Lenses

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Final project

Overview

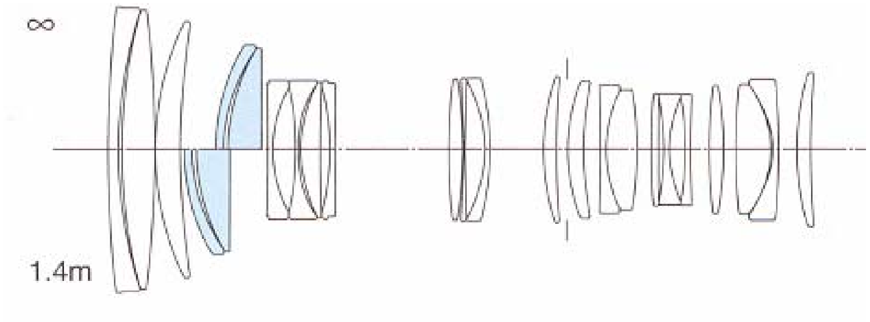
- **So far, we have mostly taken the input image for granted**
- **Today, we focus on the optics side & image formation**

Important question

- **Why is this toy so expensive**
  - EF 70-200mm f/2.8L IS USM

- **Why is it better than this toy?**
  - EF 70-300mm f/4-5.6 IS USM

- **Why is it so complicated?**

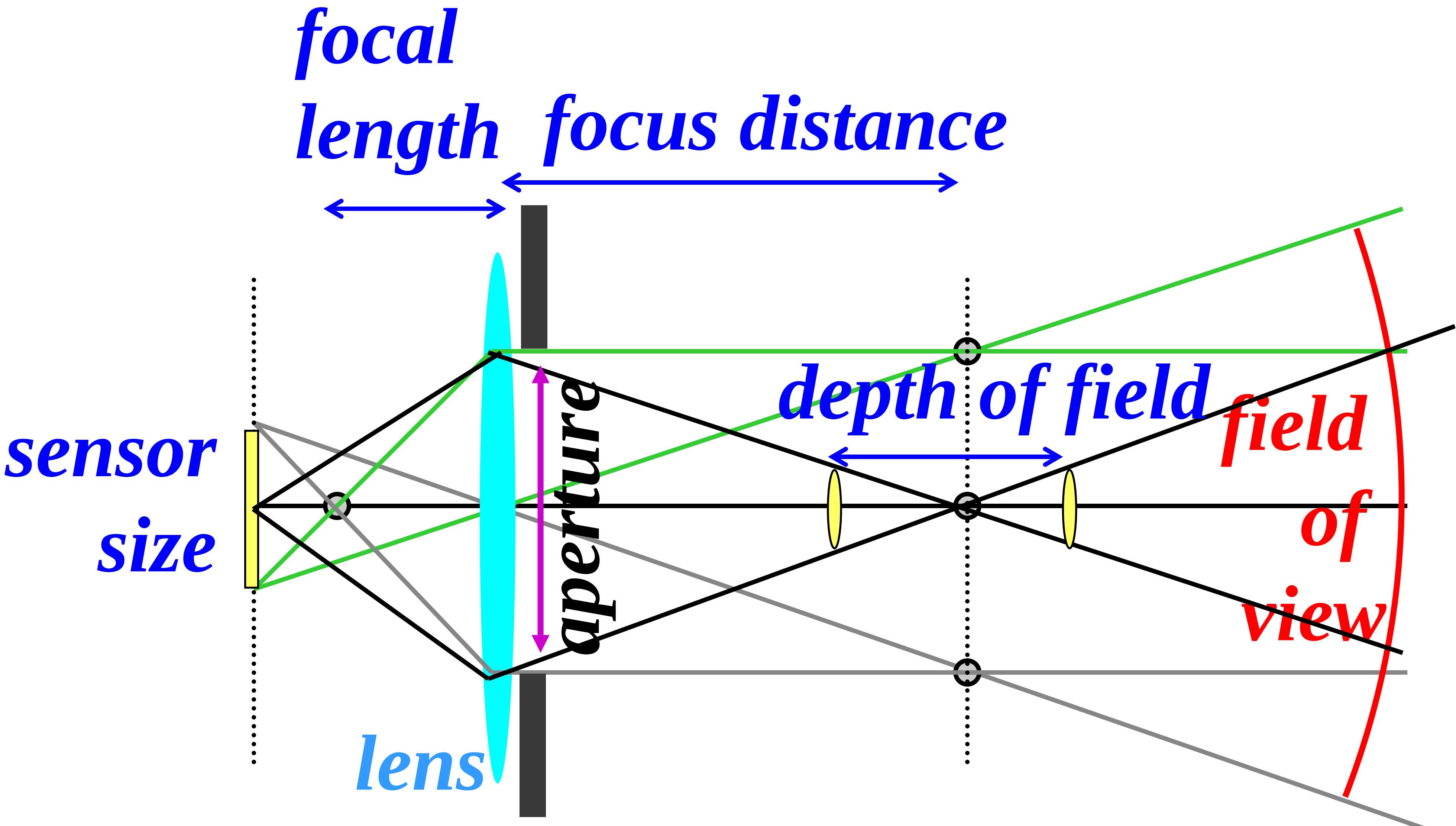


- **What do these buzzwords and acronyms mean?**

Lens 101 review

- **Focal length (in mm)**
  - Determines the field of view.
    wide angle (<30mm) to telephoto (>100mm)
- **Focusing distance**
  - Which distance in the scene is sharp
- **Depth of field**
  - Given tolerance, zone around the focus distance that is sharp
- **Aperture (in f number)**
  - Ratio of used diameter and focal lens.
    Number under the divider ➔ small number = large aperture
    (e.g. f/2.8 is a large aperture, f/16 is a small aperture)
- **Shutter speed (in fraction of a second)**
  - Reciprocity relates shutter speed and aperture
- **Sensitivity (in ISO)**

Quantities


focal length
focus distance
sensor size
aperture
depth of field
field of view
lens

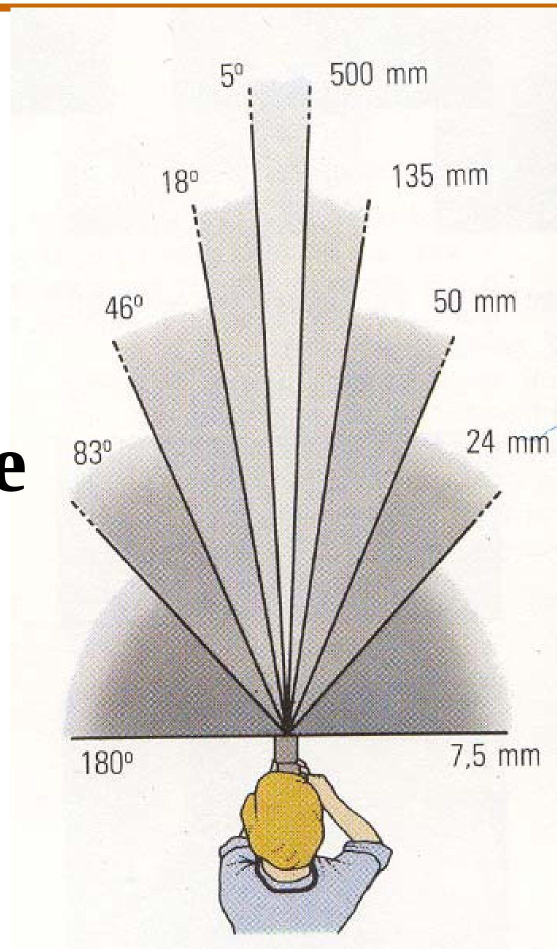
Focal length

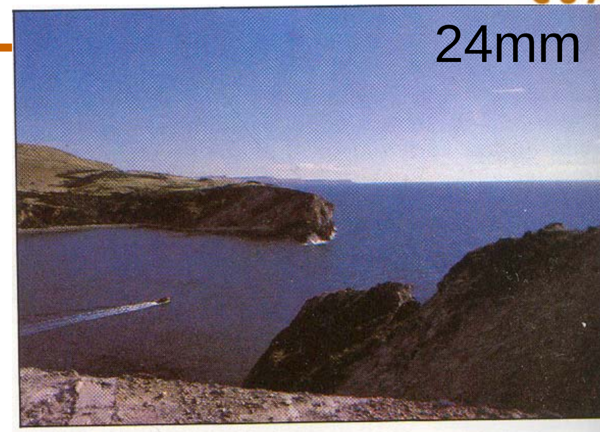
<30mm: wide angle

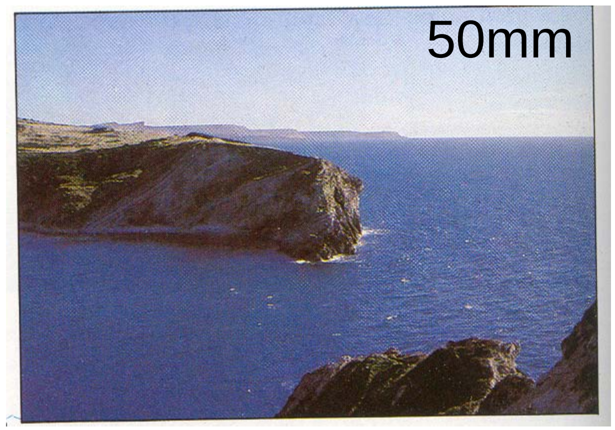
50mm: standard

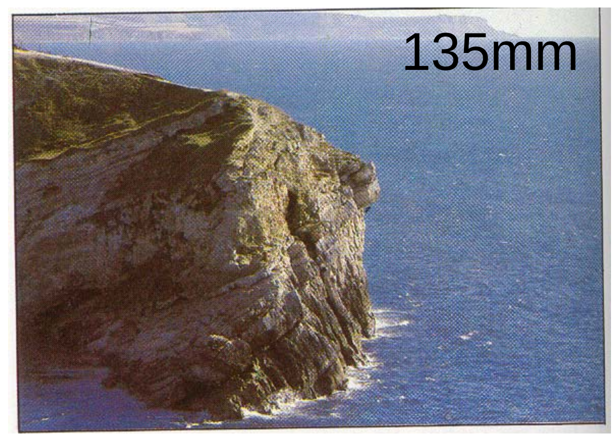
>100mm telephoto

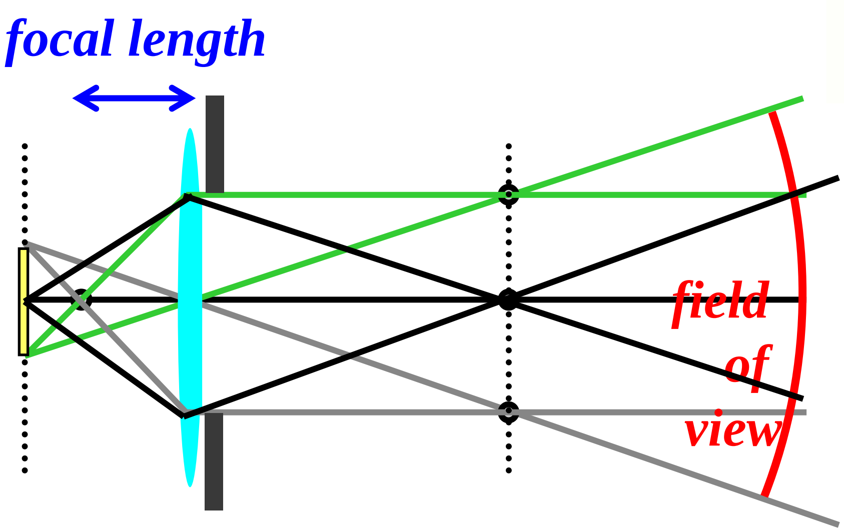
Affected by sensor size (crop factor)











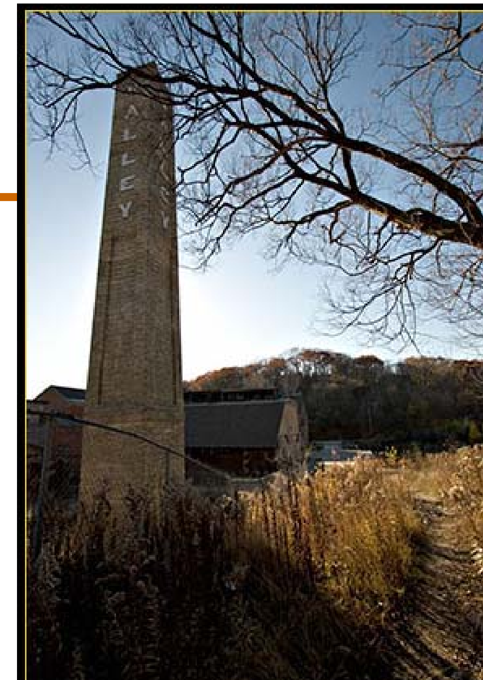
Lenses

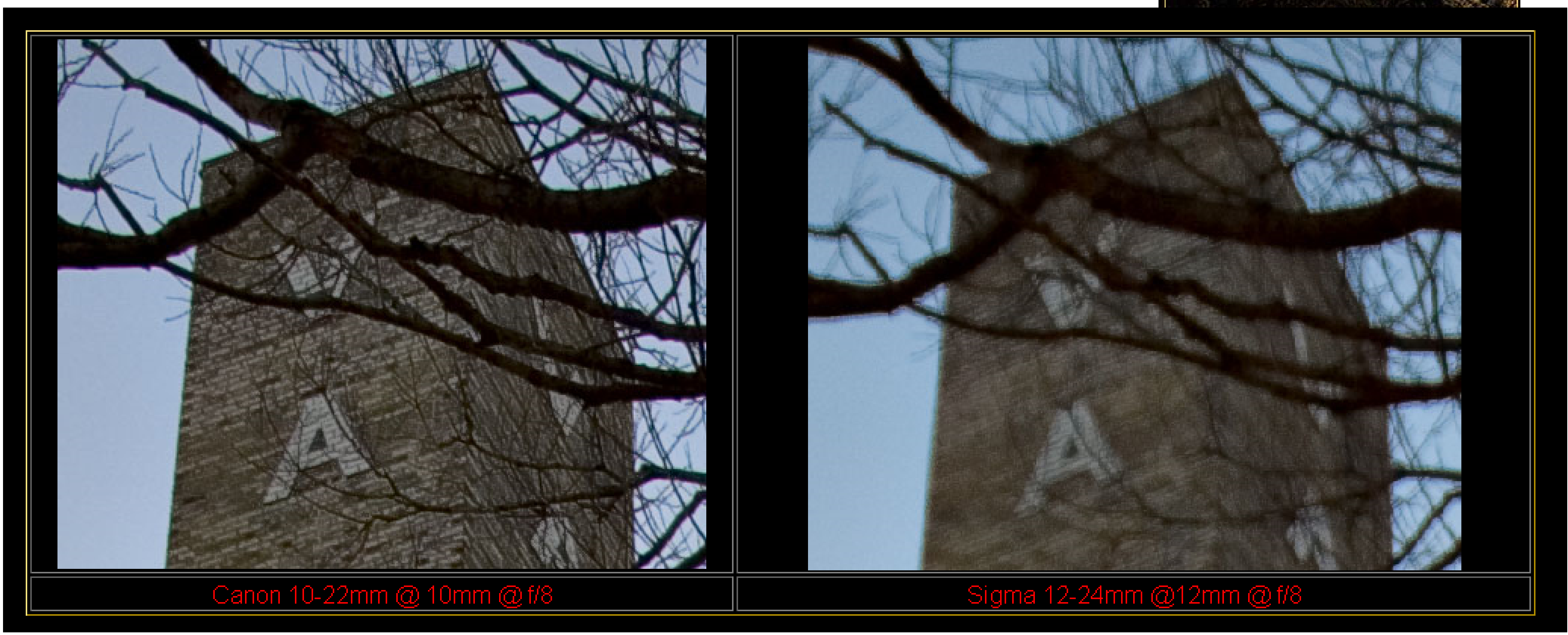
- **In a photo system, the lens is most critical**
- **Lenses are characterized by**
  - Prime vs. zoom
  - Focal length (field of view)
  - Maximum aperture (the f number like f/2.8)
  - Various gizmos (e.g. image stabilization, faster autofocus)
  - More complex quality issues
  - Minimum focusing distance
- **Max aperture is usually correlated with quality**
- **Warning: lenses are addictive**

Bottom line

- **Yes, you can get a cheap & razor sharp high-quality lens:**
  look for a prime in the 35-100mm range
  - e.g. Canon 50mm f/1.8, 85mm f/1.8, Nikon 50mm f/1.8
- **See also** **http://www.photozone.de/3Technology/lenstec4.htm**

Lens quality varies!





source: the luminous landscape

Canon 100-400mm f/3.5-f/5.6L zoom
@ f/5.6

Canon 400mm f/5.6L
@ f/5.6

source: the luminous landscape

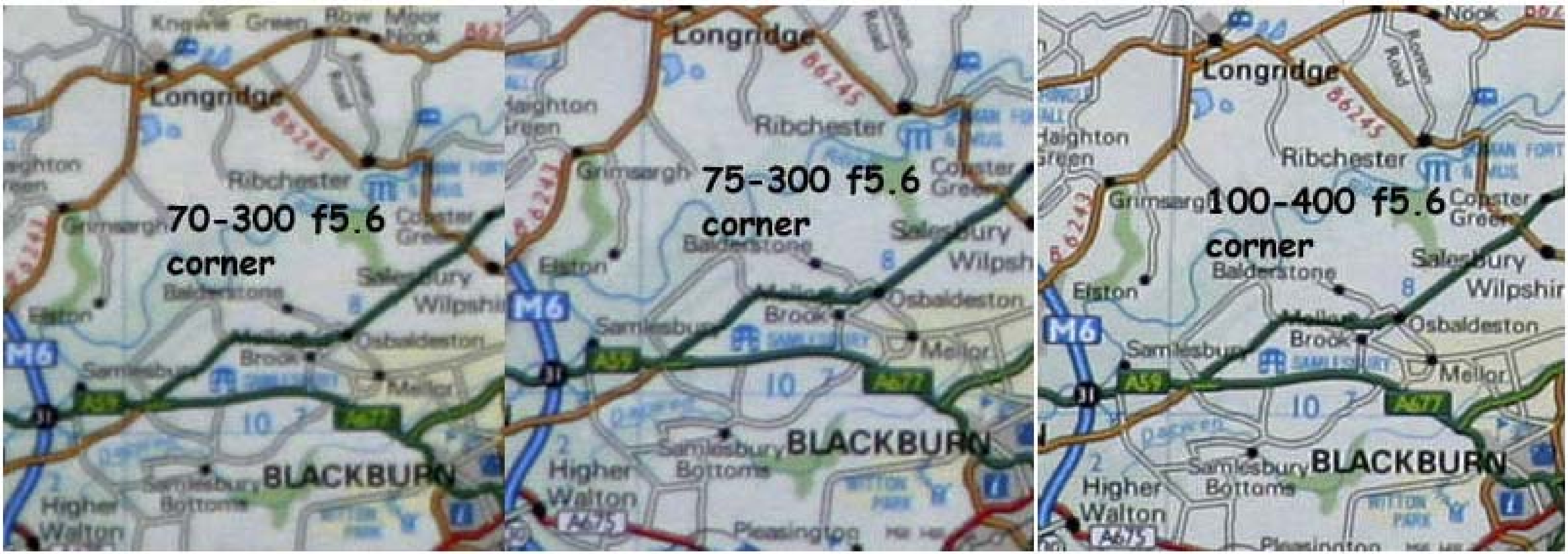
Center is usually OK

- http://www.photo.net/equipment/canon/70-300do_2/

250x500 pixel crops, centre of frame f5.6

Image corners are often sacrificed

- http://www.photo.net/equipment/canon/70-300do_2/



250x500 pixel crops, corner of frame f5.6

Max aperture is tough

- http://www.photo.net/equipment/canon/70-300do_2/

250x500 pixel crops, centre of frame f5.6

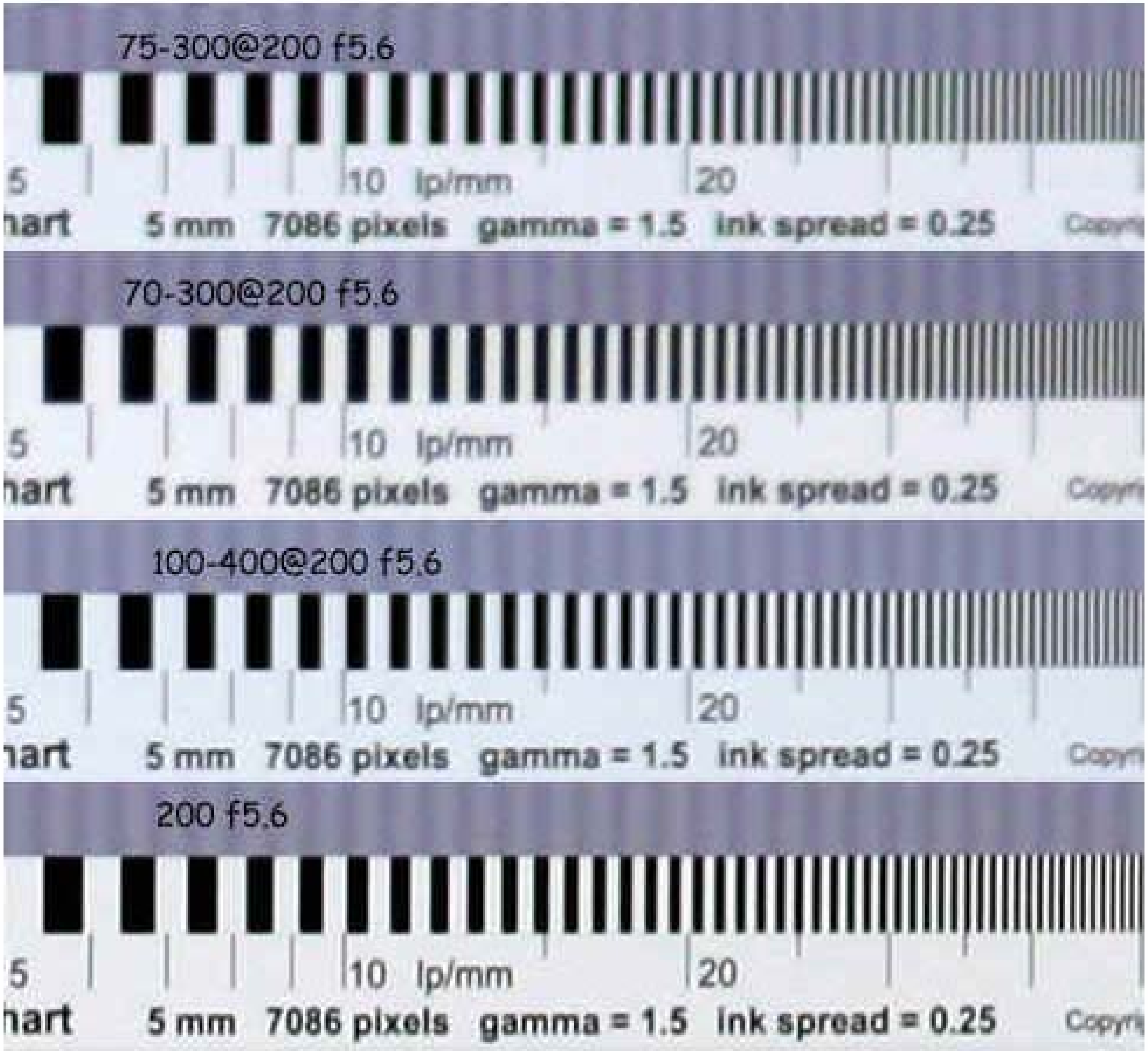
Gets better when stopped down

- http://www.photo.net/equipment/canon/70-300do_2/

250x500 pixel crops, centre of frame f11

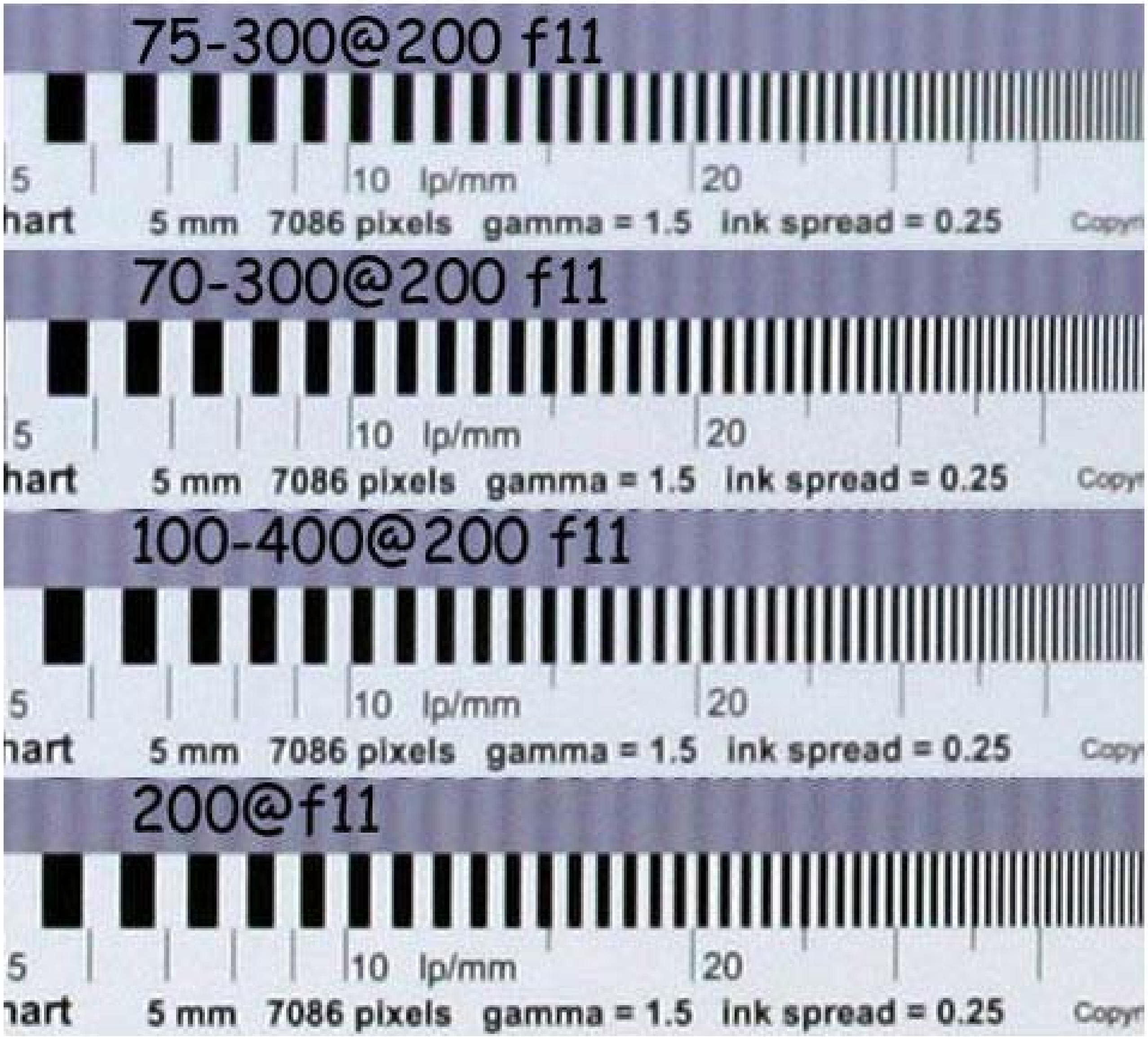
Typical test pattern

- **http://www.photo.net/equipment/canon/70-300do_2/**

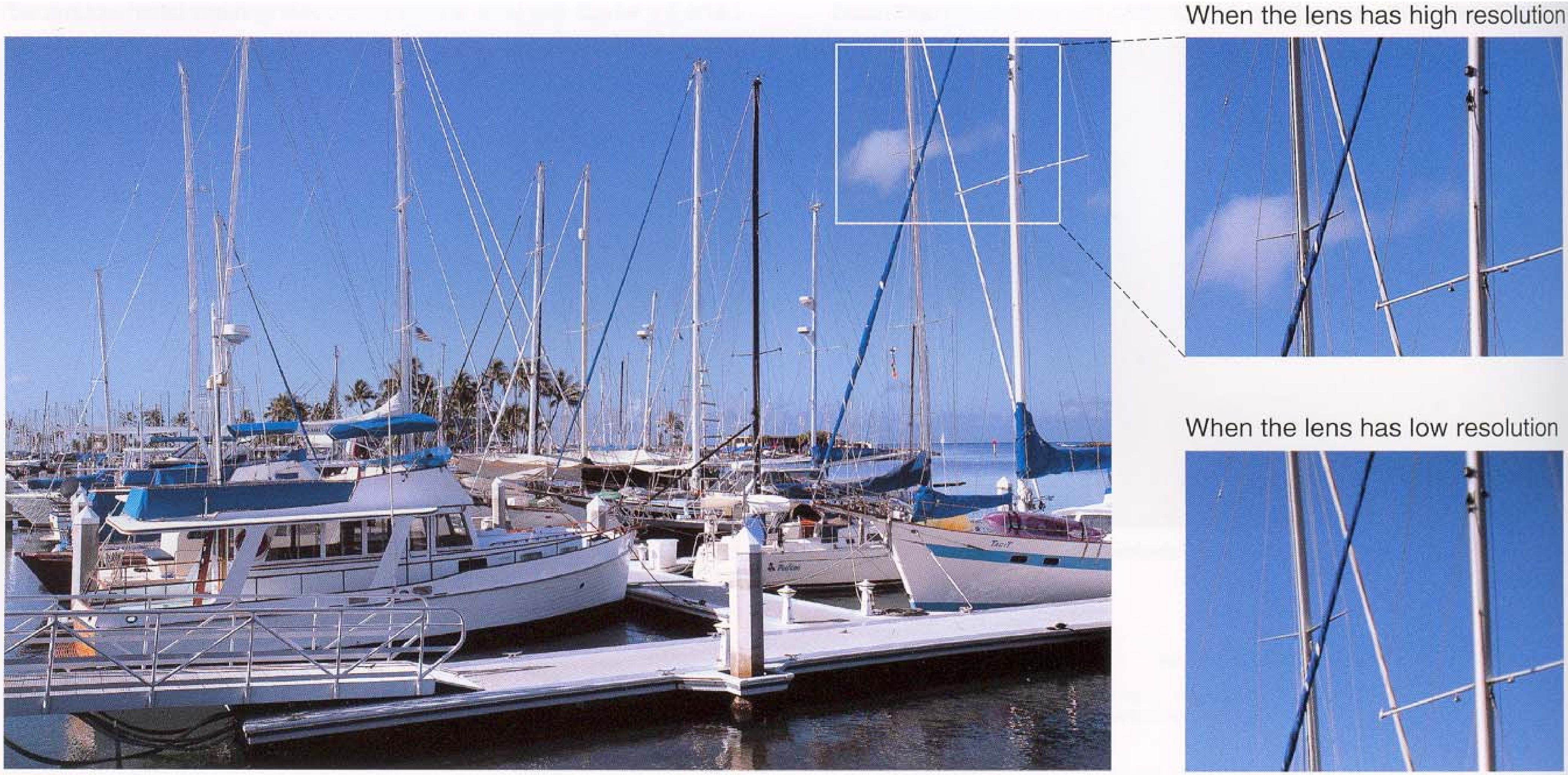


Again, better when stopped down

- **http://www.photo.net/equipment/canon/70-300do_2/**



Power of lenses

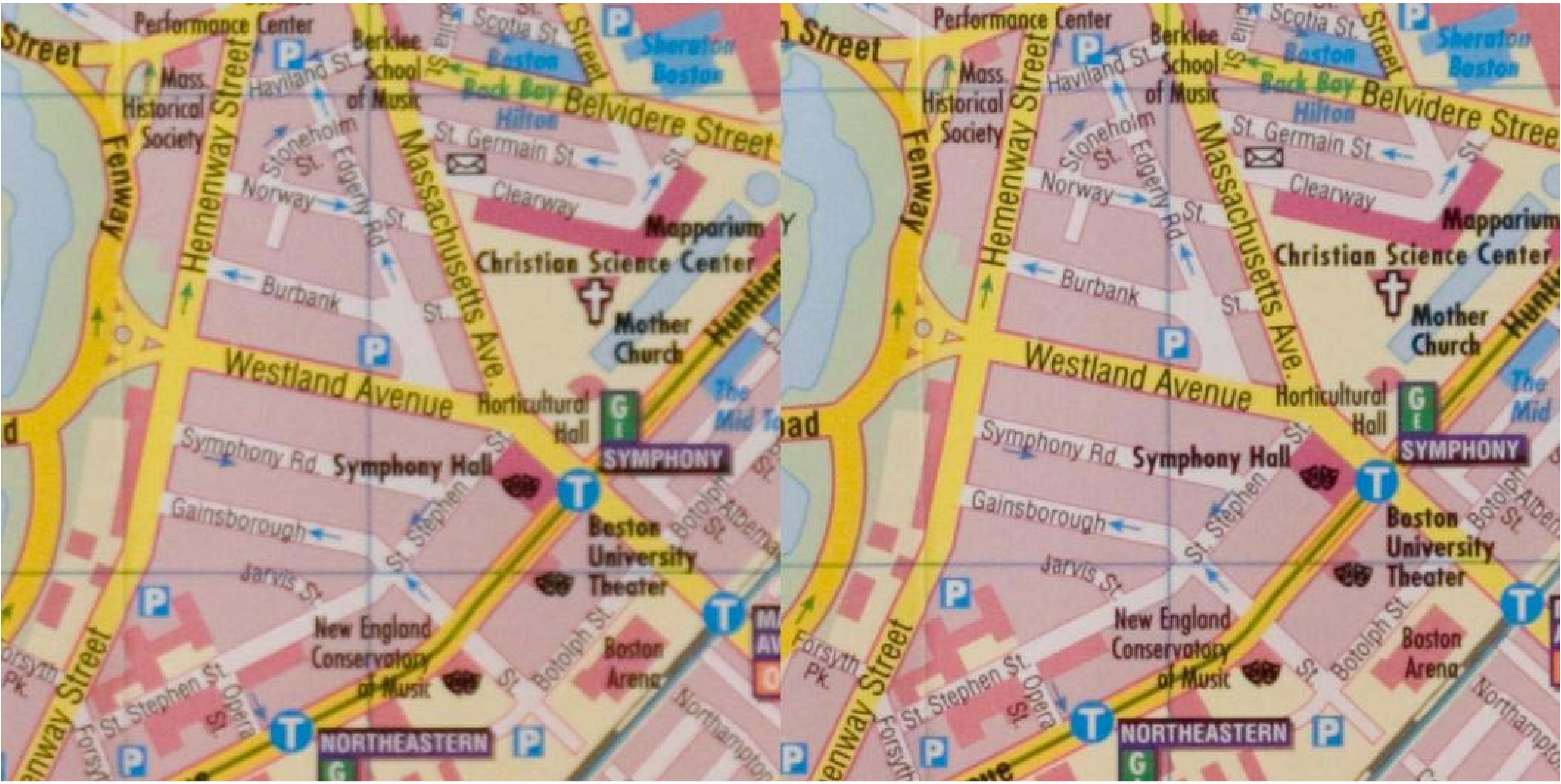


Even when combined with a digital camera, an EF lens has high potential. In this photo of a harbor crowded with yachts, high resolution reveals the fine detail in individual boats. Photographing images with detailed subject matter, such as landscapes, is possible without having to differentiate between a digital camera and a 35mm film camera.

source: canon red book

Copy variation

- **Left: Addy's 100-400; Right: Frédo's**
- **(full aperture, 135mm)**



Why are lenses so complex?

- **It's not so easy to send light where it should go**

Figure-11
Cross-Section of the EF24-70mm f/2.8L USM

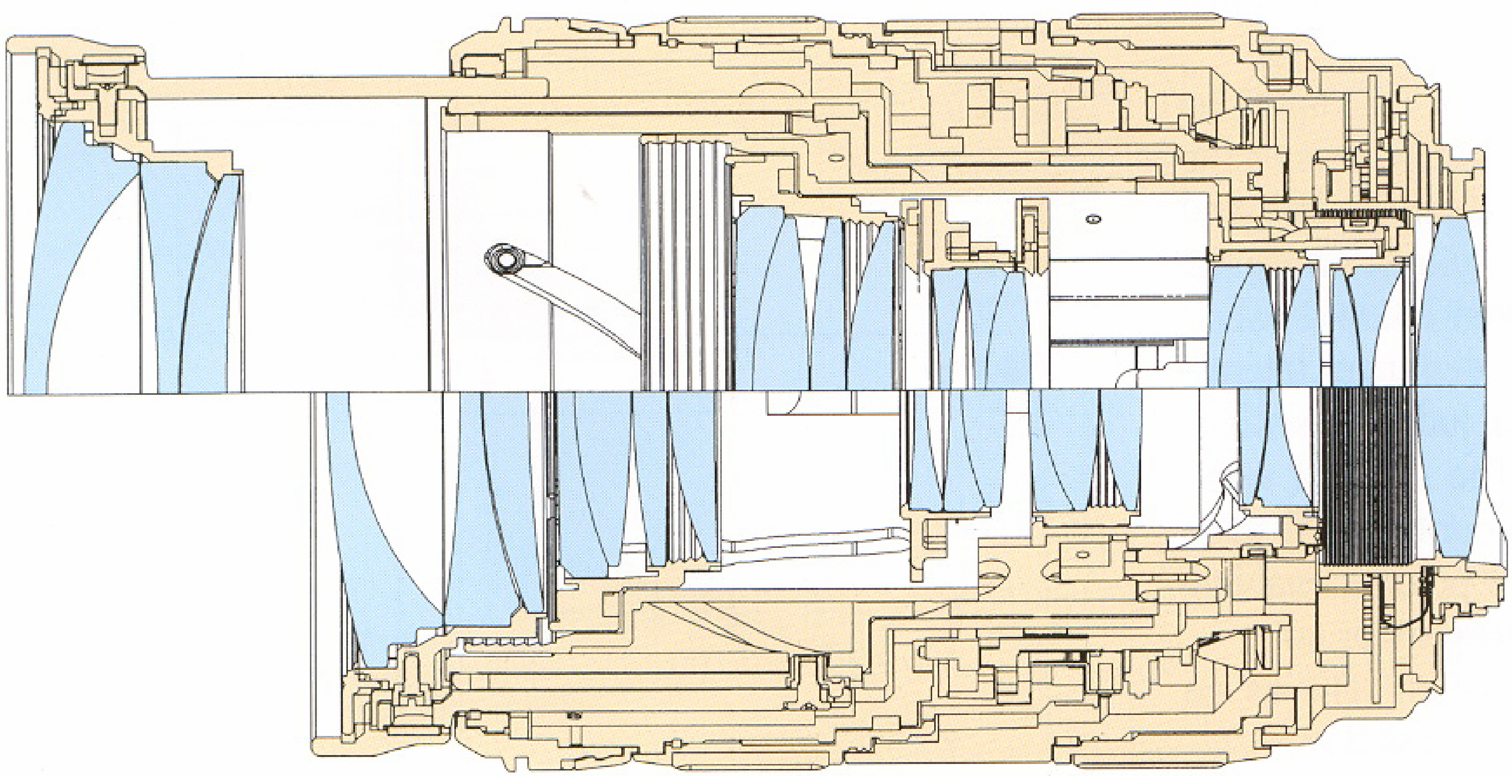


Photo-4 Precision Zoom Cam Lens Barrel

source: canon red book

Simple lenses are not so good

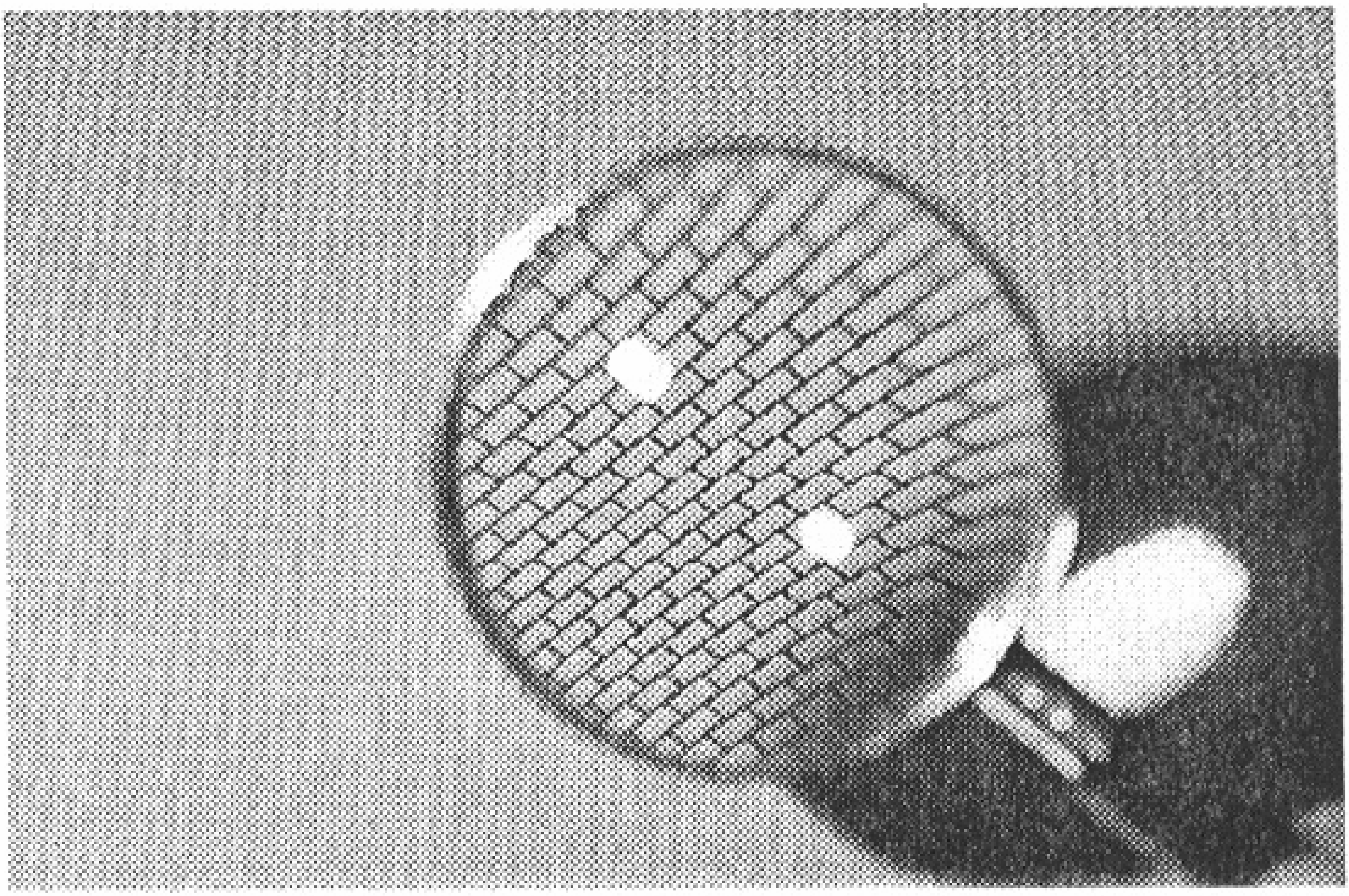
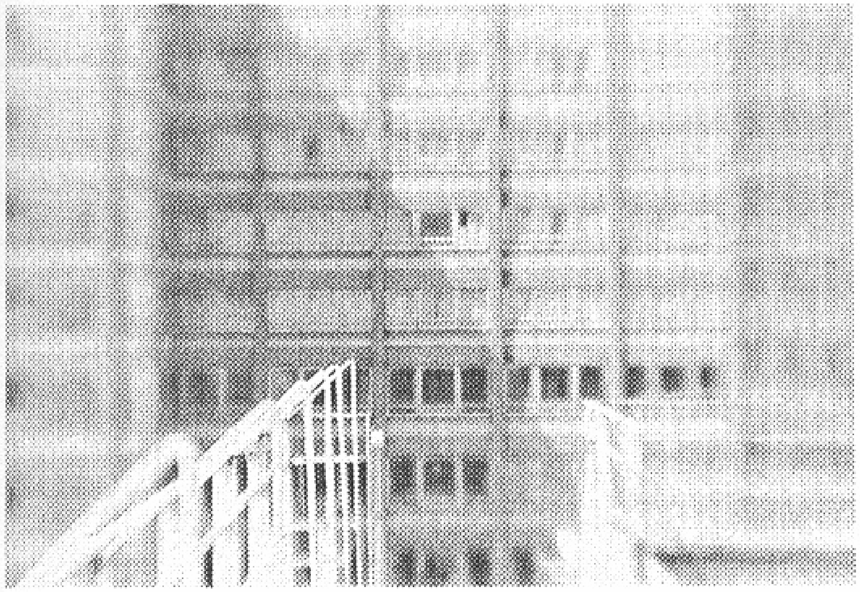


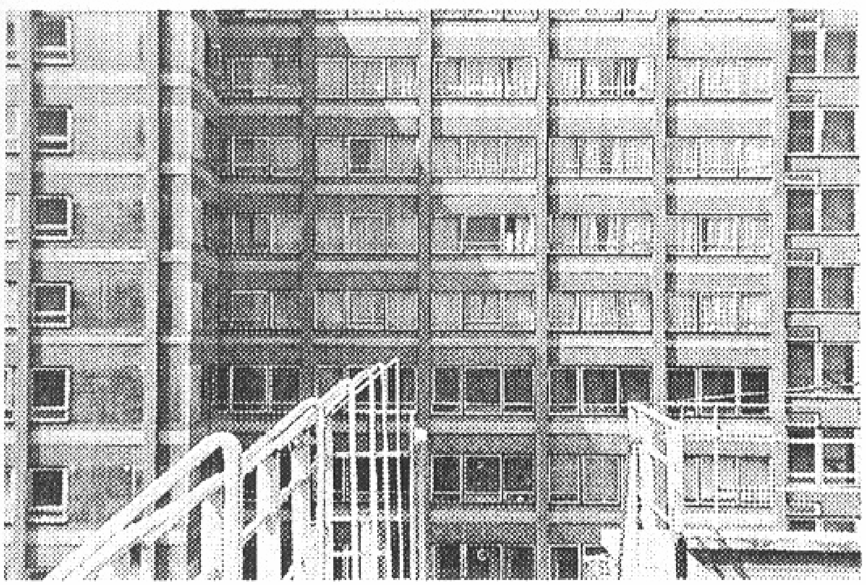
Plate 11.2 Aberrated imagery from a simple biconvex lens
The image of simple regular patterned subject shows increasingly poor quality off axis and the two uncoated surfaces of the lens both reflect the light source.

From Ray's Applied Photographic Optics

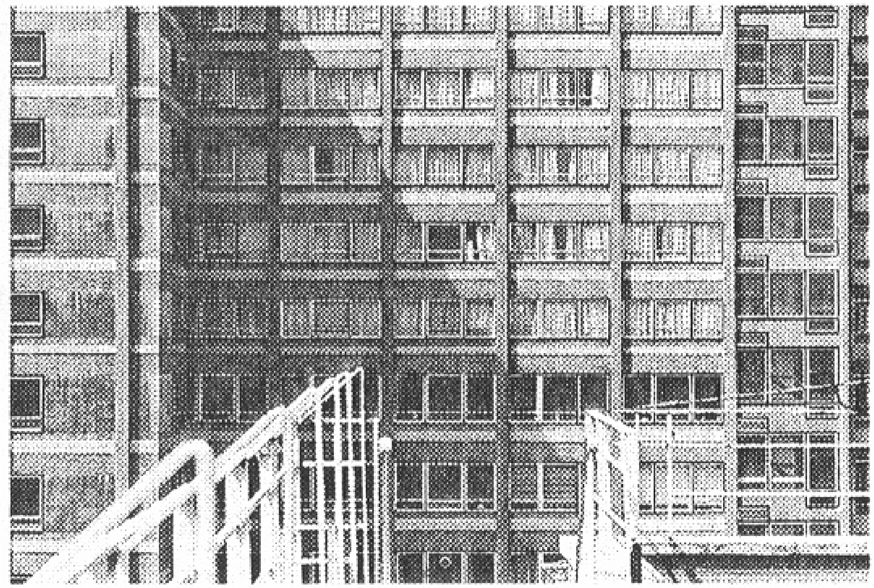
Complex lenses are better!



(a)



(b)



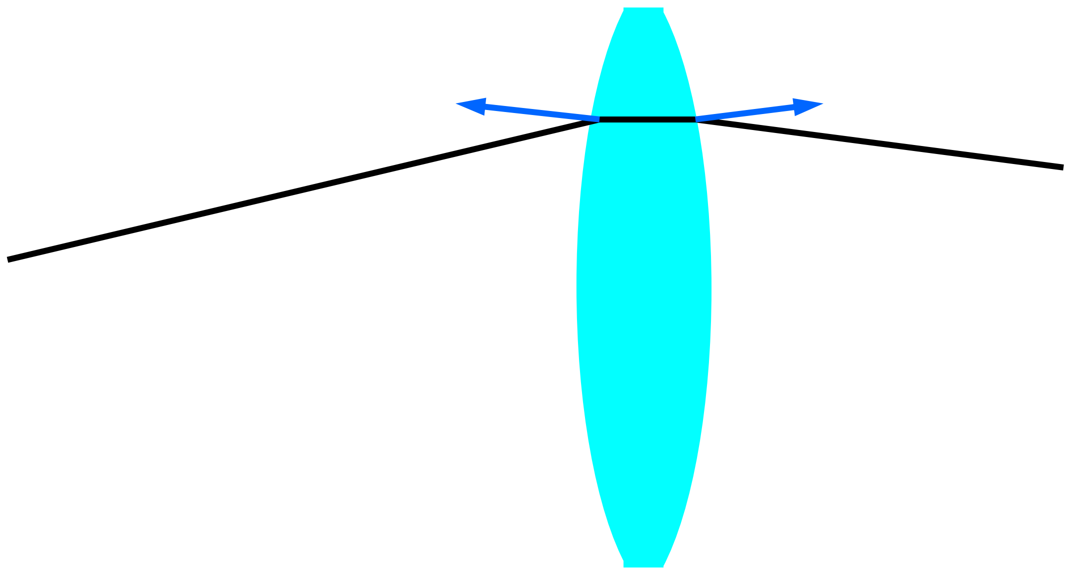
(c)

Plate 11.1 Imaging by simple and compound lenses
(a) Simple biconvex one element lens of focal length 100 mm and diameter 50 mm giving *f*/2. Note poor edge detail and low overall contrast. (b) Same lens stopped down to *f*/11. Quality and contrast have improved. (c) A well-corrected five-element 105 mm lens used at *f*/11.

From Ray's Applied Photographic Optics

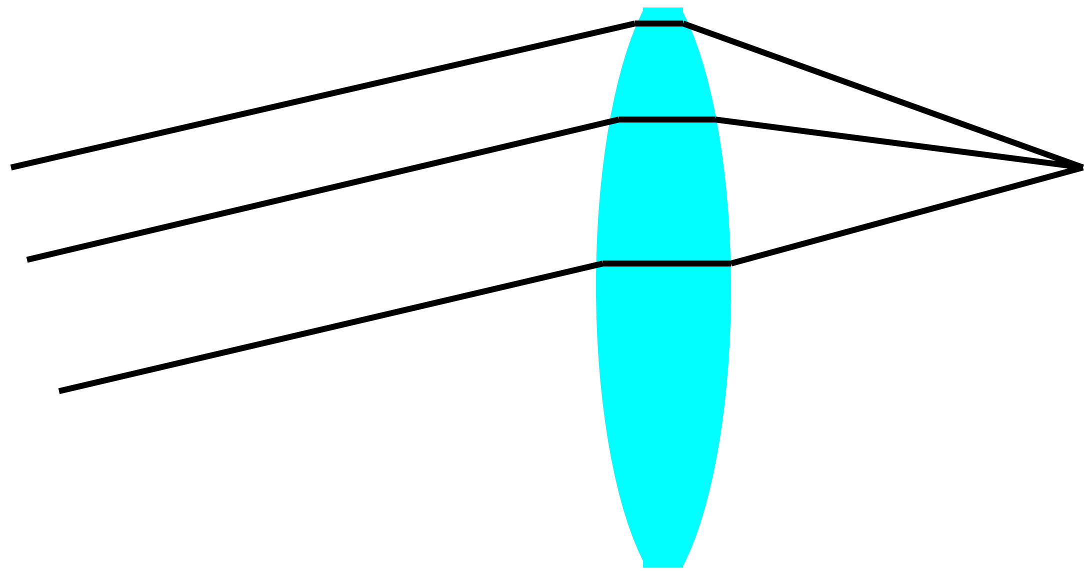
View #1 of lenses: Geometrical

- **Snell's law bends geometrical rays**
- **Most aberrations can be expressed in this framework**



View #2 of lenses (Fermat/wave)

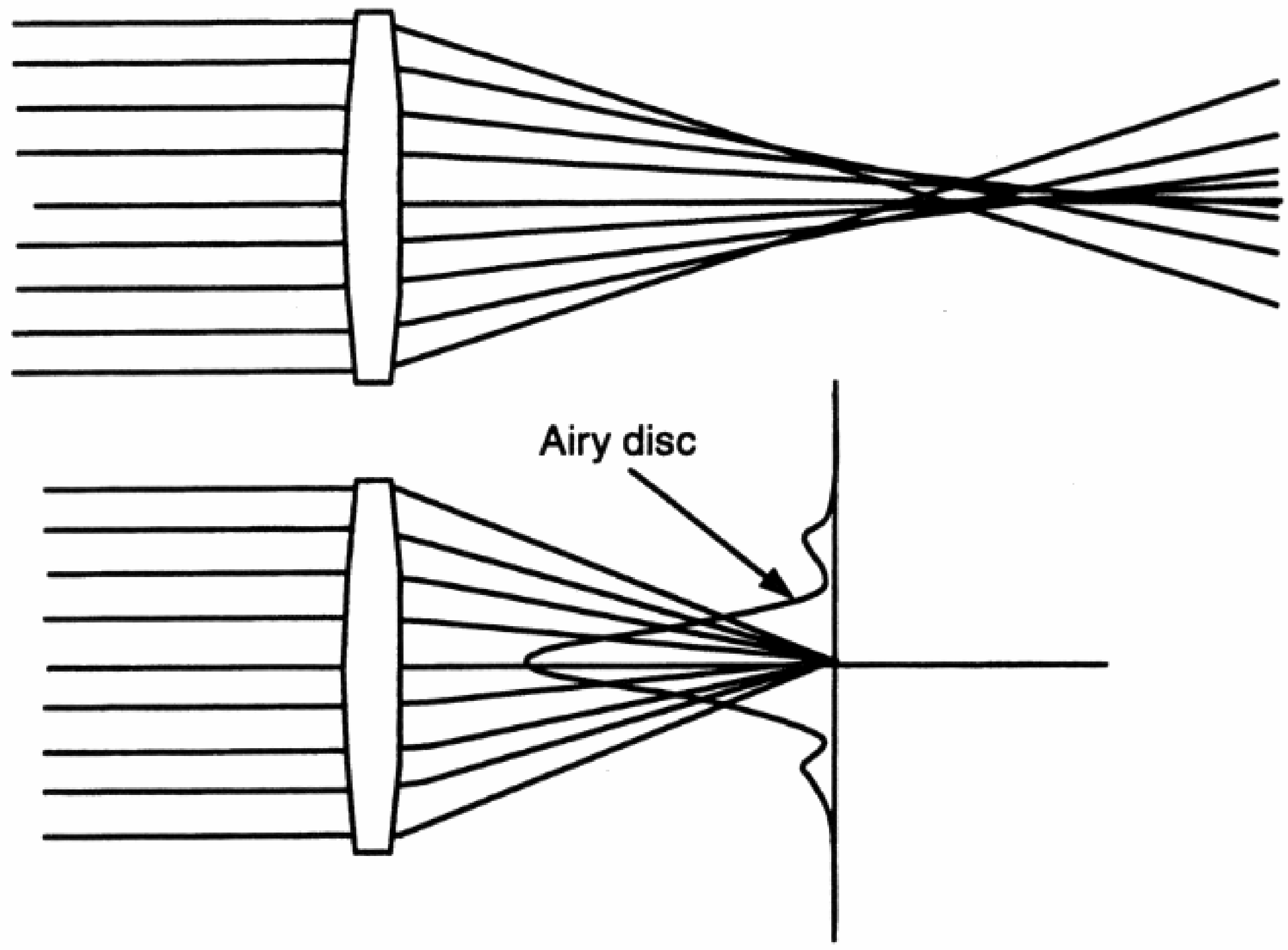
- **Light is focused because all paths have same length**
  - Higher index of refraction (speed of light) compensates for length
  - Constructive interference



Consequences on image quality

- **Geometrical optics: hard to focus all rays**
- **Wave optics: diffraction problems**

Figure 3.1
Image Quality, Geometrical Aberrations (Top) and Diffraction Limited (Bottom)



From Optical System Design by Fisher and Tadic

Diffraction

Geeky joke

At first God said

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

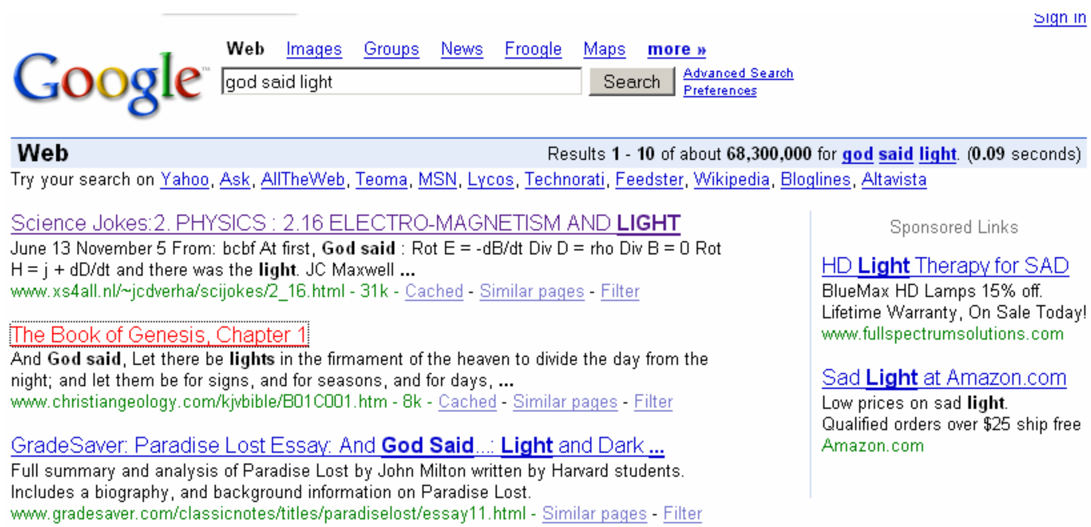
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t},$$

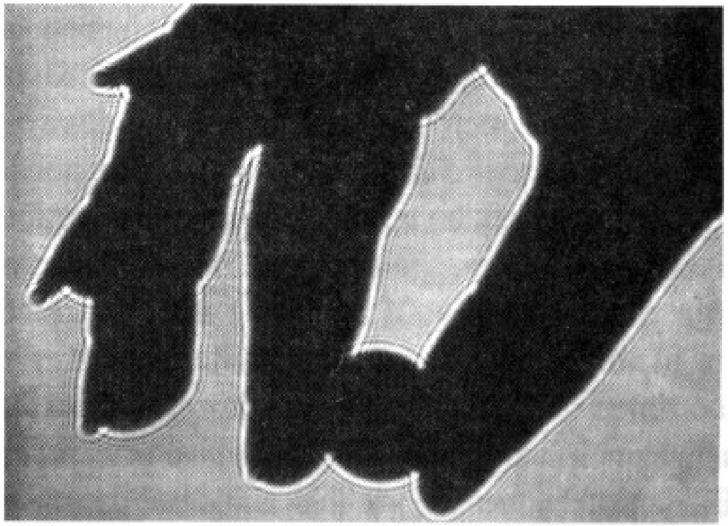
and there was light

(interestingly, the joke has a higher Google rating than the actual book of Genesis)

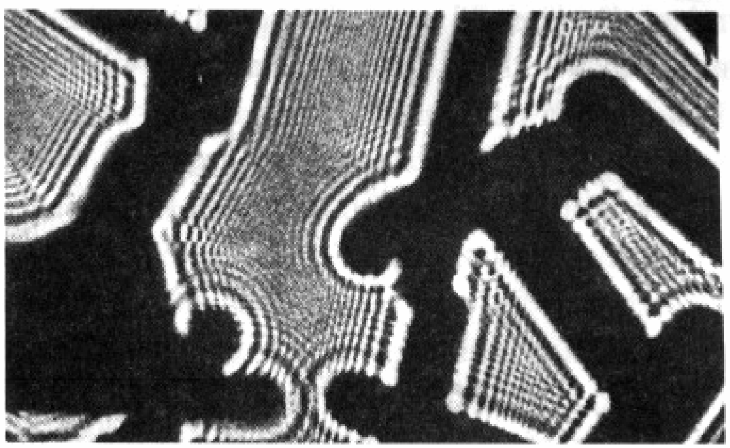


Equations from http://scienceworld.wolfram.com/physics/MaxwellEquations.html

Diffraction

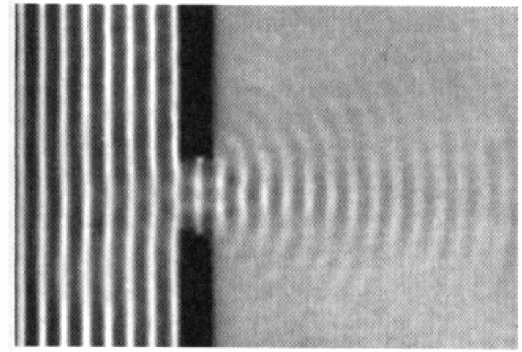


(a)

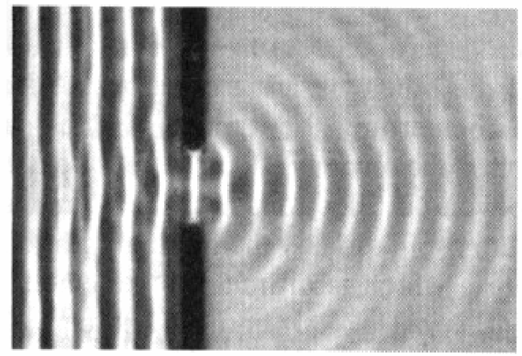


(b)

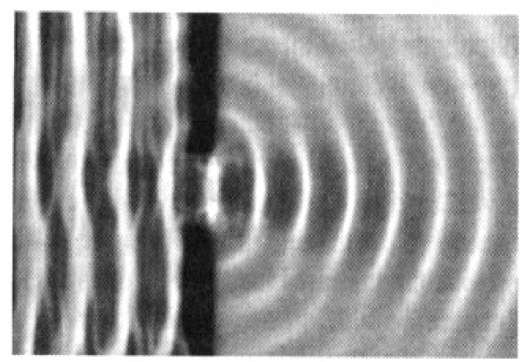
(*a*) The shadow of Mary's hand holding a dime, cast directly on 4 × 5 Polaroid A.S.A. 3000 film using a He–Ne beam and no lenses. (Photo by E. H.) (*b*) Fresnel diffraction of electrons by zinc oxide crystals. (After H. Boersch from *Handbuch der Physik*, edited by S.Flügge, Springer-Verlag, Heidelberg.)



(a)



(b)



(c)

Diffraction through an aperture with varying λ as seen in a ripple tank. (Photo courtesy PSSC *Physics*, D. C. Heath, Boston, 1960.)

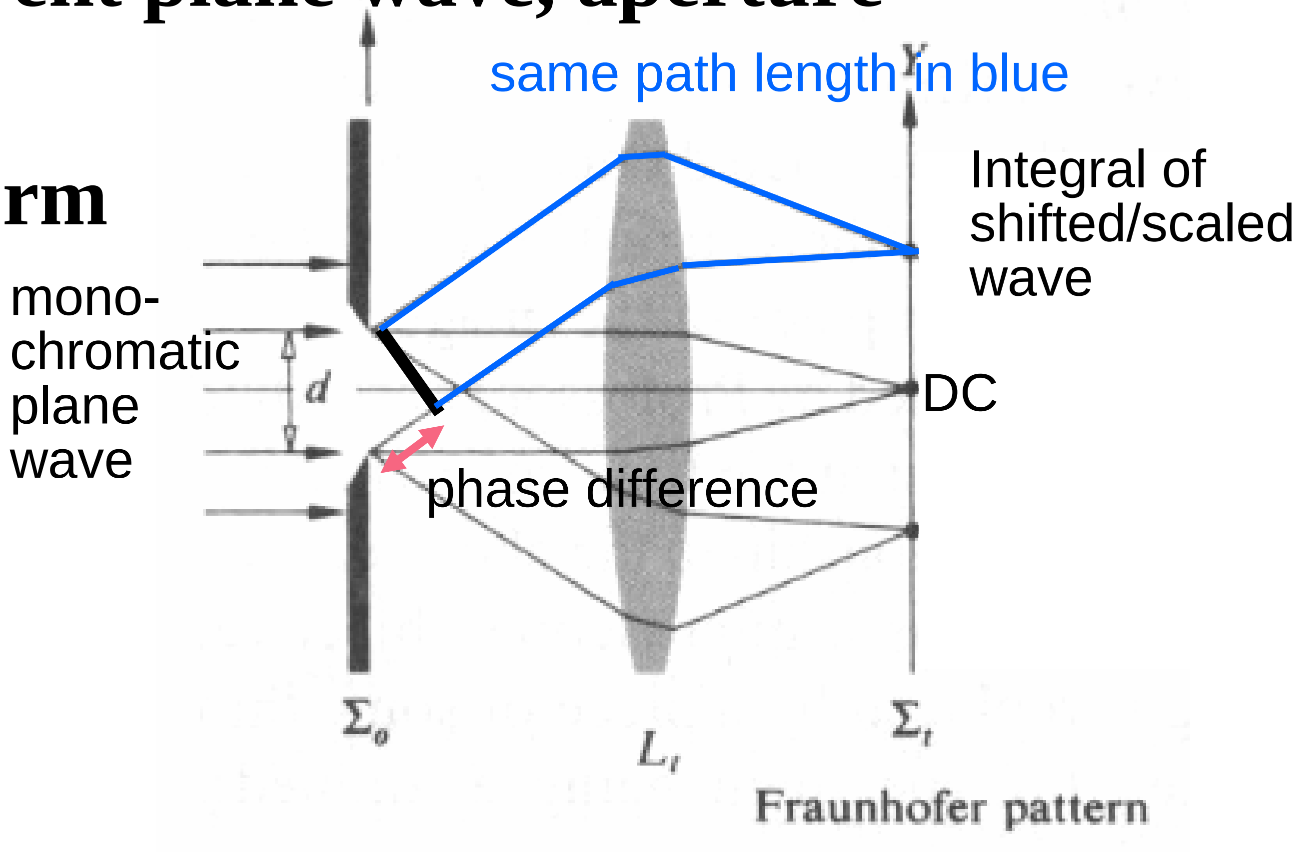
From Hecht's Optics

Fraunhofer diffraction

- **Far from aperture (ideally at infinity)**

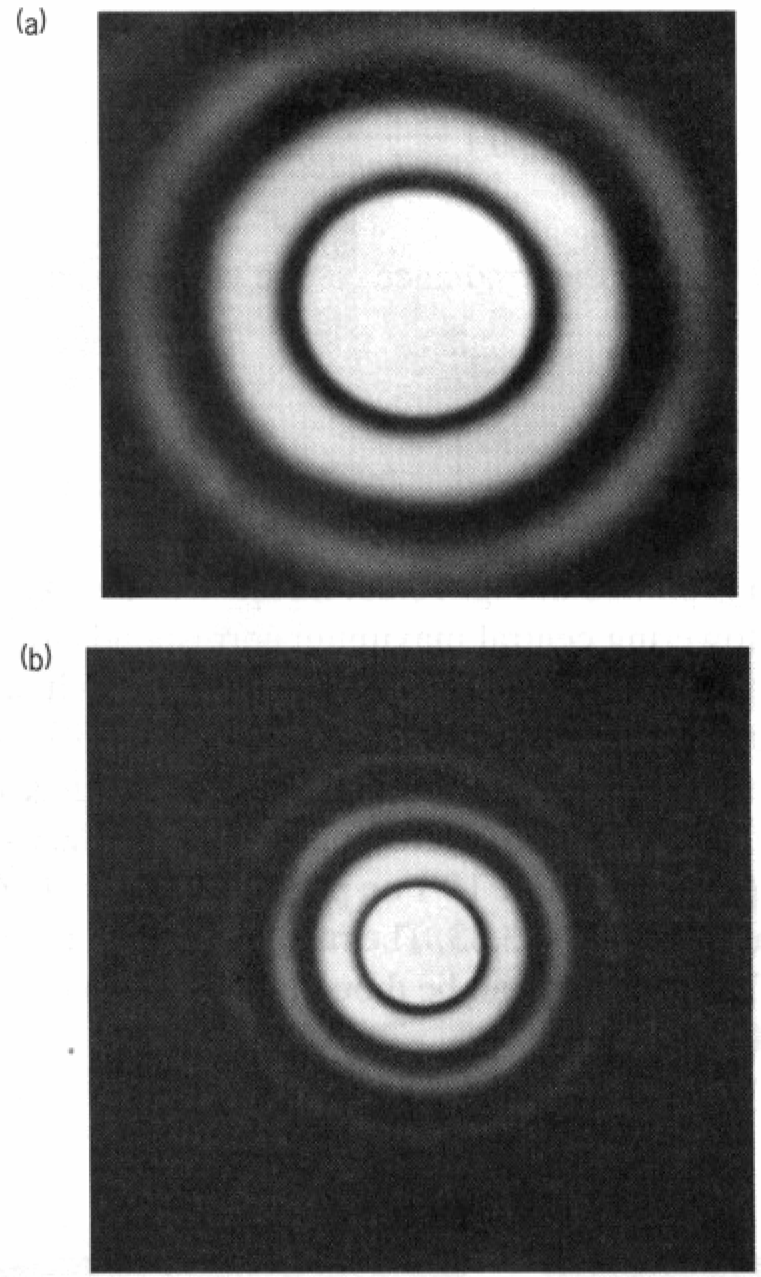
  Lots of things get linearized

- **Incoming coherent plane wave, aperture**
- **Diffraction = Fourier transform of aperture**
- **Works because**
  - wave in time & space
  - coherent



Airy patterns

- **Absolute limit on lens resolution**
- **Important at small aperture**



Airy rings using (a) a 0.5-mm hole diameter and (b) a 1.0-mm hole diameter. (Photo by E. H.)

From Hecht's Optics

Lens diffraction

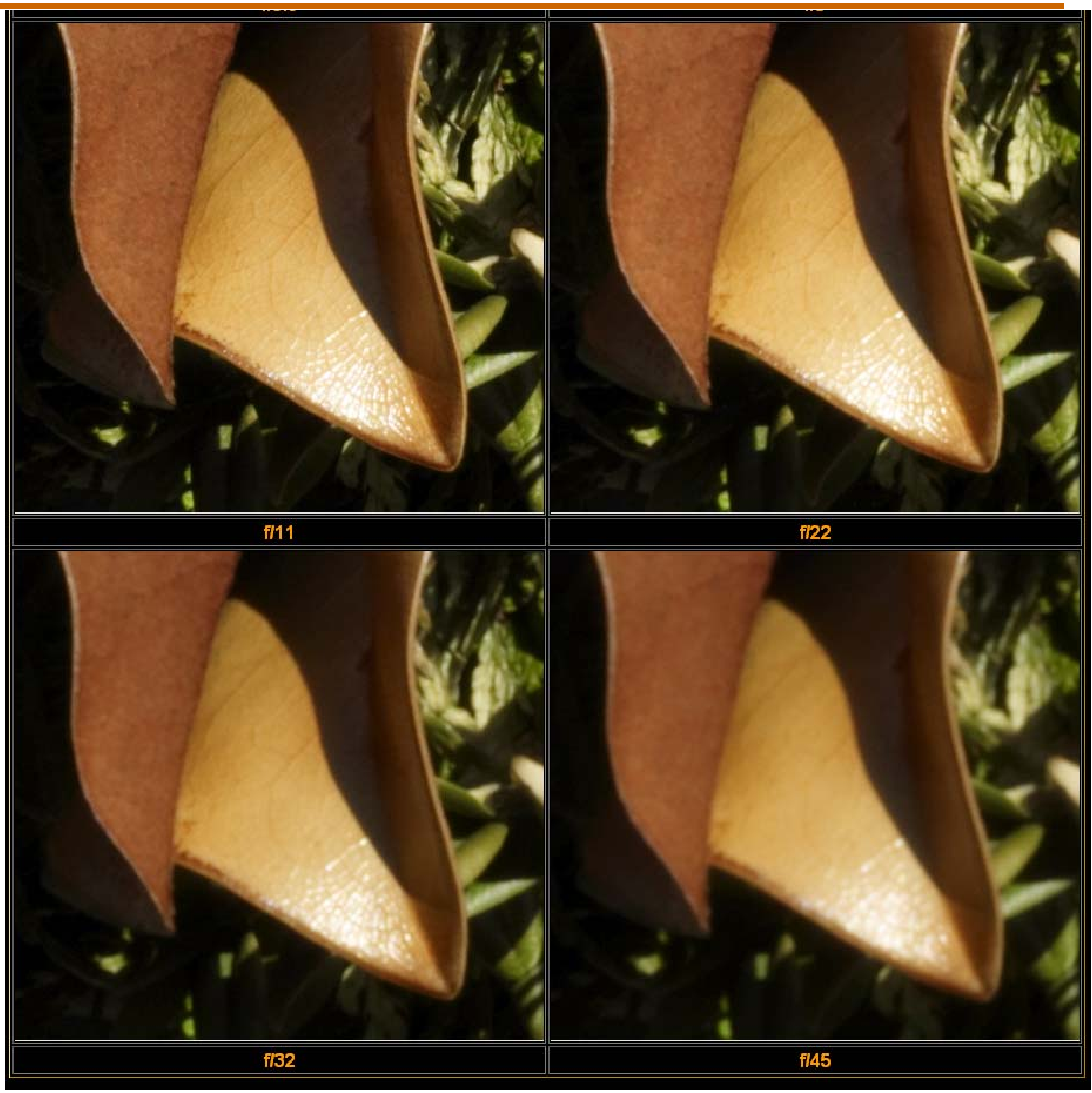
- **http://luminous-landscape.com/tutorials/understanding-series/u-diffraction.shtml**

  (heavily cropped)

Lens diffraction

- http://luminous-landscape.com/tutorials/understanding-series/u-diffraction.shtml

  (heavily cropped)
- See also http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm



Diffraction & Fourier

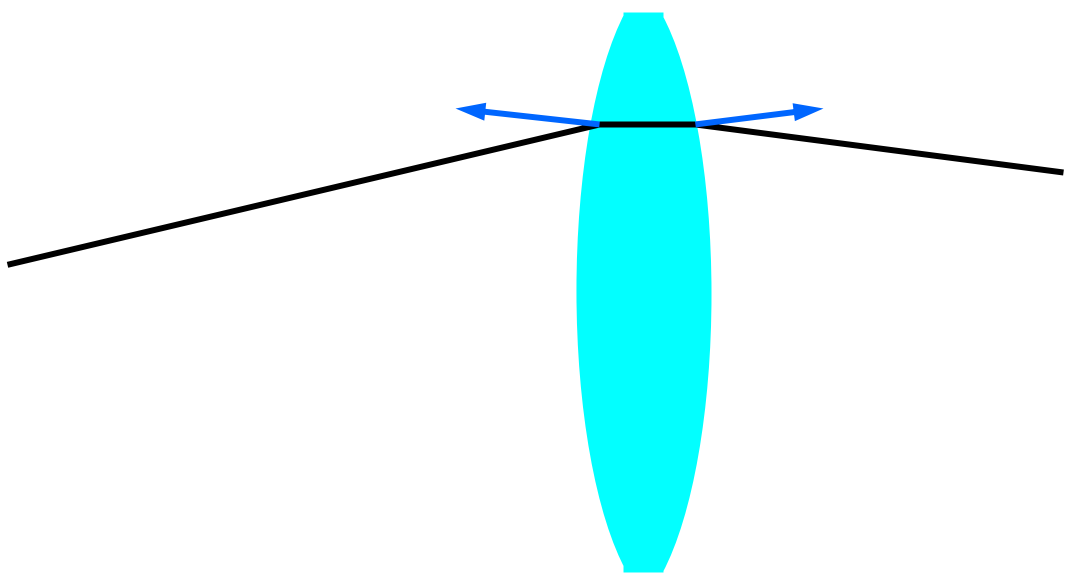
- **Aperture Fourier transform**

Photo by Eric Chan

Geometrical perspective

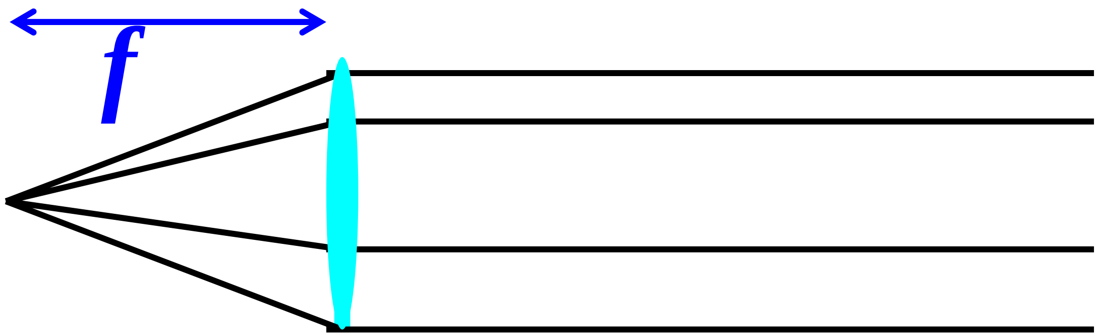
Back to View #1 of thin lenses

- **Snell's law bends geometrical rays**

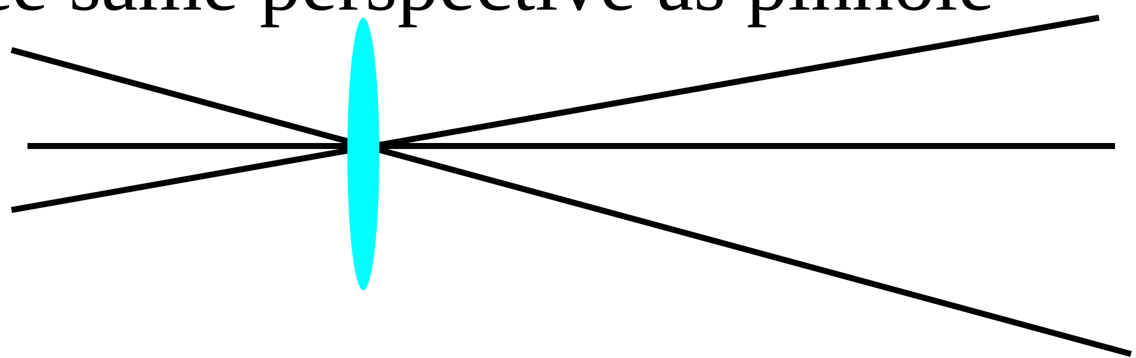


Thin lens optics

- **Simplification of geometrical optics for well-behaved lenses**
- **All parallel rays converge to one point on a plane located at the focal length *f***

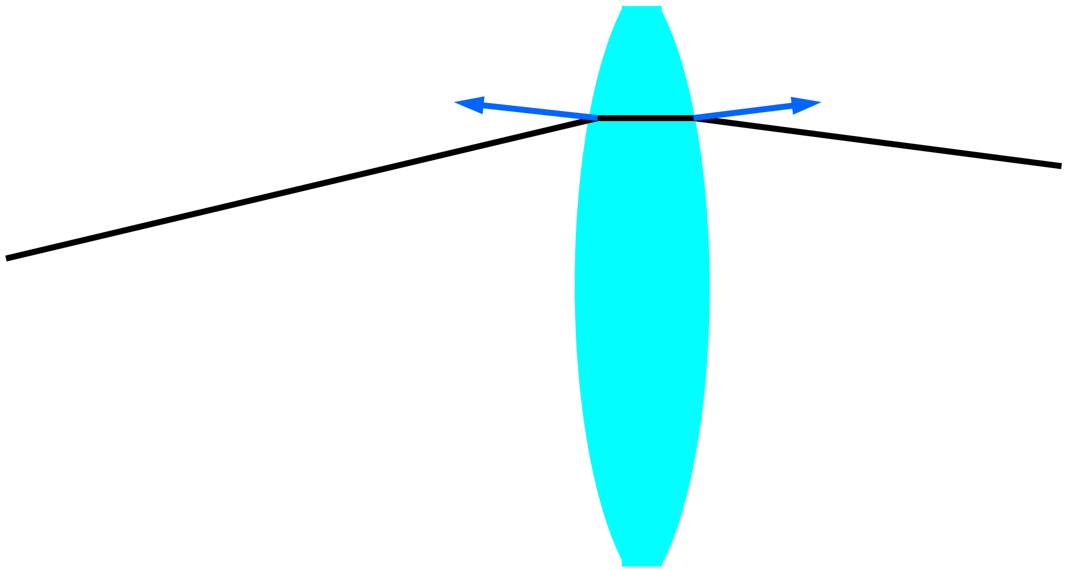


- **All rays going through the center are not deviated**
  - Hence same perspective as pinhole



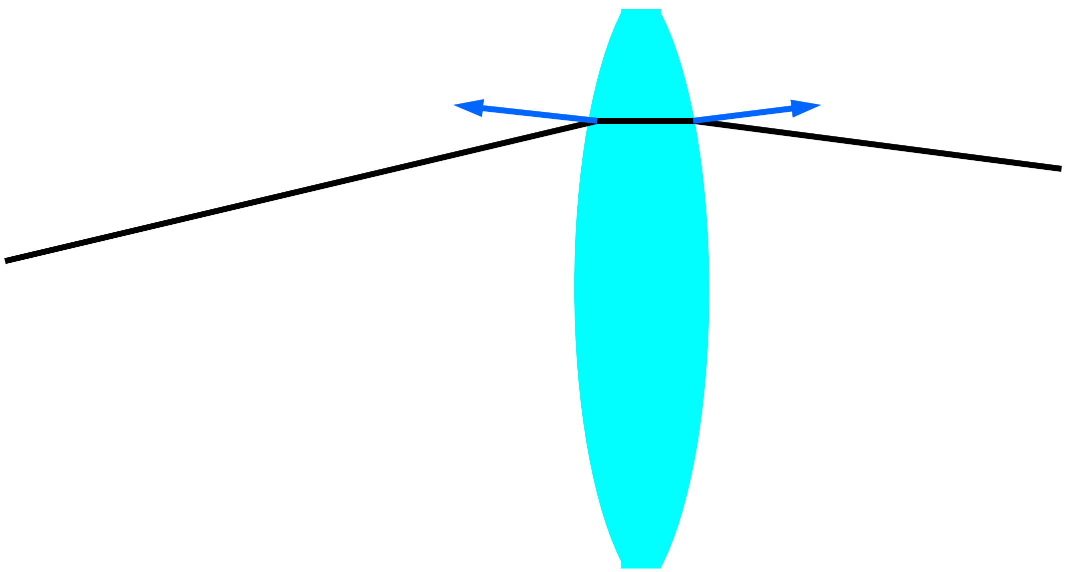
Simplification of first-order optics

- **Snell's law: $\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$**
- **First order/thin lens optics: use $\sin\theta = \theta$**



Third-order optics

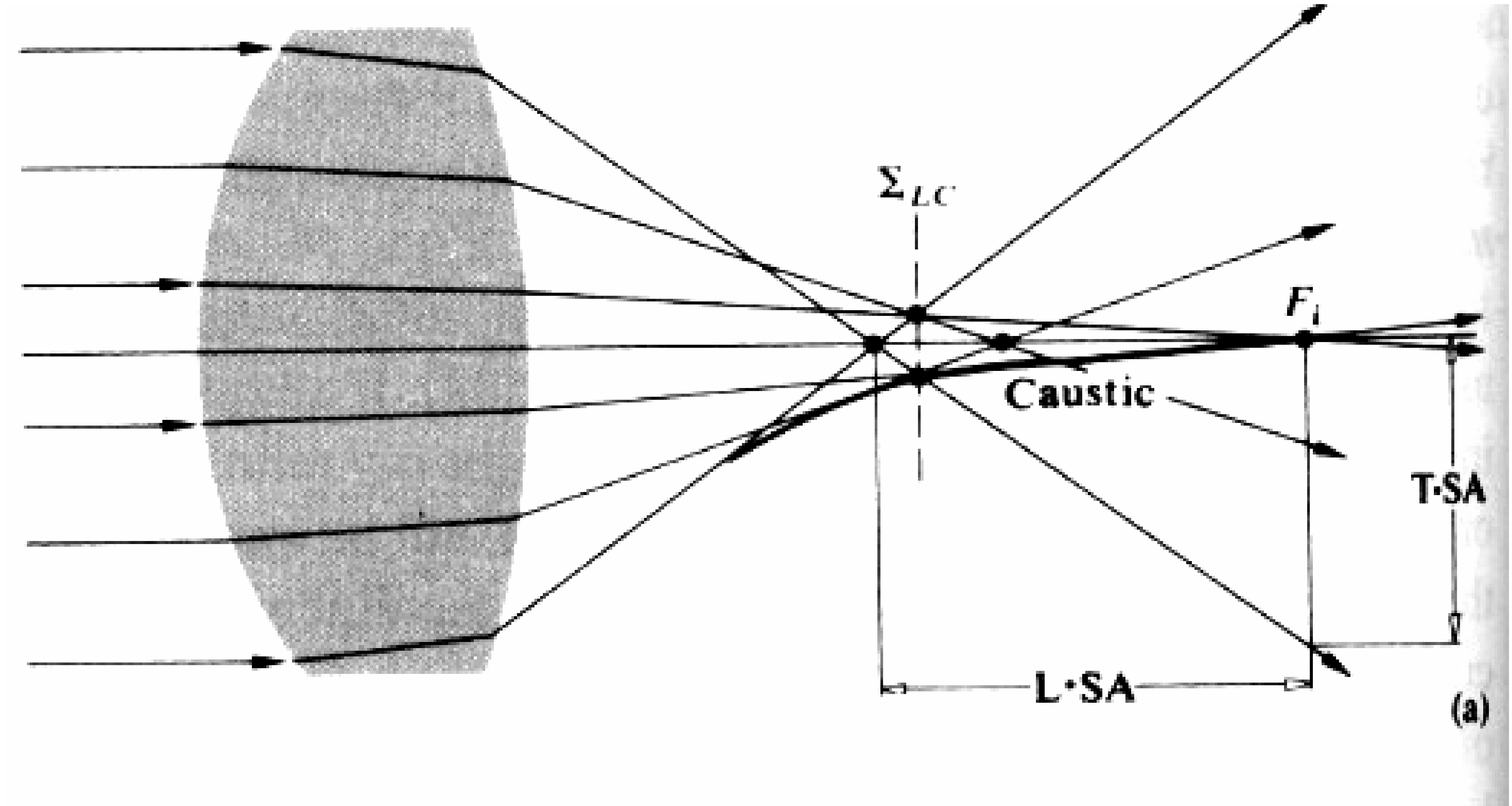
- $\sin\theta = \theta - \theta^3/6$
- **The extra term leads to third-order aberrations**



Third-order aberrations

Spherical aberration

- **Rays don't focus at same position**

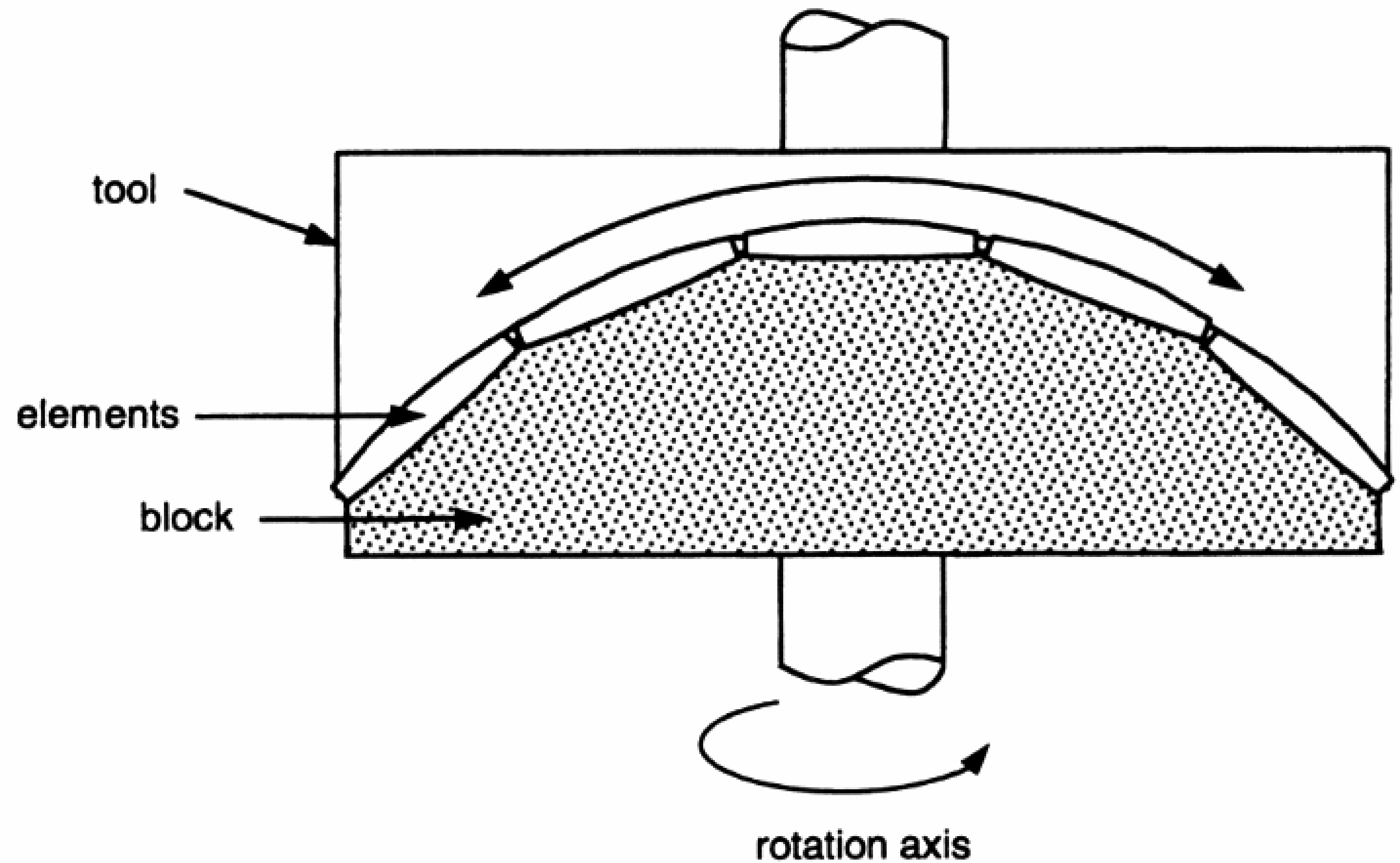


source: Hecht Optics

Why spherical lenses?

- **Because they are easy to manufacture**
- **(Start from whatever shape, if you grind enough, it will become spherical)**

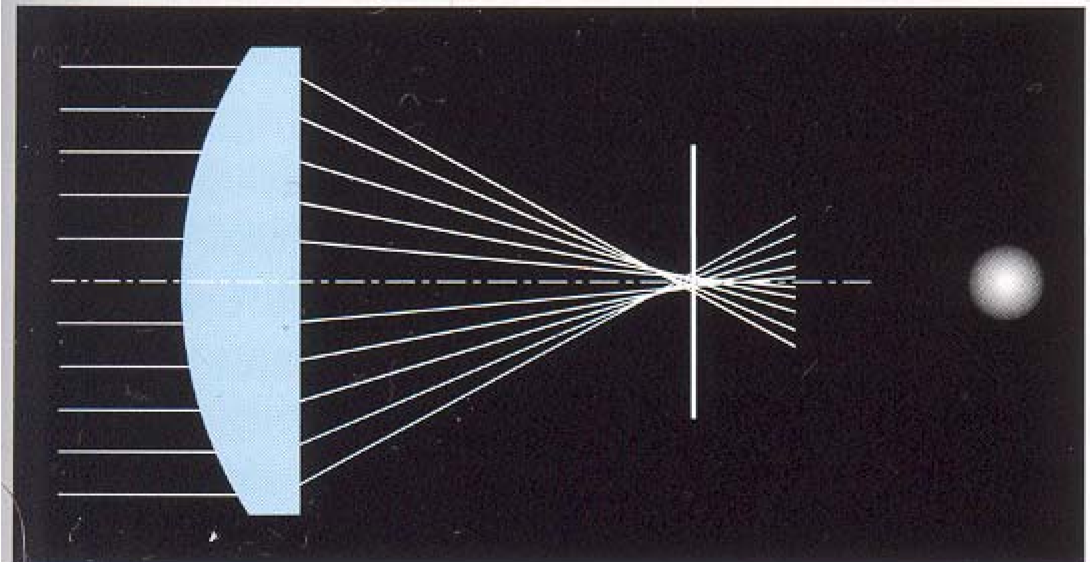
Figure 3.2
Manufacture of Spherical Lens Surfaces



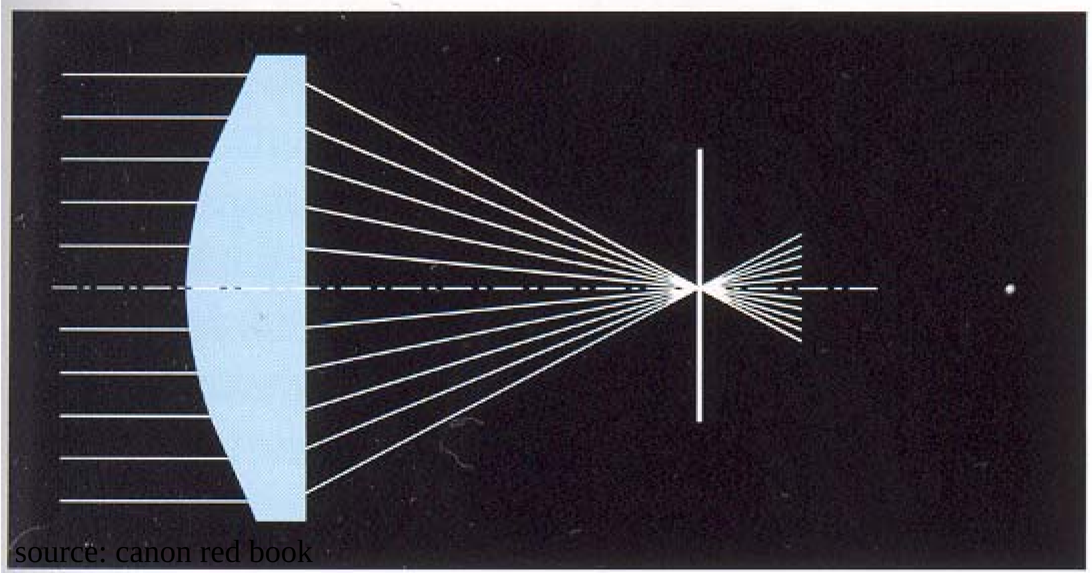
From Optical System Design by Fisher and Tadic

Aspherical lenses

Spherical aberration of spherical lens



Focal point alignment with aspherical lens



source: canon red book

Photo-9 Spherical Lens Example

Photo-10 Aspherical Lens Example

Aspherical lenses

- **Harder to manufacture → used with parsimony**

Figure -14 EF85mm f/1.2L USM Optical System - Ray Tracing Diagram

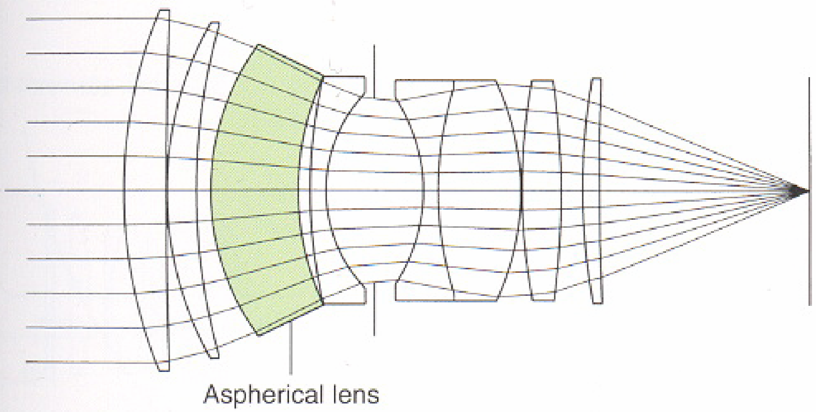
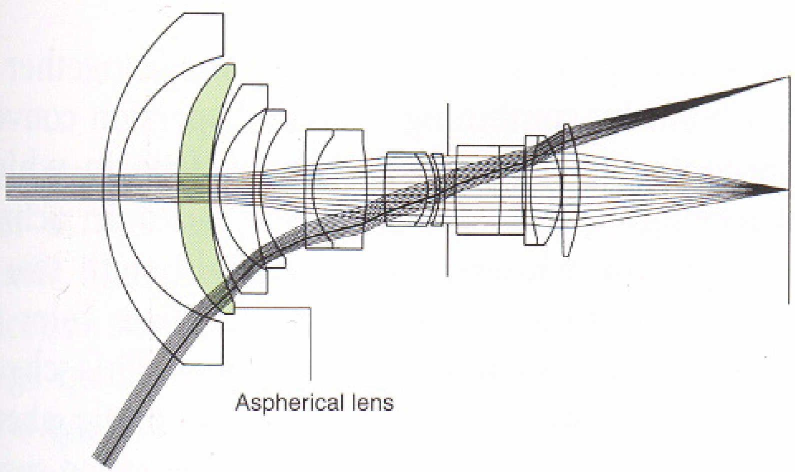


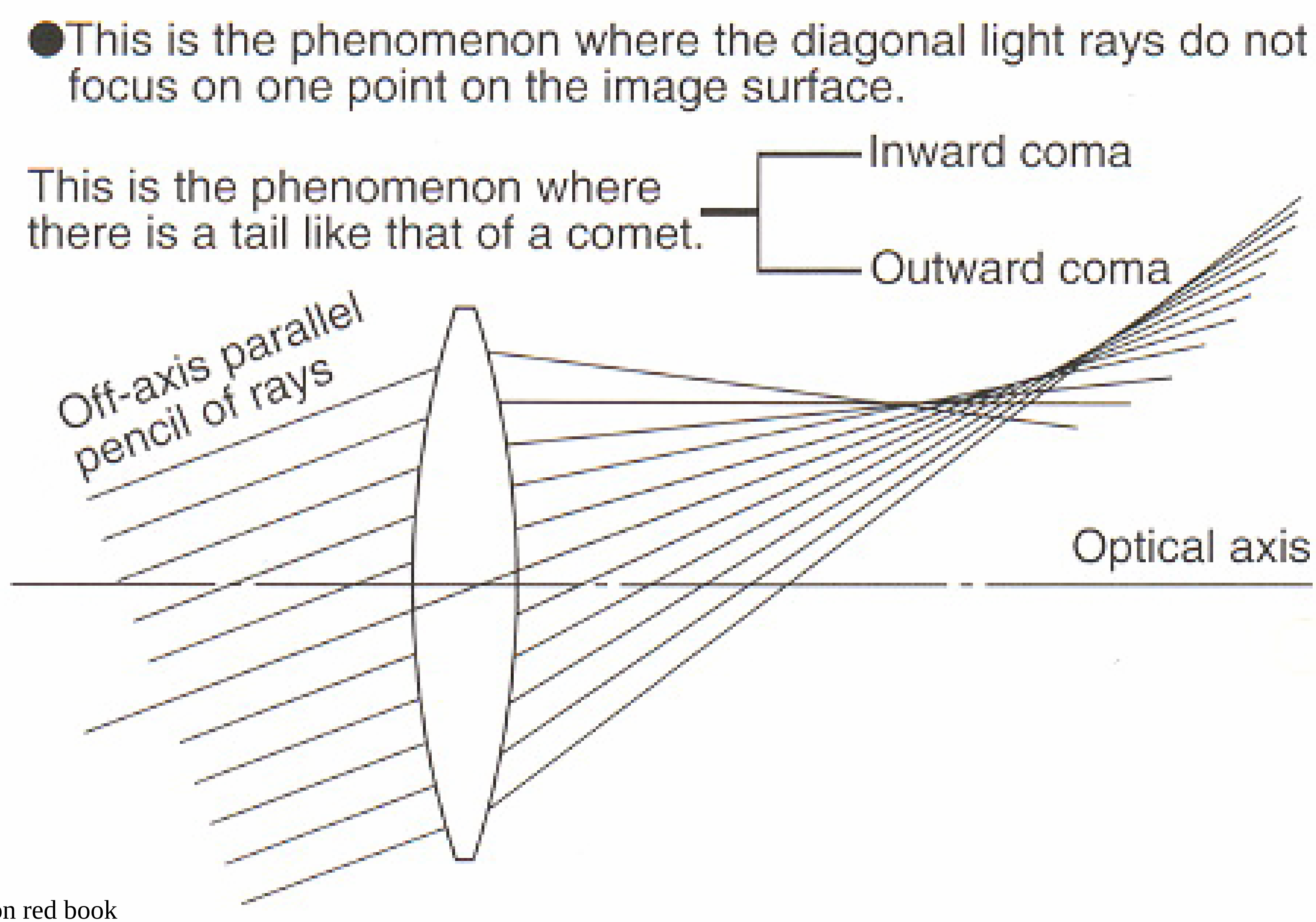
Figure-15 EF14mm f/2.8L USM Optical System - Ray Tracing Diagram



source: canon red book

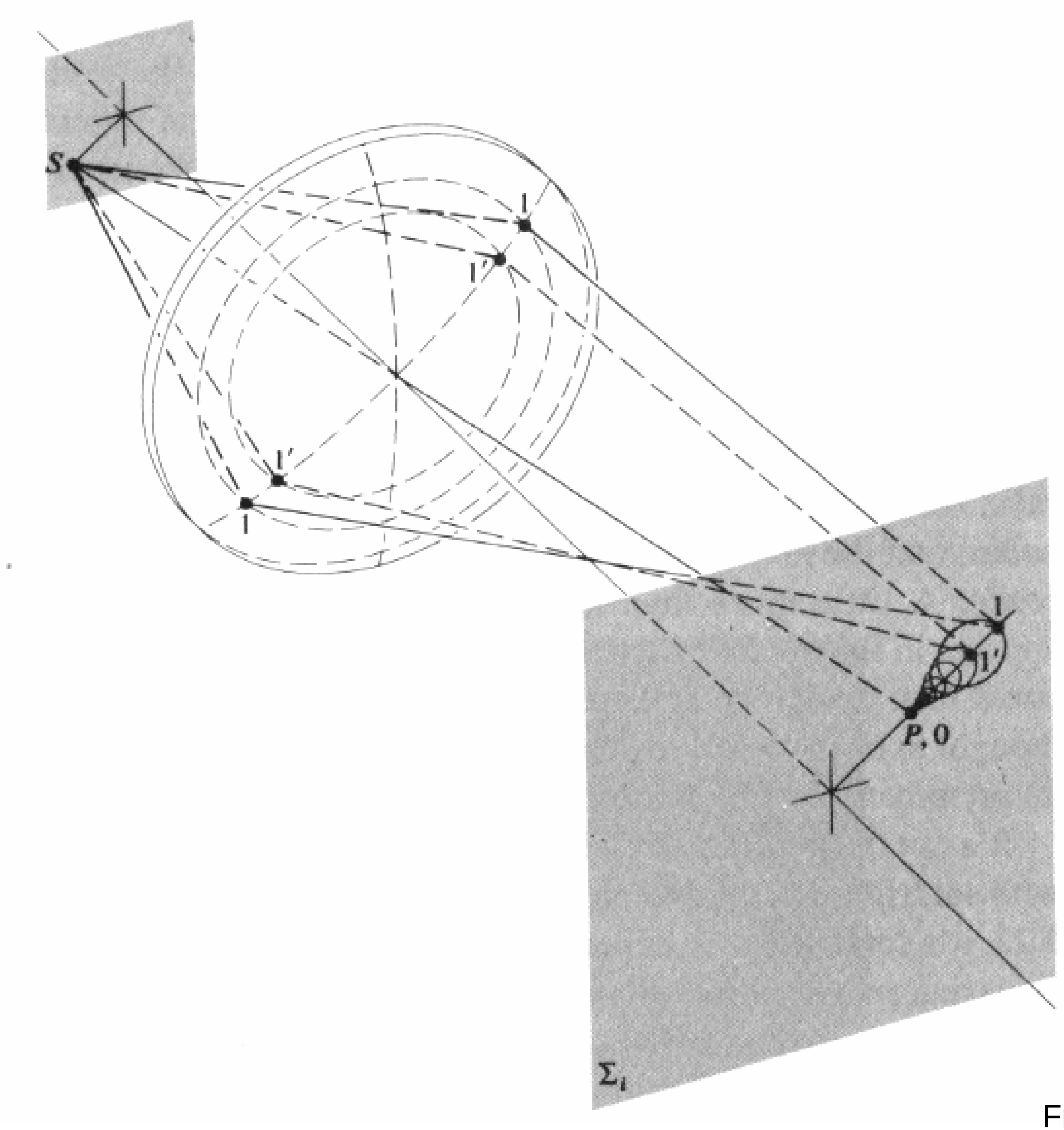
Comatic aberration

Figure-20 Comatic Aberration



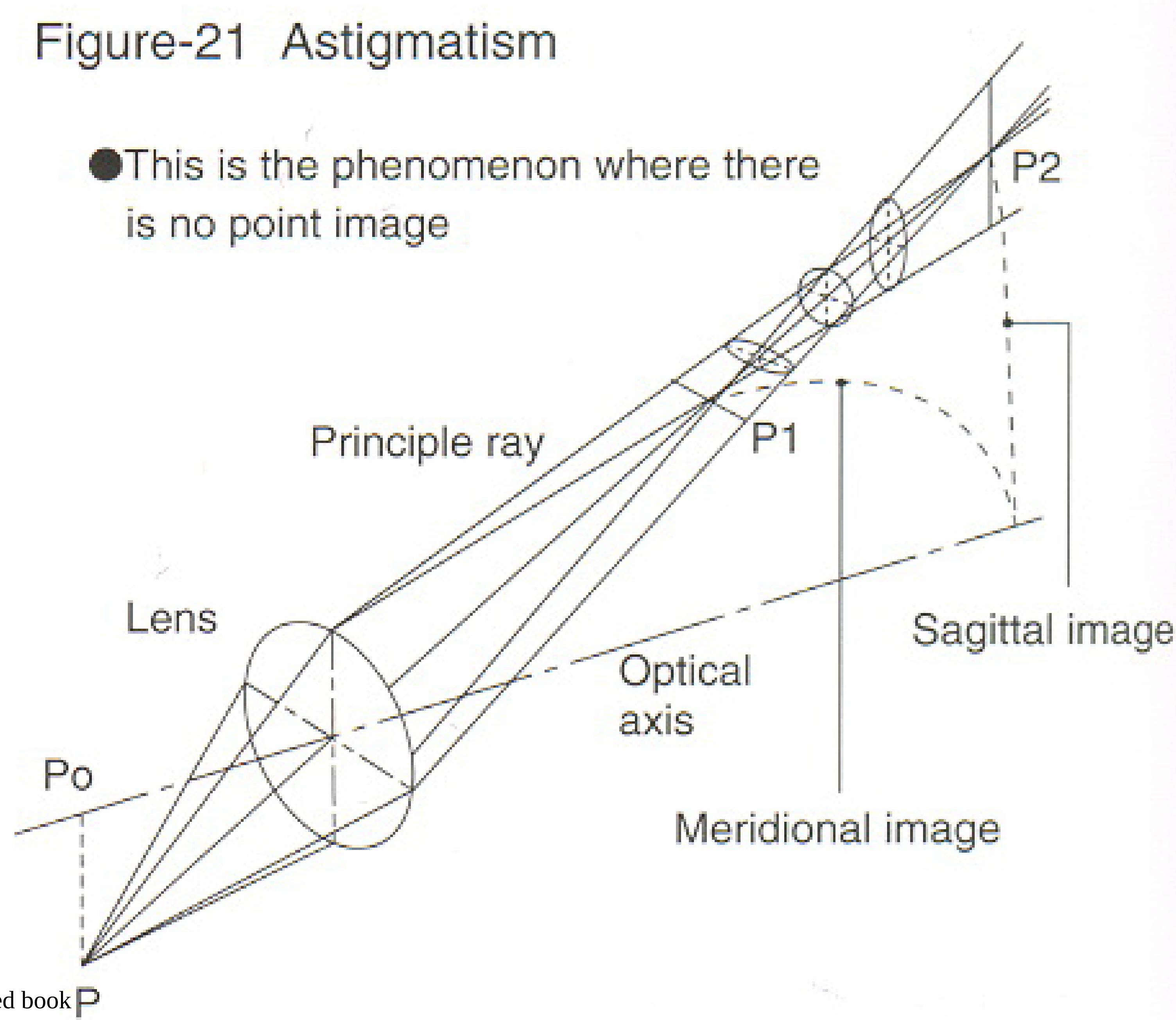
source: canon red book

Comatic aberration



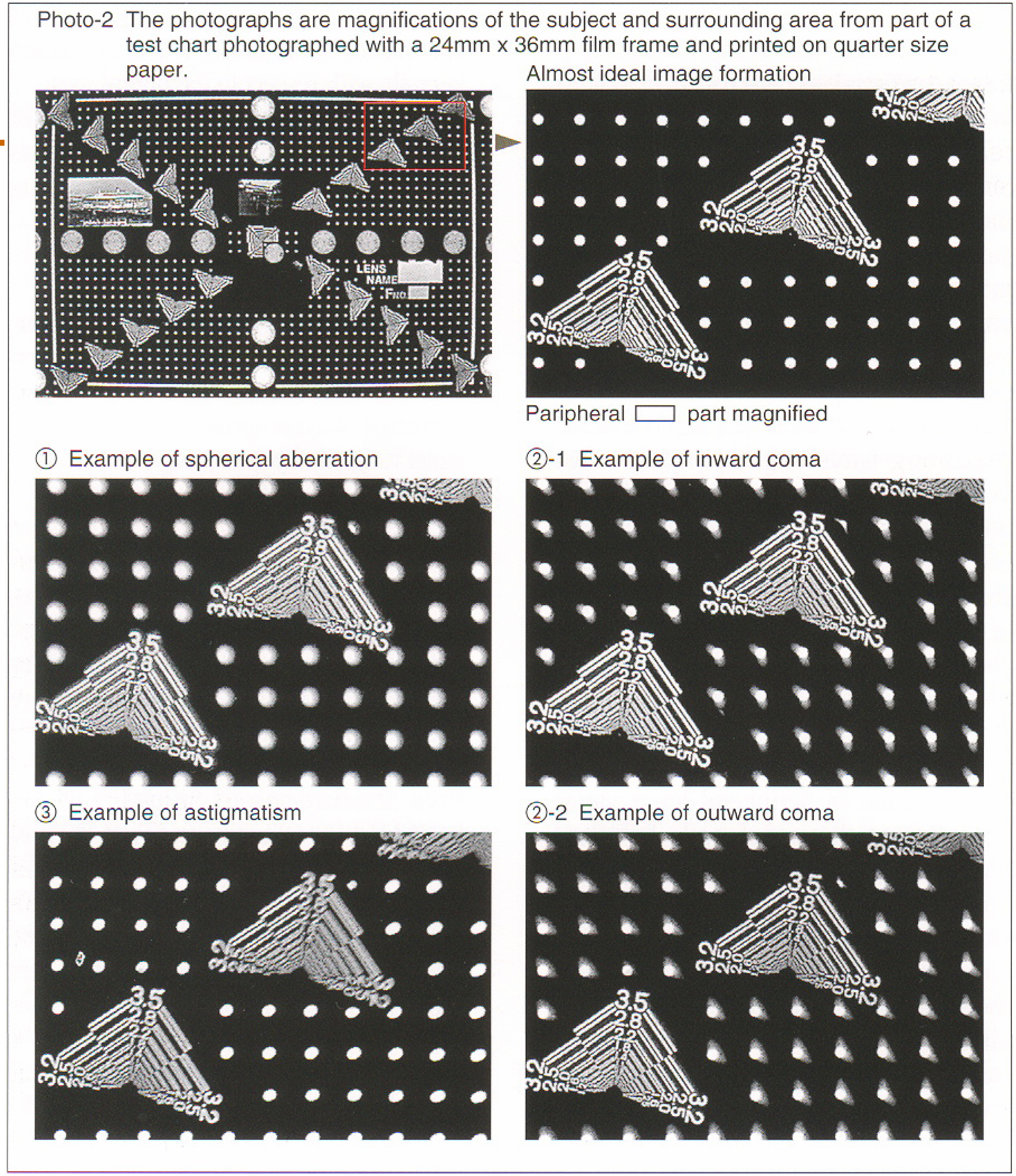
From Hecht's Optics

Astigmatism



source: canon red book

Defects



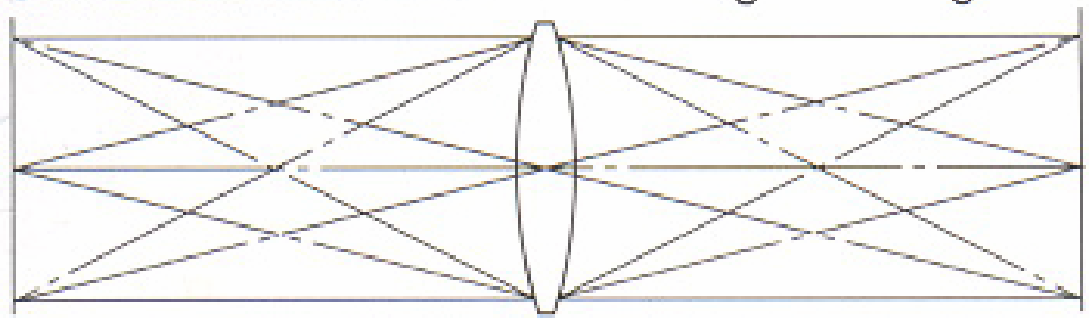
source: canon red book

Curvature of field

Figure-22 Curvature of field

This is the phenomenon where a good image focus surface is bent.

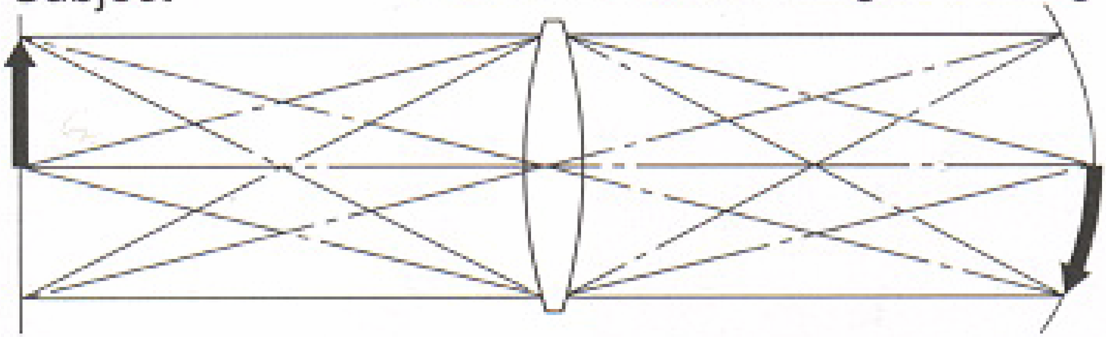
●This is an ideal lens with no image bending.



Subject surface　　Focus surface

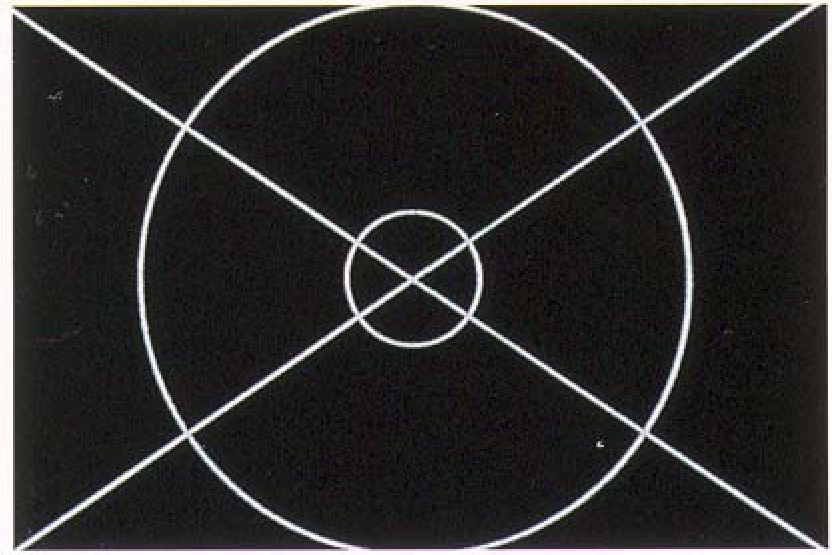
Subject

●Occurrence of image bending



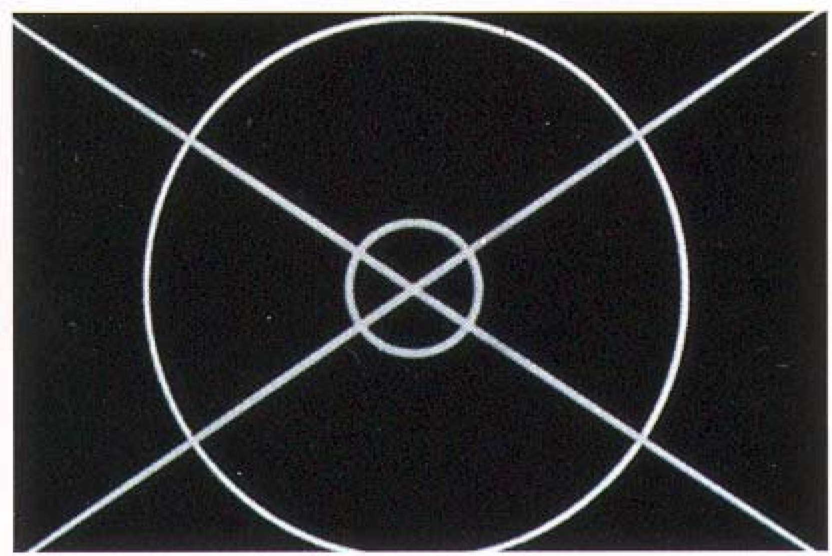
source: canon red book

Photo-5 Example of curvature of field



Focusing on center of screen causes corners to go out of focus.

Photo-6 Example of curvature of field



Focusing on corners of screen causes center to go out of focus.

Curvilinear distortion

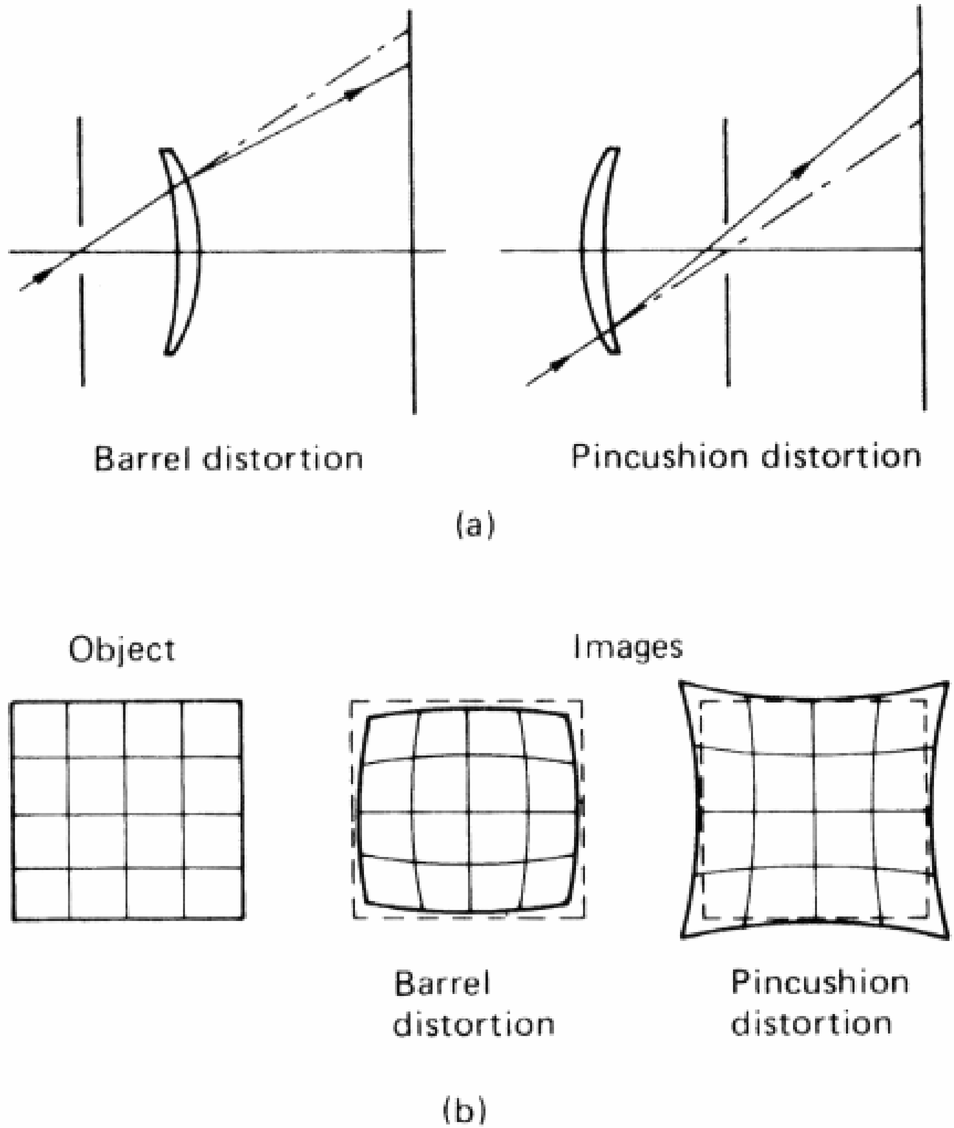


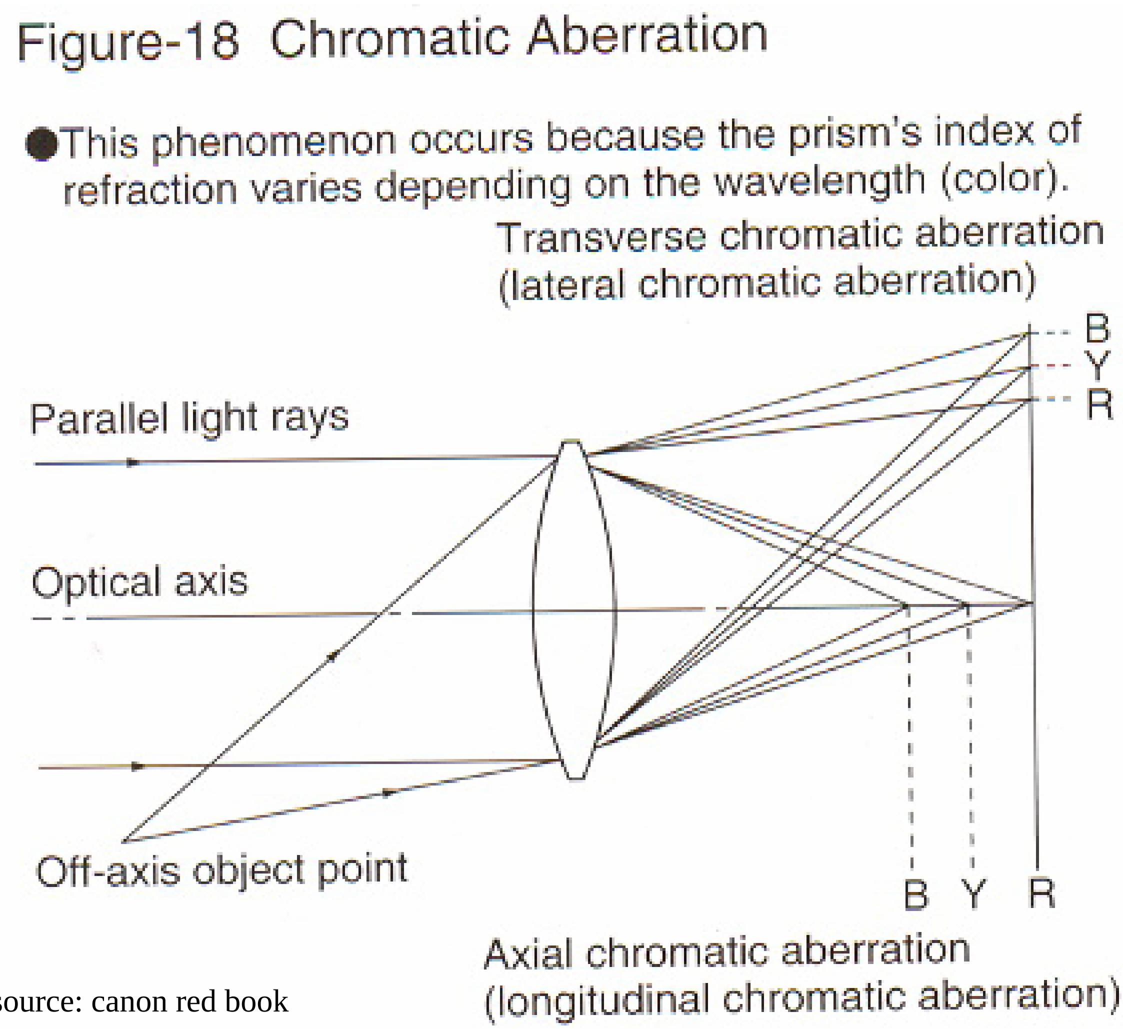
Figure 6.10 The effects of curvilinear distortion. (a) The selection of a geometrically incorrect ray bundle by asymmetric location of the aperture stop. (b) Image shape changes caused by barrel and pincushion distortion

From "The Manual of Photography" Jacobson et al

Chromatic aberrations

Chromatic aberration

- **The previous aberrations depend on wavelength (because of varying index of refraction)**



source: canon red book

Photo-3 Axial chromatic aberration

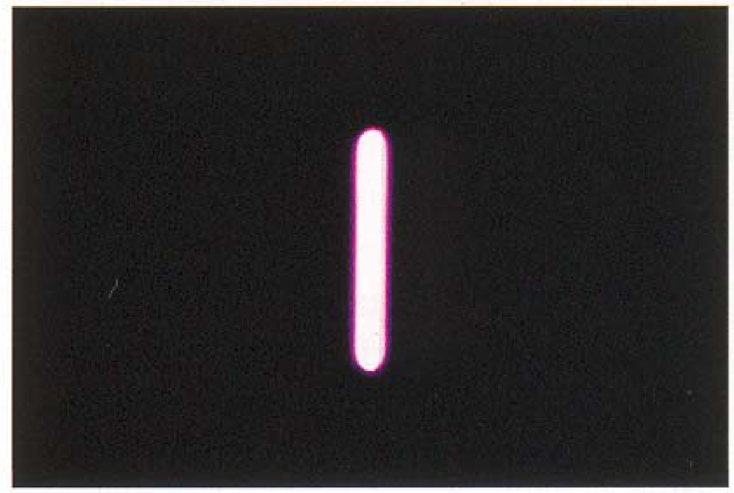
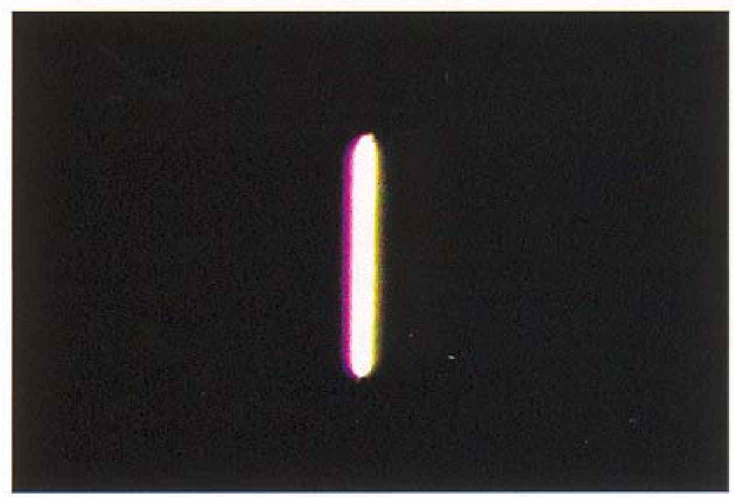


Photo-4 Transverse chromatic aberration



Achromatic doublet

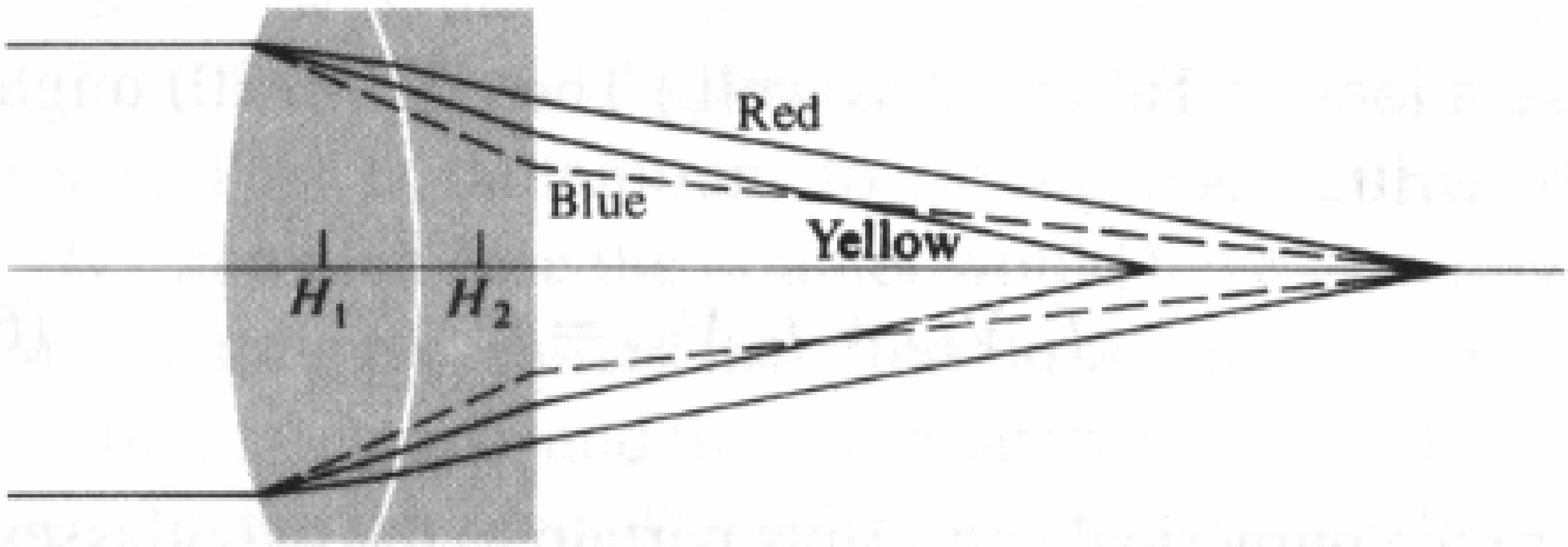


Figure 6.38 An achromatic doublet. The paths of the rays are much exaggerated.

From Hecht's Optics

Apochromatic & others

- **Optimize for multipel wavelengths**
- **http://www.vanwalree.com/optics/chromatic.html**

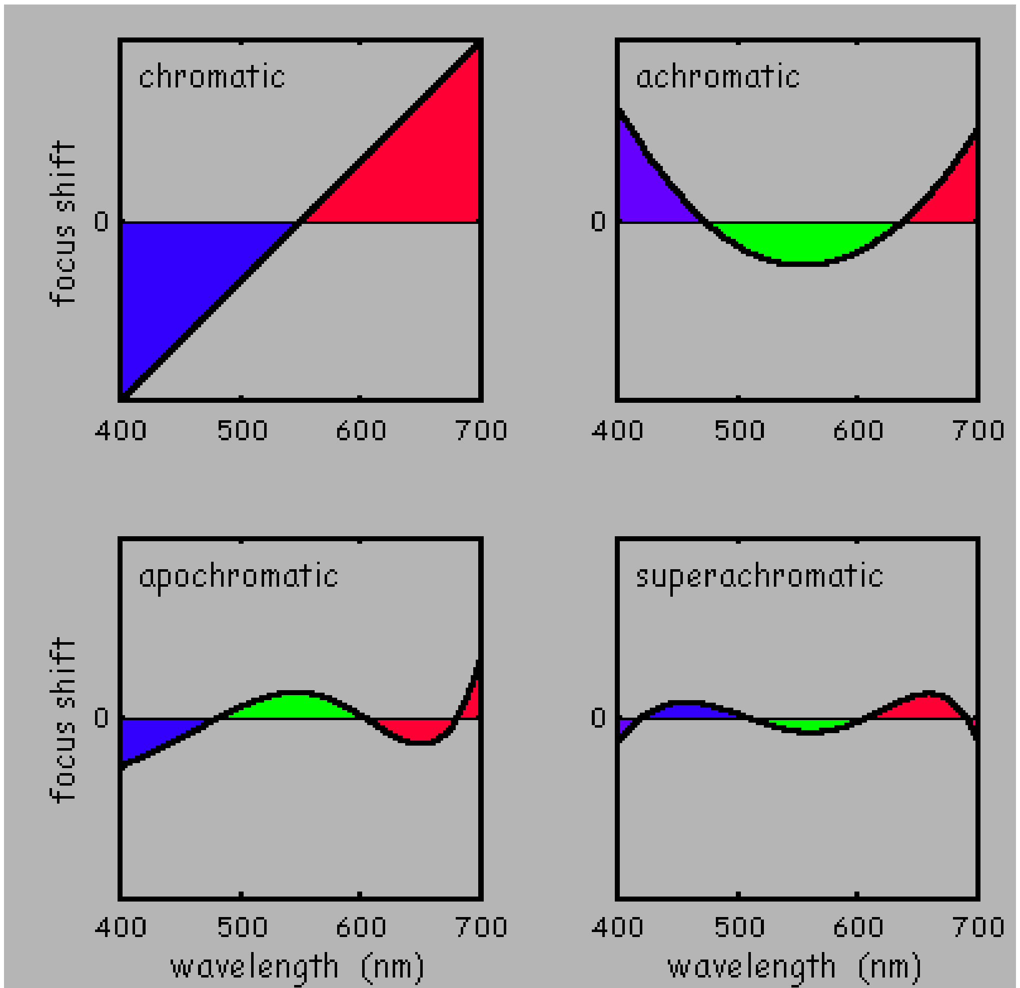
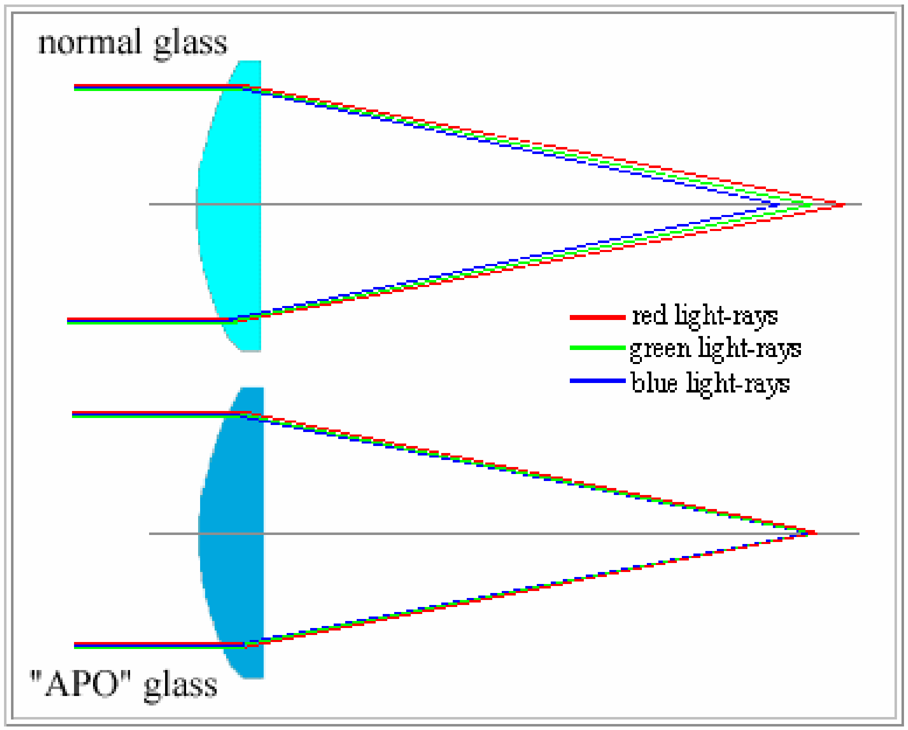


Figure 5. Principles of color correction. The colored faces are known as the secondary spectrum.

Apochromatic glass

APO" elements (UD, SUD, CaF2, LD, SLD, ED etc.) improve contrast and sharpness by reducing chromatic aberration (color defects) that usually occur in tele lenses. These elements are able to focus different wave lengths of one light ray in one point (see picture below). These elements are quite expensive and usually not used for cheaper lenses. The problem is however that the quality of these special elements varies heavily so the effect is often downgraded to a marketing gag - this is especially true for some third-party manufacturers!
As a rule-of-thumbs a good long tele lens will always feature two or more of these special elements. Recently the first ultra-wide and wide-angle lenses emerged using APO elements besides aspericals in order to reduce problems with lateral color shifts.

http://www.photozone.de/3Technology/lenstec8.htm



Fluorite

- **Low dispersion**

Figure-22 Comparison of Color Aberration Correction

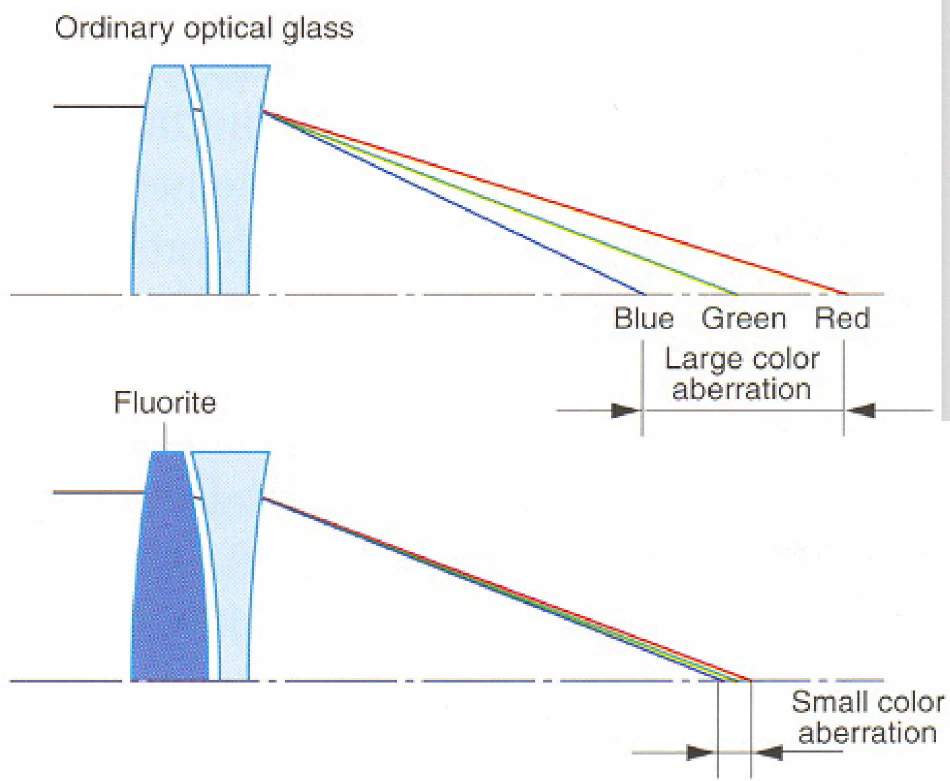


Figure-21 Secondary Spectrum

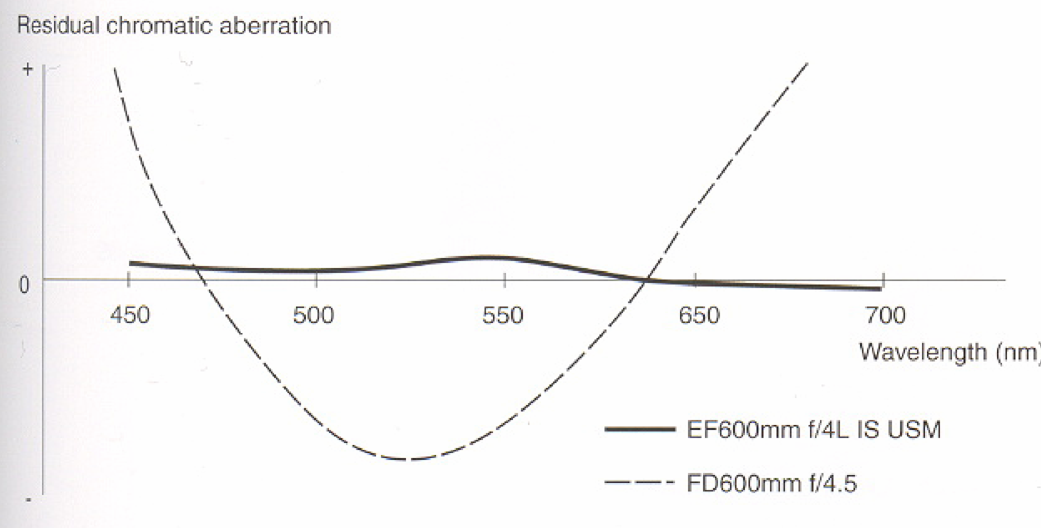


Photo-12 Artificial Fluorite Crystals and Fluorite Lenses

source: canon red book

Diffractive optics (DO)

Figure-58 Diffraction

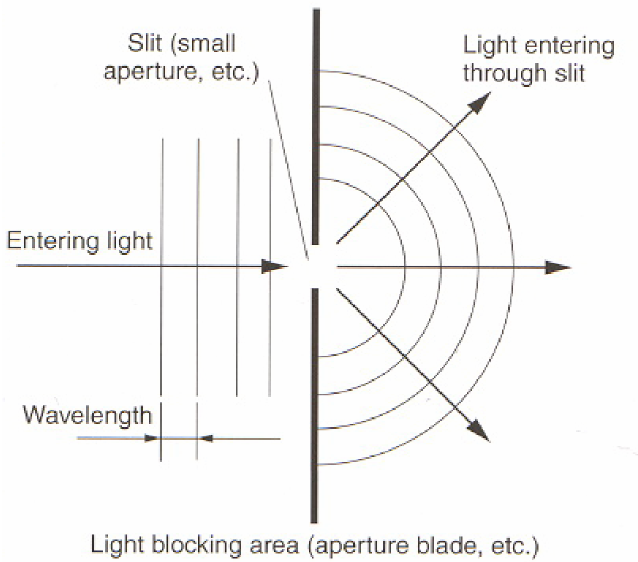
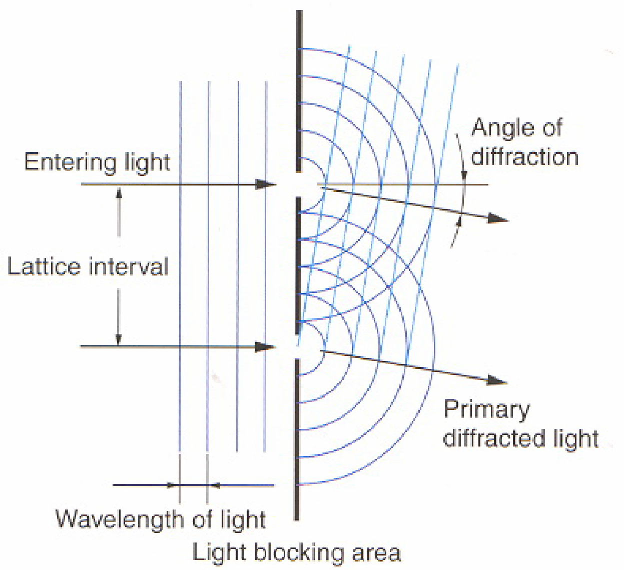
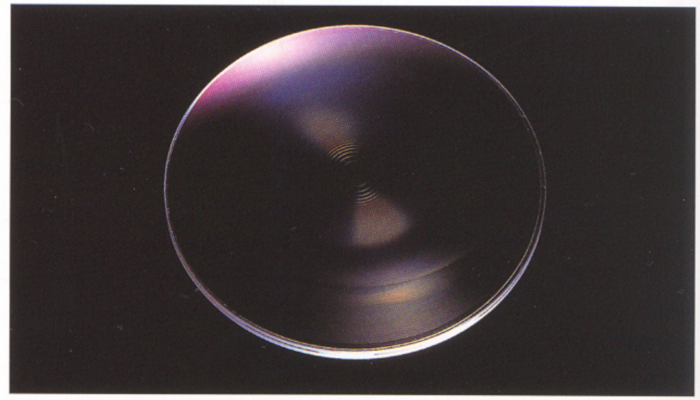


Figure-59 Principle of diffracted light generation

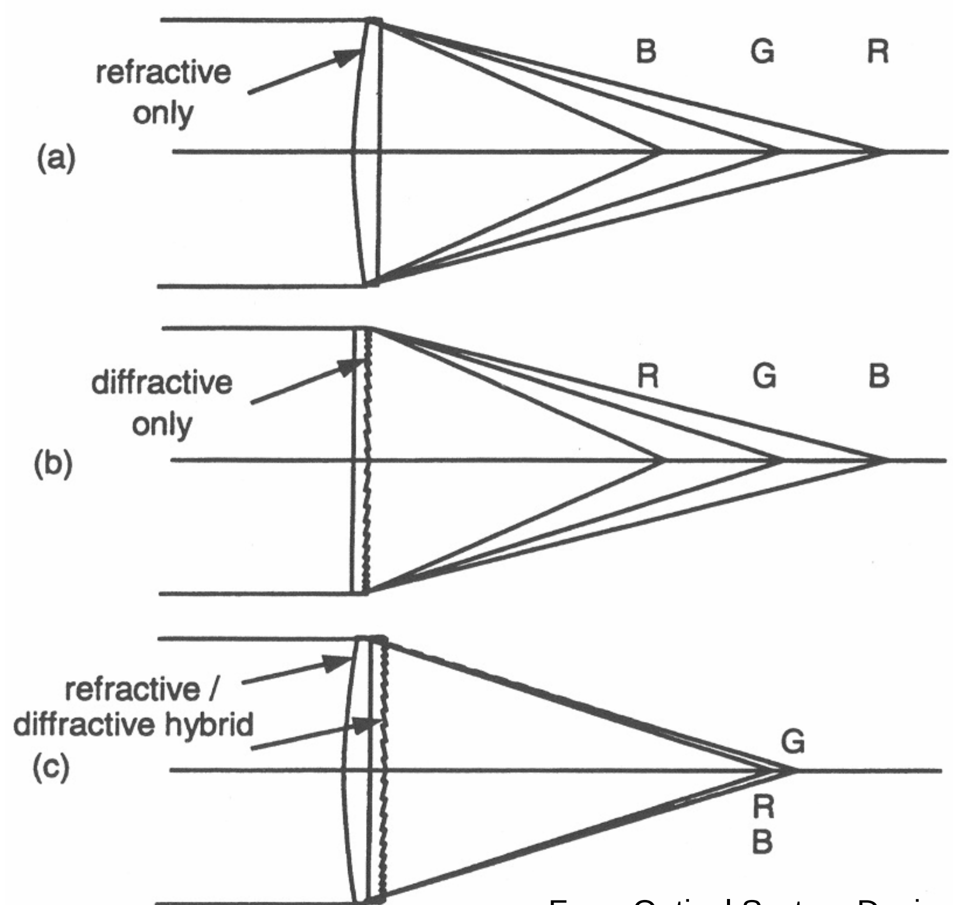


source: canon red book

Photo-23 Multi-Layered Diffractive Optical Element (DO lens)



source: canon red book

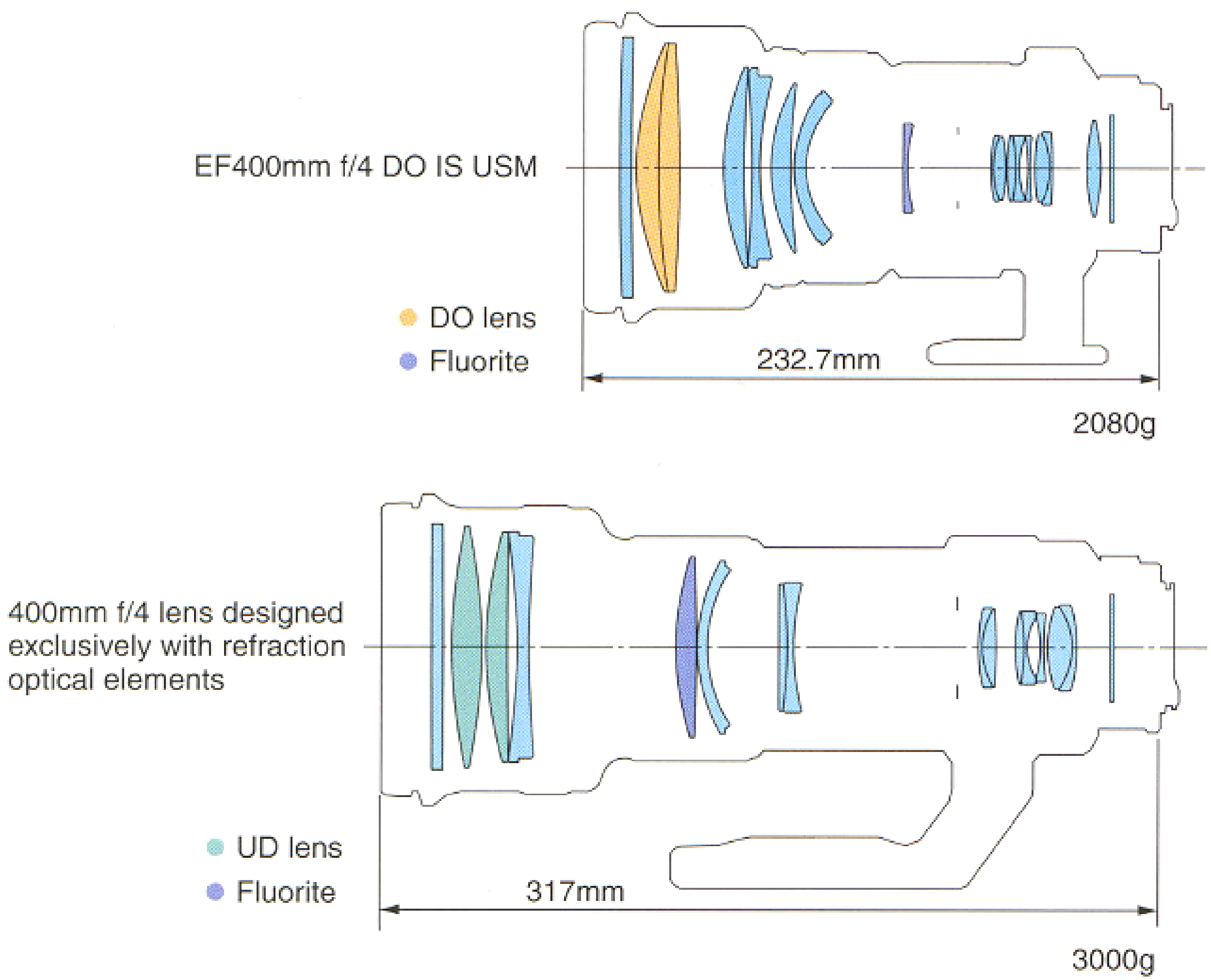


From Optical System Design by Fisher and Tadic

Diffractive optics

- **Enables smaller lenses**

Figure-64 Compact and Lightweight Lenses Thanks to Multi-layered Diffractive Optical Element



source: canon red book

Purple fringing

- http://www.dpreview.com/learn/?/key=chromatic+aberration

"Purple Fringing" and Microlenses

Although the above chromatic aberrations can be purple in color under certain circumstances, "Purple Fringing" usually refers to a typical digital camera phenomenon that is caused by the microlenses. In simplified terms purple fringing is "chromatic aberration at microlens level". As a consequence, purple fringing is visible throughout the image frame, unlike normal chromatic aberration. Edges of contrasty subjects suffer most, especially if the light comes from behind them, as shown in the example below. Blooming tends to increase the visibility of purple fringing.

Example of purple fringing

Software post-processing

Recall Radial distortion

- **Correct for "bending" in wide field of view lenses**

$$
\begin{aligned}
\hat{r}^2 &= \hat{x}^2 + \hat{y}^2 \\
\hat{x}' &= \hat{x}/(1 + \kappa_1 \hat{r}^2 + \kappa_2 \hat{r}^4) \\
\hat{y}' &= \hat{y}/(1 + \kappa_1 \hat{r}^2 + \kappa_2 \hat{r}^4) \\
x &= f\hat{x}'/\hat{z} + x_c \\
y &= f\hat{y}'/\hat{z} + y_c
\end{aligned}
$$

Use this instead of normal projection

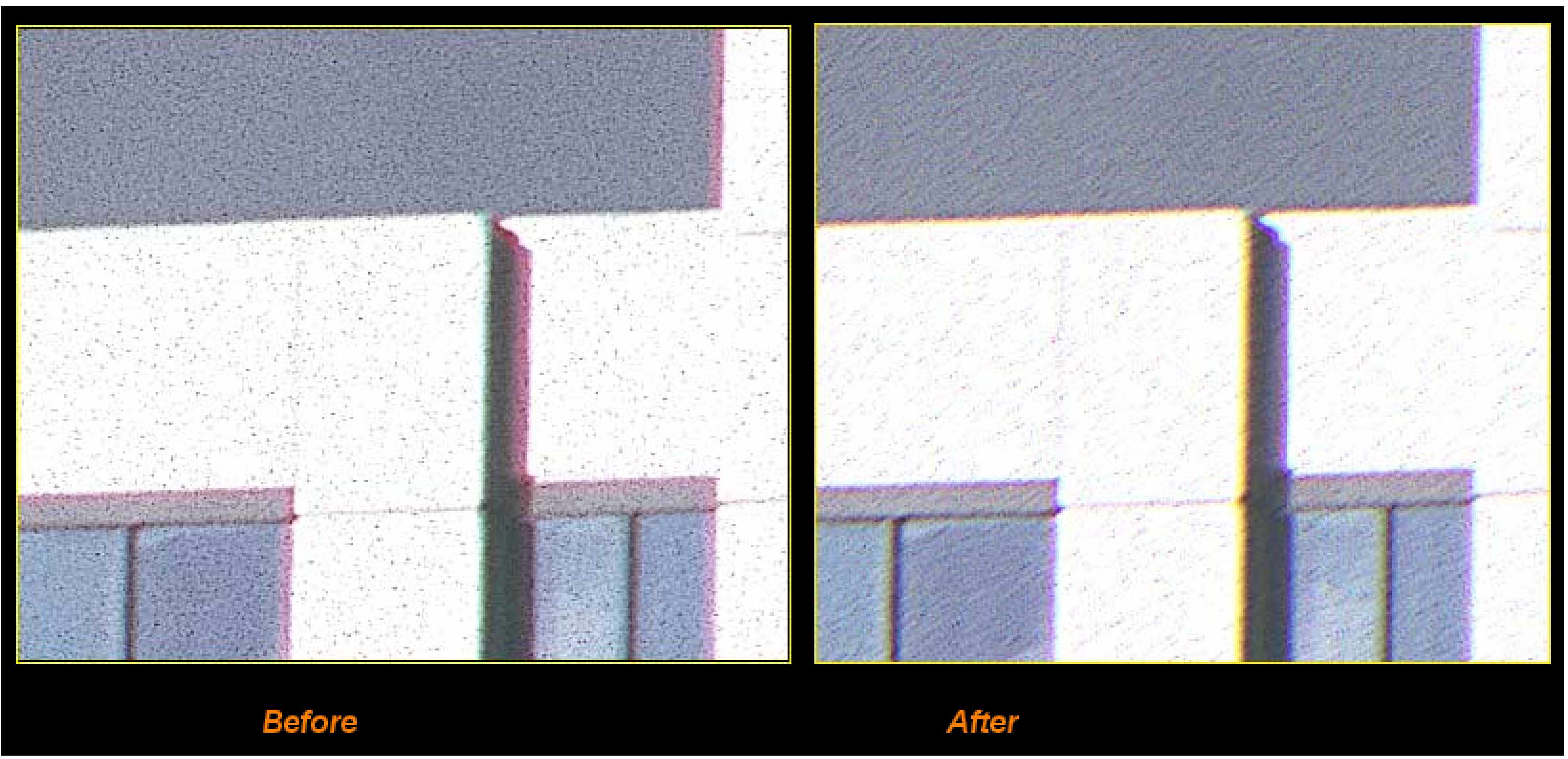
Courthouse in Santa Barbara with Leica Digilux 2

Fix with PTLens 4.1

source digital outback

General principle

- **Calibrate lens**
- **Perform image warp**
- **Perform different warps for various color channels**

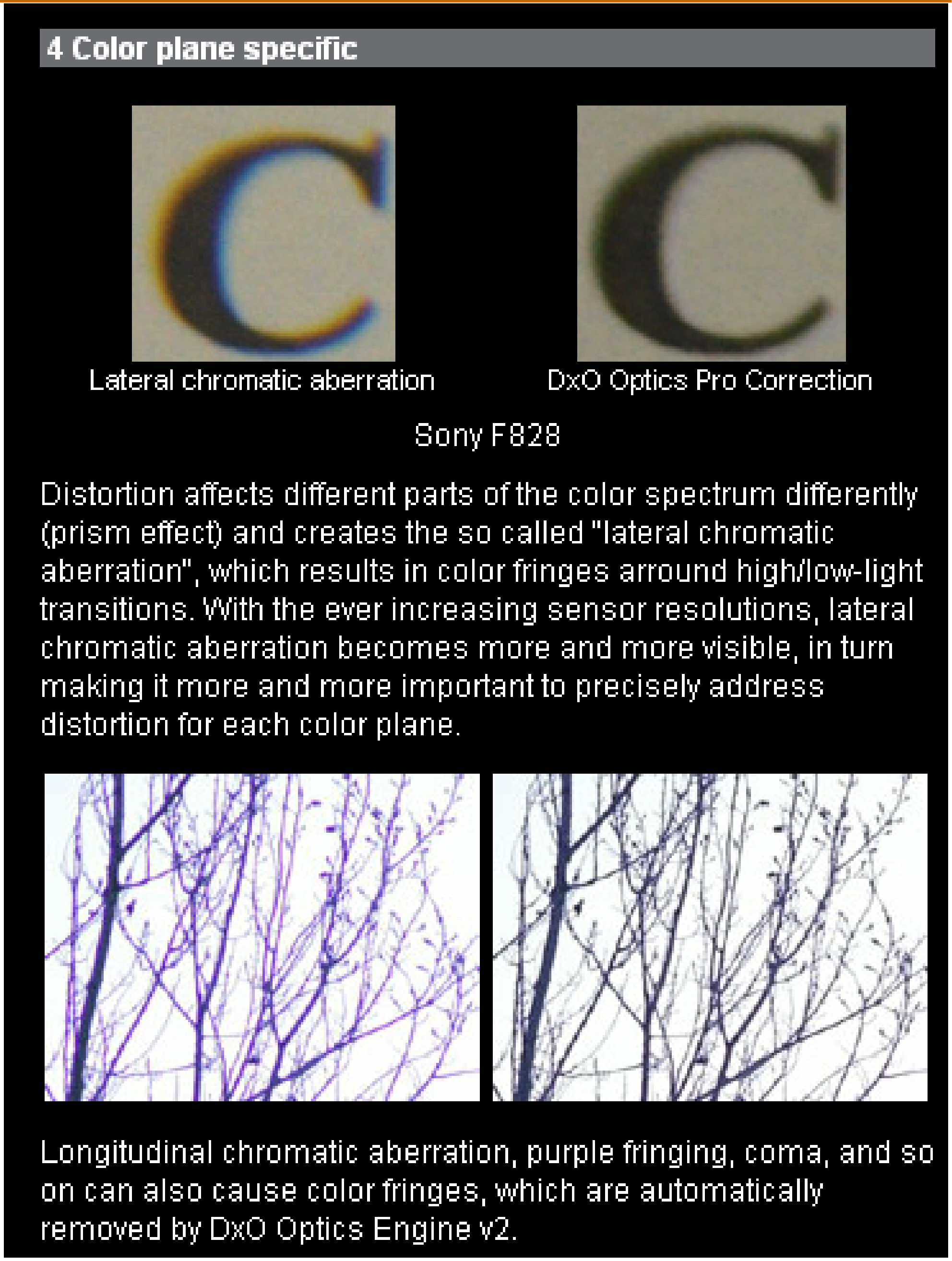


Corrected with
Picture Window 3.1

From the luminous landscape http://www.luminous-landscape.com/reviews/chromatic.shtml

Software

- http://www.dxo.com/en/photo/dxo_optics_pro/technology_distortion.php
- http://www.dl-c.com/Temp/
- http://www.tawbaware.com/maxlyons/pano12ml.htm



From DXO

Other quality issues

Flare

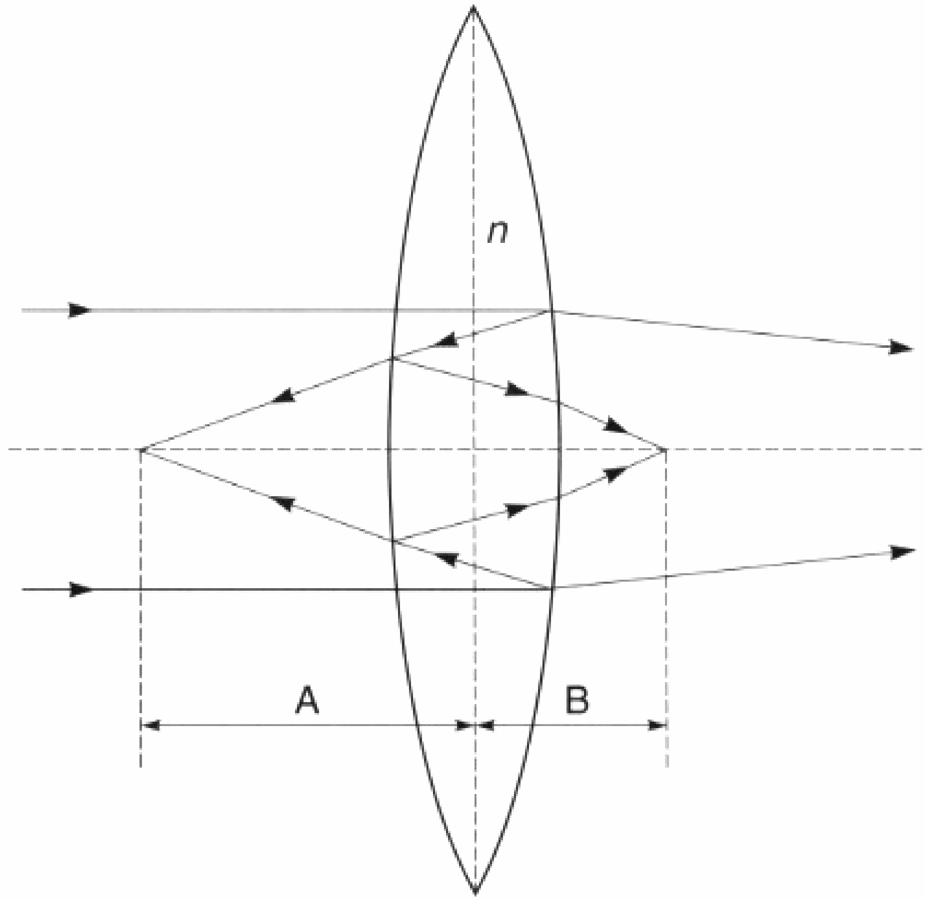


Figure 5.6 Formation of flare spots by a simple lens. Images of the source are formed at distances A and B, where:

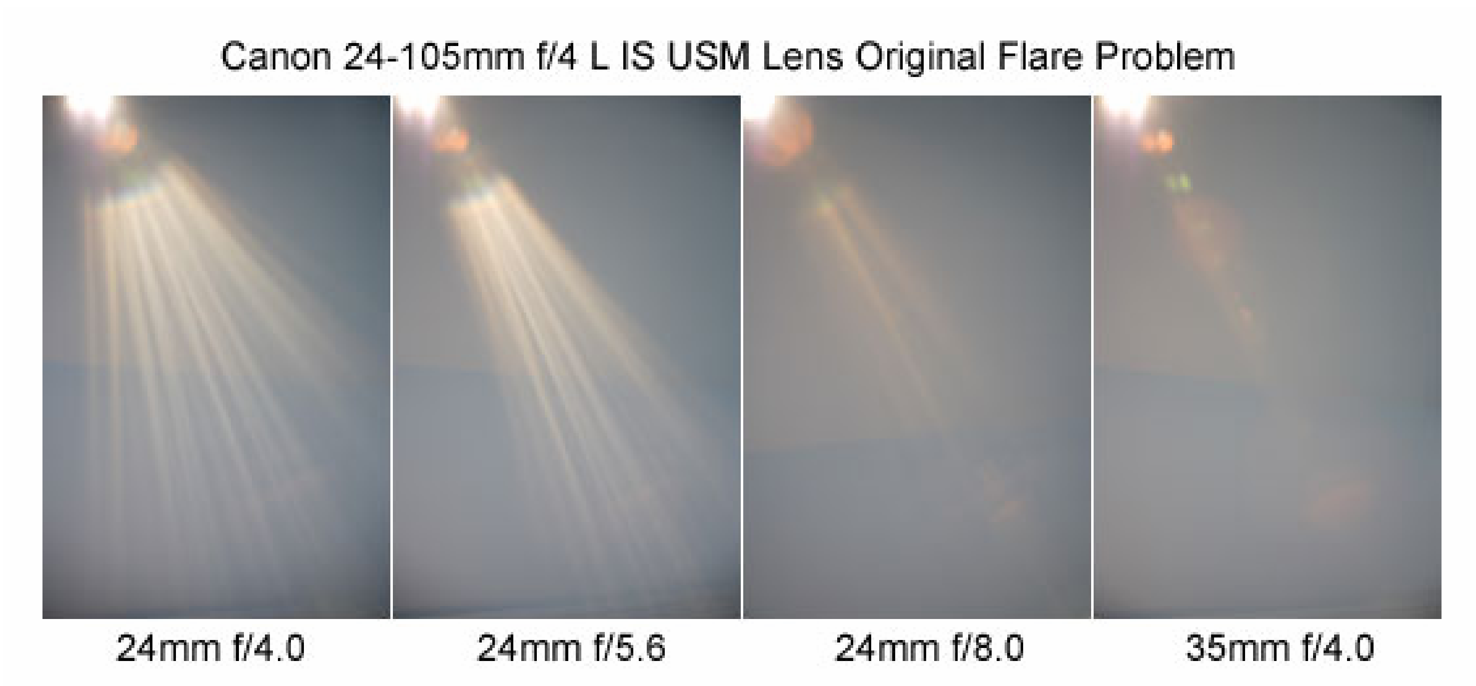
$$A = \left(\frac{n-1}{an-1}\right)f \qquad B = \left(\frac{n-1}{bn-1}\right)f$$

and $a = 2, 4, 6 \ldots$, $b = 3, 5, 7, \ldots$ For $n = 1.5$, $A = f/4$, $f/10$, $f/16$ etc. and $B = f/7$, $f/13$, $f/19$ etc.

From "The Manual of Photography" Jacobson et al

Example of flare "bug"

- **Some of the first copies of the Canon 24-105 L had big flare problems**
- **http://www.the-digital-picture.com/Reviews/Canon-EF-24-105mm-f-4-L-IS-USM-Lens-Review.aspx**
-



Coating

- **Use destructive interferences**
- **Optimized for one wavelength**

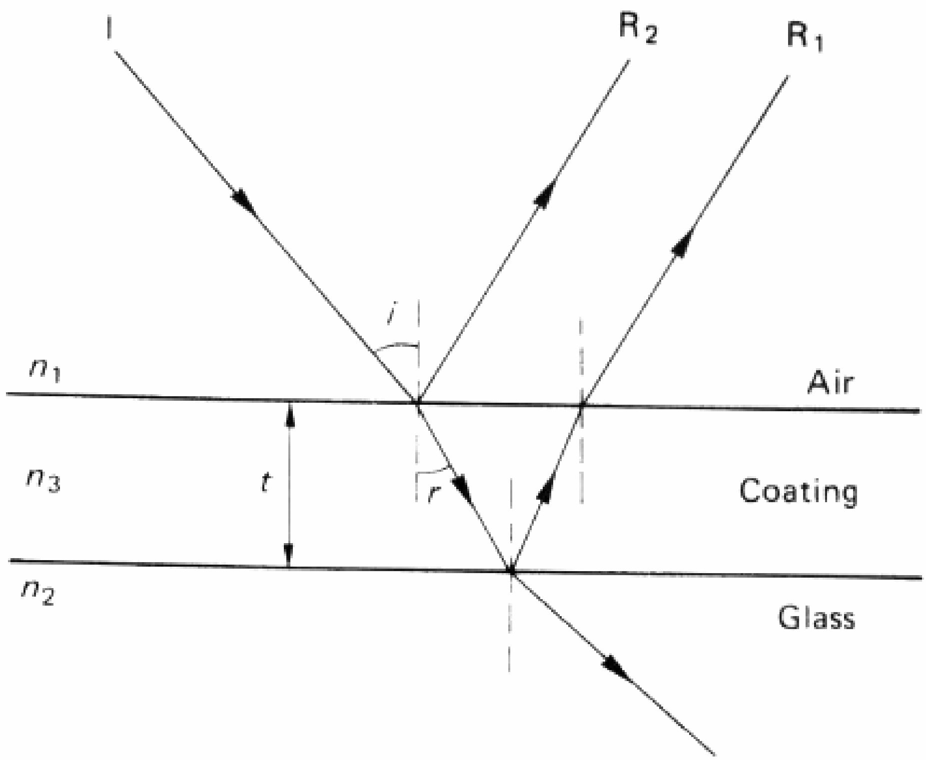


Figure 5.7 An anti-reflection coating on glass using the principle of destructive interference of light between reflections R_1 and R_2

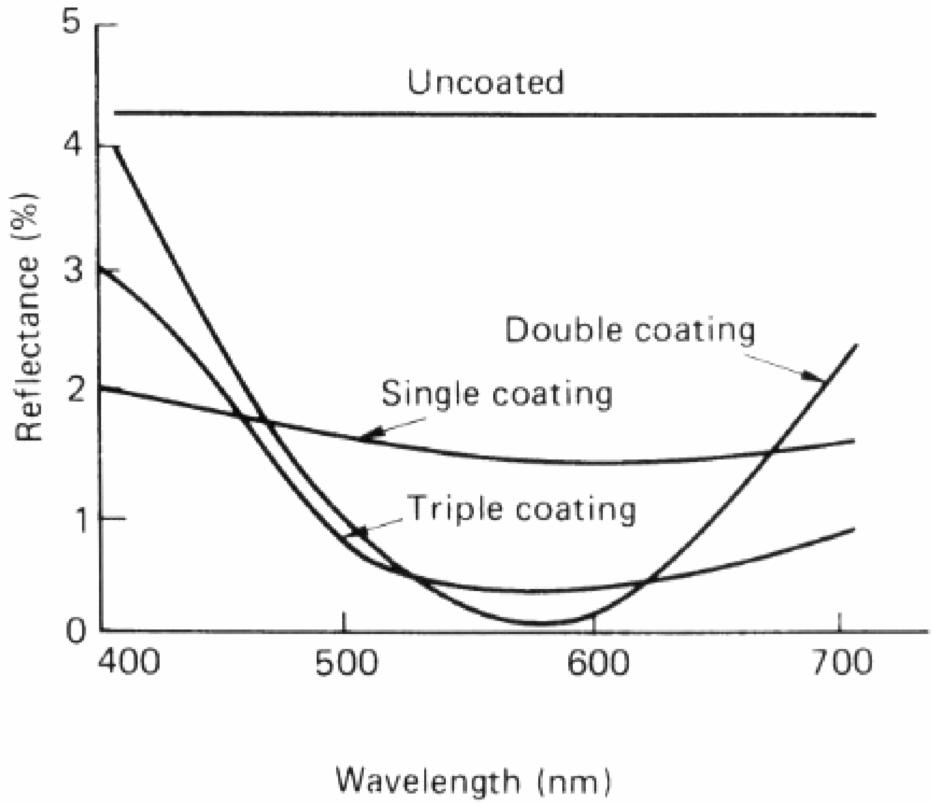
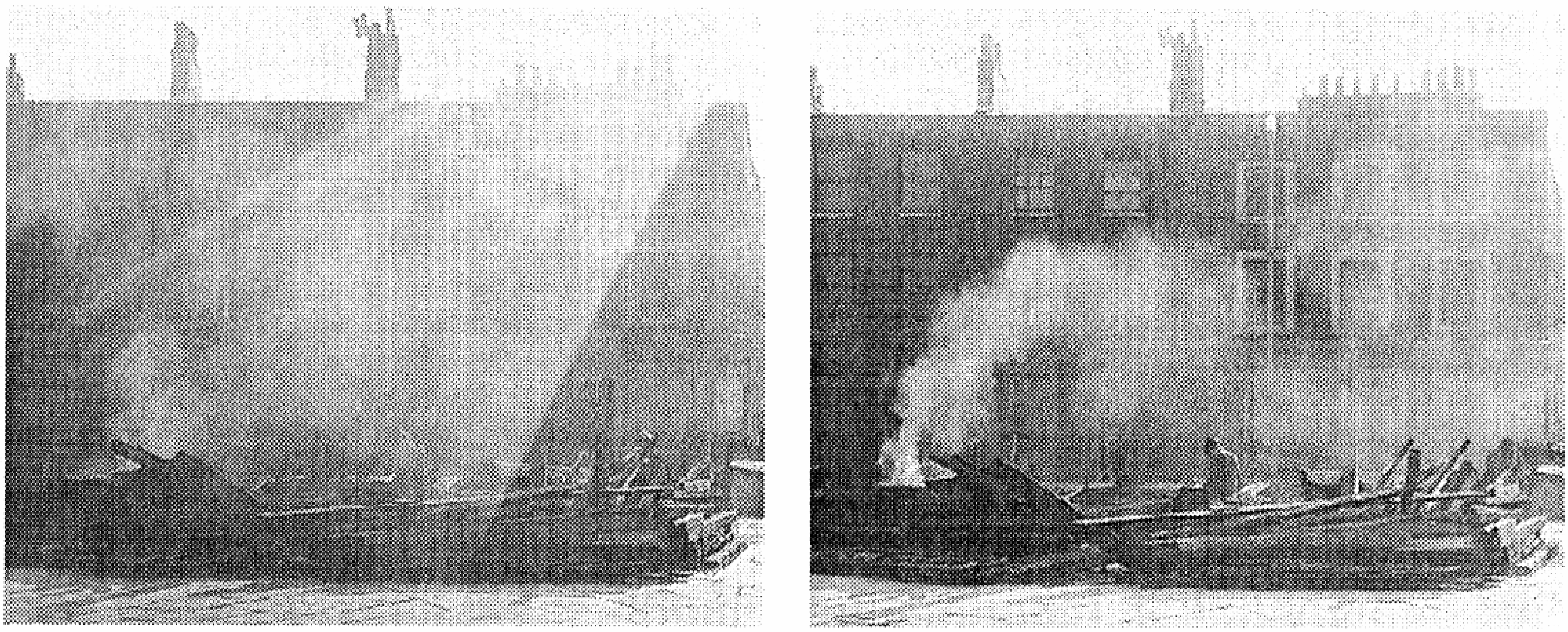


Figure 5.8 The effects on surface reflection of various types of anti-reflection coatings as compared with uncoated glass (for a single lens surface at normal incidence)

From "The Manual of Photography" Jacobson et al

Coating



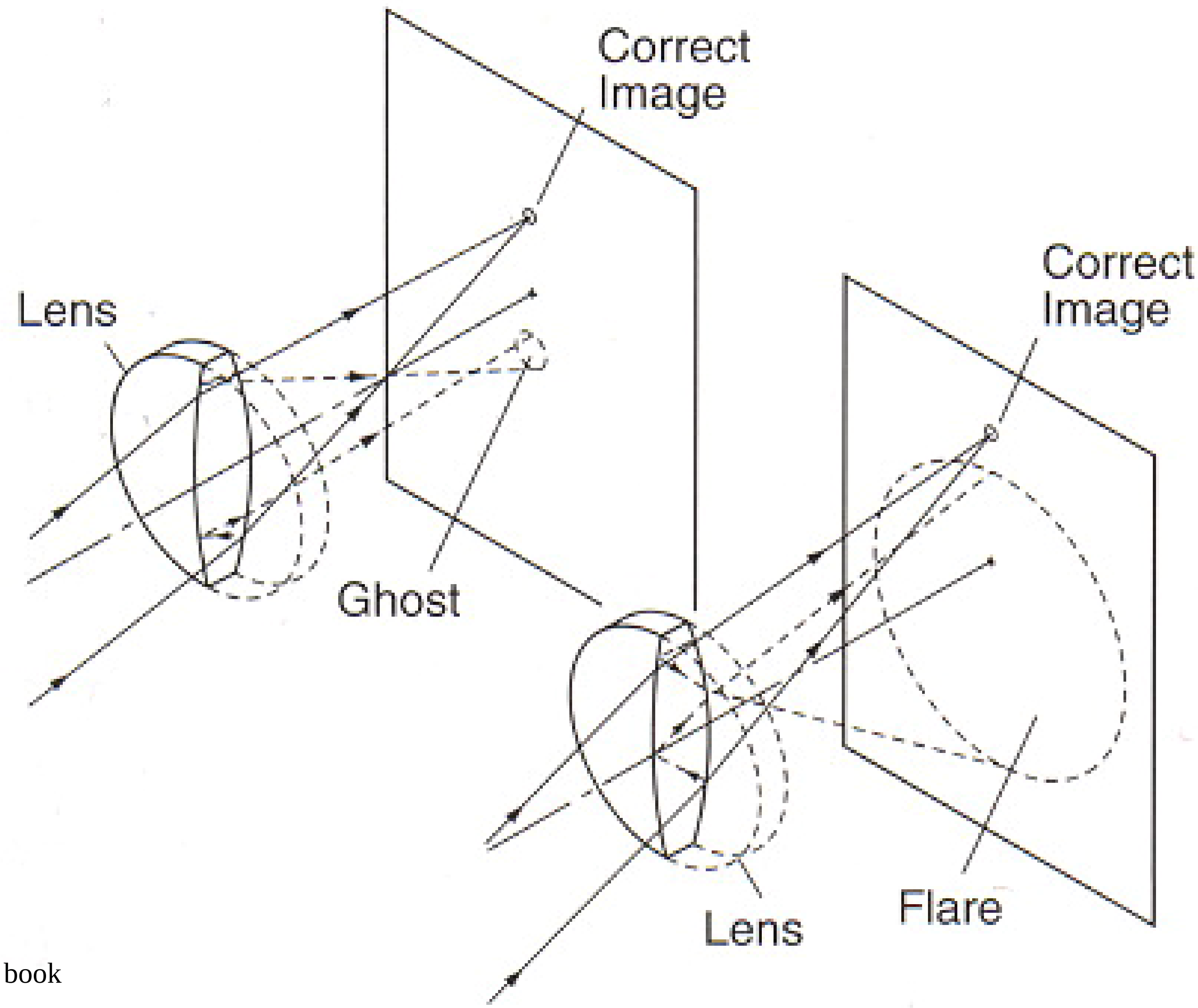
(a) (b)

Plate 15.1 Lens flare with an uncoated lens
(a) Flare effects. (b) Reduction of flare by use of a lens-hood.

From Ray's Applied Photographic Optics

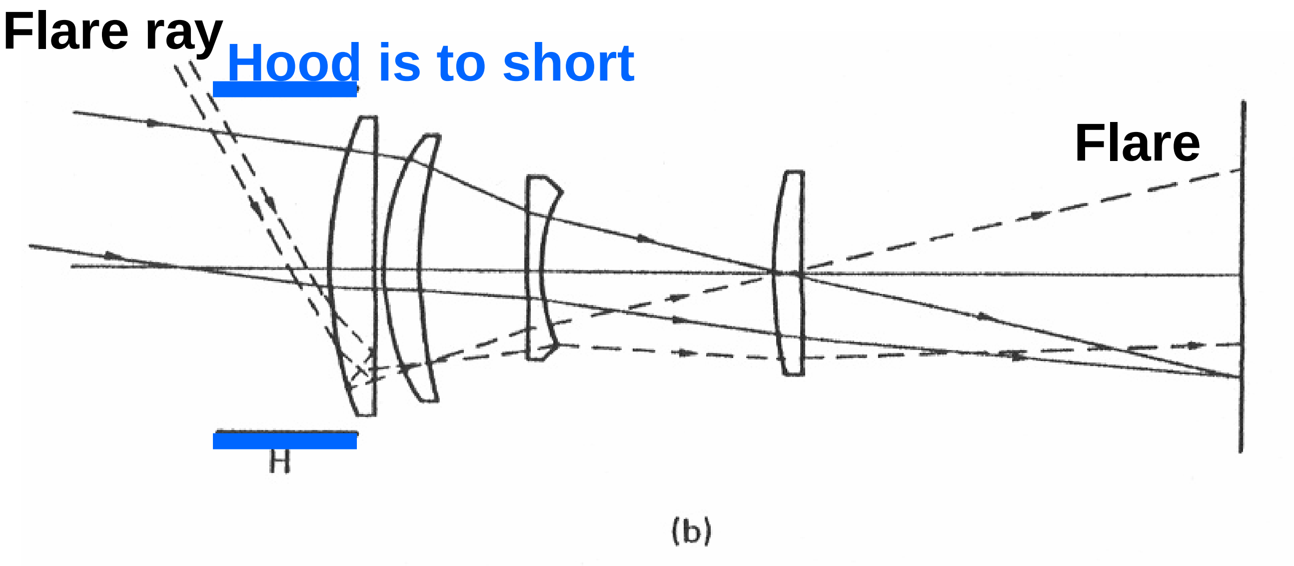
Flare and Ghosting

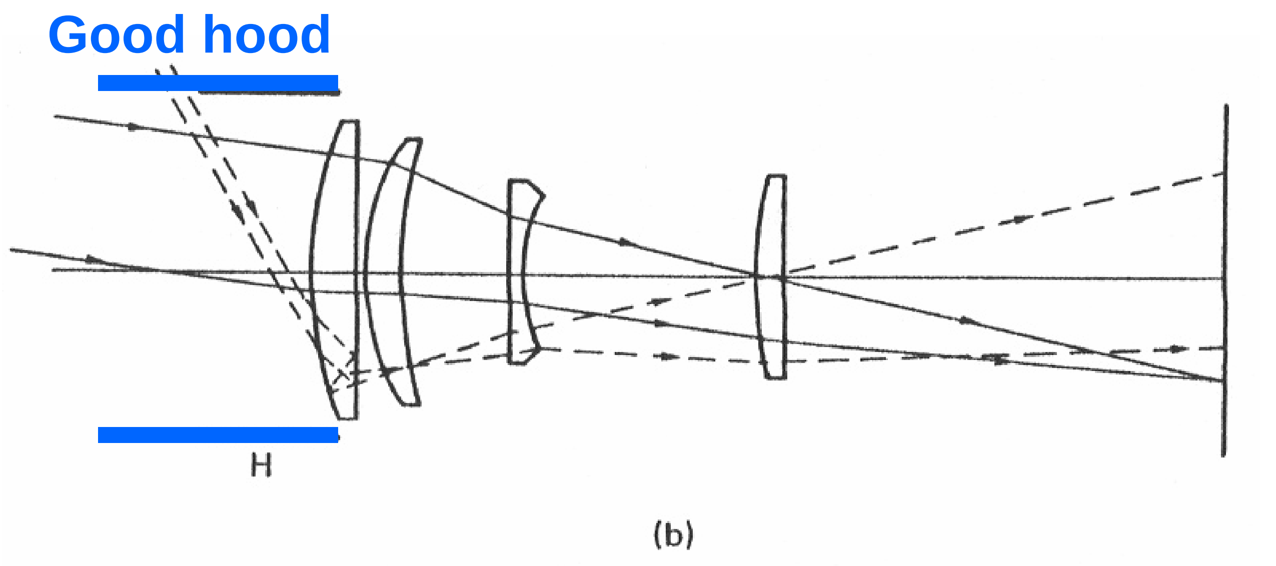
Figure-29 Flare and Ghosting



source: canon red book

Use a hood! (and a good one)





Adapted from Ray's Applied Photographic Optics

Fighting reflections

Figure-35 EF28-135mm f/3.5-5.6 IS USM flare cut moving aperture diaphragm

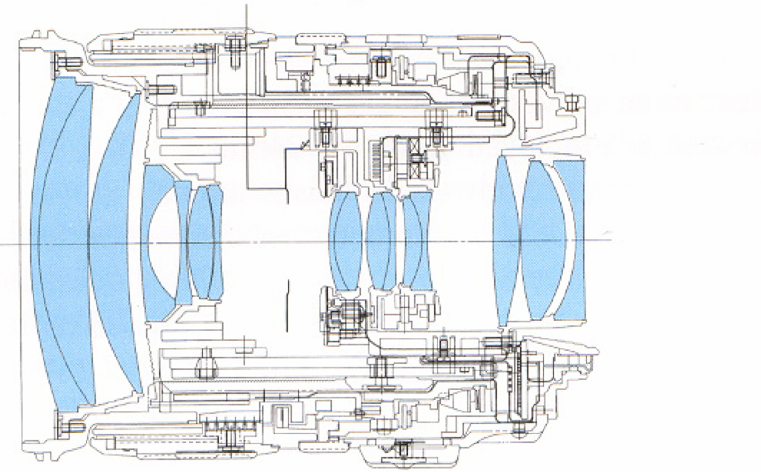
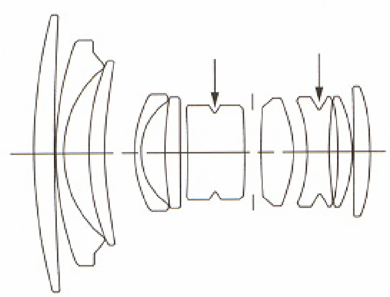


Photo-15 EF300mm f/4L IS USM Flocking Process

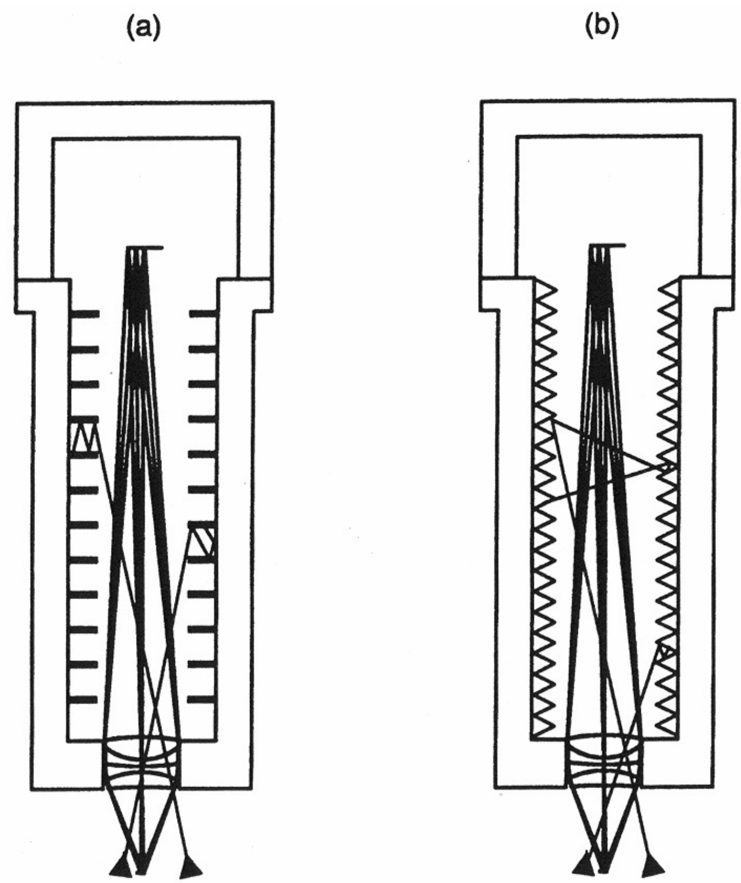
Figure-36 EF24mm f/2.8 Internal Light Blocking Grooves



① Anti-Reflection Coating Techniques

This method employs a special paint on angled surfaces and joining surfaces where the lens elements are held in place by the lens barrel to stop light entering the lens from reflecting from these parts. If a standard coating is used, reflections

Figure 20.7
Use of Threads and Baffles for Stray-Light Attenuation in Machine Vision System

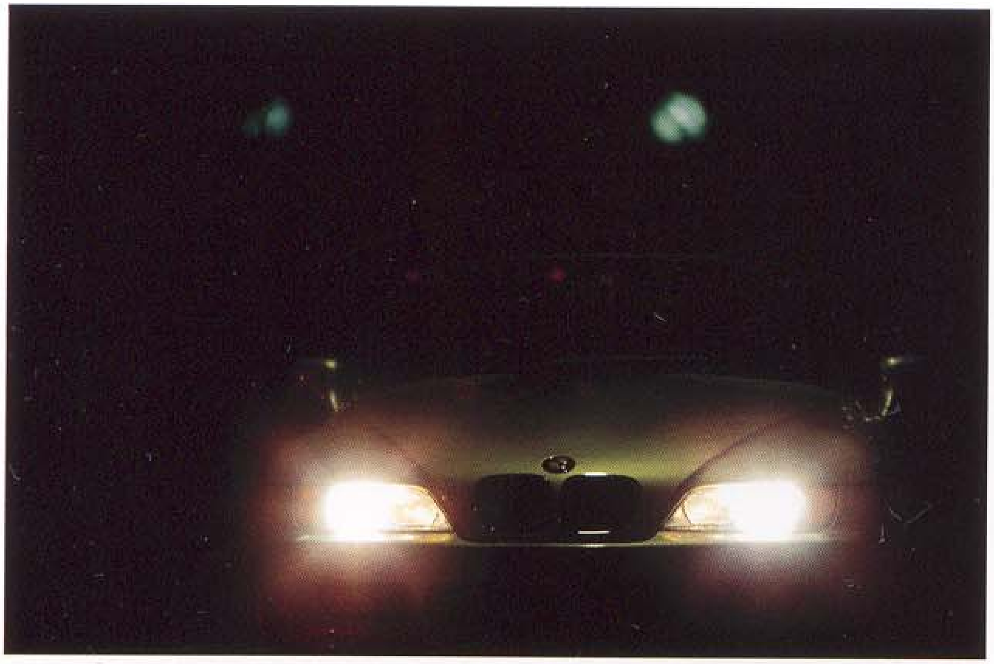


source: canon red book

From Optical System Design by Fisher and Tadic

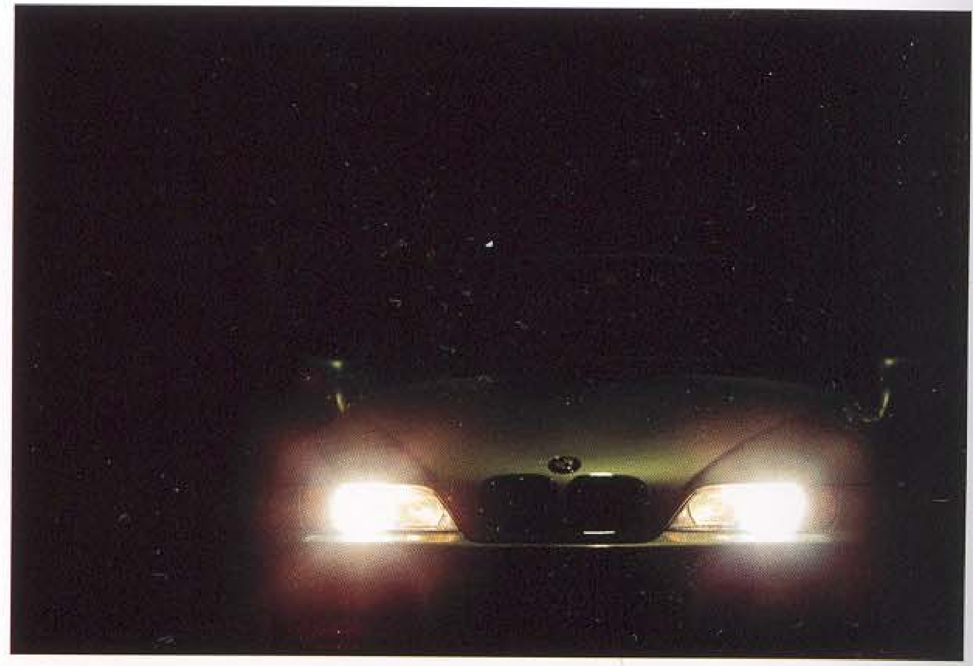
Flare/ghosting special to digital

For flat protective glass

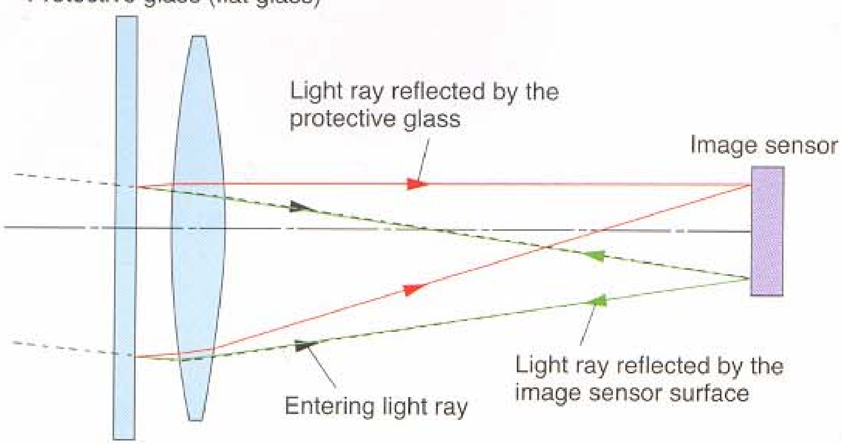


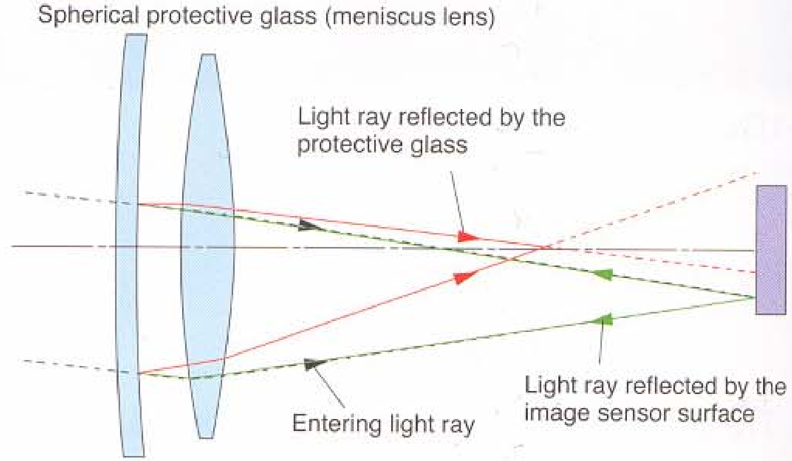
In lenses employing flat protective glass, a reflection occurs between the image sensor and the protective glass, which causes the subject to be photographed in a position different from the actual position.

For a meniscus lens



In lenses employing a meniscus lens, no reflection like that seen to the left occurs.

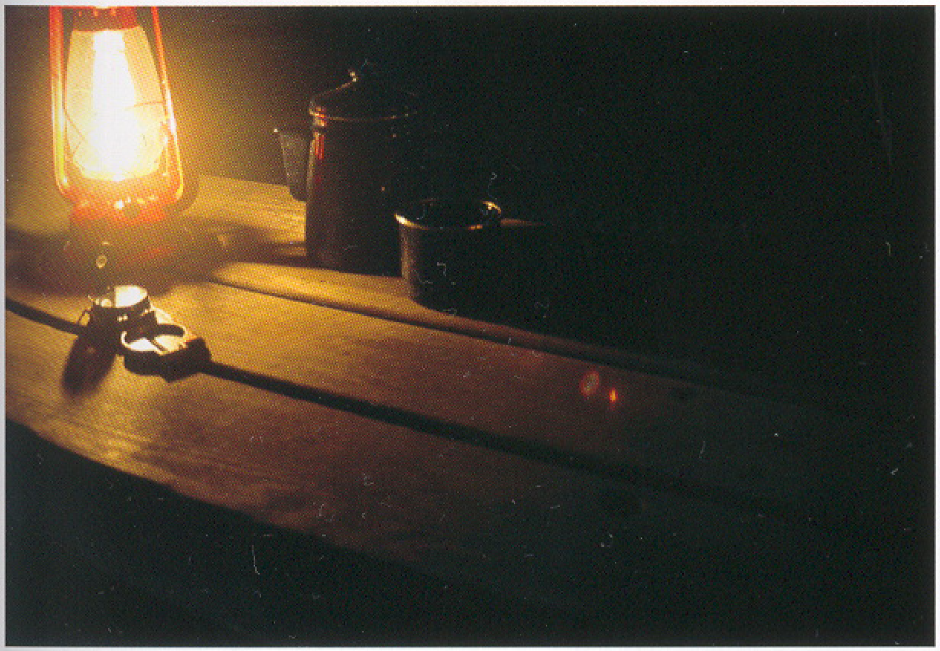




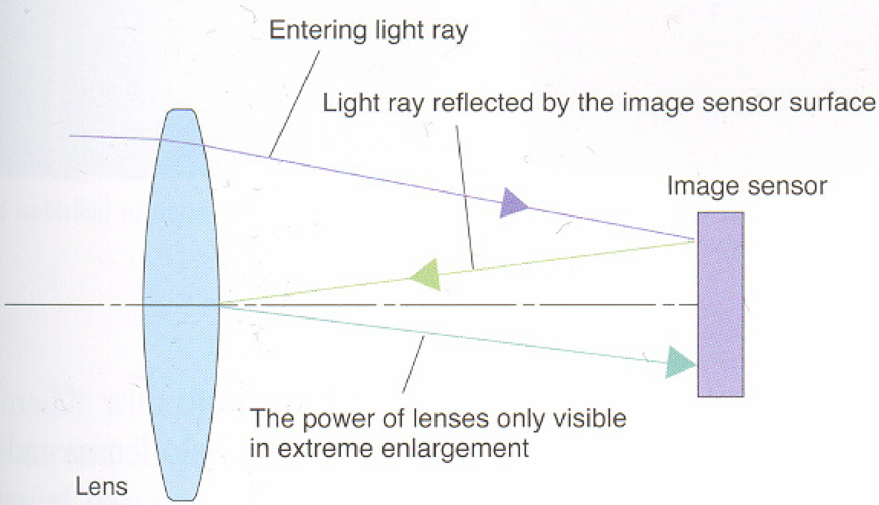
source: canon red book

Coating for digital

Lens for which the lens shape and coating have not been optimized

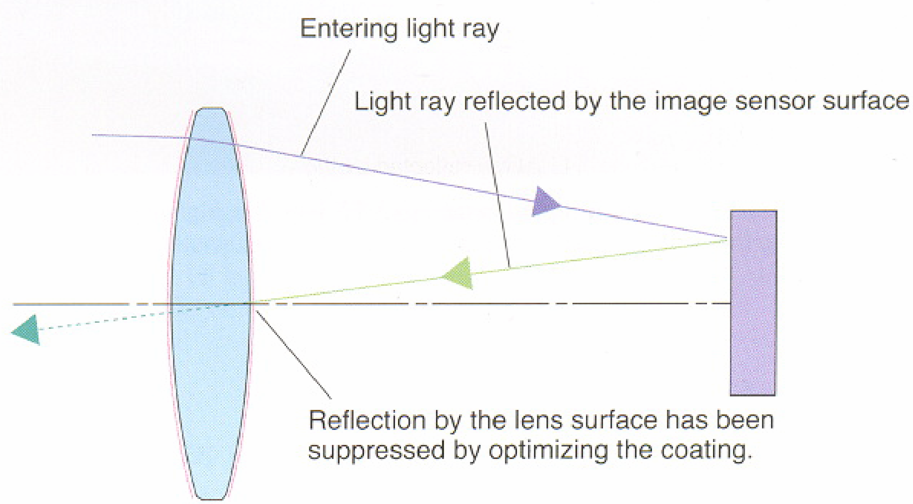


Flaring and ghosting occurs with lens for which the lens shape and coating have not been optimized.



Lens for which the lens shape and coating have been optimized

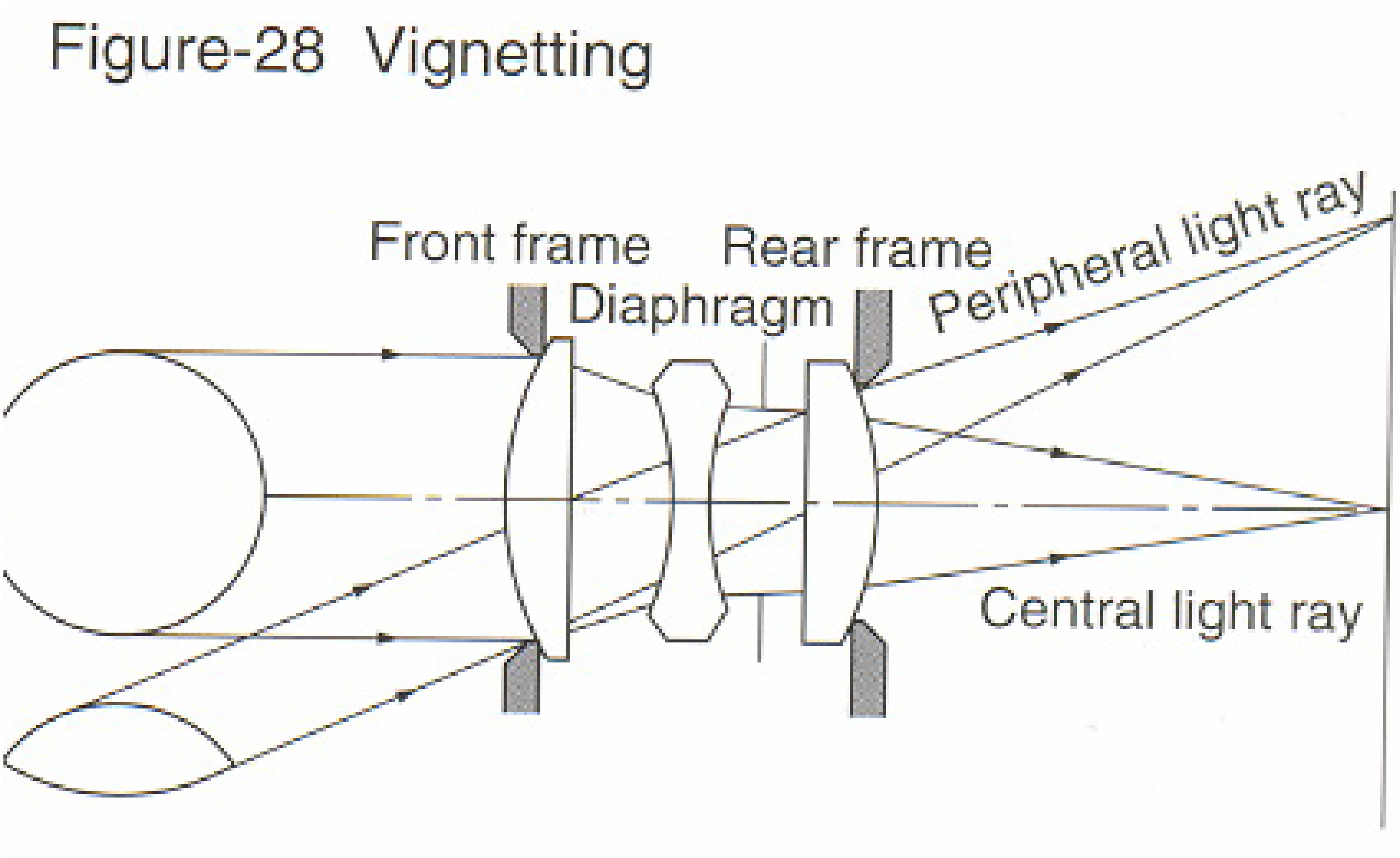
Flaring and ghosting are suppressed with lens for which the lens shape and coating have been optimized.



source: canon red book

Vignetting

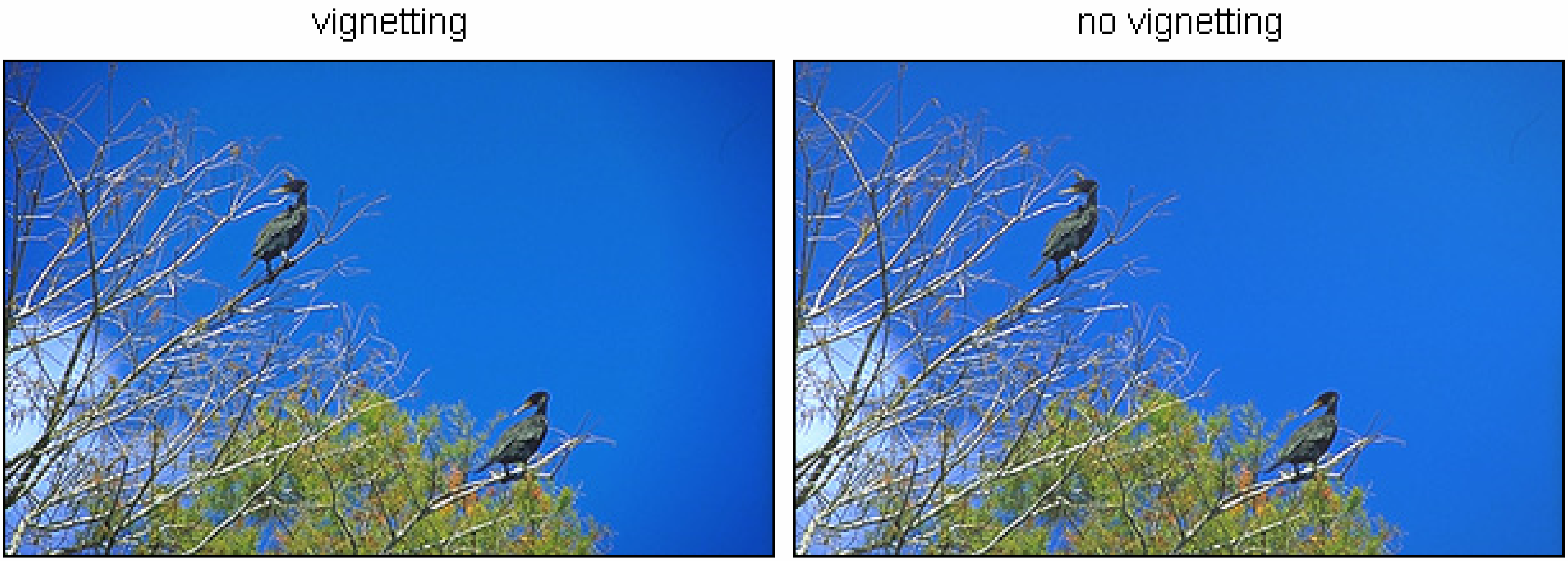
- **The periphery does not get as much light**



source: canon red book

Vignetting

- **http://www.photozone.de/3Technology/lenstec3.htm**

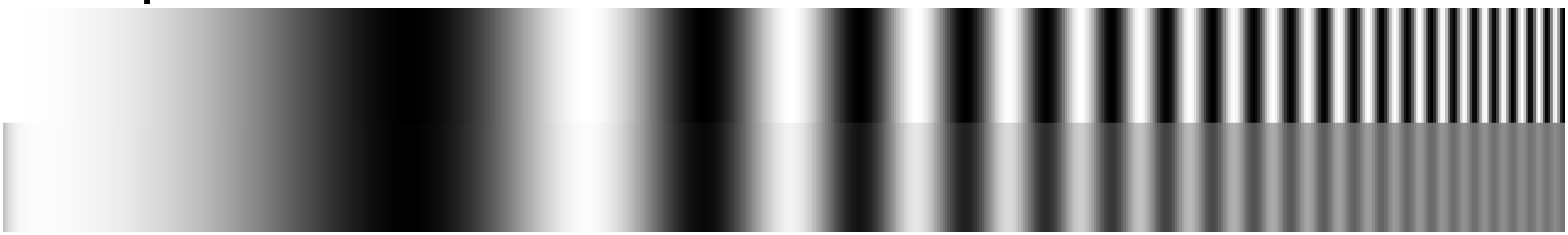


Quality evaluation

LPIs

- **Line pair per inch**

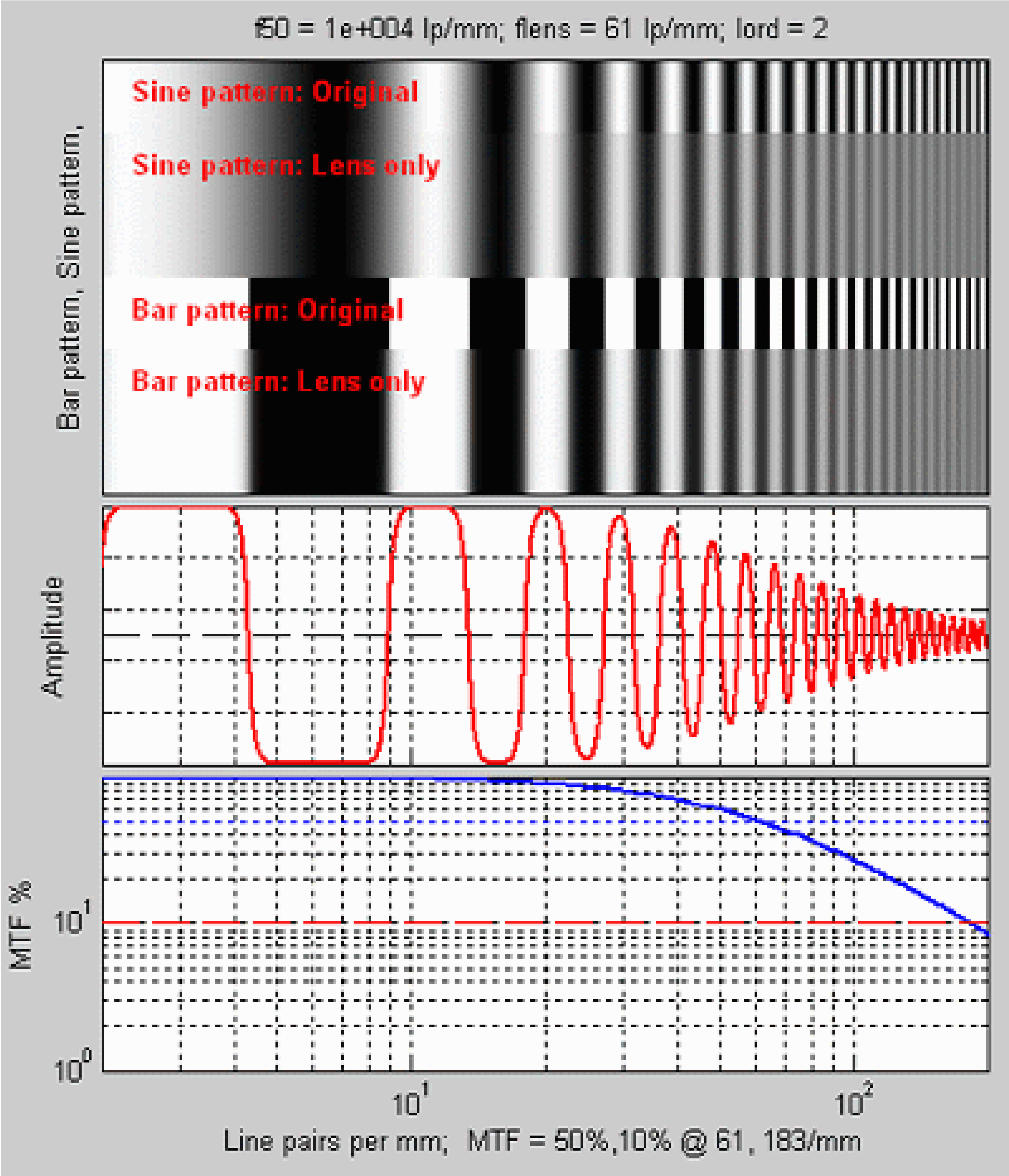
Input



After lens http://www.imatest.com/docs/sharpness.html

- **http://www.optikos.com/Pdf_files/how_to_measure_mtf.pdf**
- **http://www.imatest.com/docs/tour.html**

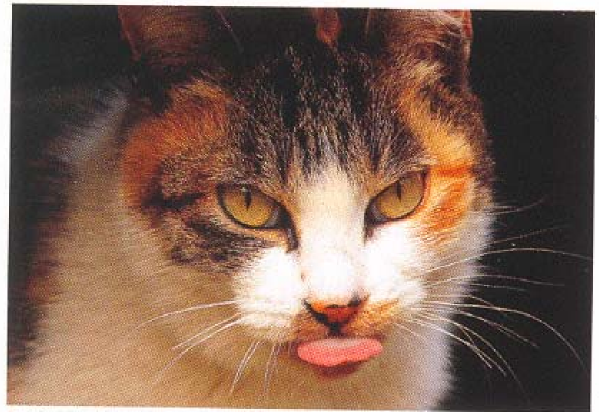
Sharpness



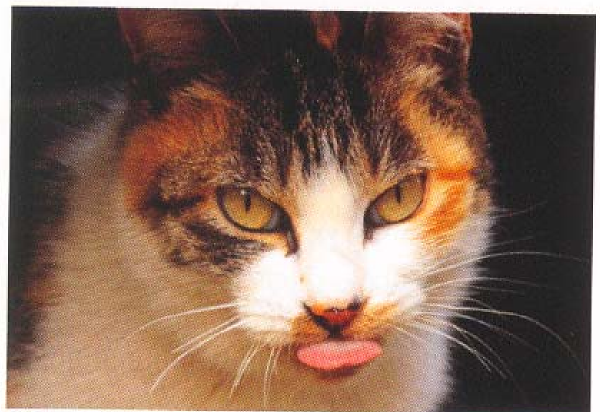
f50 = 1e+004 lp/mm; flens = 61 lp/mm; lord = 2
Sine pattern: Original
Sine pattern: Lens only
Bar pattern: Original
Bar pattern: Lens only
Bar pattern, Sine pattern,
Amplitude
MTF %
10^1
10^0
10^1
10^2
Line pairs per mm; MTF = 50%,10% @ 61, 183/mm

MTF

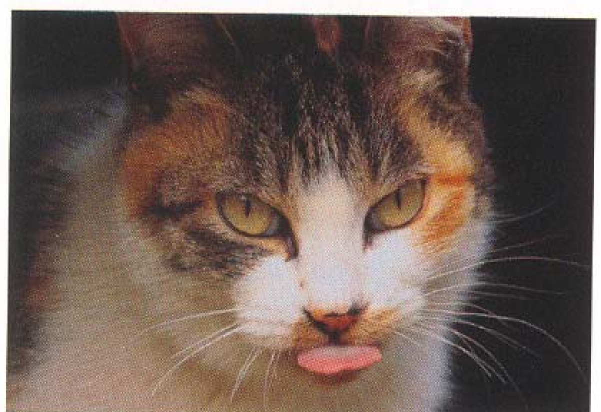
- **Modulation Transfer Function**
- **Preety much Fourier transform of lens response**
- **Complex because needs to be measured at multiple location**



A:Resolving power and contrast are both good



B:Contrast is good and resolving power is bad

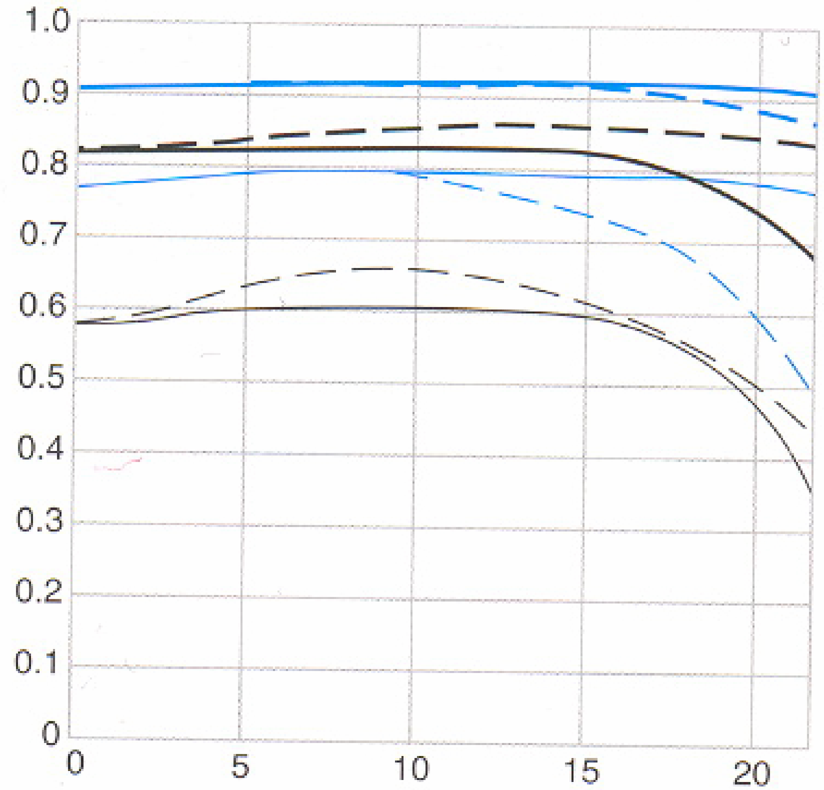


C:Resolving power is good and contrast is bad

Table-3

Spatial frequency	Maximum aperture		F 8	
	S	M	S	M
10 lines/mm	━━━	━ ━ ━	━━━	━ ━ ━
30 lines/mm	———	– – –	———	– – –

Graph-5 MTF Characteristics



Here the x axis is image location

source: canon red book

Blur index based on Photoshop!

- **Lens sharpness (or lack thereof) expressed as the amount of Photoshop blur that would blur the image similarly**
- **http://www.imatest.com/**
- **http://www.dxo.com/en/measure/dxo_analyser/default.php**
- **Cool vis at http://www.slrgear.com/reviews/index.php**

- **100 macro: http://www.slrgear.com/reviews/showproduct.php/product/157/sort/2/cat/10/page/1**
- **50mm f/1.4 http://www.slrgear.com/reviews/showproduct.php/product/140/sort/2/cat/10/page/2**
- **16-35mm http://www.slrgear.com/reviews/showproduct.php/product/142/sort/2/cat/11/page/1**
- **55-200 http://www.slrgear.com/reviews/showproduct.php/product/141/sort/2/cat/11/page/1**
- **28-135 http://www.slrgear.com/reviews/showproduct.php/product/139/sort/2/cat/11/page/1**
- **18-55 http://www.slrgear.com/reviews/showproduct.php/product/137/sort/2/cat/11/page/1**
- **17-85 http://www.slrgear.com/reviews/showproduct.php/product/136/sort/2/cat/11/page/1**
- **10-22 http://www.slrgear.com/reviews/showproduct.php/product/135/sort/2/cat/11/page/1**

Lens design

Optimization software

- **Has revolutionized lens design**
- **E.g. zooms are good now**

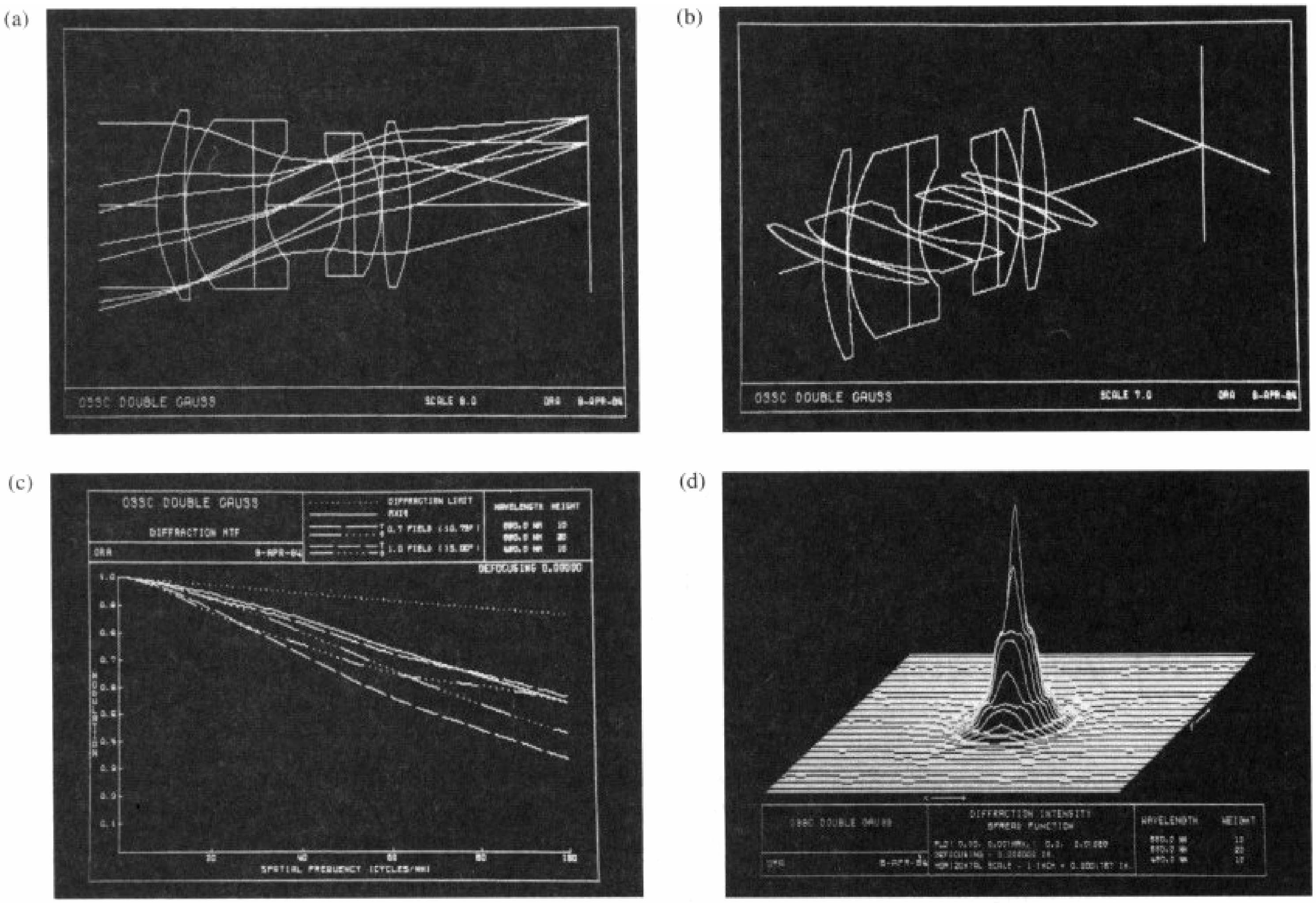


Figure 11.50 An example of the kind of lens design information available via computer techniques. (Photos courtesy Optical Research Associates.)

From Hecht's Optics

Lens design, ray tracing

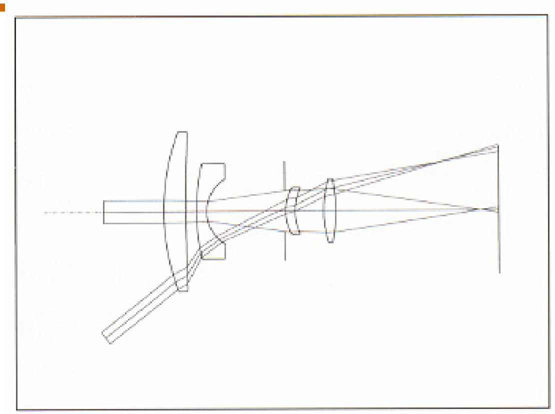


Figure-5

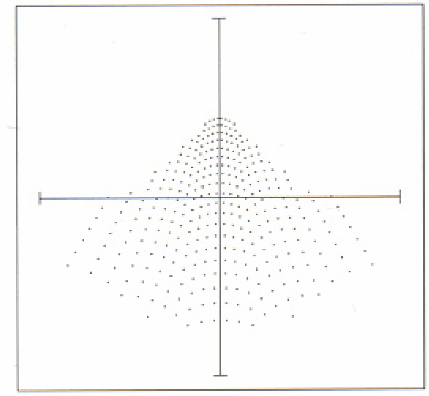


Figure-8

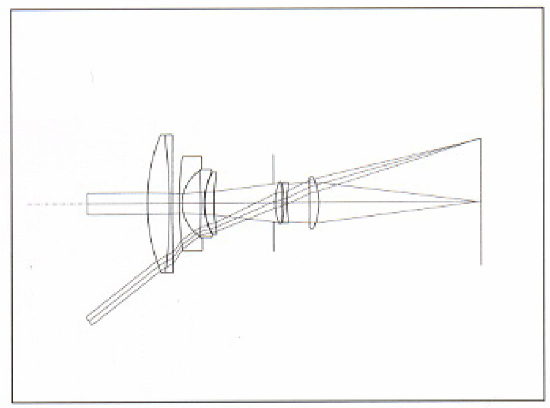


Figure-6

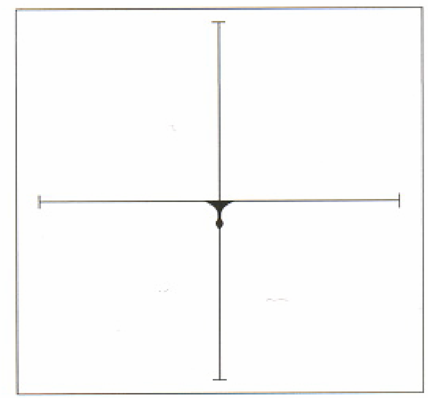


Figure-9

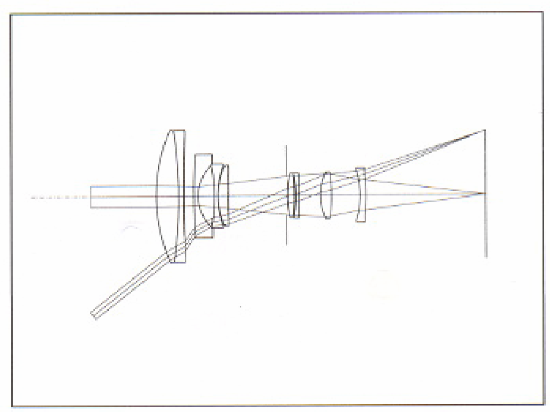


Figure-7

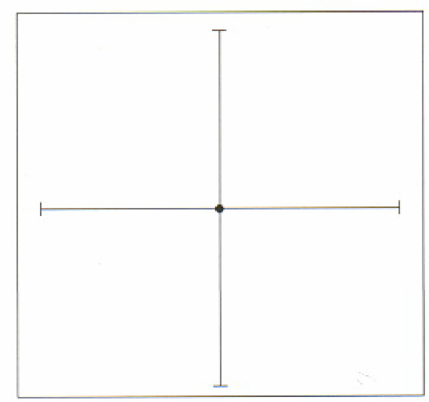


Figure-10

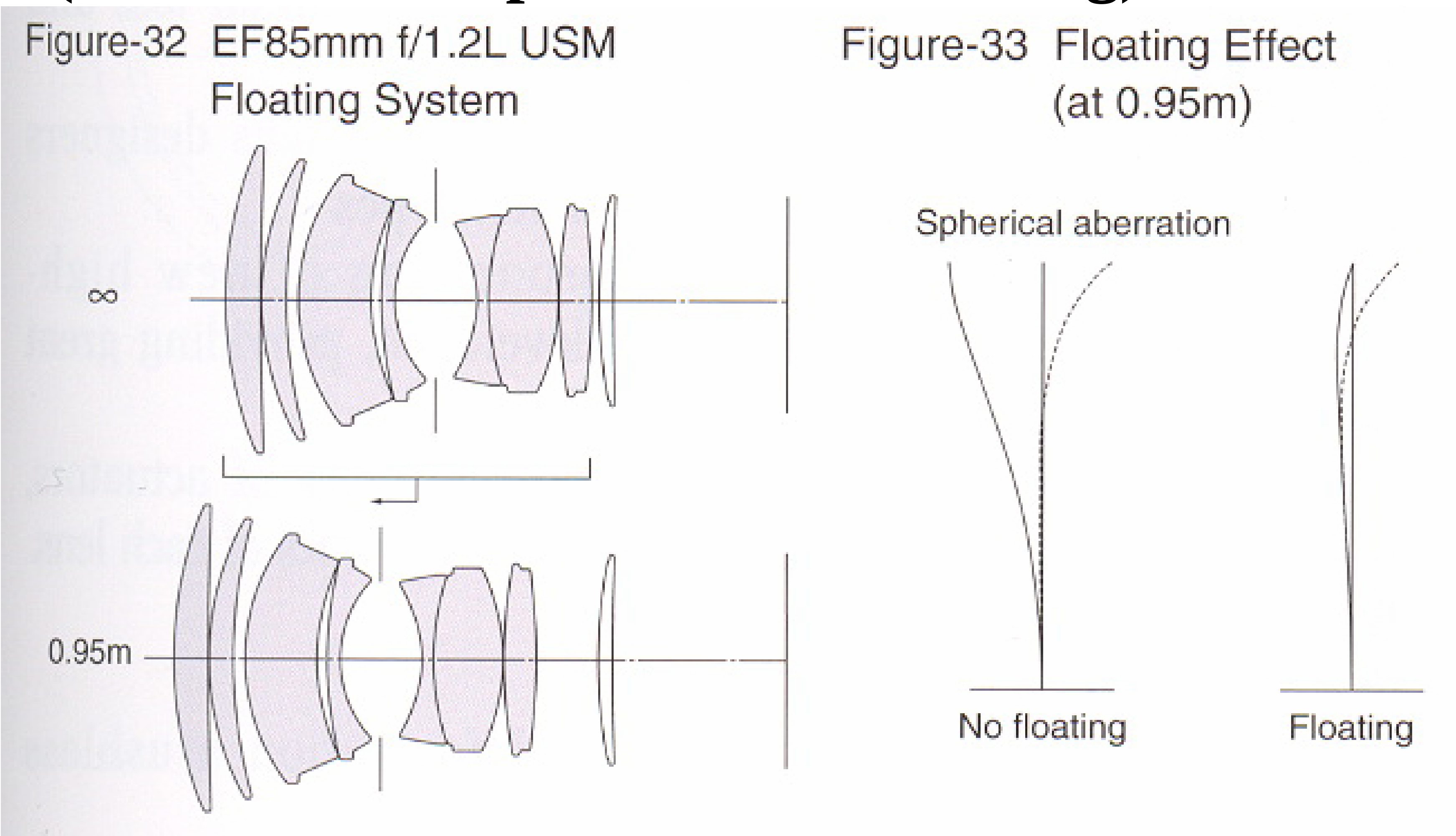
source: canon red book

Optimization

- **Free parameters**
  - Lens curvature, width, position, type of glass
  - Some can be fixed, other vary with focal length, focus (e.g. floating elements)
  - Multiplied by number of lens elements
- **Energy/merit function**
  - MTF, etc.
  - Black art of massaging the merit function
- **Optimize for**
  - All image locations
  - All wavelengths
  - All apertures
  - All focusing distances
  - All focal lengths (zoom only)

Floating elements

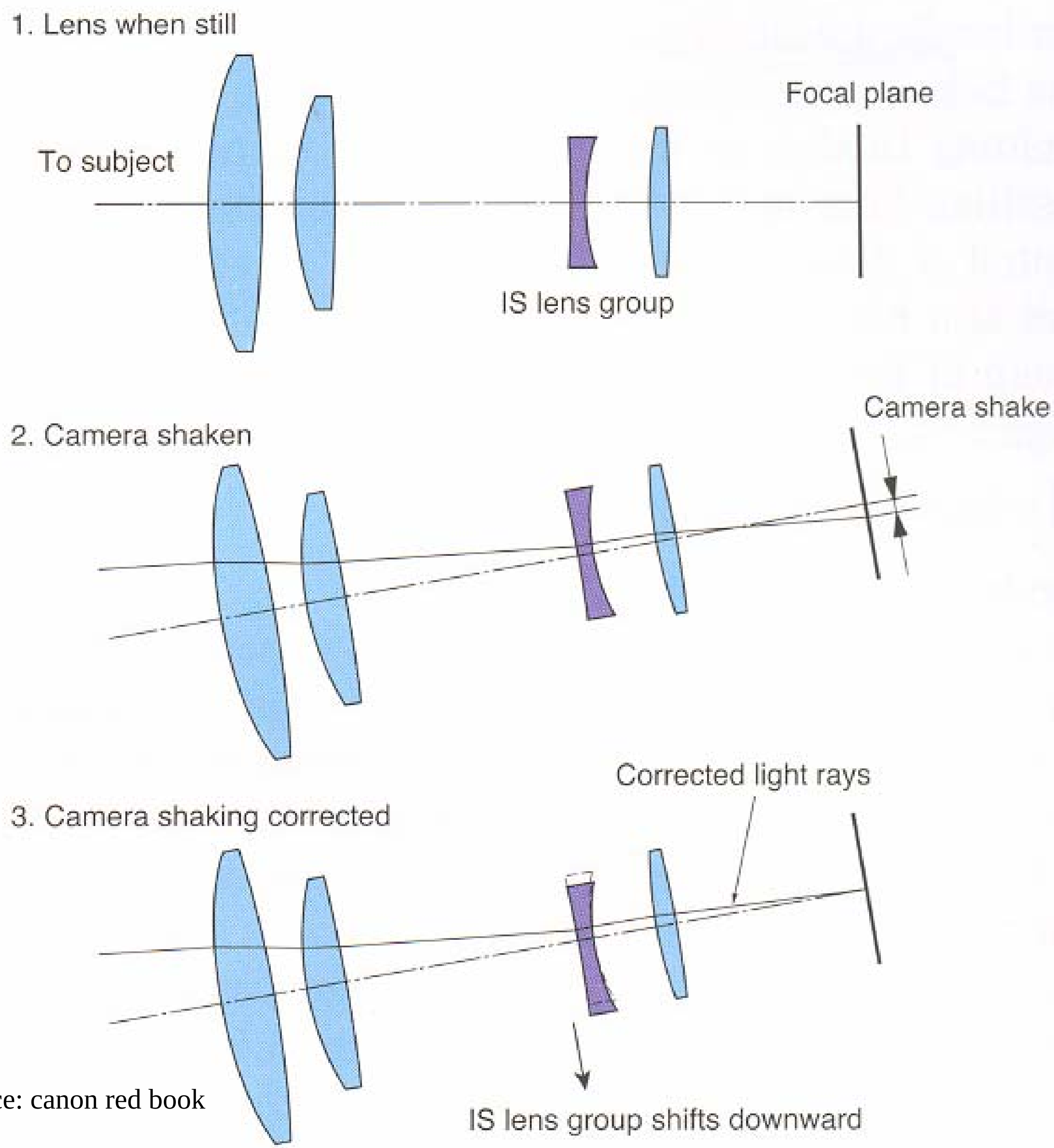
- **Move with focus to optimize response (but are not responsible for focusing)**



source: canon red book

Image stabilization

Image stabilization



source: canon red book

Photo-21 Shake-detecting gyro sensor

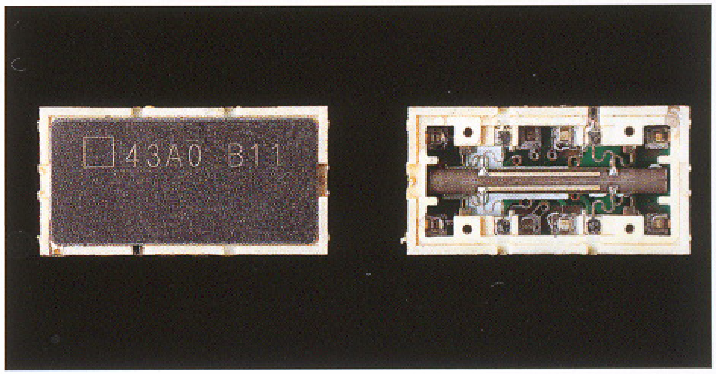
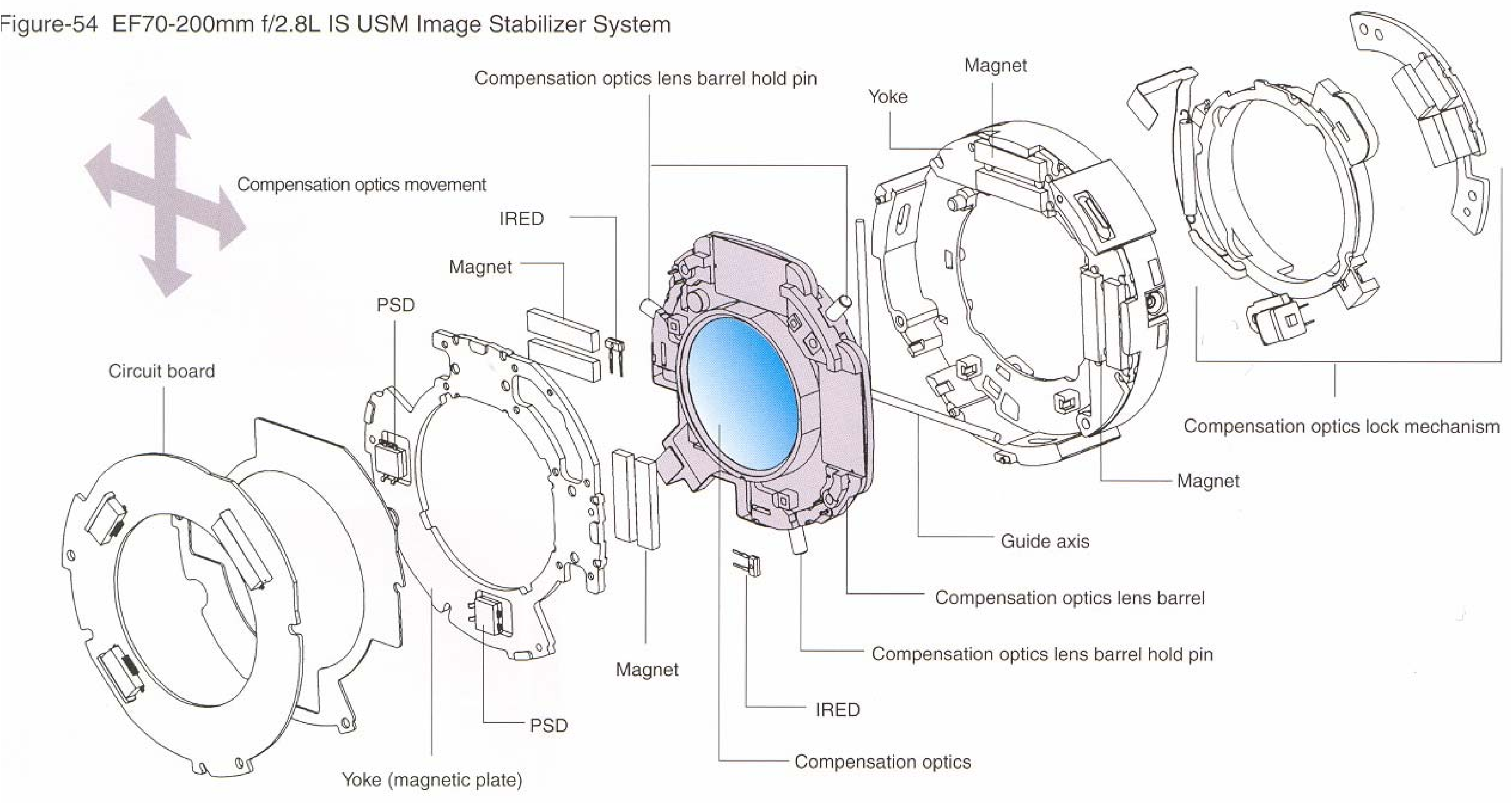
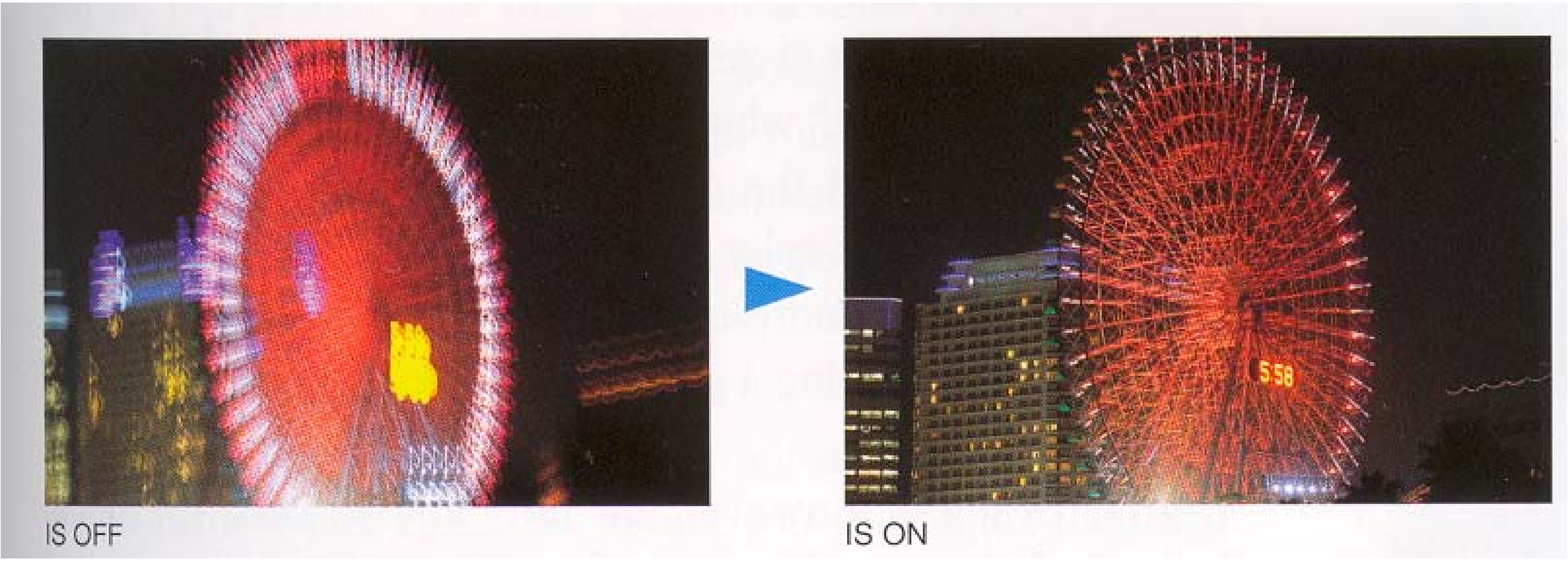


Image stabilization



source: canon red book

Image stabilization



source: canon red book

1000mm, 1/100s, monopod, IS

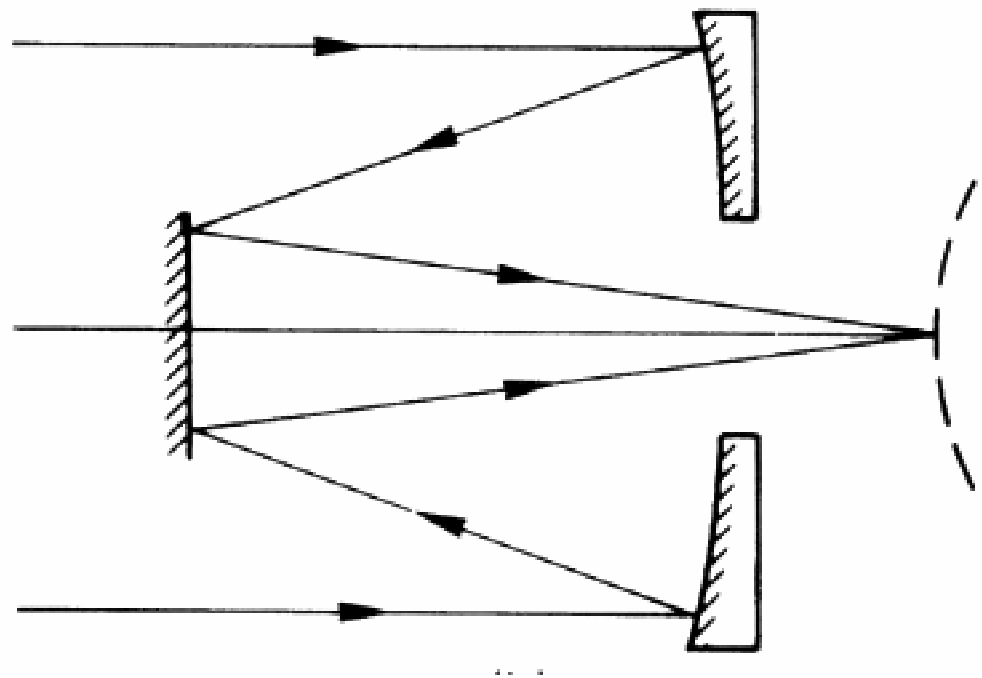
Different versions

- **Canon, Nikon: in the lens**
- **Panasonic, Konica/Minolta: move sensor**

Special lenses

Some special lenses

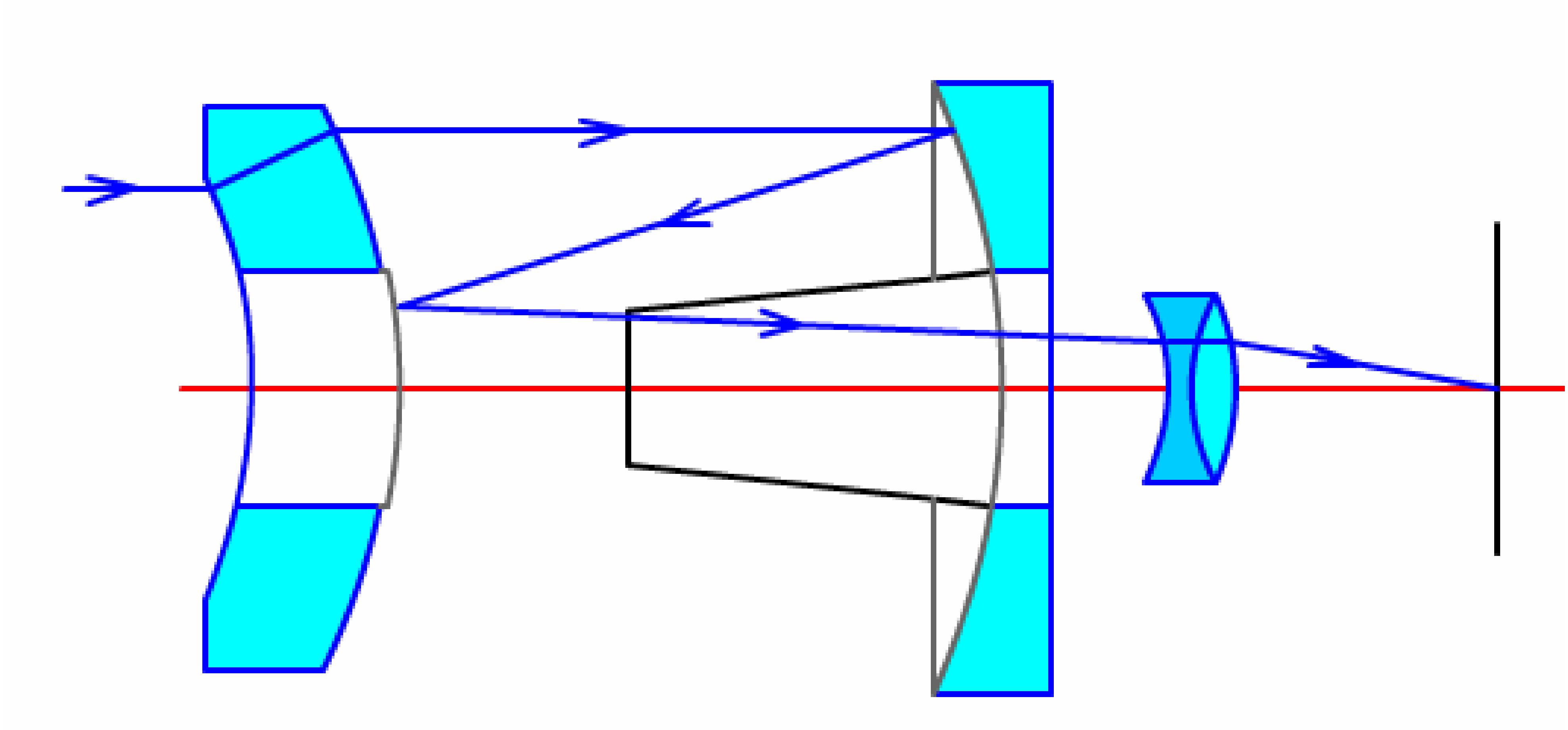
- **Mirror lenses**
- **Tilt-shift lenses**
- **Macro lenses**
  - Why sharpness is always great (thanks Gauss)
  - Why you lose light



From "The Manual of Photography" Jacobson et al

catadioptric (mirror)

- http://www.digit-life.com/articles2/rubinar/



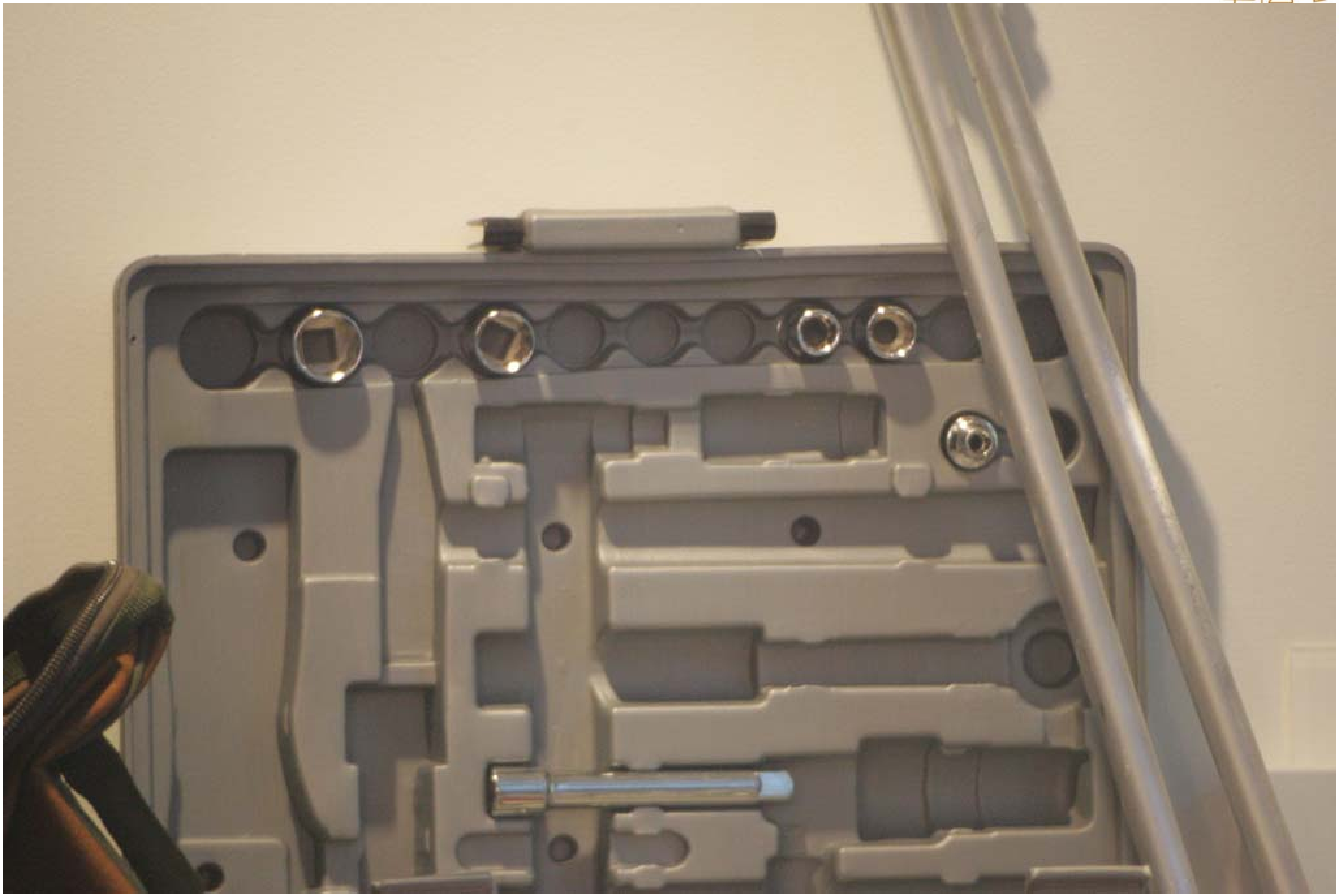
500mm vivitar ($100)

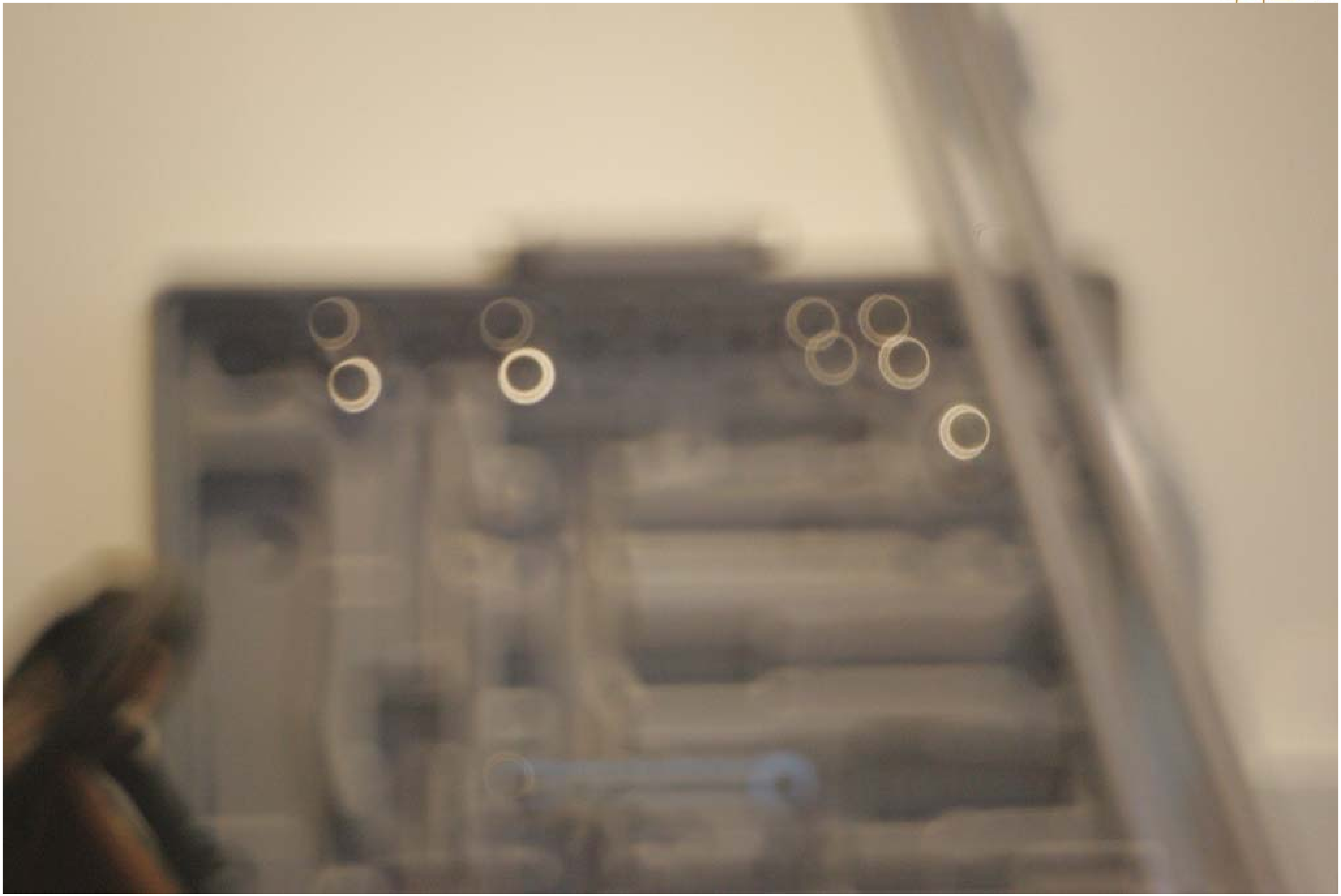
500mm Canon (5k)


rachet


rachet

Mirror lens

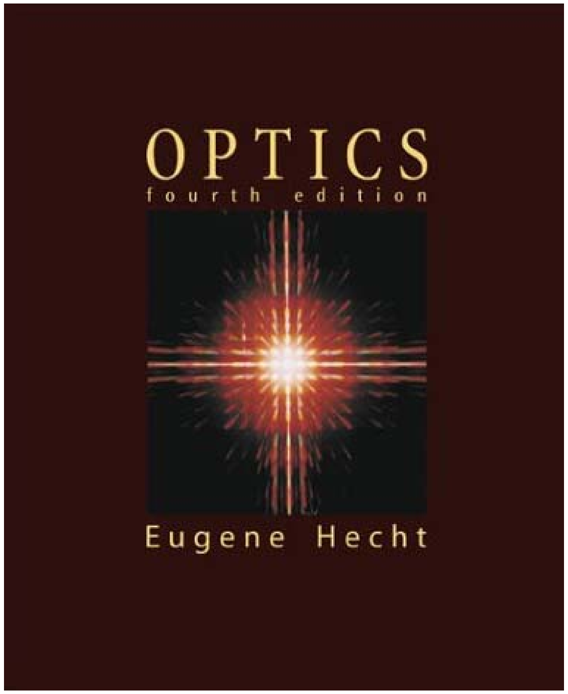




http://www.digit-life.com/articles2/rubinar/

Canon D60 + MS Rubinar - 8/500

References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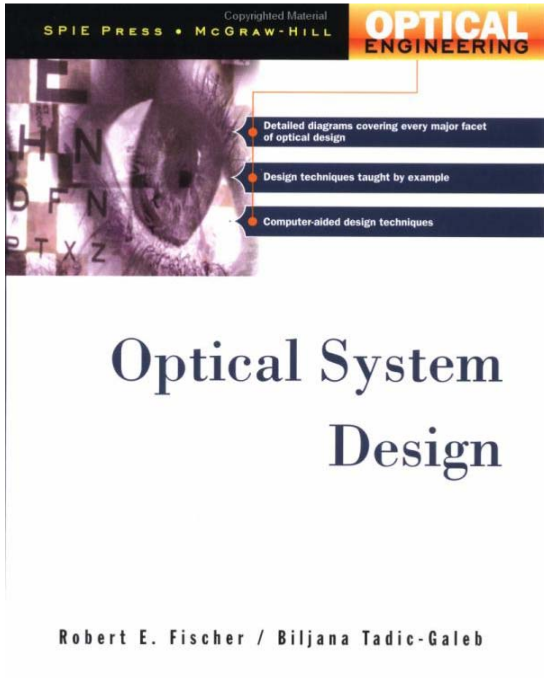
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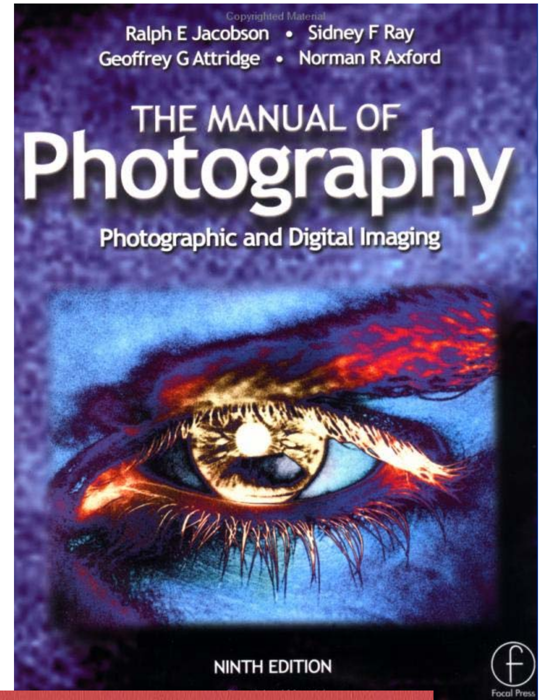
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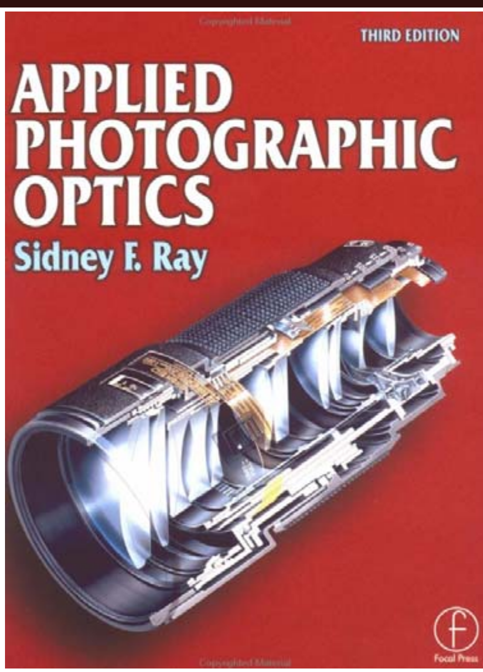
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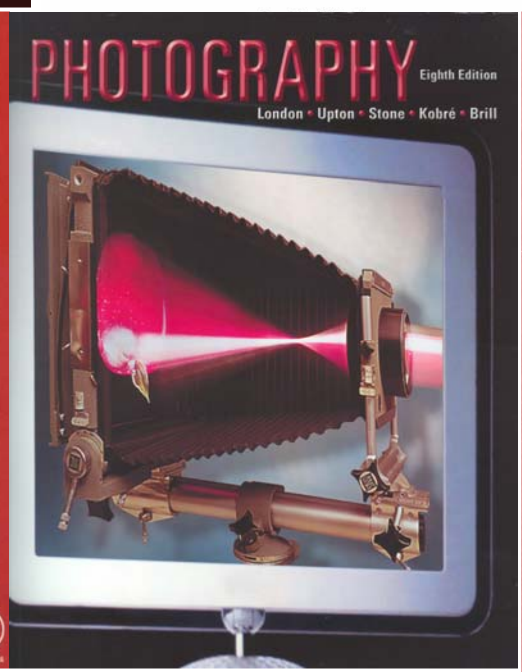
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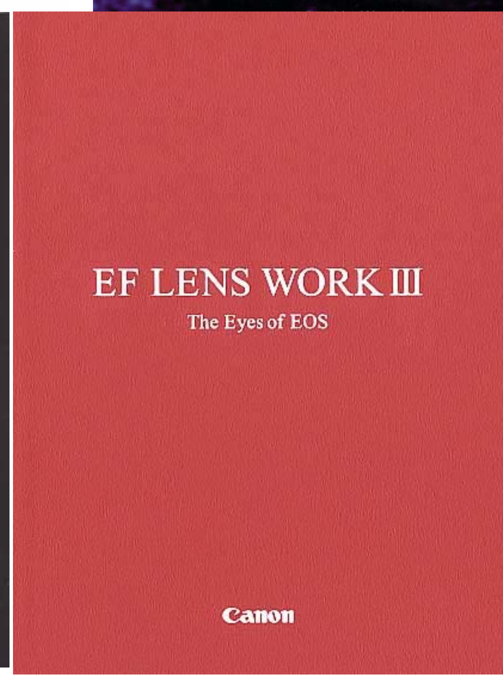
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The Eyes of EOS
Canon

Links

- http://en.wikipedia.org/wiki/Chromatic_aberration
- http://www.dpreview.com/learn/?/key=chromatic+aberration
- http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/aberrcon.html#c1
- http://en.wikipedia.org/wiki/Spherical_aberration
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