MASSACHVSETTS INSTITVTE OF TECHNOLOGY
6.946J, 8.351J, 12.620J

Classical Mechanics: A Computational Approach

Problem Set 5—Fall 2024

Issued: 2 October 2024 Due: 11 October 2024

Project 2 is also due on Friday, 11 October 2024;
Project 3 announcement on next page.

Reading: SICM2 all of Chapter 2

Introduction

The motion of a free body appears paradoxical in that the motion is stable around the axes of minimum and maximum moments of intertia, but it wobbles around the intermediate axis. This is clarified by an understanding of the conserved momenta and energy.

The axisymmetric top involves a potential energy, but it is an easy study because the two symmetries lead to conserved quantities that reduce the essential problem to one degree of freedom.

A more interesting problem is that of spin-orbit coupling. In this case the potential energy function is essential and must be cleverly approximated, reducing the problem to a driven-pendulum–like system. This system exhibits resonances and chaotic behavior.

Since project 2 is due, this problem set is a bit shorter.

Exercises

- Exercise 2.16: Precession of the top. SICM2 page 164
- Exercise: Width of Resonances.

  Consider a periodically-driven pendulum. A nice Lagrangian for the driven pendulum is:

  $$L(t,\theta,\dot{\theta}) = \frac{1}{2}ml^2\dot{\theta}^2 + ml(g + D^2 y_s(t))\cos\theta. \tag{1}$$

  This is equation (1.120) on page SICM2 page 66. A nice periodic drive is:

  $$y_s(t) = A\cos(\omega t) \tag{2}$$

  1. Express this Lagrangian as a Poisson series. A Poisson series is a multiply periodic Fourier expansion where products of sines and cosines are expressed in terms of sines and cosines of sums and differences of their arguments. For example, $(\sin(a))^2 = (1/2) - (1/2)\cos(2a)$.
  2. The Lagrangian will have three potential-energy terms. For each of these determine the "width" and center of the oscillation region. The "width" is the extent of the region in angular velocity. The center is the angle and angular velocity of the state that has zero oscillation amplitude. Hint: Look at how this was done for the spin-orbit problem in Section 2.11.3.

Project 3

Project 3 will be due on 1 November 2024. There will also be a problem set due on that date, but it will be shorter. Project 3 will be about evolution in phase space and observation of surfaces of section. You should choose one of the following for that project

- Exercise 3.14: Periodically Driven Pendulum SICM2 page 282
- Exercise 3.15: Spin-orbit Surfaces of Section SICM2 page 282