MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002x – Circuits and Electronics
Spring 2004

Problem Set 9

Issued: 21 April 2004

Quiz 3 will be on Friday, May 7 in your recitation section

Reading: Finish reading chapter 4 from Bose and Stevens *Introductory Network Theory* (copies were distributed in lecture on April 5)

To write up and hand in at lecture on Monday, May 3.

This is the last problem set of the semester. (Hooray!)

For this assignment, you are to write up and hand in the following problems from chapter 4 of Bose and Stevens:

- Problem 4-20. Note: The notation "cps" means "cycles per second". This notation was replaced by "Hertz" in the late 70's.
  - *Part a*

$$H(s_p) = \frac{V_2}{V_1} \quad (1)$$

$$H(s_p) = \frac{R}{R + \frac{1}{sC}} \quad (2)$$

$$H(s_p) = \frac{s}{s + \frac{1}{RC}} \quad (3)$$

    There is a pole at $\frac{-1}{RC}$ *and a zero at zero.*

    The magnitude of $H(s_p)$ *starts from 0 and increases by 20db per decade until it hits 1. The breakpoint frequency is* $\omega = \frac{1}{RC}$.
    The phase starts at 90 degrees and decreases to 0, intersecting 45 degrees at $\omega = \frac{1}{RC}$.
  - *Part b*
    The half-power frequency is the frequency for which the magnitude of the system function is $\frac{1}{\sqrt{2}}$ *its maximum. Since the maximum magnitude of the system is 1, we need to find the frequency where* $H(j\omega) = \frac{1}{\sqrt{2}}$. *This happens at* $w = \frac{1}{RC}$.

    Setting $f = \frac{\omega}{2\pi} = \frac{\frac{1}{RC}}{2\pi} = \frac{1}{2\pi RC} = 5000$, *we get* $C = 1.9\mu F$
- Problem 4-24.

- *Part a*

  The voltage transfer ratio $H(s_p)$ can be obtained as follows:

$$H(s_p) = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL + R} \tag{4}$$

$$H(s_p) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \tag{5}$$

  The system function has 2 poles and no zeros. The two poles are given by the roots of the denominator, where $p_1 = \frac{-R}{2L} + j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and $p_2 = \frac{-R}{2L} + j\sqrt{\frac{1}{LC} + \frac{R^2}{4L^2}}$

- *Part b*

$$\alpha = \frac{R}{2L} \tag{6}$$

$$\omega_d = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{7}$$

$$\omega_o = \sqrt{\frac{1}{LC}} \tag{8}$$

- *Part c*

  We find the frequency at which $|H(j\omega)|$ is maximum.

$$H(j\omega) = \frac{\frac{1}{LC}}{(-\omega^2 + \frac{1}{LC}) + \frac{R}{L}j\omega} \tag{9}$$

$$|H(j\omega)| = \frac{\frac{1}{LC}}{\sqrt{(-\omega^2 + \frac{1}{LC})^2 + (\frac{R}{L}\omega)^2}} \tag{10}$$

  Thus, $|H(j\omega)|$ will be maximum when $w^2 = \frac{1}{LC}$. The frequency is $\frac{1}{2\pi\sqrt{LC}}$.

  For systems where $\alpha \ll \omega$ (stated in this problem), the half power points are at $\omega_d + \alpha$ and $\omega_d - \alpha$. The frequencies are $\frac{\omega_d+\alpha}{2\pi}$ and $\frac{\omega_d-\alpha}{2\pi}$

- *Part d*

  Looking at the pole-zero diagram, the magnitude of the system starts at a constant and peaks at the resonance frequency of $\omega = \frac{1}{\sqrt{LC}}$. As ω increases further, the magnitude decreases at the rate of 40db per decade.

  The phase of the system starts at 0 degrees and hits -90 degrees at the resonance frequency. At large ω, the phase approaches -180 degrees.

  Figure 1 shows the magnitude and phase of such a system.

- *Part e*

$$Q = \frac{\omega}{2\alpha} \tag{11}$$

$$Bandwidth = 2\alpha \tag{12}$$

- *Part f*

  $v_2(t) = V_o(1 - \frac{\omega_o}{\omega_d}e^{-\alpha t}cos(\omega_d t - tan^{-1}1(\frac{\alpha}{\omega_d})))$*, where $V_o = 1$. See page 889-893 in Agarwal and Lang for detailed derivation.*

  As Q changes from low to high, the amount of ringing changes accordingly.

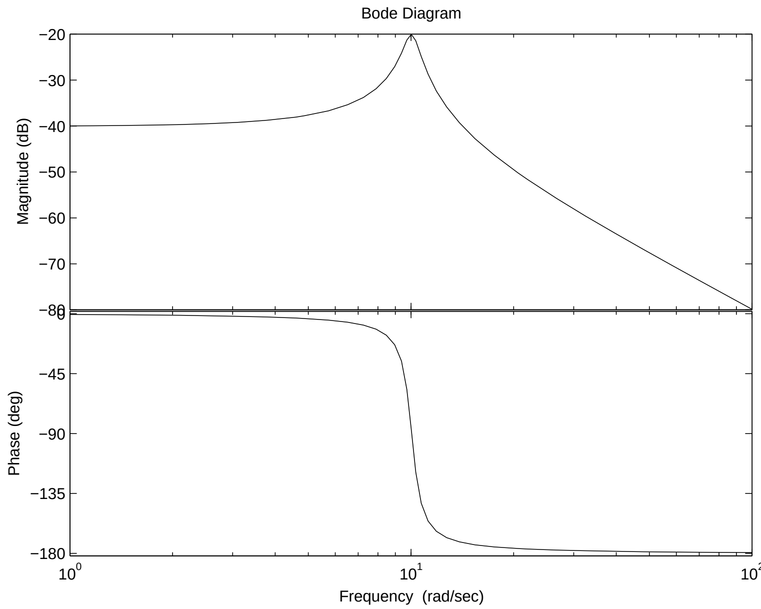

Figure 1: Bode Plot

- Problem 4-31.
  - $Z(s_p) = \frac{s_p}{s_p+1}$
    For circuit shown in Figure 2, $Z(s_p) = \frac{s_pLR}{s_pL+R} = \frac{s_p}{s_p+1}$

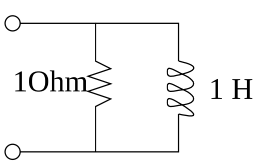

Figure 2: Circuit with Z(sp) for 4-31, Part 1

  - $Z(s_p) = \frac{s_p+2}{s_p+1}$
    Expressing the impedance as sum of two impedances in series, a we get $Z(s_p) = \frac{s_p+2}{s_p+1} = \frac{s_p}{s_p+1} + \frac{2}{s_p+1}$
    Circuit shown in Figure 3 shows the correct component values which will result in the above Z(sp).
  - $Z(s_p) = 2\frac{s_p^2+2s_p+2}{s_p(s_p+1)}$
    The circuit shown in Figure 4 will give the correct impedance.
  - $Z(s_p) = \frac{s_p^2+1}{s_p(s_p^2+2)}$
    The circuit shown in Figure 5 will give the correct impedance.

$$Z(s_p) = (s_pL_1 + \frac{1}{s_pC1})||\frac{1}{s_pC_2} \tag{13}$$

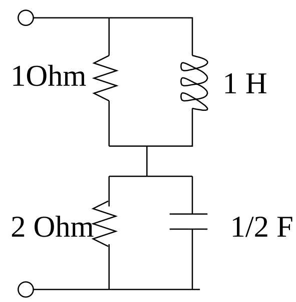

Figure 3: Circuit with Z(sp) for 4-31, Part 2

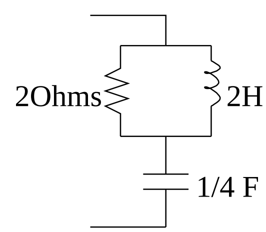

Figure 4: Circuit with Z(sp) for 4-31, Part 3

$$Z(s_p) = \frac{s_p^2 L_1 C_1 + 1}{s_p(s_p^2 C_1 C_2 L_1 + (C_1 + C_2))} \tag{14}$$

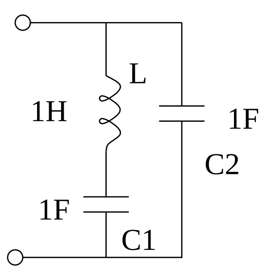

Figure 5: Circuit with Z(sp) for 4-31, Part 4

- Problem 4-44.
  We start by writing the system function for the three networks.
  - *Network 1*

$$Z(s_p) = R_1 + \frac{s_p L R_2}{s_p L + R_2} \tag{15}$$

$$Z(s_p) = \frac{s_p(LR_2 + LR_1) + R_1 R_2}{s_p L + R_2} \tag{16}$$

  - *Network 2*

$$Z(s_p) = s_p L || (R + \frac{1}{s_p C}) \tag{17}$$

$$Z(s_p) = \frac{s_p L(s_p CR + 1)}{s_p^2 LC + RCs_p + 1} \tag{18}$$

– *Network 3*

$$Z(s_p) = \frac{1}{s_p C_1} || (R + \frac{1}{s_p C_2}) \tag{19}$$

$$Z(s_p) = \frac{s_p RC_2 + 1}{s_p(C_1 + C_2) + s_p^2 C_1 C_2 R} \tag{20}$$

1. *Property 1*
   Looking at the system functions, the 2 pole, 1 zero system corresponds to Network 3.
2. *Property 2*
   This does not correspond to any network.
3. *Property 3*
   The voltage response of $v(t) = Ae^{-3t} - Be^{-t}$ suggests that the system has 2 complex poles, resulting in a response in the form of $Ae^{st} + Be^{st}$. Thus, network 2 could have this property.
4. *Property 4*
   $|Z(j\omega) = \sqrt{\frac{\omega^2+1}{\omega^2+2}}$ implies a single pole and single zero system. Thus, network 1 could have this property.
5. *Property 5*
   None of the networks will have this property.
6. *Property 6*
   Voltage across the inductor is $L\frac{di}{dt}$, when the current changes from 0 to 1 (step current source), the voltage across the inductor will have a discontinuity. Network 1, 2 could have this property.