

## A Subtle Fallacy

- |     |                                                                                                                                                                                                                                        |             |       |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-------|
| 1.  | $  \begin{aligned}  &(\text{All } (x) \\  &\quad (\text{implies } (\text{Natnum } x) \\  &\quad\quad (\text{Exists } (y) \\  &\quad\quad\quad (\text{and } (\text{Natnum } y) \\  &\quad\quad\quad\quad (< x y))))))  \end{aligned}  $ | Premise     | {1}   |
| 2.  | $(\text{Natnum } *A)$                                                                                                                                                                                                                  | Premise     | {2}   |
| 3.  | $  \begin{aligned}  &(\text{implies } (\text{Natnum } *A) \\  &\quad (\text{Exists } (y) \\  &\quad\quad (\text{and } (\text{Natnum } y) \\  &\quad\quad\quad (< *A y))))  \end{aligned}  $                                            | (US 1 *A x) | {1}   |
| 4.  | $  \begin{aligned}  &(\text{Exists } (y) \\  &\quad (\text{and } (\text{Natnum } y) (< *A y)))  \end{aligned}  $                                                                                                                       | (MP 3 2)    | {1 2} |
| 5.  | $(\text{and } (\text{Natnum } a) (< *A a))$                                                                                                                                                                                            | (ES 4 a y)  | {1 2} |
| 6.  | $(\text{Natnum } a)$                                                                                                                                                                                                                   | (-&:L 5)    | {1 2} |
| 7.  | $(< *A a)$                                                                                                                                                                                                                             | (-&:R 5)    | {1 2} |
| 8.  | $(\text{implies } (\text{Natnum } *A) (< *A a))$                                                                                                                                                                                       | (CP 7 2)    | {1}   |
| 9.  | $  \begin{aligned}  &(\text{All } (x) \\  &\quad (\text{implies } (\text{Natnum } x) (< x a)))  \end{aligned}  $                                                                                                                       | (UG 8 x *A) | {1}   |
| 10. | $  \begin{aligned}  &(\text{and } (\text{Natnum } a) \\  &\quad (\text{All } (x) \\  &\quad\quad (\text{implies } (\text{Natnum } x) \\  &\quad\quad\quad (< x a))))  \end{aligned}  $                                                 | (+& 6 9)    | {1 2} |
| 11. | $  \begin{aligned}  &(\text{Exists } (y) \\  &\quad (\text{and } (\text{Natnum } y) \\  &\quad\quad (\text{All } (x) \\  &\quad\quad\quad (\text{implies } (\text{Natnum } x) \\  &\quad\quad\quad\quad (< x y))))))  \end{aligned}  $ | (EG 10 x a) | {1 2} |

## Skolem Functions Patch

|     |                                                                                  |                                  |       |
|-----|----------------------------------------------------------------------------------|----------------------------------|-------|
| 1.  | (All (x)<br>(implies (Natnum x)<br>(Exists (y)<br>(and (Natnum y)<br>(< x y))))) | Premise                          | {1}   |
| 2.  | (Natnum *A)                                                                      | Premise                          | {2}   |
| 3.  | (implies (Natnum *A)<br>(Exists (y)<br>(and (Natnum y)<br>(< *A y))))            | (US 1 *A x)                      | {1}   |
| 4.  | (Exists (y)<br>(and (Natnum y) (< *A y)))                                        | (MP 3 2)                         | {1 2} |
| 5.  | (and (Natnum (a *A))<br>(< *A (a *A)))                                           | (ES 4 a y)                       | {1 2} |
| 6.  | (Natnum (a *A))                                                                  | (-&:L 5)                         | {1 2} |
| 7.  | (< *A (a *A))                                                                    | (-&:R 5)                         | {1 2} |
| 8.  | (implies (Natnum *A)<br>(< *A (a *A)))                                           | (CP 7 2)                         | {1}   |
| 9.  | (All (x)<br>(implies (Natnum x)<br>(< x (a x))))                                 | (UG 8 x *A)                      | {1}   |
| 10. | (and (Natnum (a *A))<br>(All (x)<br>(implies (Natnum x)<br>(< x (a x)))))        | (+& 6 9)                         | {1 2} |
| 11. | (Exists (y)<br>(and (Natnum y)<br>(All (x)<br>(implies (Natnum x)<br>(< x y))))) | (EG 10 x (a x))<br>*NOT ALLOWED* | {1 2} |