years which work well in certain cases, but not always, and the assumptions behind the models are not fully understood (Hirschberg [1], Verge [2] and references therein).

In this paper, a soprano recorder is simulated using the compressible Navier Stokes equations. As far as we know, these are the first simulations of flue pipes using the compressible Navier Stokes equations to be presented (also see Hirschberg [1]). Further, the predictions of the simulations are compared against physical measurements of the acoustic signal generated by the recorder at different blowing speeds. The comparison is encouraging and points towards ways of improving the simulations.

Some of the difficulties of simulating subsonic hydrodynamics with acoustic waves are summarized in Hardin [3]. Most importantly, one must choose the time step of integration very small in order to follow the fast-moving acoustic waves. Second, one must control the slow-growing numerical instabilities which arise in simulations of subsonic flow. Third, one must impose boundary conditions around the simulation region that do not reflect acoustic waves.

The requirement of small time steps in simulating subsonic flow makes the computation expensive, but it also simplifies the computation. In particular, explicit numerical methods can be used which are simple to program and ideal for parallel computing. In many other PDE-based problems, explicit methods are usually avoided because they require very small time steps to remain numerically stable. In the present problem however, the time step must be kept very small anyway to follow the acoustic waves, so there is a match between the problem and the use of explicit methods.

The remaining parts of this paper are as follows. First, we describe the physical model and the geometry of the soprano recorder which we have simulated numerically. Then, we outline our numerical methods. Subsequently, we present representative results from the simulations of the recorder, and a comparison with physical measurements of the acoustic signal generated by the recorder. Finally, we comment on our results and on future directions.

2. PHYSICAL MODEL

The flow of air is modeled using the adiabatic approximation [4] so that the fluctuations of pressure are proportional to the fluctuations of density times the square of the speed of sound. The density ρ and the components of the flow velocity in the x,y directions V_x, V_y describe the flow (ρ, V_x, V_y are functions of space and time). The following Navier Stokes equations are used,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + \frac{c_s^2}{\rho} \frac{\partial \rho}{\partial x} = \nu \nabla^2 V_x \qquad (2)$$

$$\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + \frac{c_s^2}{\rho} \frac{\partial \rho}{\partial y} = \nu \nabla^2 V_y \qquad (3)$$

The constants $\nu = 0.15$ cm^2/s and $c_s = 34400$ cm/s are the kinematic viscosity and the speed of sound corresponding to air flow at 22 degrees centigrade.

The simulations are based on the two-dimensional model of the soprano recorder which is shown in figure 7. A steady supply of air is forced through the flue channel and exits through the outlet at the top of figure 7. The walls around the flue pipe (gray areas in figure 7) are non-slip walls which reflect acoustic waves. By contrast, the outlet and the two vertical "walls" above the flue pipe do not reflect acoustic waves so that the region above the flue pipe resembles an infinite space as much as possible and not another resonant cavity.

Non-reflecting boundary conditions are implemented by imposing both the density and the velocity at the boundary. For example, ambient pressure is imposed at the outlet, and a pressure which is slightly larger than the pressure drop of Hagen-Poiseuille flow through the narrow flue channel, is imposed at the inlet. Further, a parabolic velocity profile is imposed both at the outlet and the inlet. These boundary conditions are overconstrained; however, they do not cause any problems in the discrete numerical system (see [4]). They are used to prevent the reflection of acoustic waves.

Regarding the initial blowing of air into the flue channel, it is chosen to rise smoothly to a final steady value. The following formula is used both for the velocity and the density at the inlet and the outlet,

$$V(t) = V_{\text{final}} \left[1 - 10^{-(t/T)^2} \right] \qquad (4)$$

where T is 3 ms.

3. NUMERICAL METHODS

The Navier Stokes equations are solved numerically using a uniform grid and two different explicit numerical methods: the lattice Boltzmann method and a traditional finite difference method [4, 5]. The simulation results obtained with the lattice Boltzmann method and the finite difference method are very similar to each other; and for the sake of brevity, only the lattice Boltzmann simulations are presented here.

A fourth-order artificial viscosity filter is used in combination with the numerical methods in order to control slow-growing nonlinear instabilities [4, 6]. The filter does not alter the fluid dynamics at long wavelengths, but provides a small amount of additional dissipation at short wavelengths comparable to the grid spacing. Similar filters have been very common in transonic and supersonic flow (Peyret&Taylor [7]), and appear to be necessary for simulating compressible flow in general.

In the present simulations, the spatial resolution is $\Delta x = 0.01$ cm so that 10 fluid nodes fit along the width of the flue channel. About 0.8 million fluid nodes are used in the simulation, and they are computed in parallel by 22 non-dedicated

Hewlett-Packard HP9000/715 workstations (see [6]). The time step is $\Delta t = 2.17 \times 10^{-7}$ s so that about 150000 integration steps are needed to produce about 32 ms of simulated time. Each integration step takes about 1 s of runtime, and a typical simulation of 150000 integration steps lasts approximately 2 days.

4. RESULTS

Figure 7 shows the 2D geometry of the soprano recorder which is used in the simulations. The real recorder is cylindrical and looks like figure 7 when sliced vertically [4]. The recorder is 20 cm long and is closed at the far end. Thus, the recorder behaves as an open-closed pipe with major frequencies in odd ratios $1 : 3 : 5$ etc. The two lowest modes of an ideal open-closed pipe 20 cm long are 430Hz and 1290Hz, and the present recorder produces frequencies around 400Hz and 1100Hz as can be seen from the simulations and the physical measurements below.

Figures 1 to 6 show acoustic signals obtained from simulations and from physical measurements at different blowing speeds. The figures are matched in pairs (1&2, 3&4, 5&6) based on the resemblance between the simulated and the physical signals. A matching between simulated and physical signals based on the blowing speed is not possible probably because the simulations are two-dimensional, and also because the blowing speed in the physical setup is only known very roughly. Pair 1&2 shows the behavior at low blowing speed where the fundamental mode 400Hz dominates. Pair 3&4 shows the behavior at medium blowing speed where the fundamental 400Hz and the "third" 1100Hz have approximately equal size. Finally, pair 5&6 shows the behavior at high blowing speed where the "third" 1100Hz dominates.

5. CONCLUSION

We have presented representative results from the simulation of a soprano recorder using the compressible Navier Stokes equations. Physical measurements of the acoustic signal generated by the recorder at different blowing speeds are in reasonable agreement with the predictions of the simulations. However, there are also important differences between the simulated and the physical behavior. One source of differences is that the simulations are the initial transient behavior, and the physical measurements are the steady-state behavior. The present simulations can not be continued much longer than shown here because vorticity accumulates above the flue and affects the operation of the flue pipe. In the physical world, the generated vorticity quickly moves away from the flue because of air entrainment and also because of the available volume along the third dimension.

In figure 3 (medium blowing speed) we can see that good acoustic oscillations are only achieved for a few cycles, and then are lost. We suspect that this is a result of the accumu-

lation of vorticity above the flue because the presence of vortices changes the ambient pressure level outside the flue pipe. Work is in progress to find ways of removing the generated vorticity, and to understand better the simulation results with regard to boundary conditions and the two-dimensionality of the simulations versus the three-dimensionality of the physical world.

References

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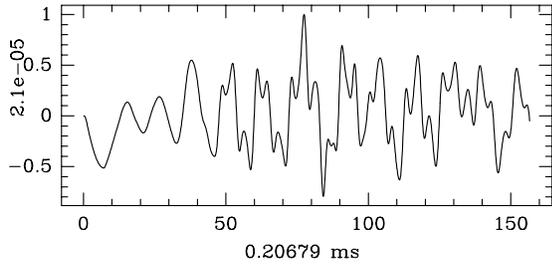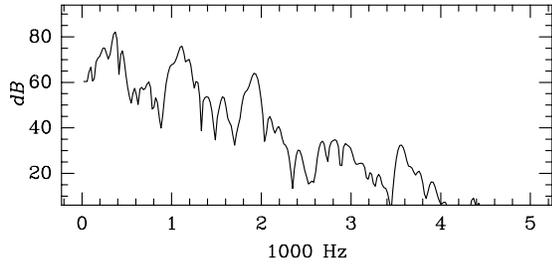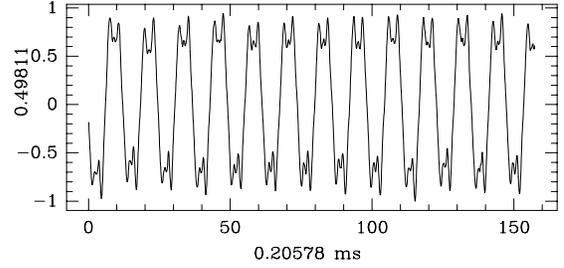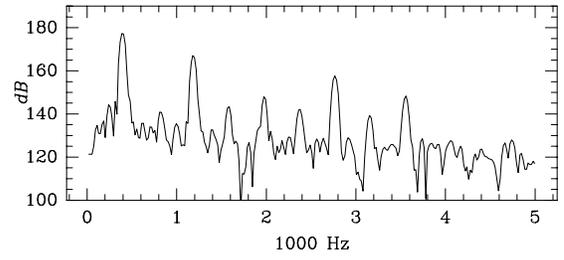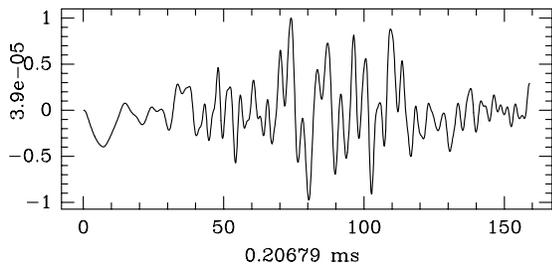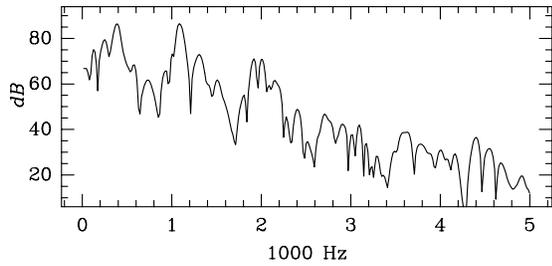
Figure 1: Simulation, 811 cm/s
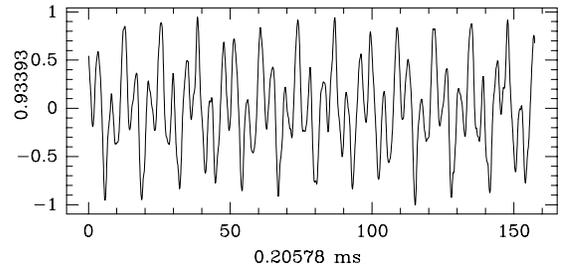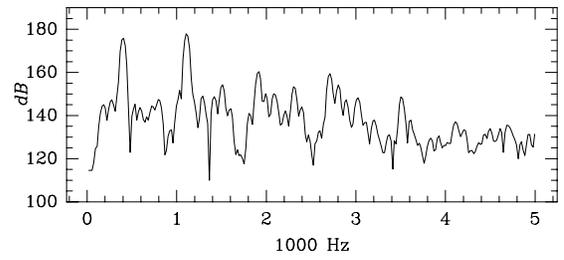

Figure 2: Physical, 734 cm/s

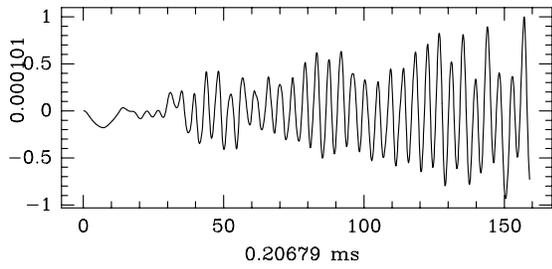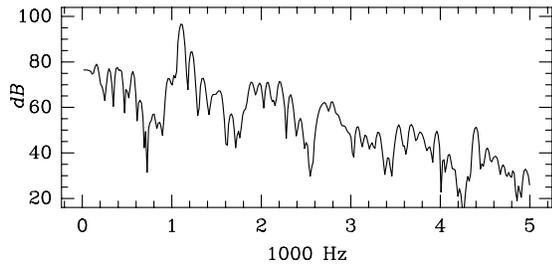Figure 3: Simulation, 953 cm/s

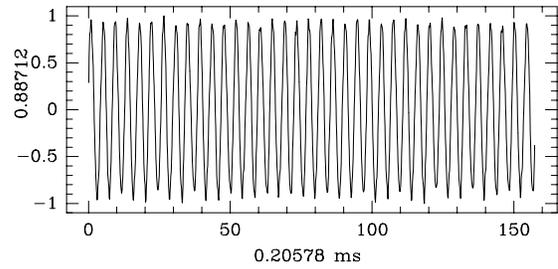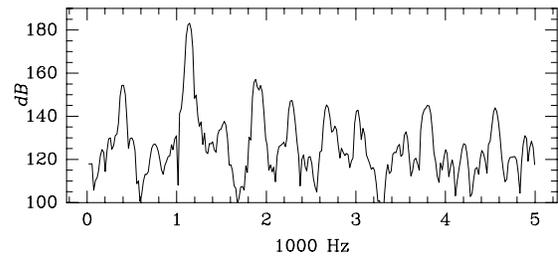Figure 4: Physical, 1140 cm/s

Figure 5: Simulation, 1108 cm/s

Figure 6: Physical, 1558 cm/s

The above figures (1 to 6) show acoustic signals obtained from simulations and from physical measurements of the 20 cm closed-end recorder at different blowing speeds (mean flow speed through the flue channel). The simulation data is sampled 4 cm above the labium, and the physical data is sampled 100 cm above the labium. Each figure plots the time series of the normalized density fluctuation at the bottom, and the frequency spectrum (decibels with respect to standard pressure level) at the top. The physical measurements have arbitrary units of amplitude because the measurements were not calibrated.
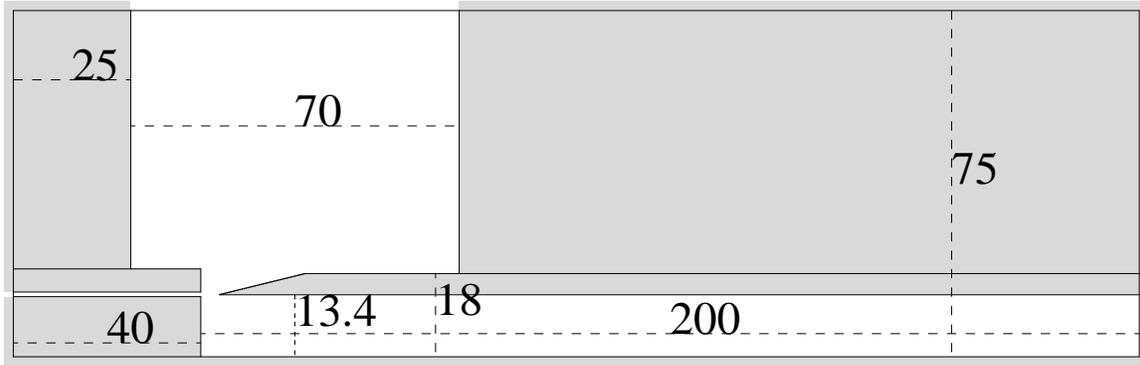
Figure 7: The two-dimensional geometry of the soprano recorder which is used in the simulations. The numbers shown correspond to millimeters. The pipe is located at the bottom of the picture, and measures 20 cm long and 1.34 cm wide. The flue channel is located at the bottom left corner, and measures 4 cm long and 0.1 cm wide. At a distance of 0.4 cm in front of the flue channel, there is a sharp labium which measures an angle of 14 degrees approximately, and is positioned slightly below the midline of the flue channel. Namely, the tip of the labium is located at 1.34 cm from the bottom of the pipe, and the flue channel is located between 1.3 cm and 1.4 cm.
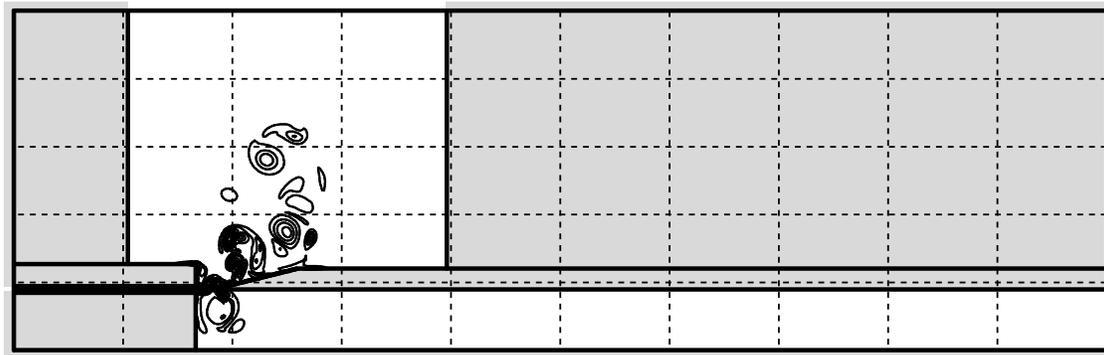

Figure 8: Simulation of a 20 cm closed-end soprano recorder. Picture is 30 ms after startup. Iso-vorticity contours are plotted. The dashed lines show the decomposition of the problem into 22 workstations. The gray-shaded areas are not simulated.