

# Encoding formulas as deep networks: Reinforcement learning for zero-shot execution of LTL formulas

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**Abstract**—We demonstrate a reinforcement learning agent which uses a compositional recurrent neural network that takes as input an LTL formula and determines satisfying actions. The input LTL formulas have never been seen before, yet the network performs zero-shot generalization to satisfy them. This is a novel form of multi-task learning for RL agents where agents learn from one diverse set of tasks and generalize to a new set of diverse tasks. The formulation of the network enables this capacity to generalize. We demonstrate this ability in two domains. In a symbolic domain, the agent finds a sequence of letters that is accepted. In a Minecraft-like environment, the agent finds a sequence of actions that conform to the formula. While prior work could learn to execute one formula reliably given examples of that formula, we demonstrate how to encode all formulas reliably. This could form the basis of new multi-task agents that discover sub-tasks and execute them without any additional training, as well as the agents which follow more complex linguistic commands. The structures required for this generalization are specific to LTL formulas, which opens up an interesting theoretical question: what structures are required in neural networks for zero-shot generalization to different logics?

## I. INTRODUCTION

Robots must execute commands that are extended in time while being responsive to changes in their environments. A popular representation for such commands is linear temporal logic, LTL [1]. Commands expressed in LTL encode both spatial and temporal constraints that should be true while executing the command. Executing such commands is particularly difficult in robotics because integration is required between the complex symbolic reasoning that finds satisfying sequences of moves for an LTL command and data-driven perceptual capabilities required to sense the environment. While individual formulas can be learned by deep networks with extensive experience, we demonstrate how to compose together tasks and skills to learn a general principle of how to encode all LTL formulas and follow them without per-formula experience. We demonstrate how to integrate the learning abilities of neural networks with the symbolic structure of LTL commands to achieve a new capability: learning to perform end-to-end zero-shot execution of LTL commands.

Given a command represented as an LTL formula, our approach turns that formula into a specific recurrent deep network which encodes the meaning of that command; see fig. 1. The resulting network takes as input the current map state, extracts the environment around the robot, processes it with

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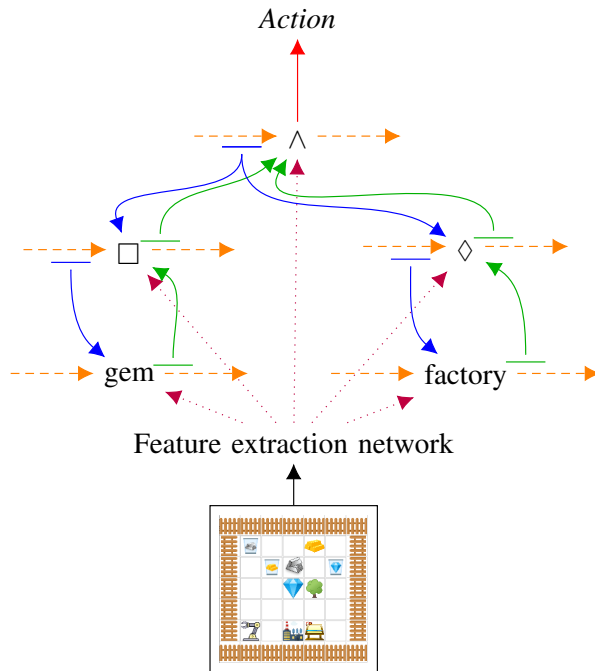


Fig. 1. A structured compositional deep network encoding one formula,  $\square \text{GEM} \wedge \diamond \text{FACTORY}$ . This formula corresponds to a command such as “Always hold the gem and get to the factory”. Each operator and predicate in the formula is represented by a network, shown in black, selected from a trained collection of sub-networks. Sub-networks are RNNs which maintain state over time, shown in orange. Each sub-network takes as input features extracted from the surroundings of the robot using a co-trained network, shown in dotted purple. The next state of each sub-network is decoded by a linear layer and passed to its parents, shown in green. The previous state of each sub-network is decoded using a linear layer and passed to its children, shown in blue. Finally, the state of the root node is decoded into a distribution over the value of the actions the robot can take at the current time step. Crucially, due to the compositional representation employed, novel formulas that were never seen at training time can be encoded and followed on novel never-before-seen maps. This couples the power of deep networks to learn to extract features and engage in complex behaviors with knowledge about the structure of formulas, to perform zero-shot execution. See fig. 2 for an example of the model in action.

a co-trained feature extraction network, and predicts which actions will satisfy the formula. This compositional approach ties together neural networks and symbolic reasoning allowing any LTL formula to be encoded and followed, even if it has never been seen before at training time.

In our experiments, we generate random LTL formulas and train an RL agent to follow those formulas. We develop a mechanism for generating hard and diverse LTL formulas, as random instances tend to be homogeneous and trivially solved.

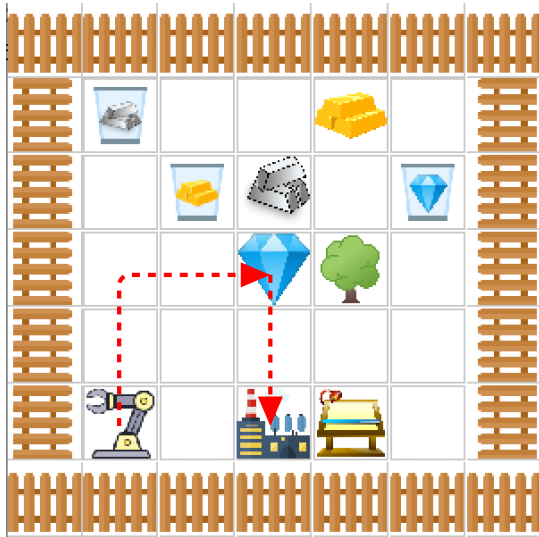
$\square \text{GEM} \wedge \diamond \text{FACTORY}$ 

Fig. 2. The model from fig. 1 executing a novel command that was not shown at training time on a novel map. Maps and formulas are randomly generated. In this case the robot executes  $\square \text{GEM} \wedge \diamond \text{FACTORY}$  by going to the gem, picking it up, and, while holding on to the gem, going to the factory.

This is generally useful for other large-scale experiments on following commands that can be encoded as LTL formulas. In two different domains, *Symbol* and *Craft*, we show that this approach can learn to execute never-before-seen formulas. The *Symbol* domain is more akin to boolean satisfiability, where an accepting string must be generated for an LTL formula. The *Craft* domain is a simplified Minecraft introduced by Andreas *et al.* 2017 [2] to test the integration with robotics; see fig. 2 for an example of the network in fig. 1 executing a command in the *Craft* domain. In all cases, we compare against baselines to demonstrate that each part of our model plays a key role in encoding temporal structures. All components of our networks are learned end-to-end, in a process that automatically isolates the meaning of each sub-network allowing us to compose sub-networks together in novel ways.

This work makes four contributions:

1. a trained deep network for following LTL commands,
2. end-to-end zero-shot execution of LTL commands,
3. co-training the feature extraction with the LTL controller,
4. an investigation of the properties of random LTL formulas.

This work can be seen as a novel approach to composing the policies of multi-task reinforcement learning agents in a principled manner according to a particular logic. While we only discuss LTL here, this approach suggests how other logics might similarly be encoded to create new powerful zero-shot deep approaches to reinforcement learning. The best aspects of symbolic reasoning in robotics are compatible with deep networks when both are correctly formulated. Perhaps in the future, such approaches could be used for model checking with LTL formulas.

## II. RELATED WORK

The most related work to what we present here is in the area of multi-task learning and task composition. Previous work has learned finite state machines in conjunction with robot controllers [3]. The focus in that prior work is on training extensively with one LTL formula and then following that formula; here we show how to zero-shot follow novel LTL formulas on new maps.

A related line of research shows how to accelerate learning of new LTL formulas by shaping rewards according to the structure of the formulas [4]. Here we provide no example of novel formulas whereas this prior work requires tens of thousands to millions of training steps, in the *Craft* domain, for each new formula.

Compositions of LTL sub-formulas have been investigated before [5]. Sahni *et al.* (2017) show how to compose together controllers for LTL formulas. The formulas considered are small: the largest is well below the mean size of our formulas. Only 4 formulas are tested thoroughly. Some zero-shot generalization is achieved to 4 formulas which have the same structure as those in the training set or which are carefully stitched together by hand given knowledge of their semantics. No general algorithm is given for how to encode an arbitrary LTL formula, nor is it possible to automatically train on a set of generated formulas.

LTL has been recently used to decompose tasks into sub-tasks, learn policies for each of the subtasks, and to improve the reliability and generalization capabilities of robots [6]. Their approach must re-learn each subtask; in essence, the structure of the LTL formulas themselves plays no role. Here we show a more extreme approach where tasks encoded in LTL and composed together and directly followed without any task-specific training. Moreover, only 10 tasks which are paired with training data are considered, whereas here we execute thousands of new tasks. We adopt a variant of the *Craft* domain in Andreas *et al.* [2]. The *Craft* domain was originally developed for multi-task RL.

Safety-critical systems benefit from following constraints expressed as LTL formulas [7]. Shielded reinforcement learning prevents agents from entering states which violate constraints that can be expressed in LTL. This is critical for many real-world applications such as autonomous cars that can adapt to new environments. The approach presented here is complementary; it could provide an effective compositional shield that encodes new constraints. In the future, one might even be able to learn what constraints are required to shield a system. Several publications investigate combining Markov decision processes and Q-learning with constraints specified in LTL [8, 9, 10].

Prior work has attempted to augment learning to satisfy constraints with policies derived from those constraints [11, 12]. This is a finer-grained analysis of the formulas than we perform here. While this speeds up learning, each formula requires significant training data.

### III. MODEL

The model we introduce is presented in fig. 1. It is inherently compositional, i.e., the structure of the model reflects the parsing structure of the input LTL formula. Each LTL formula is parsed into a tree where the nodes are the operators or predicates. Operator and predicates are replaced by recurrent networks and are connected with one another according to the parse tree of the LTL formula. Every LTL formula generates a unique network that encodes the meaning of that specific formula.

We employ an Advantage Actor-Critic, A2C [13, 14], agent. Actor-critic models learn a value function, the critic that determines the score of a state, and a policy, the actor which determines what to do at a given state. These could in principle be two separate networks. The critic network evaluates state,  $s$  at time  $t$ ,  $V_v(s_t)$  with learned parameters  $v$ . The actor network evaluates the effect of action,  $a$ , at time  $t$ , given a state  $s_t$ ,  $A_\theta(s_t, a_t)$ , with learned parameters  $\theta$ . Advantage Actor-Critic models use the fact that the actor is computing an advantage, a change in value between two states, to estimate the actor using the value function of the critic. Practically, this means that a single network is required from which both the actor and the critic can be computed. The compositional model shown in fig. 1 is this shared network between the actor and the critic. The parameter updates of this network follow the standard methodology for training recurrent networks with A2C presented in Minh *et al.* (2016) [14].

In prior work, when learning two different policies, the networks for those two policies would be unrelated to one another. The model presented here can be seen as a principled way to share weights between these networks informed by the structure of LTL formulas. All operators and symbols share weights both within a formula and between updates, i.e., there is only a single model for  $\circ$  or *gem*.

At training time, we supervise the agent with random LTL formulas. Each formula is converted to a Buchi automaton using Spot [15]. Note that we only consider and evaluate LTL with finite traces [16] here, meaning that these automata are interpreted as having the semantics of finite automata. Each training episode proceeds with the agent taking a sequence of actions. Each action is evaluated against the automaton. The predicates in the possible transitions of the automaton are evaluated. If the predicates hold, the agent is given a small, 0.1, reward. Staying in the same non-accepting state lowers the reward at a reward decay rate 0.8 for both *Symbol* and *Craft* domains. If the predicates do not hold, the agent has violated the semantics of the LTL formula and it receives a large negative, -1, reward. If the predicates hold and the agent is in an accepting state, it receives a large positive, 1, reward. This reward structure encodes the notion that agents should follow an automaton which encodes a particular LTL formula, although agents do not have access to the automaton directly. We employ curriculum learning to sort generated formulas and provide shorter formulas first. Short formulas are more likely to accept more strings and their shallower corresponding models make error assignment more reliable.

The model is structured in such a way that knowledge flows between operators and predicates; see fig. 1. The current state of all operators is fed to children after being decoded by a linear layer. This allows models to communicate information about the current sequence of steps to their children. The next state of each child is passed to its parents after being decoded by a linear layer. This is the only form of communication between models. Parents let their children know about the current state sequence and children let the parent update their representations based on observations. The next state of the root of the tree is decoded by the actor and the critic and used to predict the next action.

We use the intuition developed in Kuo *et al.* (2020) [17] that models which are composed of sub-networks can disentangle the meaning of words in a sentence without direct supervision. In other words, the agent is never told what  $\circ$  is supposed to mean as opposed to *gem*. Yet this can be done automatically, since from the definition of LTL formulas we understand that the computation required for every single operator or predicate should be the same. A learning method that attempts to find the most parsimonious explanation of the meaning of these shared sub-networks should then hone in on their meaning as components of LTL formulas by virtue of being forced to share computation when the definition of LTL formulas demands it. This intuition makes clear why the model presented here can recombine the sub-networks into new formulas: it has the capacity to execute recurrent computations, it shares computations in a way that the definition of LTL requires, and it is rewarded when it replicates the computations that are required to satisfy LTL formulas.

### IV. EXPERIMENTS

We demonstrate the model in two domains.<sup>1</sup> The first, *Symbol*, is designed to stress the symbolic reasoning abilities of RL agents. An LTL formula is provided to an agent which must immediately produce a satisfying assignment to that formula as a series of symbols. The second, *Craft*, is designed to stress the multi-task execution capabilities of RL agents in a robotic environment. An LTL formula and a map containing a robot are provided to an agent which must immediately find a sequence of moves that result in behavior of the robot that satisfies the LTL command.

Four datasets are generated for each domain and each is tested against the model and three ablations of the model. A training dataset is first generated. Note that even in the case of the training set, our method has a significant advantage over prior work: our training set contains thousands to tens of thousands of formulas which are all learned, as opposed to learning one or a small handful of formulas. Then three test sets are generated of increasing difficulty. A test set which has roughly the same statistics as the training set in terms of formula length, i.e., 1 to 10 predicates with an average of 8, one which has 10-15 predicates, with an average of 13, and one which has 15-20 predicates with an average of 18.

<sup>1</sup>Source code is available at <https://github.com/ykkuo/tl-zero-shot>

These test sets stress the generalization capabilities of the model and demonstrate that even when faced with formulas that are well beyond any that have been seen before in terms of complexity, the compositional nature of the model often leads to correct executions.

### A. Symbol domain

The `Symbol` domain is introduced here as a new challenge for multi-task LTL-capable agents. It removes the map and focuses on generating accepting strings for an LTL formula. The map can be a crutch for agents, e.g., crowded maps can have few paths making even random exploration efficient. The absence of particular resources on the map can also significantly simplify the problem, e.g., if there is no gem on the map, the agent can't mistakenly pick up a gem. In the `Symbol` domain, given a fixed inventory of symbols, the agent predicts the assignment of symbols at each time step, a sequence of fixed length that will be accepted by the LTL formula.

In the experiments reported here, we use an environment that has 5 symbols and requires satisfying sequences of length 15. Changing the number of symbols does make the problem more difficult but not substantially so. This is because the approach presented here separately learns networks for each symbol making it robust to increasing the number of symbols; adding a symbol corresponds to adding a single sub-network.

Data generation is complicated by the fact that generating random LTL formulas produces uninteresting and easily satisfied instances. In a related domain, this is a well-known property of random instances of boolean satisfiability, SAT, problems [18]. We adapt solutions from this community to generating interesting collections of LTL formulas that are diverse and difficult to satisfy.

The generation process begins by sampling a formula from a uniform distribution over trees with one parameter: the prior distribution over the number of elements (predicates and symbols). This formula is converted to a Buchi automata using Spot [15]. For a fixed  $n$ , the number of strings up to length  $n$  that the formula accepts is computed using a dynamic programming algorithm. This recursively computes the number of accepted strings for each time step,  $N$ . All formulas which have an acceptance ratio of more than 0.0001% are rejected as being too easy. Most randomly generated formulas tend to accept almost all strings. This provides hardness but does not guarantee variety.

In a second step, a second formula is generated as a function of the first formula. This uses a random process that picks a random subtree of the formula and replaces it with a new, also random, subtree. If the second formula is rejected, the process restarted based on the acceptance criteria above. If accepted, random accepted strings are sampled from the first formula and verified against the second. If more than 10% of strings that satisfy the first formula also satisfy the second, it is rejected as not diverse enough.

This may appear to be a rather laborious process, but without ensuring both hardness and diversity, we found that the generated formulas are uninteresting and fairly



Fig. 3. A  $7 \times 7$  `Craft` domain example showing all the various elements available in the domain. The robot is in the top right corner and is controlled by the RL agent presented here. Four trash cans contain different resources: gold, grass, silver, and gems. A work bench, a tree, a tool shed, and a factory provide additional destinations. The general intent is to model a Minecraft-like environment in which resources are processed through several stages.

homogeneous in what strings or robot moves they accept. This is evidenced by the fact that without this process, there often existed a single string or sequence of robot moves which was accepted by the majority of generated formulas. After this more complex generation process, the resulting dataset is far more challenging, as is reflected in the experiments section by the low performance of baseline methods. The statistics of the generated datasets are shown in fig. 4. The generation process is heavily biased against short formulas as they tend to be overly permissive and overlap with one another in meaning.

### B. Craft domain

We use the Minecraft-like world already used by several prior multi-task learning reinforcement learning publications [2, 6]. See 3 for an example map that contains all of the elements of the `Craft` domain. It mixes together unmovable objects (trees, tool sheds, workbenches, and factories) with resources that can be manipulated (silver, grass, gold, and gems). We generate random `Craft` maps with random initial positions for the robot arm. The environment is 4-connected with an additional “use” action for the arm which picks up or drops the resource. The overall intent is that resources can be manipulated through multiple processing stages resulting in a range of interesting tasks.

We extend this environment with a recycling bin so that agents can discard items. This allows the agents to satisfy formulas that require owning an item for only part of the action.

Unlike in the `Symbol` domain, where most random formulas were trivially satisfied without our procedure to find hard and diverse LTL instances, in the `Craft` domain,

most formulas are trivially unsatisfiable. Constraints on where the robot is at any one time and the fact that it must traverse the map step by step, make even basic formulas such as  $\square \text{GEM}$ , the gem must always be held, unsatisfiable in all but the most fortunate situations. If the robot happens to be next to the gem, this formula can be satisfied by picking the gem up. If the robot is not next to the gem, no one action the robot can take will satisfy this formula.

We could employ rejection sampling and resample formulas until they can be satisfied. This would be wasteful and produce the same results as the following transformation. Since formulas are unsatisfiable in *Craft* largely because the robot does not have time to satisfy them, we introduce a transformation that gives the robot time. Whenever the robot needs to satisfy a predicate, we transform that predicate into “ $\text{closer}(\text{predicate}) \cup \text{predicate}$ ”, which states that the robot must get closer to its goal until it is reached. To make following this command feasible, the robot is given an additional feature which is the Manhattan distance to the predicate. This does not otherwise change the semantics of any of the formulas, nor does it make zero-shot generalization easier as the robot must still understand the command as a whole. The transformation also does not count toward the formula lengths we produce; it merely gives the robot some time. Indeed, this transformation actually makes learning harder because the agent does not receive any meaningful feedback if it runs out of time. It has no way of knowing if an episode failed because it could not reach a goal in time or because some constraint was violated. In the future, we intend to investigate how to make the causes of failure more transparent to the agents, thereby hopefully speeding up learning.

### C. Generated formulas

Four sets of formulas are generated for the experiments in the *Symbol* and the *Craft* domains; see fig. 4. The training set for each has between 1 and 10 elements, i.e., operators and symbols. An out of domain test set has the same number of elements but does not overlap; this tests zero-shot execution. To further demonstrate that this approach generalizes, we produce two additional test sets of even longer formulas than those seen in the training set. Note that these latter test sets are both longer and more difficult than the training sets.

### D. Hyper-parameters

We train the Advantage Actor Critic, A2C, model with RMSprop [19] optimizer for both domains. Agents must explore the space of symbols and moves thoroughly since our data generation process ensures that the LTL formulas are strict and few operations resulting in accepting states. To encourage this, we set a higher entropy weight of 0.1 and set the reward decay,  $\gamma$ , to 0.9 during training. While training, the next move is an action from a distribution over possible actions. While testing, a deterministic policy chooses the optimal move. We observed a significant performance drop if the agents chose deterministically at training time or stochastically at test time.

Each sub-network, which represents an operator or predicate, is represented using a gated recurrent unit, GRU [20]. The hidden sizes are 64 for both domains. The *Symbol* domain has only one hidden layer and the *Craft* domain has two hidden layers since it is significantly more complex. A single linear layer per sub-network is always used as the decoder that transmits information from children to parents and a single linear layer transmits information from parents to children in the model structure.

We consider only LTL formulas with finite traces [16] here, although nothing in our method prevents executing non-finite LTL. The interpretation of the results for LTL without a finite horizon is considerably more complex and requires new metrics because it must consider when a model fails, not just if it fails.

We perform 15 rollouts at every iteration. If at the end of 15 steps the model is not in an accepting state, it is considered to have failed and given zero reward as if it had taken an action that violated the semantics of the LTL formula.

### E. Baselines

In addition to our model, we test three baselines. The “no structure, no language” baseline takes the current state of the world and attempts to predict what action the agent should take next. It is a standard recurrent A2C agent that does not observe the LTL formula at all. This is the performance an off-the-shelf agent would have knowing only the environment. The “no structure” baseline takes as input the LTL formula, learns an embedding of that formula into a single 32-dimensional vector, and then attempts to follow it. To create the embeddings, each operator and predicate in the formula is encoded as a one-hot vector. Sequences of these one-hot vectors are passed to an RNN which is co-trained to produce the embedding of the formula, much like the CNN produces an embedding of the environment. The formula is provided to every sub-network just like the observations of the environment. This is the performance an off-the-shelf agent would have that does not understand compositionality. The “no time” baseline is an ablation of our model; it is structured but is missing any recurrent connections, i.e., all symbols are feed-forward networks instead of GRUs. This is the performance an off-the-shelf non-recurrent A2C agent would have because it cannot keep track of its progress through the formula.

### F. Results

The performance of the *Symbol* domain is reported on formula sets of 10,000 formulas each, while the *Craft* domain, which is considerably slower, is reported on 4,000 formulas each; see fig. 5. In domain, a difficult task where one of 10,000 LTL formulas that have been seen before is provided and must be executed correctly, has 97% accuracy for *Symbol* and 73% accuracy for *Craft*. Ablations of our method show that without the particular compositional structure imposed the performance is two-third for “no structure” and even lower for the other ablations. Every part of our model is critical to performance as ablating any part

Formula set	symbols	tree nodes	tree depth	automata states
Symbol (train)	3.28 ± 0.49	9.14 ± 1.16	4.49 ± 0.96	4.21 ± 0.52
Symbol (test, 1-10)	3.36 ± 0.50	9.18 ± 1.13	4.70 ± 0.94	4.20 ± 0.45
Symbol (test, 10-15)	4.64 ± 0.77	13.05 ± 1.49	6.04 ± 1.14	5.32 ± 1.75
Symbol (test, 15-20)	6.11 ± 0.73	17.99 ± 1.42	8.05 ± 1.56	6.69 ± 3.44
Craft (train)	2.76 ± 0.63	7.94 ± 1.49	4.35 ± 1.04	3.40 ± 0.77
Craft (test, 1-10)	2.78 ± 0.64	7.83 ± 1.46	4.22 ± 0.98	3.37 ± 0.74
Craft (test, 10-15)	4.33 ± 0.82	12.94 ± 1.54	6.62 ± 1.05	3.73 ± 0.96
Craft (test, 15-20)	6.06 ± 0.75	18.14 ± 1.31	7.94 ± 1.37	4.44 ± 1.96

Fig. 4. Statistics of the generated formulas used in each of the two kinds of experiments, on the `Symbol` domain and the `Craft` domain. The training set is generated with the same mechanism as (test, 1-10), i.e., containing formulas that have been 1 and 10 elements. Training and test sets are randomly generated but verified to be disjoint. Two additional more difficult test sets are created, 10-15 and 15-20 which have longer formulas with, 10 to 15 elements and 15 to 20 elements, respectively. Each test set is summarized using number and standard deviation of the number of symbols in the formula, total tree nodes in the parse of the formula, depth of the parse of the formula, and the states in the Buchi automata for those formulas. Note that while formulas are generated randomly, very short formulas are unlikely to appear; see section IV-A and section IV-B for a discussion on how to generate difficult random collections of LTL formulas in each of the two domains.

	In domain	Out of domain (1-10)	Out of domain (10-15)	Out of domain (15-20)
<b>Symbol domain</b>				
no structure, no language	0.12	0.15	0.10	0.12
no structure	0.65	0.74	0.39	0.23
no time	0.01	0.01	0.02	0.04
ours	0.97	0.90	0.60	0.39
<b>Craft domain</b>				
no structure, no language	0.43	0.48	0.38	0.31
no structure	0.54	0.59	0.36	0.31
no time	0.43	0.31	0.19	0.19
ours	0.73	0.67	0.52	0.57

Fig. 5. The performance of our model finding satisfying actions for LTL formulas. Formulas which are perfectly executed are reported as successes, any errors are considered a failure. Each formula set contains 10,000 formulas for the `Symbol` domain, and 4,000 formulas for the `Craft` domain. “In domain” refers to training and testing on the same formulas. Note that this task is already far more complex than what existing methods can do as the model must learn to execute thousands of LTL formulas given the formula as input. We then test out of domain, to demonstrate that our model can zero-shot execute new formulas. Finally, two increasingly complex out of domain scenarios are tested. One in which has formulas that are 1.5 times as long, and one with formulas that are twice as long. Note the poor performance of the baseline methods and ablations; our method performs far better.

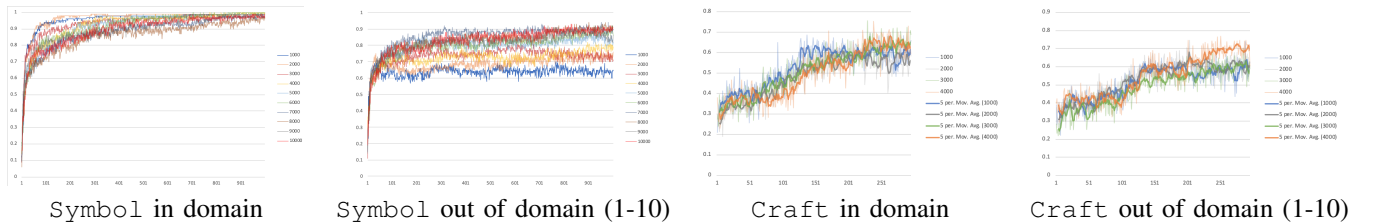


Fig. 6. Performance of the model as the number of training and test formulas increases. The x-axis is the number of model updates performed, each unit is 200 updates for `Craft` and 500 updates for `Symbol`. The `Symbol` domain on 1,000 and 10,000 formulas can reach similar performance but a 25% performance difference for out of domain (65% vs. 90% accuracy). The `Craft` domain on 1,000 to 4,000 formulas shows a similar performance improvement. Seeing on the order of 10,000 formulas allows good generalization to new formulas. Note that any minor error made while executing the formulas was considered a failure.

away hurts performance tremendously. No previous method can learn to execute such formulas.

The performance on formulas that have similar statistics to the ones in the training degrades only slightly when testing on new formulas, and is shown in the second column of fig. 5, “Out of domain (1-10)”. This shows zero-shot generalization and that in all cases our method generalizes well.

Absolute performance is a function of how many formulas our model is trained with. Several thousand formulas must be seen before generalization to new formulas and longer

formulas is reliable, see fig. 6. In the `Symbol` domain, 1,000 training formulas result in 65% accuracy out of domain while 10,000 training formulas result in 90% out of domain accuracy. As the number of training formulas increases, generalization improves. To extrapolate how many formulas would be needed for a given level of performance, we used a least squares fit to a logarithmic function, which had  $R^2$  of 0.99. This predicts that performance would approach 100% at around 24,000 formulas. In other words, seeing 24,000 formulas that contain up to 10 predicates and operators is likely to be

enough to perfectly generalize to all formulas of that length. To put this into context, with 5 symbols in the `Symbol` domain presented here, there are approximately  $10^{10}$  formulas of length 10. This means that our network sees only one-millionth of all possible formulas before generalizing, not including its ability to generalize to longer formulas. Similarly in the `Craft` domain, 1,000 training formulas result in 59% accuracy out of domain, while 4,000 training formulas result in 70% accuracy out of domain. Together, these results show that our method learns to generalize formulas and to execute them zero-shot.

## V. CONCLUSION

We created a principled network that is able to encode the dynamics required to solve LTL formulas in a compositional manner. This represents a new and powerful type of multi-task learning, where learning occurs on one set of tasks and generalizes to all others. Coupled with a semantic parser, this model could execute linguistic commands that refer to temporal relations; we intend to pursue this in the future. While we believe that such network architectures are useful for other logics, such as first-order logic, how precisely they should be extended to even more exotic modal logics or second-order logics is unclear. We would additionally like to encompass more powerful logics like CTL\*. Theoretically characterizing the structures required in neural networks to generalize out of domain to new problems generated from particular logics is a new and interesting problem that may impact our understanding of other types of generalization.

Perfect, 100% generalization, performance on both `Symbol` and `Craft` appears to be within reach, although computationally this remains rather expensive. Around 10 to 100 times more computationally intensive than the results reported here. We intend to investigate what additional priors, curricula, or training algorithms can speed up learning to saturate performance in a more effective time frame. Overall, depending on the experiment and number of formulas, the results here took between 13 hours to 3 days to generate with each run executing on an Nvidia Titan X. Note that while relatively computationally intensive, such networks need only be trained once for any domain; due to their ability to zero-shot generalize to new formulas.

As it stands, the reward function designed here to supervise the RL agent is powerful but lacks some critical feedback. For example, the agents do not know what went wrong. Some feedback about the constraint that was violated could localize errors within particular sub-networks. This would be akin to telling someone that they had picked up gold instead of silver; clearly very useful information. Providing such targeted feedback to a compositional network seems possible in theory, one would need to more carefully determine which sub-networks could be at fault and emphasize updates to those sub-networks for a failed trial. Turning this intuition into a practical learning mechanism remains an open problem.

In the future, we intend to combine this work with that presented by Kuo *et al.* 2020 [17] and create a single

agent capable of following complex linguistic commands in continuous environments.

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