MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Circuits and Electronics
Spring 2004

Problem Set 7
Readings and exercises for March 29 through April 4

Issued: 29 March 2004

Reading: This week, read

- From Agarwal and Lang, Chapter 13, through section 13.6

Quiz 2 will be on Friday, April 9, in recitation.

To do and turn in on line before lecture on Monday, April 5

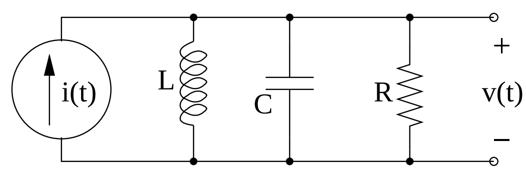

Figure 1: A parallel RLC circuit driven by a current source

1.1: Step responses for second-order systems This problem deals with two circuits. Each circuit is a parallel RLC circuit driven by a current source, as shown in figure figure 1. The circuits both have $L = 0.1$ henrys, but they different values of C and R, with the result that one of the circuits has higher Q than the other.

Assume the circuits start in an initial state where the capacitor voltages and the inductor currents are both 0, and suppose we drive both circuits with a unit step in current $i(t) = 0$ for $t < 0$, $i(t) = 1$ for $t \geq 0$. Consider the resulting response of the circuit quantities i_R, i_C, i_L, and v_C.

1. The on-line system will show you eight graphs (also shown on the following page) for these circuit quantities—-four for the lower Q circuit and four for the higher Q circuit. Your job is to say which graph corresponds to which quantity for each circuit.
2. By examining the graphs, you are to estimate the Q for each circuit, and also estimate R and C for each circuit.

Answers and explanation

Graph identification:

- The higher Q circuit is the one that ocillates longer, i.e., graphs 2, 5, 7, and 3.
- Graphs 2 and 8 do not start at zero: these must be the capacitor currents.
- Graphs 1 and 7 end up at 1 rather than 0. These must be the inductor currents.

This leaves graphs 3, 4, 5, 6 unidentified. You need to determine which two are the resistor currents and which are the capacitor voltages (which are the same as the resistor voltages). Note that the current and voltage graphs have to have the same shape since $v = iR$. In doing more analysis on the circuits to answer the questions below, you'll find that R is on the order of thousands for both circuits. Therefore, graphs 3 and 4, which have the larger values, are the voltages, and graphs 5 and 6 are the resistor currents.

Estimating Q: We can estimate Q from the graphs, using the rule of thumb that the circuit states decay to about 4% of their peak values in about Q cycles (so long as Q is not very small). This gives Q=5 for the higher Q circuit and Q=3 for the lower Q circuit.

Estimating C: By looking at the graphs, we see that both circuits have a frequency of about 1000 cycles/sec, or about 6300 radians/sec. Thus we have that 1 divided by the square root of LC is about 6300. We know that L=0.1, so solving for C gives C=0.25 microfarads (same for both circuits).

Estimating R: If we know Q, L, and C, we can use the fact that $Q = R\sqrt{C/L}$ (see the book, section 13.2.1) to solve for R.

R equals 3150 Ω for the higher Q circuit and 1890 Ω for the lower Q circuit.

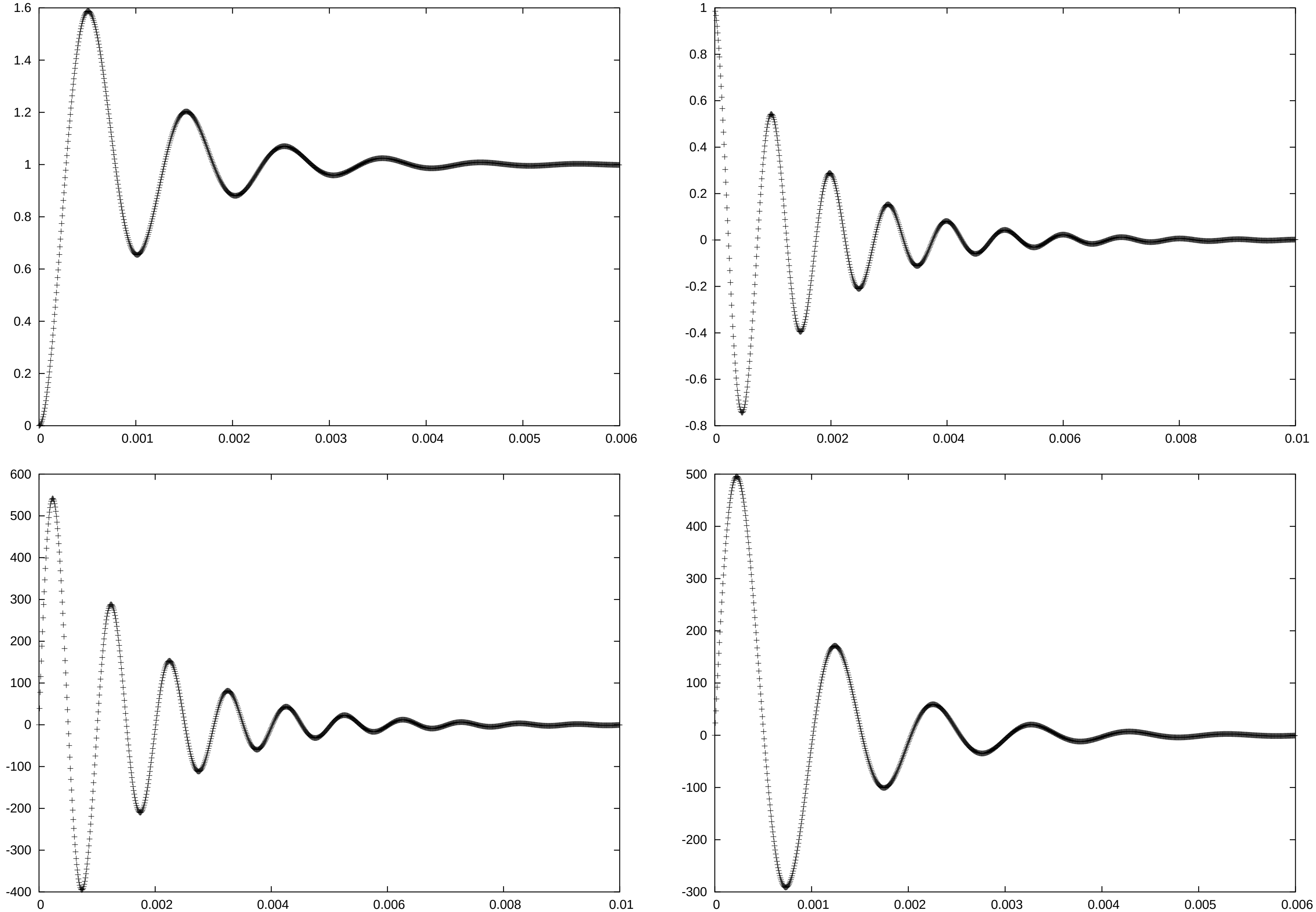

Figure 2: Graphs 1–4 for this problem

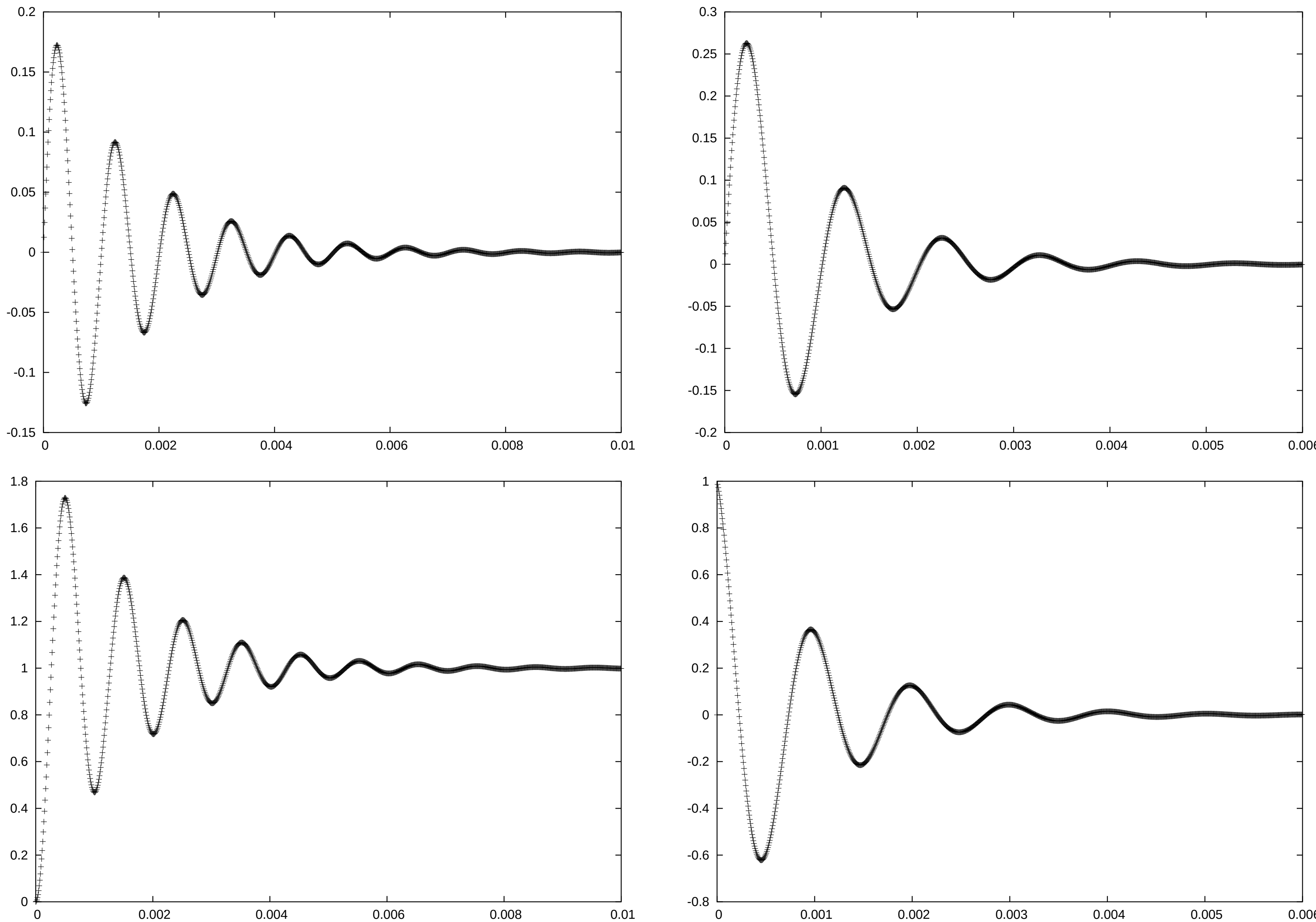

Figure 3: Graphs 5–8 for this problem