

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.5160J, 8.351J, 12.620J

Classical Mechanics: A Computational Approach

Problem Set 9—Fall 2024

Issued: 30 October 2024

Due: Friday, 15 November 2024

Remember: there are no classes on Monday, 11 November 2024.

Reading: SICM2 Chapter 4 through Section 4.5

Introduction

When we examine a surface of section for a dynamical system we see various features. There are often “islands” in a “chaotic sea.” Here we begin to understand the nature of these features. For small perturbations we see that the chaos arises at the unstable fixed points. The homoclinic tangle is the extension of the stable and unstable manifolds derived from the separatrices emanating from the unstable fixed points. The exponentially long tendrils of the homoclinic tangle illuminate the origin of exponential divergence in the chaotic orbits. The islands of “regular” behavior are a consequence of the perturbation of orbits of rational rotation numbers in an integrable system. The Poincaré-Birkhoff construction shows how the continuum of fixed points for a commensurate motion is destroyed and replaced with an alternating sequence of stable and unstable fixed points. Each stable fixed point is surrounded by an island. There remains a mystery: Since the rationals are dense in the reals why is there enough space for an island chain for every rational rotation number? We will answer this next time!

Exercises

- Exercise: Computing Homoclinic Tangles SICM2 page 309
- Exercise: Computing the Poincaré-Birkhoff construction SICM2 page 322