MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002x – Circuits and Electronics
Spring 2004

Problem Set 2
Readings and exercises for the week of Feb. 9–16

Issued: 9 February 2004

Textbook: Be sure to do the required readings *before class* as indicated.

- From Agarwal and Lang, read sections 3.5 and 3.6 before lecture on on Tuesday, Feb. 17 (This is "virtual Monday"; There is no lecture on Monday, Feb. 16.)

Part 1: To do on line before recitation on Friday, Feb. 13

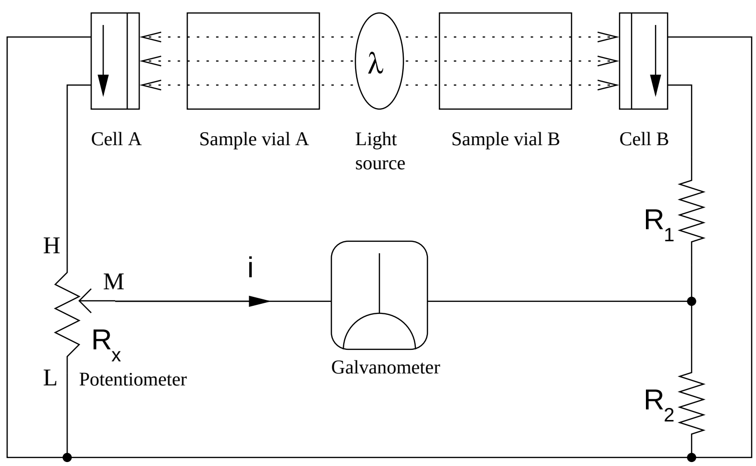


Figure 1: Tom Knight's differential photo densitometer

1.1: A broken densitometer

The story in this problem is substantially true, including the description of the flaw in the device and the attempt to isolate the flaw.

One of the instruments in Tom Knight's biology lab at the Artificial Intelligence Laboratory is a *differential photo-densitometer*, a device used to compare two vials of fluid to determine if they have the same optical density (the propensity to stop light).

In this instrument (see figure 1) the light source is passed through each of the two vials and then focused on two selenium photocells. This kind of photocell may be modeled as a current source, with a high shunt resistance, that puts out a current proportional to the intensity of the light falling on it (see figure 2). The currents are then passed into a simple bridge circuit with one variable arm and one fixed arm. A galvanometer (a very sensitive current indicator) is then used to indicate a balanced condition, when the setting of the potentiometer compensates for any difference of densities. The difference is then read off of the scale of the potentiometer.

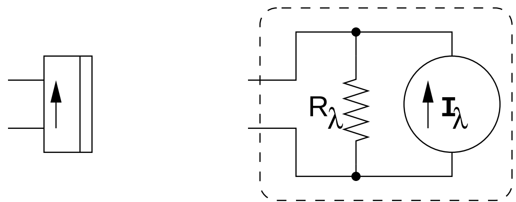


Figure 2: Model of the photocell

The potentiometer (see figure 3) is a resistor with a wiper that taps off a part of the resistive material as you turn a shaft. Our densitometer has a 10-turn potentiometer, which means that in 10 complete rotations of the shaft the wiper traverses the entire resistor. Let $x = \frac{n}{10}$ be the proportion traversed if the shaft is rotated by n turns. So, if the resistance between terminals L (low) and H (high) is R_{HL} then the resistances involving terminal M (middle) are $R_{ML} = xR_{HL}$ and $R_{HM} = (1 - x)R_{HL}$.

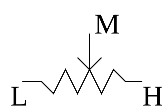


Figure 3: The potentiometer

Suppose that the fixed resistors labeled R_1 and R_2 each have a resistance of 1000 Ω and the potentiometer has a full-scale resistance $R_{HL} = 5000\ \Omega$.

1. Assume that the densitometer is working and correctly adjusted.

   (a) If the sample vials are the same, for what setting of the potentiometer (in number of turns) will the galvanometer indicate balance (no current)?

   (b) If the potentiometer balances at n, i.e., what's the ratio of the light passing through sample A to the light passing through sample B?

2. When Knight first showed his densitometer to Gerry Sussman, it was not in working order. The presenting symptom was that the galvanometer could not be balanced for any setting of the potentiometer, even when the sample vials were both filled with distilled water. Indeed, the galvanometer showed that current always flowed from right to left (i was negative). The current that flowed did in fact vary with the setting of the potentiometer, i most strongly negative when the potentiometer was at zero turns.

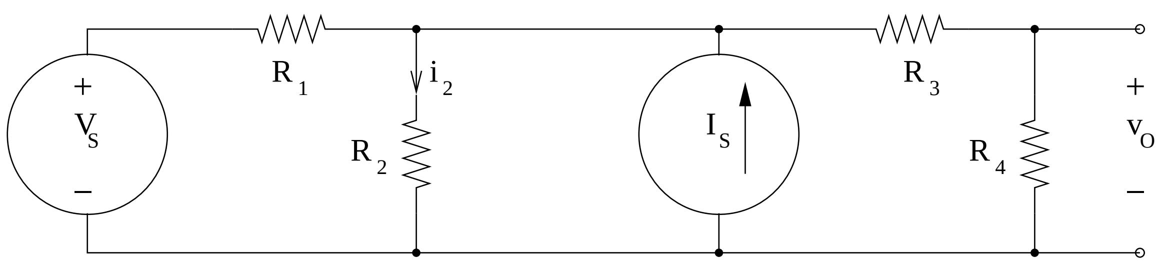
One guess might be that Cell A was broken (open circuit), but the voltage across Cell A (without taking the cell out of the circuit) varied with changes in the light. Indeed, it seemed to put out more volts than did Cell B for the same amount of light. What is more, the voltage across Cell B varied with the setting of the potentiometer, while the voltage across Cell A did not.

Can you deduce that was wrong with the densitometer? The kinds of failures you might consider are broken wires, bad connections, short circuits, fried resistors, etc.

Part 2: To do on line before lecture on Tuesday, February 17

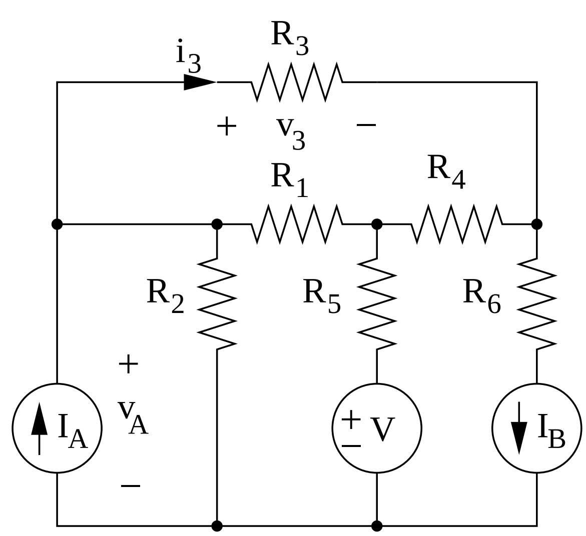
2.1: Node equations

Use the node method to determine the values of the voltage labeled v_O and the current labeled i_2, for the circuit shown below. The online system will generate specific values of V_s, I_s, R_1, R_2, R_3, and R_4 to use in your analysis.



2.2: More node equations

Determine the values of the voltages labeled v_3 and v_A, and the current labeled i_3 for the circuit shown below. The online system will generate specific values of I_A, I_B, V, R_1, R_2, R_3, R_4, R_5, and R_6 to use in your analysis.



Part 3: To write up and hand in at lecture on Tuesday, February 17

3.1: Shocking Joe

Joe is about to use an electric power saw in his back yard. Joe is going to plug the saw into an outdoor electrical receptacle with a 3-prong plug (correctly grounded), using a 100-foot power cord. Remember that neutral and ground are tied together in the house at the service entrance, and that the power cord's ground wire is connected to the saw's metal case. Joe is grounded through his wet shoes.

Unbeknownst to Joe, there been some water damage to the saw, which has connected the hot wire entering the saw to the metal case, with an effective resistance of 100 Ohms. We'd like to know whether this situation could be dangerous to Joe.

The answer depends on the amount of current that flows through Joe to the earth when he plugs in the saw. A current of 10^2 mA would likely kill him; a current of 1 mA would make him jump; a current of 0.1 mA would be noticeable but unlikely to be dangerous.

1. Draw a circuit diagram to model this situation. Make sure to include the resistance of Joe, the resistance of the saw motor (between hot and neutral), the damage resistance between the hot line and the saw case, and the resistances of the three wires in the power cord connecting the saw to the electrical receptacle.
2. Suppose first for simplicity that the power cord from the saw to the receptacle has zero resistance. Argue that Joe is safe—no current flows through him.
3. Now make the more realistic assumption that the power cord has a resistance of about 2.7 Ohms per 1000 feet (i.e., each of the three wires in the power cord has about 2.7 Ohms per 1000 feet). Starting with your circuit model from part 1, draw the Thévenin equivalent system as seen by Joe. (Assume that the saw is turned off when Joe plugs it in, so that no current passes through the motor.) Show your work.
4. Assume that Joe, as a conductor of electricity, has a resistance of about 10^4 Ohms to ground. Using your Thévenin model from part 3, about how much current will flow through Joe when he plugs in the saw? Is he safe?
5. Suppose instead that Joe has been working hard and is all sweaty, so that his resistance as a conductor of electricity drops to 100 Ohms. About how much current will flow through him now? Is this safe?
6. Suppose that the receptacle is not correctly grounded, leaving the power cord's ground wire as an open circuit. How does that change your answer to parts 4 and 5?