Review of Logic and Rules

Propositional Calculus

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General Caveat

If you think that your paper is vacuous,

use the first-order functional calculus.

It then becomes logic,

and as if by magic,

the obvious is hailed as miraculous.

Paul Halmos
c.1967

A great book:

Introduction to Logic by Patrick Suppes

If Alfred studies
then he receives good grades.

If Alfred does not study
then he enjoys college.

If Alfred does not receive good grades
then he does not enjoy college.

Therefore
Alfred receives good grades.

Do we believe this argument?

if so, why?

if not, why not?

Logic is the study of arguments

An argument is a sequence of statements beginning with premises and ending with a conclusion.

The conclusion is said to follow from (or is supported by) the premises.

Arguments may be good or bad (convincing or not convincing) A good argument is called valid.

A valid form of argument is one that never allows the deduction of a false conclusion from true premises.

An invalid form of argument is one that sometimes allows the deduction of a false conclusion from true premises.

A Valid Argument

**If Socrates is human
then Socrates is mortal.**

Socrates is human.

Socrates is mortal.

An Invalid Argument

**If Socrates is human
then Socrates is mortal.**

Socrates is mortal.

Socrates is human.

I have a dog named Socrates!

Sentential Connectives
Truth functions

Implication: a --> b

If a then b.
a implies b.

Negation: -a

Not a.
It is not the case that a.

Conjunction: a & b

a and b.

Disjunction: a V b

a or b.

Equivalence: a <--> b

a if and only if b.

Truth Tables define Truth Functions

```
A || -A
-------
T || F
-------
F || T
```

```
A | B || A & B
--------------
T | T ||   T
--------------
F | T ||   F
--------------
T | F ||   F
--------------
F | F ||   F
```

```
A | B || A V B
--------------
T | T ||   T
--------------
F | T ||   T
--------------
T | F ||   T
--------------
F | F ||   F
```

```
A | B || A --> B
----------------
T | T ||    T
----------------
F | T ||    T
----------------
T | F ||    F
----------------
F | F ||    T
```

```
A | B || A <--> B
-----------------
T | T ||    T
-----------------
F | T ||    F
-----------------
T | F ||    F
-----------------
F | F ||    T
```

Analysis of arguments
by sentential interpretation.

Primitive sentences
S = Alfred studies.
E = Alfred enjoys college.
G = Alfred receives good grades.

Premises: A = S --> G
B = -S --> E
C = -G --> -E

S	E	G	A	B	C
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	F	T
F	F	F	T	F	T

Note that for every row where the conclusion G is false, at least one of the premises (A, B, or C) is false.

Sentential Derivation

1. If Alfred studies
   then he receives good grades.
   PREMISE

2. If Alfred does not study
   then he enjoys college.
   PREMISE

3. If Alfred does not receive good grades
   then he does not enjoy college.
   PREMISE

4. Alfred does not receive good grades.
   PREMISE (hypothetical)

5. Alfred does not enjoy college.
   MP 3,4

6. Alfred studies.
   MT 2,5; NE

7. Alfred does not study.
   MT 1,4

8. Alfred receives good grades.
   RAA 4,6,7

QED

Sentential Derivation with Dependencies

The Suppes Formalism

S = Alfred studies.
E = Alfred enjoys college.
G = Alfred receives good grades.

1.	S --> G	Premise	{1}
2.	-S --> E	Premise	{2}
3.	-G --> -E	Premise	{3}
4.	-G	Premise	{4}
5.	-E	MP 3,4	{3 4}
6.	--S	MT 2,5	{2 3 4}
7.	S	NE 6	{2 3 4}
8.	-S	MT 1,4	{1 4}
9.	G	RAA 7,8;4	{1 2 3}

With Quantification

```
1.                                  Premise        {1}
    All y [(greek y) --> (human y)]

2.                                  Premise        {2}
    All x [(human x) --> (mortal x)]

3.                                  Premise        {3}
    (greek *G)

4.                                  (US 1 *G y)    {1}
   (greek *G) --> (human *G)

5.                                  (MP 4 3)     {1 3}
   (human *G)

6.                                  (US 2 *G x)    {2}
   (human *G) --> (mortal *G)

7.                                  (MP 6 5)   {1 2 3}
   (mortal *G)

8.                                  (CP 7 3)     {1 2}
   (greek *G) --> (mortal *G)

9.                                  (UG 8 z *G) {1 2}
   All z [(greek z) --> (mortal z)]
```