

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering and Computer Science

6.002x – Circuits and Electronics  
Spring 2004

Problem Set 4 — Answers

*This problem set was your first real taste of analysis with transistors and incremental models. This problem set was very challenging because it was hard to dig out the material you really need from all of the reading. We hope you found the “BJTs Without Tears” handout useful.*

*We’re providing a more detailed answer write-up for this problem set than we usually do. You should find this useful in reviewing the BJT material and in studying for the quiz on April 9. In particular, the answer to the final questions should be a helpful reference for incremental models.*

*We’ve reprinted the questions here, with the answers interpolated in a different font.*

*As always, get in touch with us for help if there are things here that aren’t clear.*

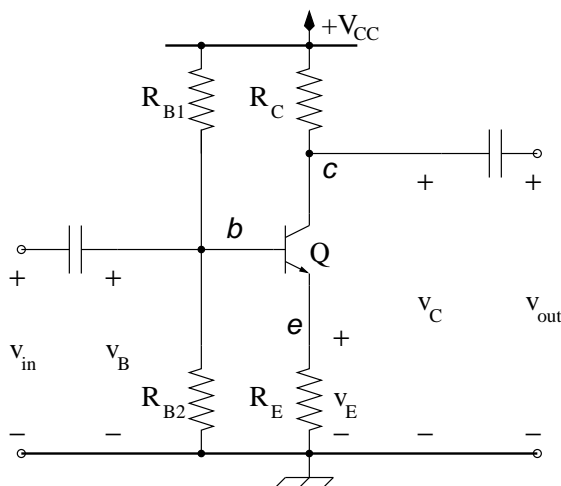


Figure 1: An essential transistor amplifier circuit.

The circuit you see in Figure 1 is one of the most common configurations of a transistor amplifier stage. We will walk through the analysis of this carefully, because it is one key to understanding how to deal with transistor amplifiers.

We will use the standard notations for bias and signal components of voltages and currents. The convention is to use upper-case letters variables and subscripts for bias values, lower-case variables and subscripts for incremental values, and lower-case letters with upper-case subscripts for total values.<sup>1</sup> For example,  $v_B$  is the total voltage on the base of the transistor, relative to the indicated ground.  $V_B$  is the bias voltage, which is present in the absence of a signal. And  $v_b$  is the incremental change to the voltage due to the imposition of a signal. So  $v_B = V_B + v_b$ .

---

<sup>1</sup>Please don't blame us for this convention.

## Part 1: To write up and hand in at lecture Monday, March 1.

Assume that  $R_C = 5\text{ k}\Omega$ , and  $R_E = 1\text{ k}\Omega$ . If the circuit is correctly biased, so that the transistor is operating in the amplification region, then the gain of this amplifier is approximately -5 (an inverting amplifier with a gain of 5). Write a paragraph explaining why this is true. Hint: Consider what happens if there is a small change in the base voltage  $v_B$  and show how this changes the collector voltage  $v_C$ .

*A small change  $v_b$  in the base voltage  $v_B$  causes a corresponding change  $v_e$  in the emitter voltage  $v_E$ . The change in emitter voltage produces a change  $i_e$  in emitter current  $i_E$ . Indeed,  $i_e = \frac{v_e}{R_E}$ . That change in the emitter current is approximately the same as the change in collector current  $i_c = i_e$ . But the collector voltage  $v_C = V_{CC} - i_C R_C$ , so the change in collector voltage  $v_c = -i_c R_C = -\frac{v_b}{R_E} R_C$ . Thus the gain is  $-\frac{R_C}{R_E} = -5$ .*

## Part 2: To do online before lecture on Monday, March 1.

In this sample analysis we assume the circuit of Figure 1 with the following parameters:  $V_{CC} = 10\text{ Volts}$ ,  $R_{B1} = 51\text{ k}\Omega$ ,  $R_{B2} = 10\text{ k}\Omega$ ,  $R_C = 5\text{ k}\Omega$ , and  $R_E = 1\text{ k}\Omega$ .

**Crude Bias Analysis** Consider the bias conditions. We will assume that there is no signal imposed at the base ( $v_b = 0$ ). We also assume that the voltages  $v_B$ ,  $v_E$ , and  $v_C$  indicated on the diagram are measurements—there is nothing connected to these terminals.

First, assume that  $\beta = \infty$  and that the base-emitter voltage is 0.7 volts. In the questions that follow, give expressions for the circuit variables in terms of the symbolic device parameters and the symbolic supply voltage, as well as the numerical values. You will find that it is easy to answer these questions in the order that they are given.

1. What is the bias voltage on the base  $V_B$ ?

*By assumption  $\beta = \infty$  so the base current is zero. Thus the base voltage is set by the voltage divider formed by  $R_{B1}$  and  $R_{B2}$ :*

$$V_B = V_{CC} \frac{R_{B2}}{R_{B1} + R_{B2}} \approx 1.64\text{ Volts}$$

2. What is the bias voltage on the emitter  $V_E$ ?

*By assumption, the emitter follows the base by 0.7 Volts so the bias voltage on the emitter is  $V_E \approx 1.64 - 0.7 = 0.939\text{ Volts}$ .*

3. What is the bias current leaving the emitter  $I_E$ ?

*The bias current leaving the emitter must go through the emitter resistor, producing the voltage drop  $V_E$ . Thus*

$$I_E = \frac{V_E}{R_E} \approx 0.939\text{ mA}$$

4. What is the bias current entering the collector  $I_C$ ?

Since we assume  $\beta = \infty$  then  $I_C = I_E \approx 0.939$  mA.

5. What is the bias voltage on the collector  $V_C$ ?

The bias voltage on the collector  $V_C = V_{CC} - I_C R_C \approx 5.30$  Volts.

6. What is the collector-emitter voltage  $V_{CE} = V_C - V_E$ ?

$V_{CE} = V_C - V_E \approx 4.36$  Volts.

### Checking Bias Assumptions

1. Is the transistor cutoff or switched on? *Your answer to this question should be “no.”* Explain how you verified this.

*The transistor is not cutoff, because there is a nonzero collector current. It is not switched on because there is more than 0.2 Volts from collector to emitter. Therefore the transistor is operating in the amplifying region.*

2. Assuming that  $\beta = 100$ , what is the bias current entering the base  $I_B$ ?

A quick estimate is  $I_B = \frac{I_C}{\beta} \approx 0.00939$  mA = 9.39  $\mu$ A.

However, this is not a self-consistent solution. Let's be a bit more careful. Let the bias voltage on the base be  $V_B$ . Then KCL at the base node is

$$\frac{V_B}{R_{B2}} + \frac{V_B - V_{CC}}{R_{B1}} + I_B = 0$$

and we know that

$$V_B = (\beta + 1)I_B R_E + 0.7$$

From these equations we conclude that  $V_B \approx 1.57$  Volts and  $I_B \approx 8.59 \times 10^{-6}$  Amperes = 8.59  $\mu$ A.

Note that this value is quite different (about 7%) from the original crude estimate, but we can accept both.

3. Using this new estimate for the base current, what is the new value of the base bias voltage  $V_B$ ?

We just got a self-consistent answer  $V_B \approx 1.57$  Volts, but let's see how the crude answer comes out.

With base current leaving the input node we can no longer use the voltage-divider formula to get the voltage. By superposition we can see that the new base bias voltage is

$$V_B = V_{CC} \frac{R_{B2}}{R_{B1} + R_{B2}} - I_B \frac{R_{B1} R_{B2}}{R_{B1} + R_{B2}}$$

This turns out to be  $V_B \approx 1.56$  Volts when using the crude  $I_B = 9.39$   $\mu$ A.  $V_B \approx 1.57$  Volts when using the better 8.59  $\mu$ A estimate, so we see that it does not make much difference.

4. Does this change the answer to question 1? *Your answer to this question should be “no.”* Explain how you verified this.

No, because lowering the base bias to 1.57 Volts decreases the emitter current to 0.939 mA, lowering the emitter voltage slightly (to 0.939 Volts) and raising the collector voltage to 5.31 Volts. This modification keeps the transistor well within the amplification region.

**Incremental Analysis** Now that we have the bias conditions, and we know that the transistor is neither switched on or cut off, we know that it is amplifying, and we can see how signals are handled. Again, assume, for bias purposes, that  $\beta = \infty$  and that the base-emitter voltage is 0.7 volts. Furthermore, assume that by some magic (not shown here) we are able to impose a signal and vary the voltage on the base by an amount  $v_b$ .

1. (a) What is the incremental voltage change on the emitter  $v_e$  caused by the change of base voltage? We say that “The emitter follows the base.”

“Following” means:  $v_e = v_b$ . If the two voltages differ by a constant value, then an incremental change in one results in the same incremental change in the other.

- (b) What is the incremental voltage change on the collector  $v_c$  caused by the change of base voltage?

An incremental change in the emitter voltage causes an incremental change in the emitter current:  $i_e = \frac{v_e}{R_E} = \frac{v_b}{R_E}$ . This increment in the emitter current is about the same as the increment of collector current:  $i_c = i_e = \frac{v_b}{R_E}$ . The increment in collector current causes a drop in collector voltage (through the collector resistor):

$$v_c = i_c R_C = -\frac{R_C}{R_E} v_b$$

- (c) What is the ratio  $v_c/v_b$ ? This is the incremental gain of the amplifier.

The incremental gain is just the ratio  $-R_C/R_E$

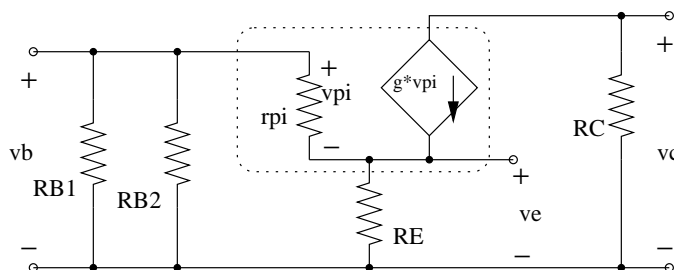


Figure 2: Incremental model of the amplifier. The transconductance  $g_m = \frac{q}{kT} I_C$ ;  $r_\pi = \frac{\beta}{g_m}$

Now, let's check this all out with an incremental model, as in Figure 2, to make sure that the crude approximations we have used are reasonable.

We see from KVL that  $v_b = v_\pi + v_e$ . But  $v_e = R_E(\frac{v_\pi}{r_\pi} + g_m v_\pi)$ , so  $i_c = g_m v_\pi = \frac{g_m v_b}{1 + R_E(g_m + \frac{1}{r_\pi})}$ .

Since  $v_c = -R_C i_c$  and  $g_m r_\pi = \beta$  we obtain

$$v_c = -\frac{R_C g_m}{1 + g_m R_E (1 + \frac{1}{\beta})} v_b$$

Now using  $g_m = 38 I_C$ ,  $I_C = 8.59 \times 10^{-4}$ , and  $\beta = 100$  we get the gain  $\frac{v_c}{v_b} \approx -4.80$ . This is not quite 5, but it shows that the crude approximation gets us within 5% of the correct gain.

2. What is the increment of base voltage from the bias point that makes the transistor switch on?

*As the base voltage rises, the emitter voltage rises (following the base) and the collector voltage falls. The transistor switches on when the base voltage increases enough so that the difference between the collector voltage and the emitter voltage falls below 0.2 Volts. This happens when*

$$i_c = \frac{V_{CC} - 0.2}{R_C + R_E} \approx 1.63 \text{ mA}$$

*At this point  $v_B = v_E + 0.7 = i_E R_E + 0.7 \approx 2.33$  Volts. This is an excess of about  $2.33 - 1.64 \approx 0.693$  Volts over the bias point.*

3. What is the increment of base voltage from the bias point that makes the transistor cut off?

*To cut the transistor off we need to lower the voltage to below 0.7 Volts. This requires an increment of about  $1.64 - 0.7 = 0.940$  Volt.*

4. What is the amplitude of the largest sinusoidal voltage signal that can be generated on the collector without distortion? We call this the “swing.”

*The largest sinusoidal signal that can be generated must not drive the transistor into either the cutoff region or the switched-on region. The closest one to the bias point is the switched-on region, so the biggest input amplitude is 0.693 Volts. The corresponding output signal has an amplitude of about  $5 \times 0.693 = 3.47$  Volts. The peak-to-peak swing is thus  $2 \times 3.47 = 6.93$  Volts.*

5. What is the input impedance of the circuit?

*If we inject an incremental current  $i_b$  into the base of the transistor it produces a voltage drop  $v_\pi = i_b r_\pi$  across the base resistor. This produces a current,  $g_m v_\pi = g_m r_\pi i_b = \beta i_b$  through the voltage-controlled current source. So the emitter resistor gets the sum of these currents and the emitter voltage is  $v_e = R_E(i_b + \beta i_b)$ . But  $v_b = v_e + v_\pi$ , so the resistance looking into the base of the transistor is  $\frac{v_b}{i_b} = r_\pi + (\beta + 1)R_E$ . The transistor is in parallel with the two base-bias resistors, so the input impedance (resistance) of the circuit is*

$$Z_{IN} = R_{B1} || R_{B2} || (r_\pi + (\beta + 1)R_E)$$

*Here  $g_m = 38I_C \approx .0326$ , with the self-consistent value  $I_C = 8.59 \times 10^{-4}$ , and  $r_\pi \approx 3060 \Omega$ , so  $\frac{v_b}{i_b} \approx 1.04 \times 10^5 \Omega$ . In this circuit  $Z_{IN} \approx 7740 \Omega$ .*

### Part 3: To write up and hand in at lecture on Monday, March 1.

#### Increasing the gain

1. Suppose we want 20% more incremental gain, which resistors should we change? How?

*To change the gain we must change the ratio of  $R_C$  to  $R_E$ . To get more gain we can make  $R_C$  larger or we can make  $R_E$  smaller.*

2. If we change the gain by changing these resistors how does the change affect the bias point? the swing?

*In either case we lower the bias voltage on the collector for a given bias voltage on the emitter, thus reducing the possible swing. We can compensate for this effect over some range by lowering the bias voltage on the emitter, by lowering the base bias.*

**Amplification** Assume again that  $\beta = \infty$  and that the base-emitter voltage is 0.7 volts, and that the values for the resistors are as in figure 1.

1. Draw the incremental model for the circuit.

*The incremental model is shown in figure 2. The part inside the dotted rectangle is the incremental model for the BJT, which was presented in lecture on February 25. This model consists of a resistor  $R_\pi$  together with a voltage-controlled current source controlled by the voltage  $v_\pi$  across  $R_\pi$ ,  $i = g_m v_\pi$ . The transconductance  $g_m$  of the source is  $g_m = \frac{q}{kT} I_C \approx 38 \text{ mA Volt}^{-1}$ . These parameters are related to the  $\beta$  of the transistor, by  $r_\pi = \frac{\beta}{g}$ . This incremental model of the BJT is called the hybrid- $\pi$  model.<sup>2</sup>*

*To obtain the incremental model for the entire amplifier, we simply redraw the amplifier circuit from figure 1, replacing each part by its corresponding incremental part. The BJT is replaced by the hybrid- $\pi$ , resistors are unchanged, and constant sources (e.g.,  $V_{DD}$ ) are replaced by zero sources (because they don't change incrementally). The one trick that people sometimes do is to redraw the circuit further so that connections to these zero sources (called "incremental grounds") actually all go visually to the same ground. This can sometimes help in doing circuit analysis on the incremental model, but it can obscure the fact that the incremental model has the same topology as the original circuit. For practice, you might want to do this process starting with the circuit in figure 1 and verifying that you really do get the incremental model shown in figure 2.*

*With the incremental model in hand, we can answer the rest of the questions.*

2. What is the incremental Thévenin resistance seen, looking into the base of the amplifier (i.e., from the pair of terminals identified in the diagram by potential difference  $v_b$ ).

*We have already computed this as the input impedance of the circuit, above.*

3. How much power is absorbed by the amplifier from an incremental voltage source  $v_b$  driving the base?

*The power is given, as usual, by  $v^2/R$ , which in this case is*

$$\frac{v_b^2}{Z_{IN}}$$

4. What is the incremental Thévenin resistance looking into the emitter?

*Here we need to do circuit analysis using the incremental model. Since we looking from the emitter, we put zero voltage sources across the input, setting  $v_b = 0$ , then we inject an*

---

<sup>2</sup>If you've been going through the course notes, you might want to note the incremental BJT model in figure 8.42c of Agarwal and Lang as a current-controlled current source. The difference between the models is the presence of the resistor  $R_\pi$ .

incremental current  $i$  at the emitter port  $v_e$  and compute the resulting incremental voltage  $v_e$ . We can do this computation by nodal analysis:

Consider the node at the bottom of the current source. Coming into this node, we have the current  $i$  (which we are injecting), the current  $g_m v_\pi$  (coming from the source), the current  $v_\pi/r_\pi$  coming from the input resistor, and the current  $v_e/R_E$  leaves the node through the emitter resistor. Therefore, we can write KCL:

$$i + \frac{v_\pi}{r_\pi} + g_m v_\pi = \frac{v_e}{R_E}$$

But now, since  $v_b = 0$  (we shorted the input), that means that  $v_\pi = -v_e$ , so our equation becomes

$$i = v_e \left( \frac{1}{r_\pi} + g_m + \frac{1}{R_E} \right)$$

and the incremental resistance is

$$\frac{v_e}{i} = \frac{1}{g_m + \frac{1}{R_E} + \frac{1}{r_\pi}}$$

5. What is the incremental Thévenin resistance looking into the collector?

We proceed similarly to the above: Short the  $v_b$  input, inject a current at collector, and compute  $v_c$ . Here, since  $v_b = 0$  we have  $v_\pi = 0$  so there is no current through the dependent source. Consequently, all the injected current goes through the resistor  $R_C$  and therefore

$$\frac{v_e}{i} = R_C$$