MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002x – Circuits and Electronics
Spring 2004

Problem Set 8
Readings and exercises for April 5–21

Issued: 5 April 2004

Reading:

- Chapter 4 from Bose and Stevens *Introductory Network Theory* (copies distributed in lecture on April 5)

Quiz 2 will be on Friday, April 9, in recitation. Be sure to read and study the *quiz announcement*, distributed in lecture on April 5.

Remember also that the next part of the radio – RF Amplifier/Mixer/Converter (lab book through page 48). Is due to be checked off in lab on April 9.

8.0: To do and turn in at lecture on Monday, April 12

Fill out and turn in your *tutorial performance evaluation form* for case 3. Copies of this form were distributed in lecture on March 29. If you don't have a copy, you can find one included with the *Tutorial evaluation sheets* on the course web site, on the handouts page.

8.1: To do and turn in on line before lecture on Wednesday April 21

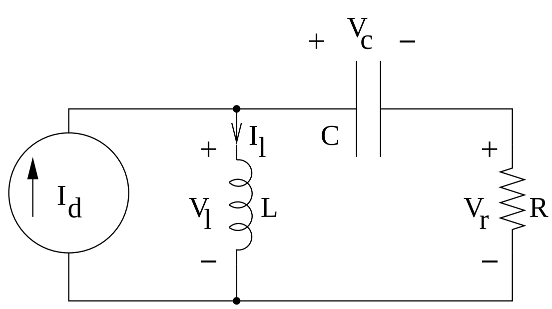

Figure 1: An RLC circuit driven by a current source

PS.8.1.1: Frequency responses for second-order systems This problem deals with the circuit shown in figure 1. The parameters of the circuit components are R=10 Ohms, L = 1 milliHenry, and C = 1 microFarad. If we drive the circuit with a current source I_d, there are four responses we could measure: I_l, V_l, V_c, and V_r.

Consider the four system functions:

$$\begin{aligned}
H_1(s) &= \frac{s^2RLC + sL}{s^2LC + sRC + 1} \\
H_2(s) &= \frac{s^2RLC}{s^2LC + sRC + 1} \\
H_3(s) &= \frac{sL}{s^2LC + sRC + 1} \\
H_4(s) &= \frac{sRC + 1}{s^2LC + sRC + 1}
\end{aligned}$$

Each of these is the system function for input I_d and one of the four responses. Which system function goes with which response?

The graphs below show show the frequency-response magnitude and phase plots for these systems. Using the online system, match the corresponding system function, magnitude plot, and phase plot to each of the responses.

Answers

- System function for I_l: 4
- Magnitude for I_l: 3
- Phase for I_l: 4
- System function for V_l: 1
- Magnitude for V_l: 2
- Phase for V_l: 1
- System function for V_c: 3
- Magnitude for V_c: 1
- Phase for V_c: 3
- System function for V_r: 2
- Magnitude for V_r: 4
- Phase for V_r: 2

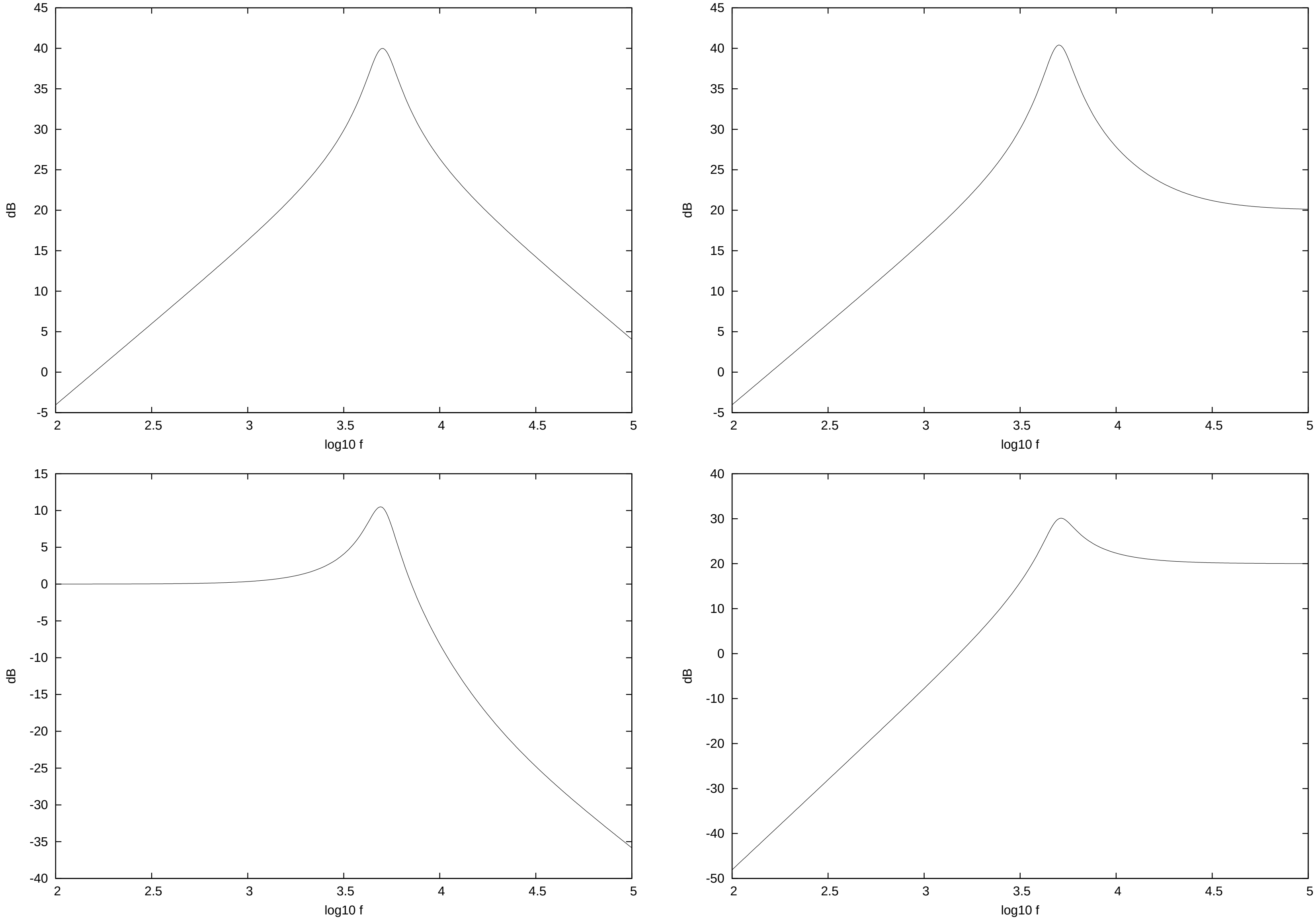

Figure 2: Magnitude plots for this problem

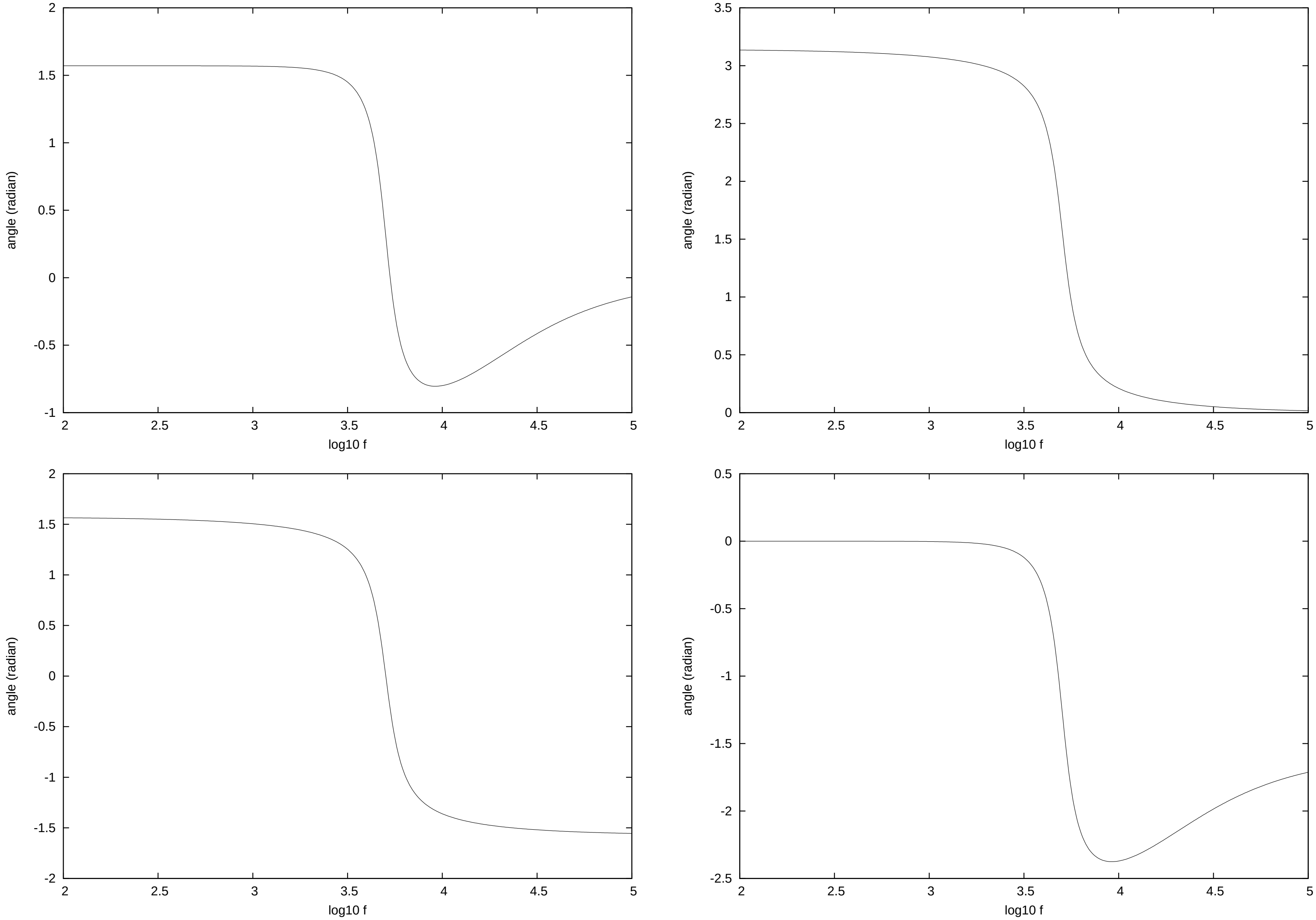

Figure 3: Angle plots for this problem

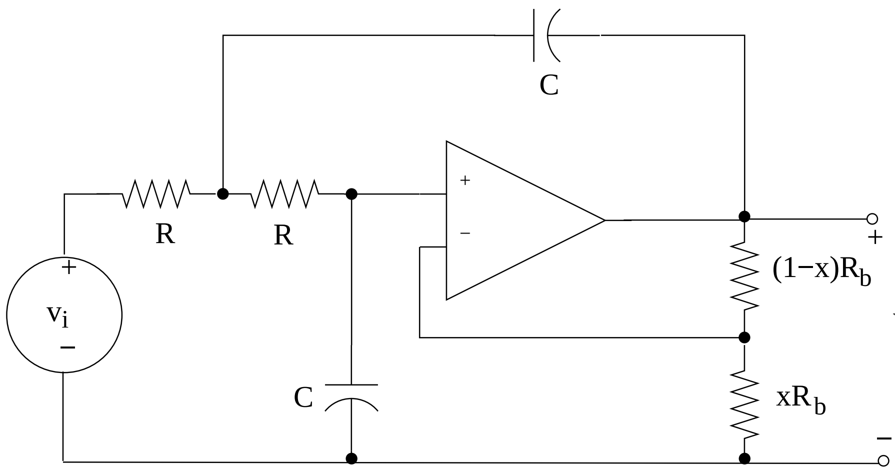

Figure 4: Sallen-Key circuit

PS.8.1.2: Sallen-Key circuit The circuit shown in figure 4 above is called a Sallen-Key circuit. The system function V_o/V_i has two poles. The Sallen-Key has the nice property that by choosing appropriate values for the capacitors and the resistors, you can place the two poles anywhere in the left half-plane. In this problem, we'll consider the more restricted case, shown in the figure, where the two capacitors have the same value C and the two resistors have the same value R. (The value of third resistor R_x doesn't affect the system function – only the ratio x matters.)

Show that the poles of the system function are given by the roots of the quadratic

$$s^2 + 2as + b^2$$

expressions for a and b in terms of R, C, and x. Check your result by answering the question posed by the on-line system.

Solution to Sallen-Key problem

We use impdedance methods, where we consider the capacitor to be a "resistor" with impedance $1/sC$.

First, note that the input voltage at the minus terminal of the op-amp is xv_o, and this is also the voltage at the plus terminal of the op-amp.

Now we can solve the circuit using the node method with two nodes: Node 1 is where the two resistors and the capactor meet. Let the voltage at node 1 be v_y. Node 2 is where the resistor and the other capacitor meet the plus terminal of the op-amp. The node voltage at node 2 is xv_o

At node 1 we have the KCL equation

$$\frac{v_i - v_y}{R} = (v_y - v_o)sC - \frac{v_y - xv_o}{R}$$

At node 2 we have the KCL equation

$$\frac{v_y - xv_o}{R} = xv_o sC$$

which we can solve for v_y to obtain

$$v_y = xv_o(1 + sRC)$$

Clearing fractions in the equation at node 1, substituting in for v_y, and simplifying gives

$$v_i = v_o \left[\frac{1}{xR^2C^2} \left[s^2 + \frac{1}{RC} \left(3 - \frac{1}{x} \right) + \frac{1}{R^2C^2} \right] \right]$$

Therefore, for the poles, we need to find the roots of

$$s^2 + 2as + b^2$$

where

$$2a = \frac{1}{RC} \left(3 - \frac{1}{x} \right)$$

and

$$b^2 = \frac{1}{R^2C^2}$$