

6.098 Digital and Computational Photography
6.882 Advanced Computational Photography

Image Warping and Morphing

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MIT - EECS

Olivier Gondry's visit

- Thursday Friday
- Contact Peter Sand: sand@mit.edu

Important scientific question

- How to turn Dr. Jekyll into Mr. Hyde?
- How to turn a man into a werewolf?
- Powerpoint cross-fading?



Important scientific question

- How to turn Dr. Jekyll into Mr. Hyde?
- How to turn a man into a werewolf?
- Powerpoint cross-fading?



From An American Werewolf in London

- or
- Image Warping and Morphing?

Digression: old metamorphoses

- http://en.wikipedia.org/wiki/The_Strange_Case_of_Dr._Jekyll_and_Mr._Hyde
- <http://www.eatmybrains.com/showtopten.php?id=15>
- http://www.horror-wood.com/next_gen_jekyll.htm
- Unless I'm mistaken, both employ the trick of making already-applied makeup turn visible via changes in the color of the lighting, something that works only in black-and-white cinematography. It's an interesting alternative to the more familiar Wolf Man time-lapse dissolves. This technique was used to great effect on Fredric March in Rouben Mamoulian's 1932 film of *Dr. Jekyll and Mr. Hyde*, although Spencer Tracy eschewed extreme makeup for his 1941 portrayal.



Averaging images

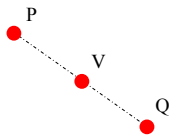
- Cross-fading
 - Pretty much the compositing equation

$$C = \alpha F + (1 - \alpha) B$$



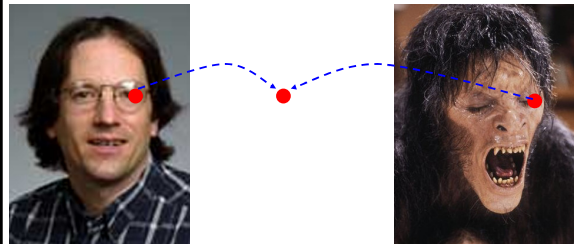
Averaging vectors

- $V = \alpha P + (1-\alpha) Q$



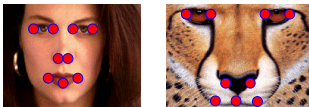
Warping & Morphing combine both

- For each pixel
 - Transform its location like a vector
 - Then linearly interpolate like an image



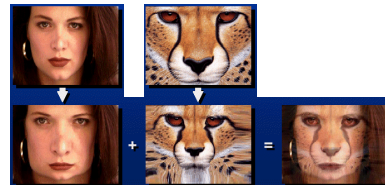
Morphing

- Input: two images I_0 and I_N
- Expected output: image sequence I_i , with $i \in 1..N-1$
- User specifies sparse correspondences on the images
 - Pairs of vectors $\{(P_i^0, P_i^N)\}$



Morphing

- For each intermediate frame I_i
 - Interpolate feature locations $P_i^t = (1-t) P_i^0 + t P_i^1$
 - Perform **two** warps: one for I_0 , one for I_1
 - Deduce a dense warp field from the pairs of features
 - Warp the pixels
 - Linearly interpolate the two warped images



Warping

Intelligent design & image warping

- D'Arcy Thompson
 - <http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html>
 - http://en.wikipedia.org/wiki/D'Arcy_Thompson
- Importance of shape and structure in evolution

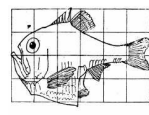


Fig. 517. *Argemonea Offici.*

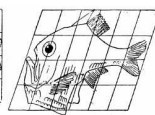
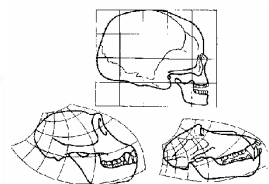


Fig. 518. *Sternopyge diaphana.*



Skulls of a human, a chimpanzee and a baboon and transformations between them

Warping

- Imagine your image is made of rubber
- warp the rubber



No prairie dogs were harmed when creating this image

Careful: warp vs. inverse warp

How do you perform a given warp:

- Forward warp**
 - Potential gap problems
- Inverse lookup the most useful**
 - For each output pixel
 - Lookup color at inverse-warped location in input

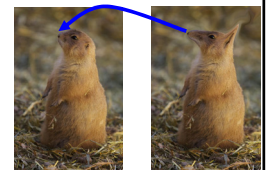
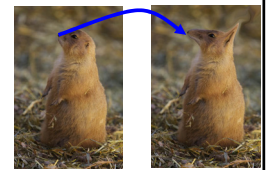
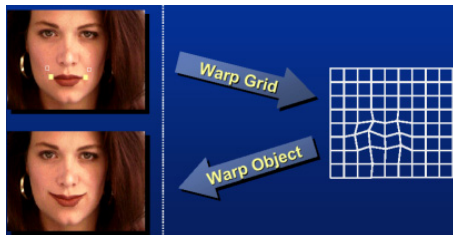


Image Warping – non-parametric

- Move control points to specify a spline warp
- Spline produces a smooth vector field



Slide Alyosha Efros

Warp specification - dense

- How can we specify the warp?
 - Specify corresponding *spline control points*
 - interpolate* to a complete warping function

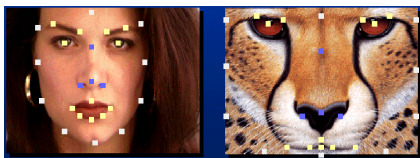


But we want to specify only a few points, not a grid

Slide Alyosha Efros

Warp specification - sparse

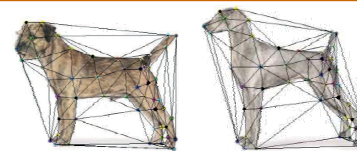
- How can we specify the warp?
 - Specify corresponding *points*
 - interpolate* to a complete warping function
 - How do we do it?



How do we go from feature points to pixels?

Slide Alyosha Efros

Triangular Mesh



- Input correspondences at key feature points
- Define a triangular mesh over the points
 - Same mesh in both images!
 - Now we have triangle-to-triangle correspondences
- Warp each triangle separately from source to destination

Slide Alyosha Efros

Problems with triangulation morphing

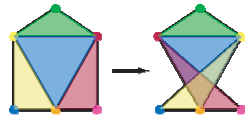
- Not very continuous

– only C^0



Fig. L. Darsa

- Folding problems

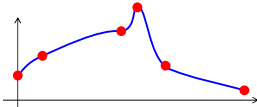


Warp as interpolation

- We are looking for a warping field
 - A function that given a 2D point, returns a warped 2D point
- We have a sparse number of correspondences
 - These specify values of the warping field
- This is an interpolation problem
 - Given sparse data, find smooth function

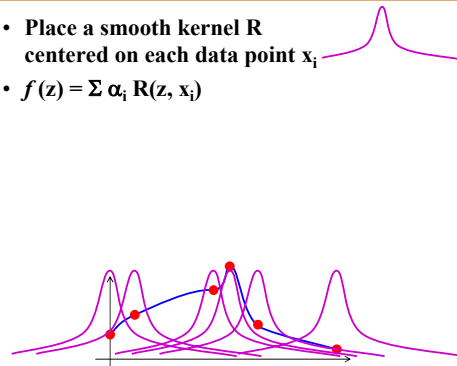
Interpolation in 1D

- We are looking for a function f
- We have N data points: x_i, y_i
 - Scattered: spacing between x_i is non-uniform
- We want f so that
 - For each $i, f(x_i) = y_i$
 - f is smooth
- Depending on notion of smoothness, different f



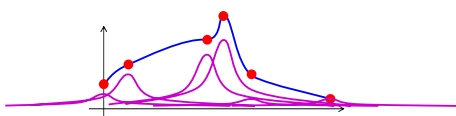
Radial Basis Functions (RBF)

- Place a smooth kernel R centered on each data point x_i
- $f(z) = \sum \alpha_i R(z, x_i)$



Radial Basis Functions (RBF)

- Place a smooth kernel R centered on each data point x_i
- $f(z) = \sum \alpha_i R(z, x_i)$
- Find weights α_i to make sure we interpolate the data for each $i, f(x_i) = y_i$



Kernel

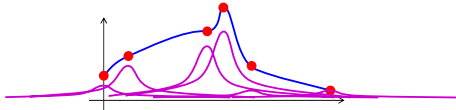
- Many choices
- In Assignment 4, we simply use inverse multiquadric

$$R(z, x_i) = \frac{1}{\sqrt{c + \|z - x_i\|^2}}$$

- where c controls falloff
- Lazy way: set c to an arbitrary constant (pset 4)
- Smarter way: c is different for each kernel. For each x_i , set c as the squared distance to the closest other x_j

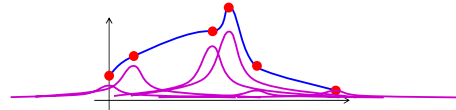
Enforcing interpolation

- $f(z) = \sum \alpha_i R(z, x_i)$
- **N equations**
for each j , $f(x_j) = y_j$
 $\sum \alpha_i R(x_j, x_i) = y_j$
- **N unknowns α_i**
- **Just inverse the matrix**



Important note

- $f(z) = \sum \alpha_i R(z, x_i)$
for each j , $\sum \alpha_i R(x_j, x_i) = y_j$
- **Note that**
the influence of each function is non-zero everywhere
at a data point, the value of the other bases is not zero
- **In contrast to e.g. various interpolation splines**

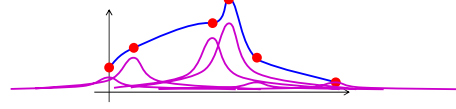


Variations of RBF

- **Lots of possible kernels**
 - Gaussians $e^{-r^2/2\sigma}$
 - Thin-plate splines $r^2 \log r$
- **Sometimes add a global polynomial term**

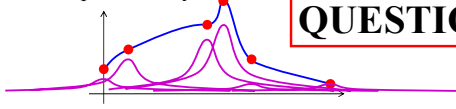
Recap: 1D scattered data interpolation

- **Sparse input/output pairs x_i, y_i**
 - non-uniformly sampled
- **RBFs (Radial Basis Functions)**
 - Weighted sum of kernels R centered on data points x_i
 $f(z) = \sum \alpha_i R(z, x_i)$
 - Compute the weights α_i by enforcing interpolation
 $f(x_j) = y_j$
 - Simple linear system



Recap: 1D scattered data interpolation

- **Sparse input/output pairs x_i, y_i**
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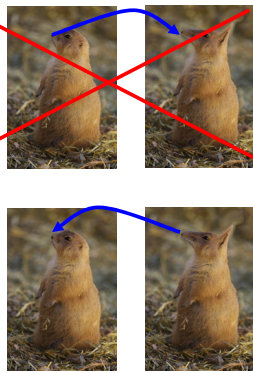
QUESTION?

RBF for warping: 2D case

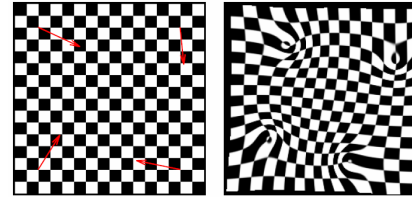
- **Instead of $f: \mathbb{R} \rightarrow \mathbb{R}$, we now deal with $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$**
 - For each 2D point, f gives us another 2D warped point
- **We have N data points**
 - Pairs of input 2D vector, output 2D vector
 - Careful: x_i is now a 2D vector, so is y_i
 - Don't be confused with coordinates (x, y)
- **Place 2D kernels at each data point**
- **The weights α_i are now 2D vectors**
- **Solve a linear system of $2N$ equations and $2N$ unknowns**

Applying a warp: USE INVERSE

- **Forward warp:**
 - For each pixel in **input** image
 - Paste color to **warped** location in output
 - Problem: gaps
- **Inverse warp**
 - For each pixel in **output** image
 - Lookup color **from** **inverse-warped** location

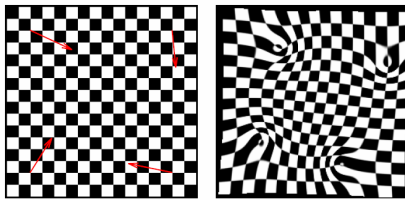


Example



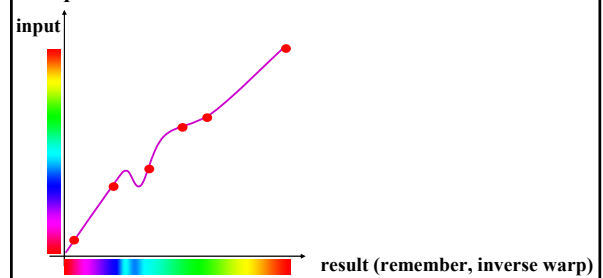
Example

- **Fold problems**
 - Oh well...



1D equivalent of folds

- There is no guarantee that our 1D RBF is monotonic
- Yes, it means that the notion of inverse of the warp is questionable.



Hardcore Photoshop for portrait



figure 9.35

figure 9.36



figure 9.37
Selecting the entire left side of the image avoids potential artifacts.

figure 9.38
Designing a Free Transform handle to narrow the selected area.



figure 9.39
The Liquify filter's Warp tool pushes pixels forward as you drag.

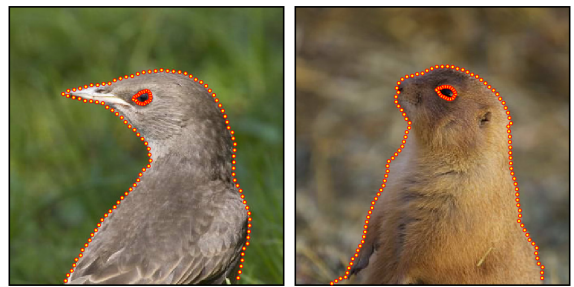


Morphing

Input images



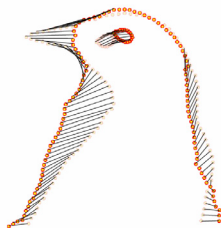
Feature correspondences



- The feature locations will be our y_i

Interpolate feature location

- Provides the x_i



Warp each image to intermediate location

Two different warps:
Same target location,
different source location
i.e. the x_i are the same
(intermediate locations),
the y_i are different (source
feature locations)

Note: the y_i do not change
along the animation, but
the x_i are different for
each intermediate image

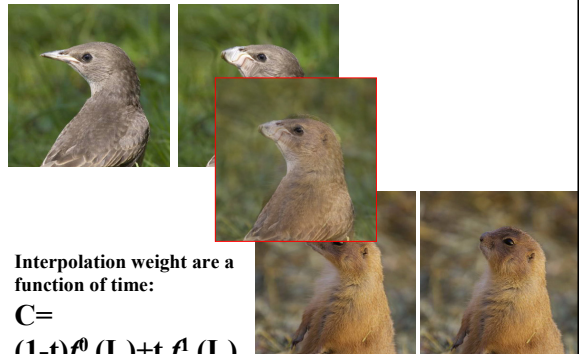
Here we show $t=0.5$
(the y_i are in the middle)



Warp each image to intermediate location



Interpolate colors linearly

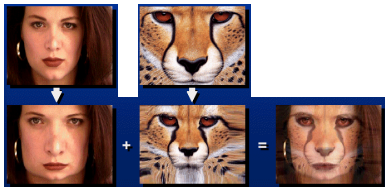


Interpolation weights are a function of time:

$$C = (1-t)f^0(I_0) + t f^1(I_1)$$

Recap

- For each intermediate frame I_t
 - Interpolate feature locations $y_t^i = (1-t)x_i^0 + t x_i^1$
 - Perform **two** warps: one for I_0 , one for I_1
 - Deduce a dense warp field from the pairs of features
 - Warp the pixels
 - Linearly interpolate the two warped images

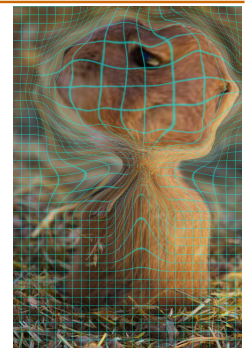


Movie time

Resampling

The sampling problem

- Parts are magnified
- Parts are minified
- Sometimes anisotropic
- Same problem for 3D texture mapping



Intuition

Plain lookup is bad

(But good news: that's all we ask for pset 4)

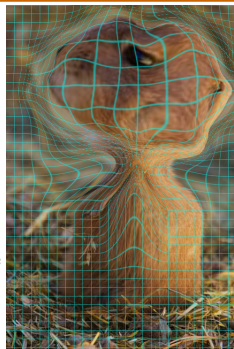
- In magnified regions, not smooth enough
- In minified regions, it creates aliasing

What we want

In magnified regions, smooth interpolation

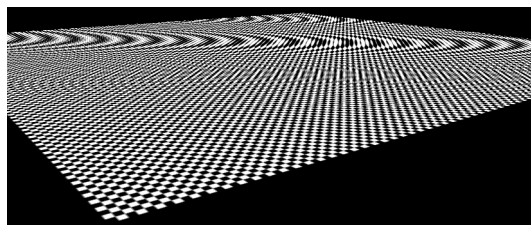
In minified regions, take the average

We need good signal processing framework to do this



Similar case: texture aliasing

- *Aliasing* is the under-sampling of a signal, and it's especially noticeable during animation



The Bible

- <http://www-2.cs.cmu.edu/~ph/textfund/textfund.pdf>

Fundamentals of Texture Mapping and Image Warping

Master's Thesis
under the direction of Carlo Séquin

Paul S. Heckbert

Dept. of Electrical Engineering and Computer Science
University of California, Berkeley, CA 94720

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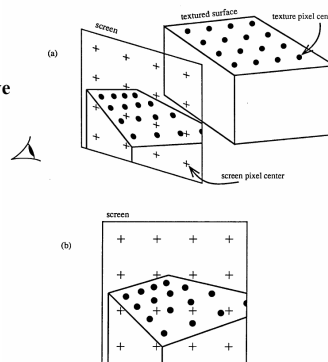
June 17, 1989

Resampling

2D texture space

warp due to perspective

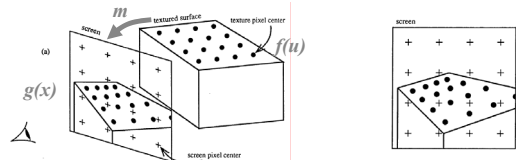
2D screen space



Notations

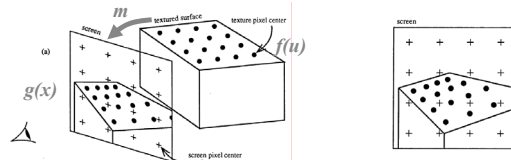
- Input signal $f(u)$
- Forward mapping (texture-to-screen) $x=m(u)$
- Output signal $g(x)$

Warning: I sloppily changed my notations:
 f is signal, warp is m



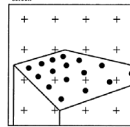
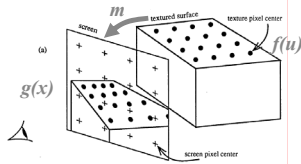
Resampling

- What do we need to do?



Resampling

1. Reconstruct the continuous signal from the discrete input signal
2. Warp the domain of the continuous signal
3. Prefilter the warped continuous signal
4. Sample this signal



Resampling

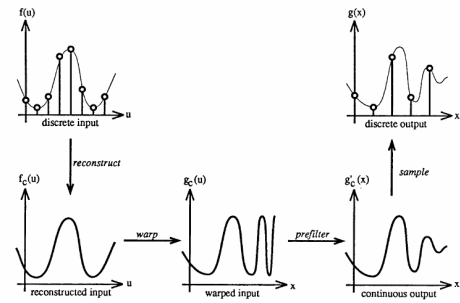
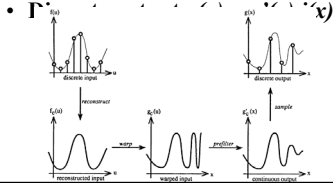


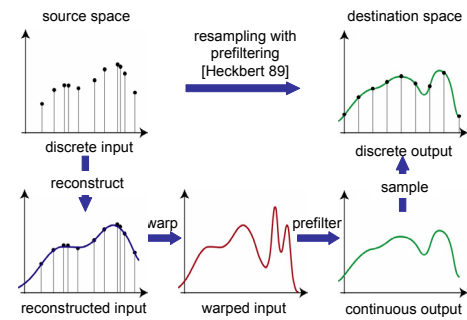
Figure 3.11: The four steps of ideal resampling: reconstruction, warp, prefilter, and sample.

Resampling: progression

- Discrete input texture $f(u)$ for integer u
- Reconstructed input texture $f_c(u) = f(u) \otimes r(u) = \sum f(k) r(u-k)$
- Warped texture $g_c(x) = f_c(m^{-1}(x))$
- Band-limited output $g'_c(x) = g_c(x) \otimes h(x) = \int g_c(t) h(x-t) dt$
- Discrete output $g(x) = g'_c(x) \otimes i(x)$



Resampling



Put it together

- Discrete input texture $f(u)$ for integer u
- Reconstructed input texture $f_c(u) = f(u) \otimes r(u) = \sum f(k) r(u-k)$
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- Discrete output $g(x) = g'_c(x) \otimes i(x)$
- $g(x) = g'_c(x)$

Put it together

- Discrete input texture $f(u)$ for integer u
- Reconstructed input texture $f_c(u) = f(u) \otimes r(u) = \sum f(k) r(u-k)$
- Warped texture $g_c(x) = f_c(m^{-1}(x))$
- Band-limited output $g'_c(x) = g_c(x) \otimes h(x) = \int g_c(t) h(x-t) dt$
- Discrete output $g(x) = g'_c(x) \otimes i(x)$
- $g(x) = g'_c(x)$

$$= \int g_c(t) h(x-t) dt$$

$$= \int f_c(m^{-1}(t)) h(x-t) dt$$

$$= \int h(x-t) \sum f(k) r(m^{-1}(t)-k) dt$$

$$= \sum f(k) \rho(x, k)$$
- Where $\rho(x, k) = \int h(x-t) r(m^{-1}(t)-k) dt$

Resampling – convolution view

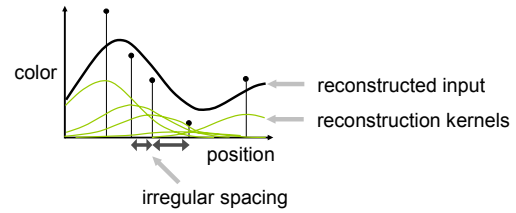
- Ignoring normalization

$$g(x) = \sum_i f(x_i) r_i(m^{-1}(x)) \otimes h(x)$$

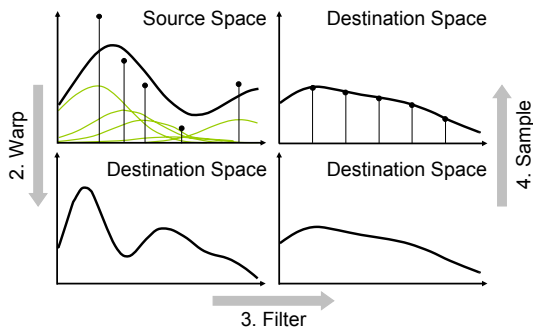
resampling filter

- The image space resampling filter combines a warped reconstruction filter and a low-pass filter

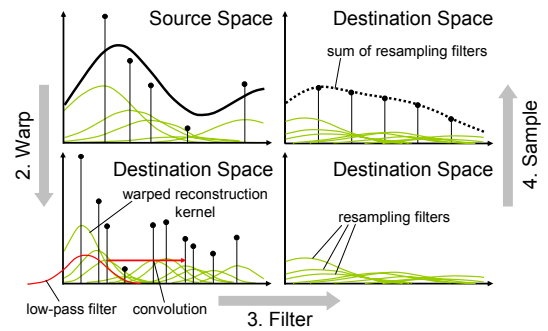
Resampling



Resampling



Resampling



Resampling – convolution view

- Ignoring normalization

$$g(x) = \sum_i f(x_i) r_i(m^{-1}(x)) \otimes h(x)$$

resampling filter

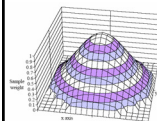
- The image space resampling filter combines a warped reconstruction filter and a low-pass filter
- This is great, but how do we warp reconstruction filters?***

Resampling

- Use local affine approximation of warp
- Elliptical Gaussian kernels [Heckbert 89]
 - Closed under affine mappings and convolution

$$g(x) = \sum_i c_i r_i(\tilde{m}_i^{-1}(x)) \otimes h(x)$$

$$= \sum_i c_i G_i(x)$$



Gaussian resampling kernel
(EWA resampling kernel)

Resampling filter

- Depends on local warp
- For perspective, approximated by local affine at center of kernel
- Not bad approximation because filter small at periphery

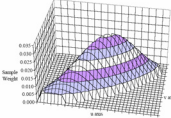
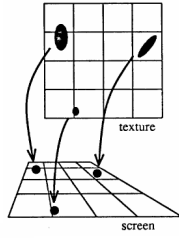


Figure 3: A perspective projection of a Gaussian filter into texture

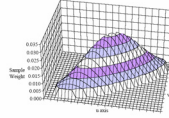
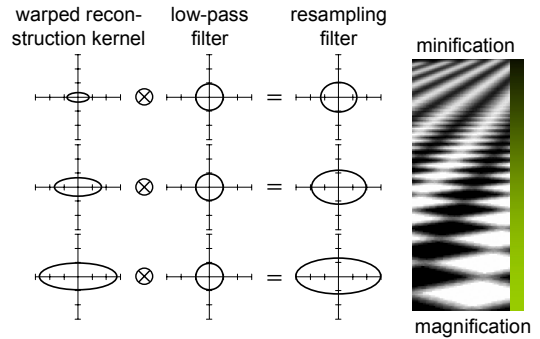


Figure 4: An affine projection of a Gaussian filter into texture space

Resampling Filter

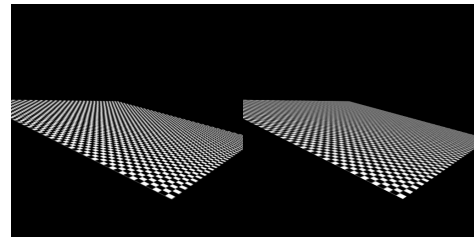


EWA resampling



Image Quality Comparison

- Trilinear mipmapping



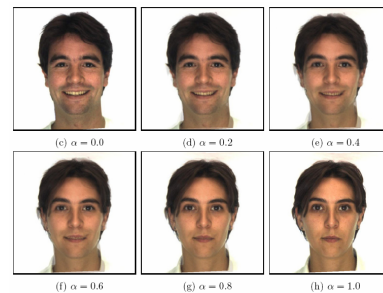
EWA

trilinear mipmapping

Bells and whistles

Morphing & matting

- Extract foreground first to avoid artifacts in the background



(c) $\alpha = 0.0$

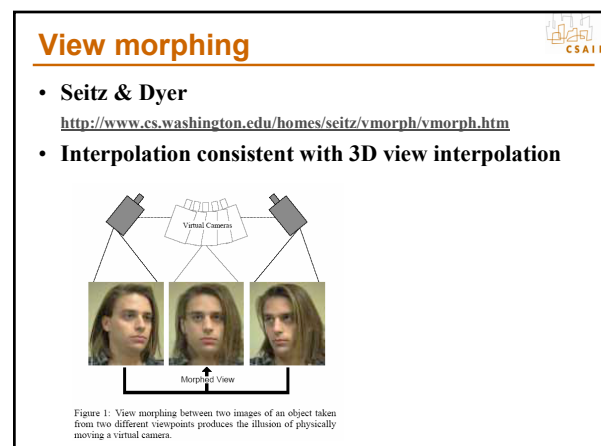
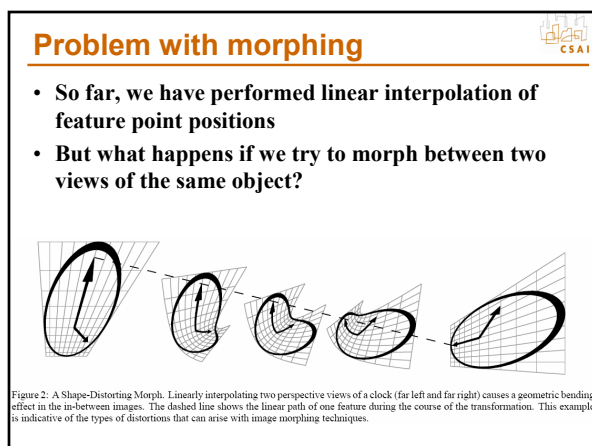
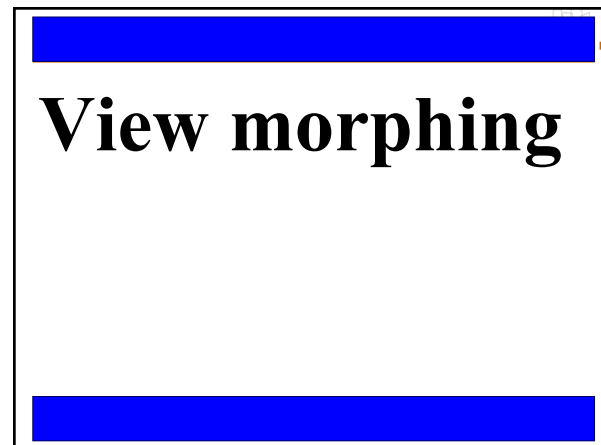
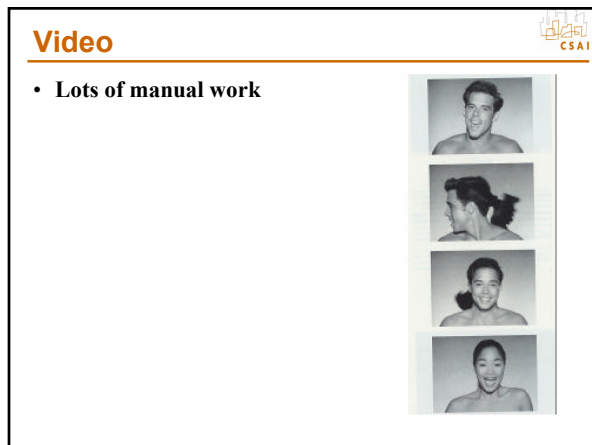
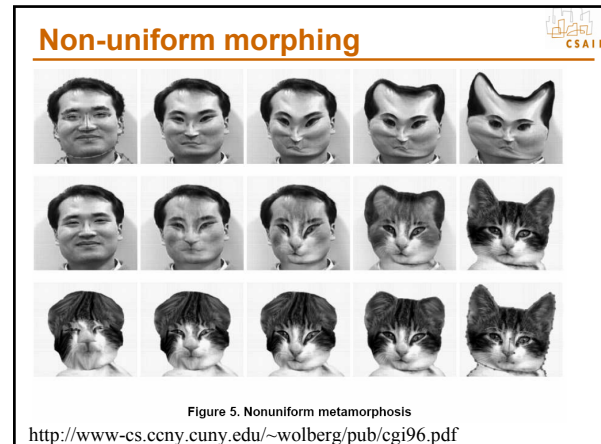
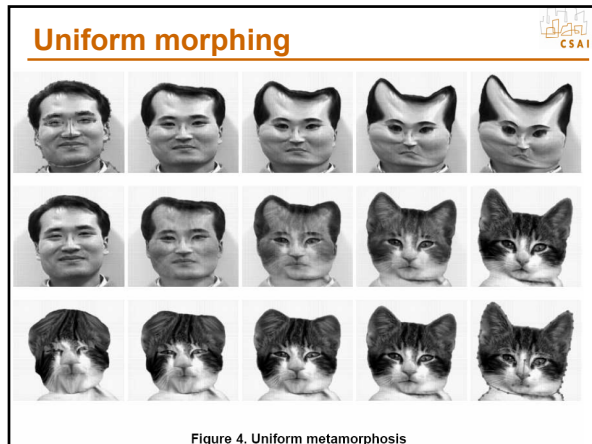
(d) $\alpha = 0.2$

(e) $\alpha = 0.4$

(f) $\alpha = 0.6$

(g) $\alpha = 0.8$

(h) $\alpha = 1.0$



Main trick

- Prewarp with a homography to "pre-align" images
- So that the two views are parallel
 - Because linear interpolation works when views are parallel

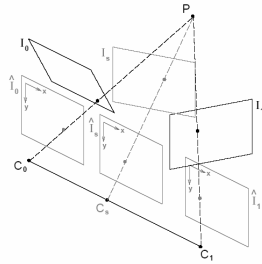


Figure 4: View Morphing in Three Steps. (1) Original images I_0 and I_1 are warped to form parallel views $\hat{I}_0$ and $\hat{I}_1$. (2) $\hat{I}_s$ is produced by morphing (interpolating) the prewarped images. (3) $\hat{I}_s$ is postwarped to form I_s .

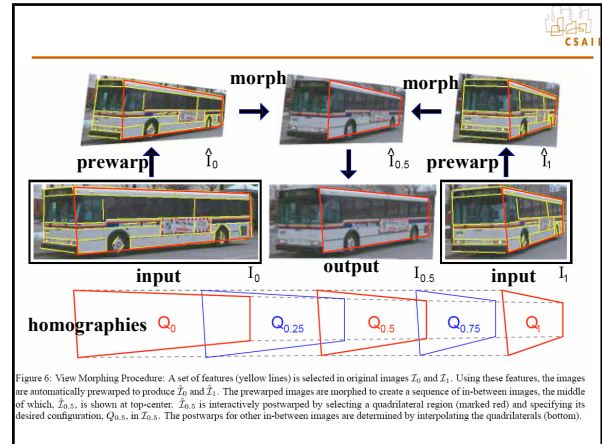


Figure 6: View Morphing Procedure: A set of features (yellow lines) is selected in original images I_0 and I_1 . Using these features, the images are automatically prewarped to produce $\hat{I}_0$ and $\hat{I}_1$. The prewarped images are morphed to create a sequence of in-between images, the middle of which, $\hat{I}_{0.5}$, is shown at top-center. $\hat{I}_{0.5}$ is interactively postwarped by selecting a quadrilateral region (marked red) and specifying its desired configuration, $Q_{0.5}$, in $I_{0.5}$. The postwarps for other in-between images are determined by interpolating the quadrilaterals (bottom).

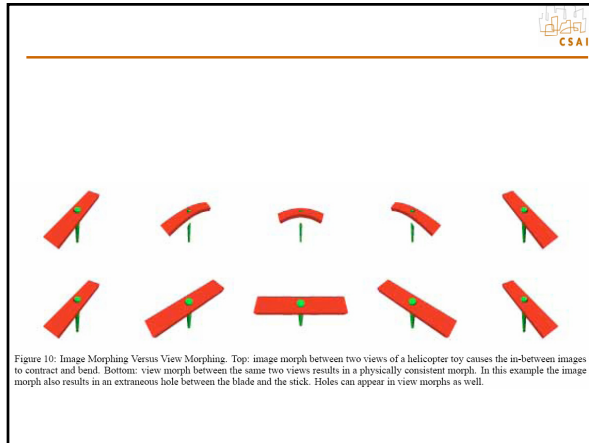


Figure 10: Image Morphing Versus View Morphing. Top: image morph between two views of a helicopter toy causes the in-between images to contract and bend. Bottom: view morph between the same two views results in a physically consistent morph. In this example the image morph also results in an extraneous hole between the blade and the stick. Holes can appear in view morphs as well.

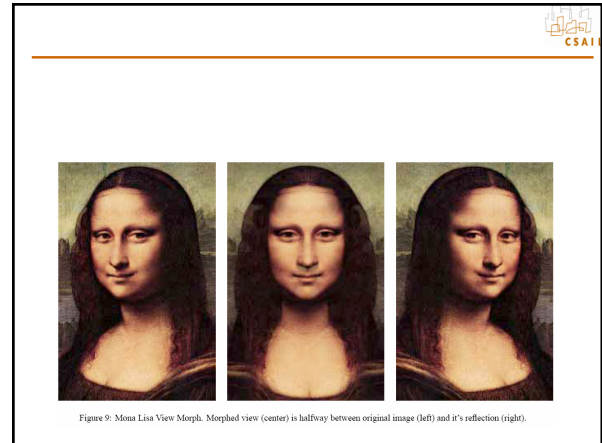


Figure 9: Mona Lisa View Morph. Morphed view (center) is halfway between original image (left) and it's reflection (right).

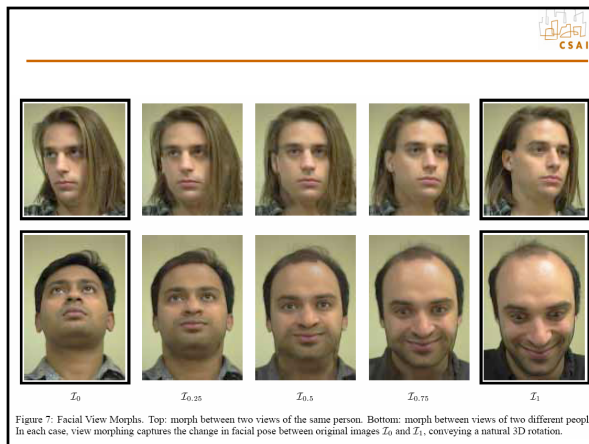
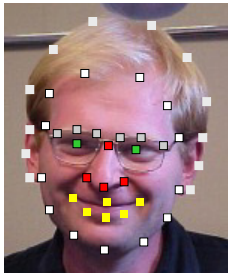


Figure 7: Facial View Morphs. Top: morph between two views of the same person. Bottom: morph between views of two different people. In each case, view morphing captures the change in facial pose between original images I_0 and I_1 , conveying a natural 3D rotation.

Extensions

Shape Vector



Provides alignment!

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Slide Alyosha Efros

The Morphable face model

- Again, assuming that we have m such vector pairs in full correspondence, we can form new shapes S_{model} and new appearances T_{model} as:

$$S_{model} = \sum_{i=1}^m a_i S_i \quad T_{model} = \sum_{i=1}^m b_i T_i$$

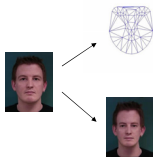
$$s = \alpha_1 \cdot \text{img}_1 + \alpha_2 \cdot \text{img}_2 + \alpha_3 \cdot \text{img}_3 + \alpha_4 \cdot \text{img}_4 + \dots = S \cdot a$$

$$t = \beta_1 \cdot \text{img}_1 + \beta_2 \cdot \text{img}_2 + \beta_3 \cdot \text{img}_3 + \beta_4 \cdot \text{img}_4 + \dots = T \cdot b$$

- If number of basis faces m is large enough to span the face subspace then:
- Any new face can be represented as a pair of **vectors**

The Morphable Face Model

The actual structure of a face is captured in the shape vector $S = (x_1, y_1, x_2, \dots, y_n)^T$, containing the (x, y) coordinates of the n vertices of a face, and the appearance (texture) vector $T = (R_1, G_1, B_1, R_2, \dots, G_n, B_n)^T$, containing the color values of the mean-warped face image.



Shape S

Appearance T

Subpopulation means

- Examples:
 - Happy faces
 - Young faces
 - Asian faces
 - Etc.
 - Sunny days
 - Rainy days
 - Etc.
 - Etc.

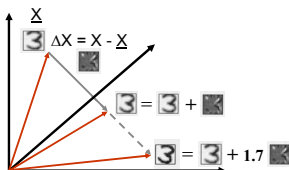


Average female

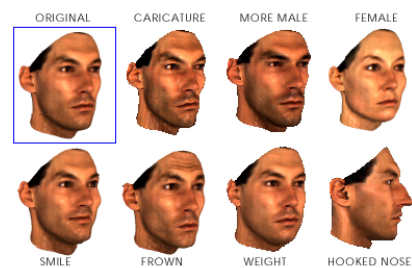


Average male

Deviations from the mean



Using 3D Geometry: Blanz & Vetter, 1999



show SIGGRAPH video

Manipulating Facial Appearance through Shape and Color

Duncan A. Rowland and David I. Perrett
St Andrews University
IEEE CG&A, September 1995

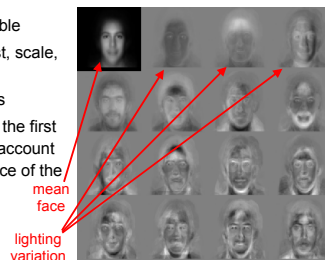
Morphable face models

- <http://citeseer.ist.psu.edu/cache/papers/cs/704/http:zSzzSzwww.ai.mit.edu/zSzprojectsSzczblzSzpublicationSzSzpsSzSzICCV98-matching2.pdf/jones98multidimensional.pdf>
- <http://www.kyb.mpg.de/publication.html?user=volker>

EigenFaces

First popular use of PCA on images was for modeling and recognition of faces [Kirby and Sirovich, 1990, Turk and Pentland, 1991]

- Collect a face ensemble
- Normalize for contrast, scale, & orientation.
- Remove backgrounds
- Apply PCA & choose the first N eigen-images that account for most of the variance of the data.



The average face

- http://www.uni-regensburg.de/Fakultaeten/phil_Fak_II/Psychologie/Psy_II/beautycheck/english/index.htm



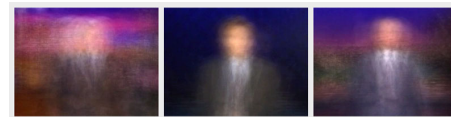
On the left: the "real" Miss Germany 2002 (= Miss Berlin) and on the right: the "virtual" Miss Germany, which was computed by blending together all contestants of the final round and was rated as being much more attractive.

Figure-centric averages



Antonio Torralba & Aude Oliva (2002)
Averages: Hundreds of images containing a person are averaged to reveal regularities in the intensity patterns across all the images.

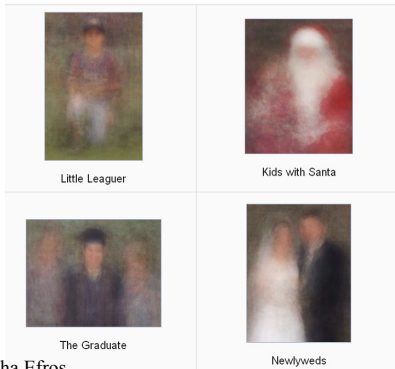
Jason Salavon



Slide Alyosha Efros

More at: <http://www.salavon.com/>

"100 Special Moments" by Jason Salavon

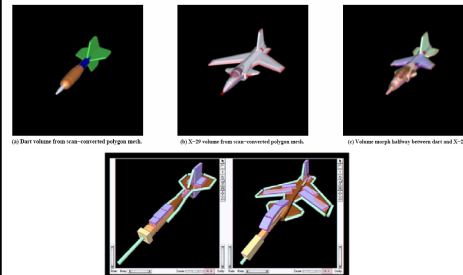


Why blurry?

Slide Alyosha Efros

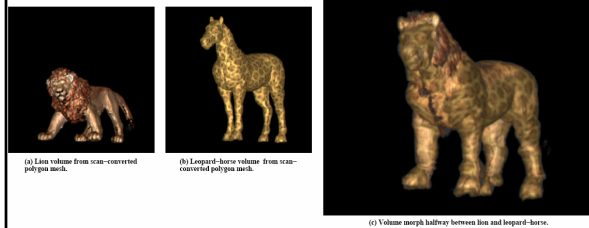
3D morphing

- Feature-Based Volume Metamorphosis Lierios, Garfinkle, and Levoy.
- <http://www-graphics.stanford.edu/~tolis/toli/research/morph.html>



3D morphing

- Feature-Based Volume Metamorphosis Lierios, Garfinkle, and Levoy.
- <http://www-graphics.stanford.edu/~tolis/toli/research/morph.html>



Automatic morphing

- <http://ccc.inaoep.mx/~fuentes/zanella.pdf>

Recap & Significance

Recap

- Idea that linear interpolation introduces blur
- Separation of shape and color
- Idea of non-rigid alignment of different images
 - Applications to medical data
- Applications, related to
 - Special effects
 - Face recognition
 - Video frame interpolation
 - MPEG
- Scattered data interpolation

References

Refs

- <http://portal.acm.org/citation.cfm?id=134003&coll=GUIDE&dl=GUIDE&CFID=72901489&CFTOKEN=24335444>
- <http://www.visgraf.impa.br/cgi-bin/morphQuery.cgi?output=html>
- <http://www.cg.tuwien.ac.at/research/ca/mrm/>
- <http://w3.impa.br/~morph/sites.html>
- <http://w3.impa.br/~morph/sig-course/slides.html>
- <http://www.owl.net.rice.edu/~elec539/Projects97/morphjks/morph.html>
- <http://www.fmrib.ox.ac.uk/~vongyue/thinplate.html>
- http://www.uoguelph.ca/~mwirth/PHD_Chapter4.pdf



Software

- <http://www.morpheussoftware.net/>
- <http://www.debugmode.com/winmorph/>
- http://www.stoik.com/products/morphman/mm1_main.htm
- http://www.creativecow.net/articles/zwar_chris/morph/index.html
- <http://meesoft.logicnet.dk/SmartMorph/>
- <http://www.asahi-net.or.jp/~FX6M-FJMY/mop00e.html>
- <http://morphing-software-review.toptenreviews.com/>
- <http://www.freedomdownloadcenter.com/Search/morphing.html>

Next time: Panoramas