Surface Reconstruction

Power Diagrams, the Medial Axis Transform and the Power Crust Algorithm

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6.838 Geometric Computation

Lecture 19 — 13 November 2001

Overview

- Introduction
- Weighted distance and power diagrams
- Medial Axis Transform
- PowerCrust Algorithm

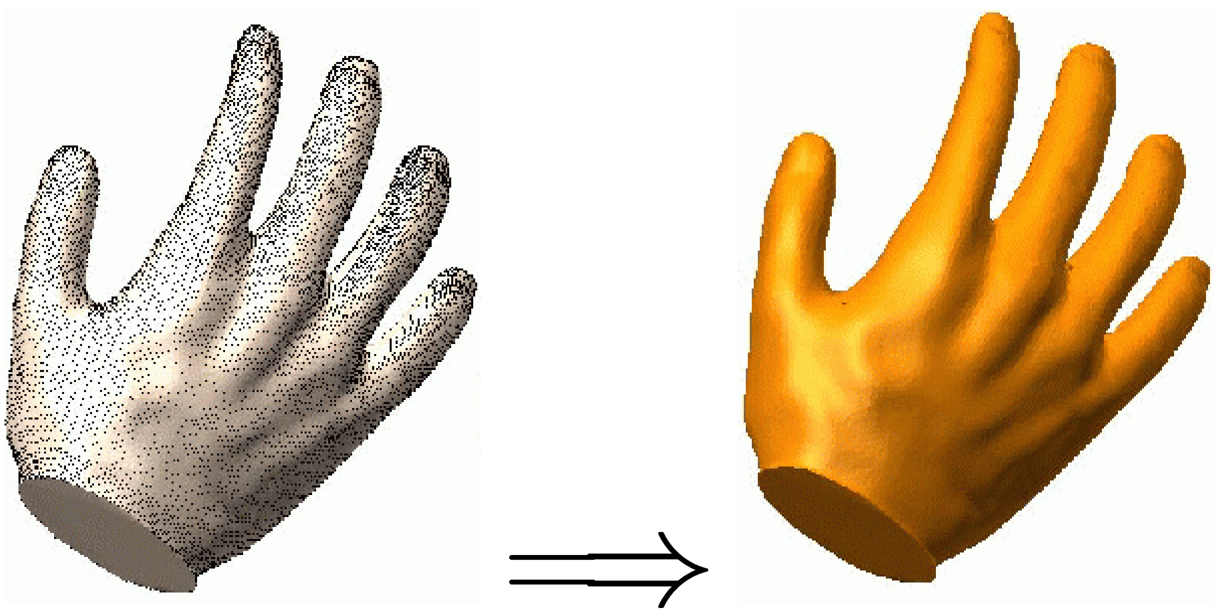
Introduction

Introduction

- What is Surface Reconstruction?
- Applications
- Difficulties
- Survey of techniques

Surface Reconstruction

Given a set of points X assumed to lie near an unknown surface U, construct a surface model S approximating U.



How it usually works

- Input points sampled from the surface either “by hand” or via a physical process (e.g. 3D scanning).
- Assume:
  - Real surface U is “nice” (= “smooth”)
  - Samples X are “dense enough”, especially near features such as edges, points, bumps, etc.
- Output S in usable format for processing
  - Triangulation of S
  - Fitted “splines” (i.e. low-dimensional surfaces)
  - CSG model

Applications of 3D Scanning

- Reverse engineering / Industrial design

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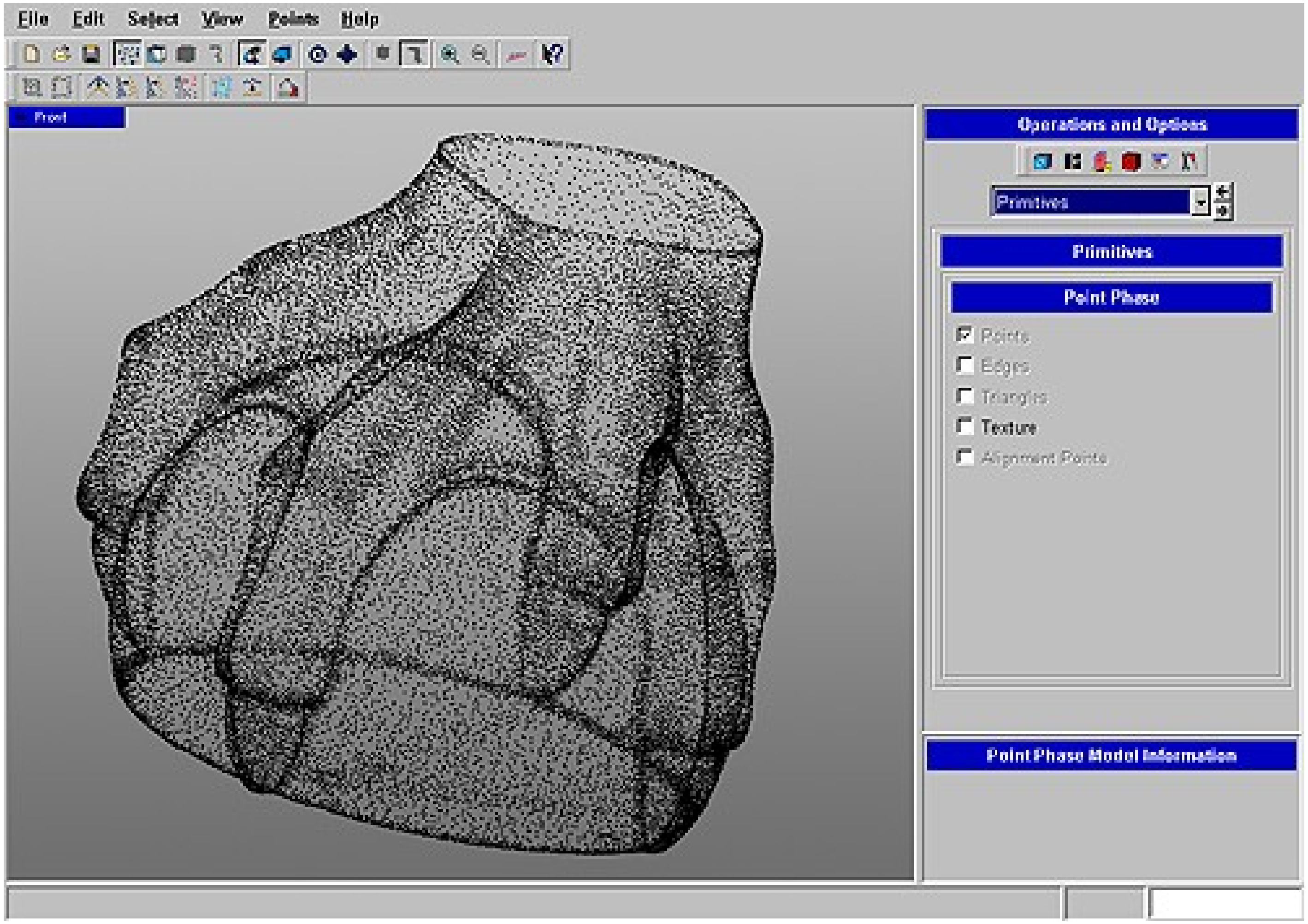
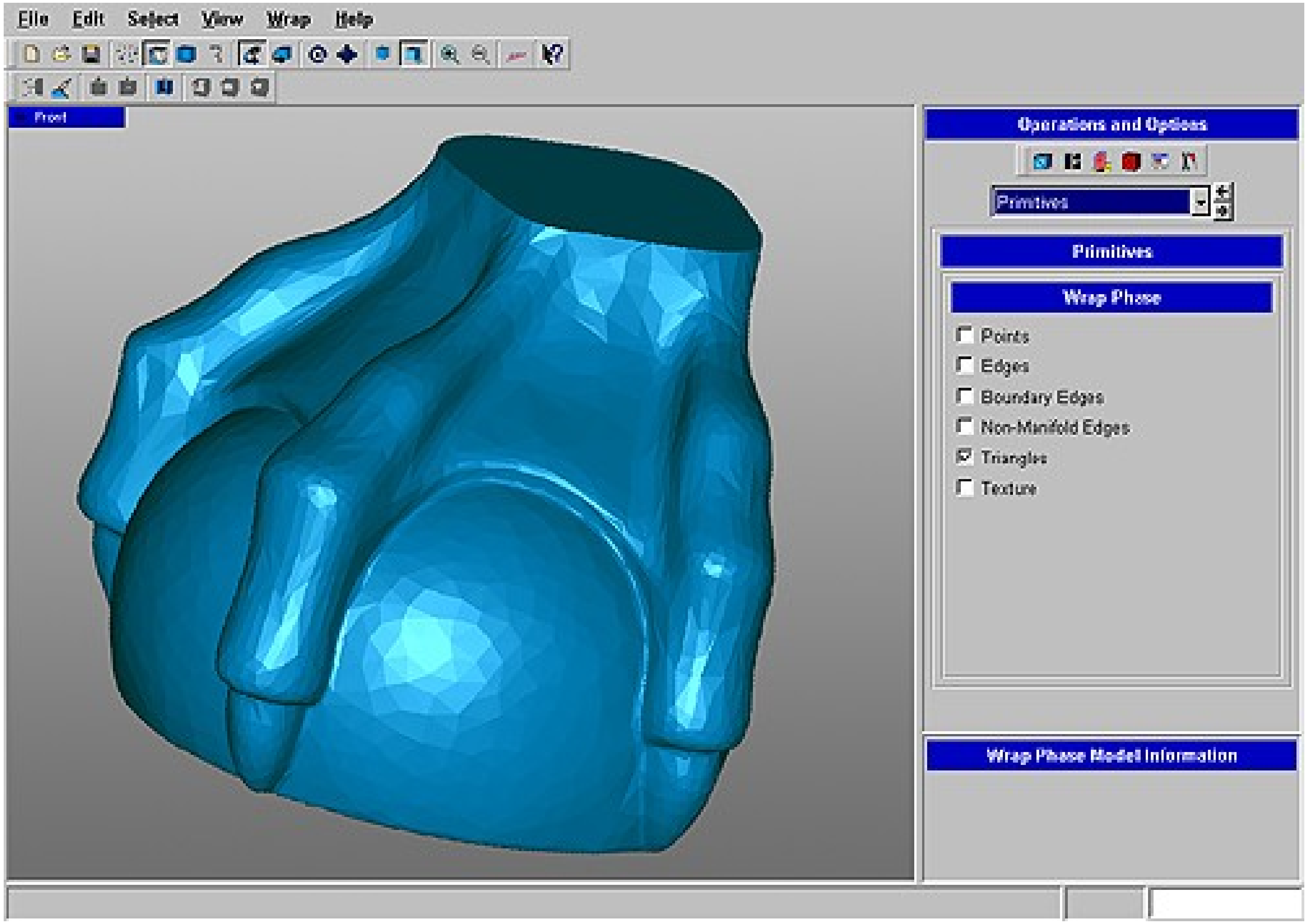
- ...

Modeling a claw I

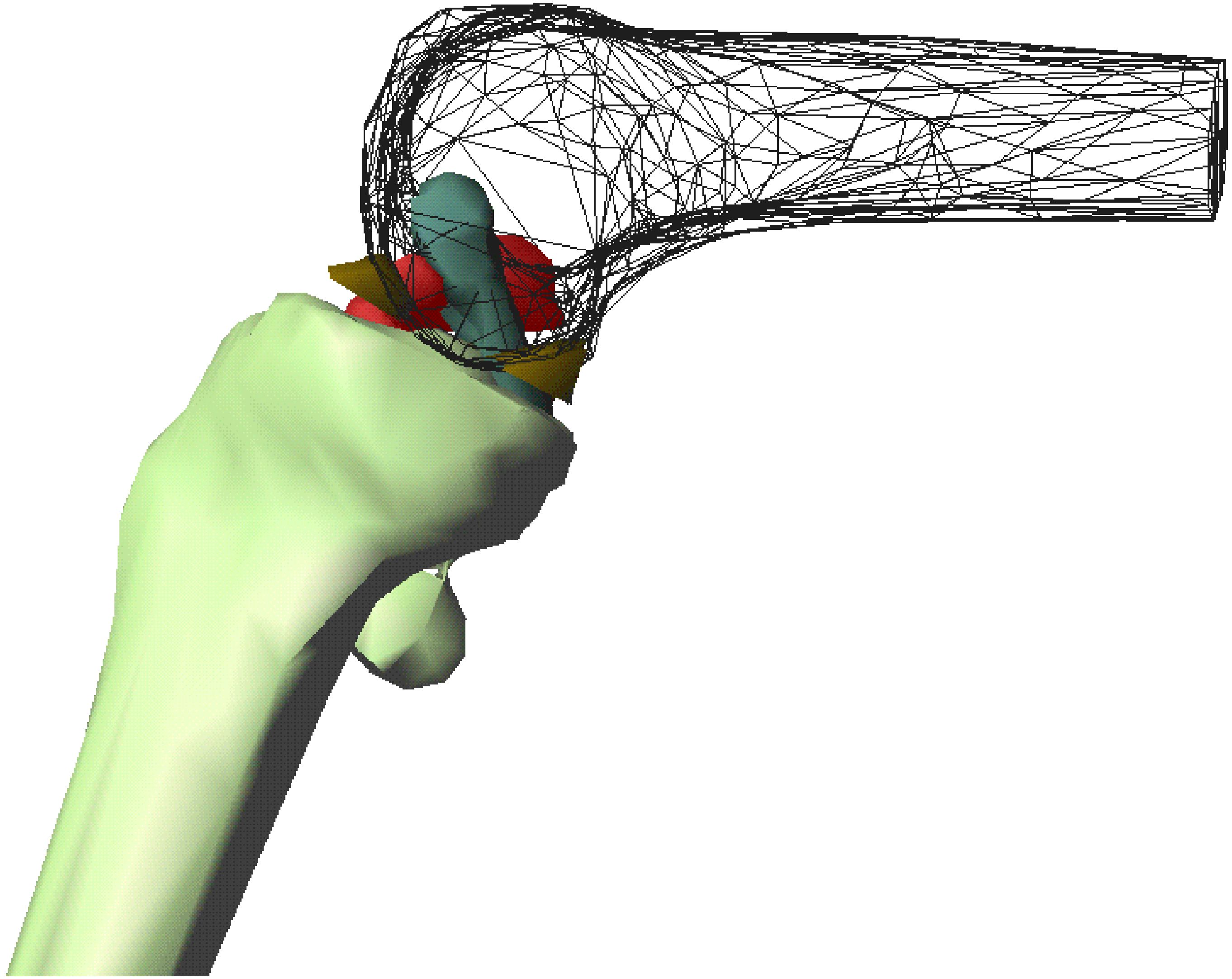


Modeling a Claw II



Modeling hand-made parts

Medical Shape Reconstruction



Difficulties

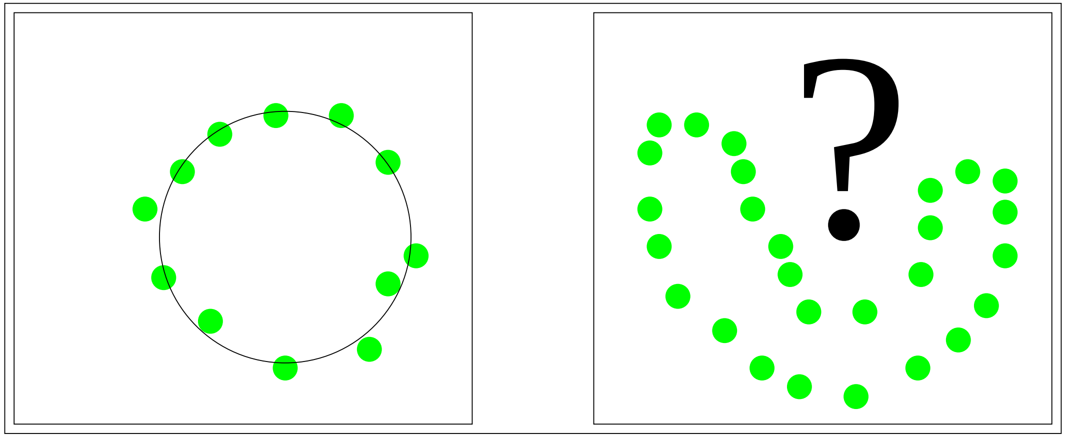
- Surface not smooth
- Noisy data
- Lack of orientation data
- Surface not watertight

Techniques

Techniques for Surface Reconstruction

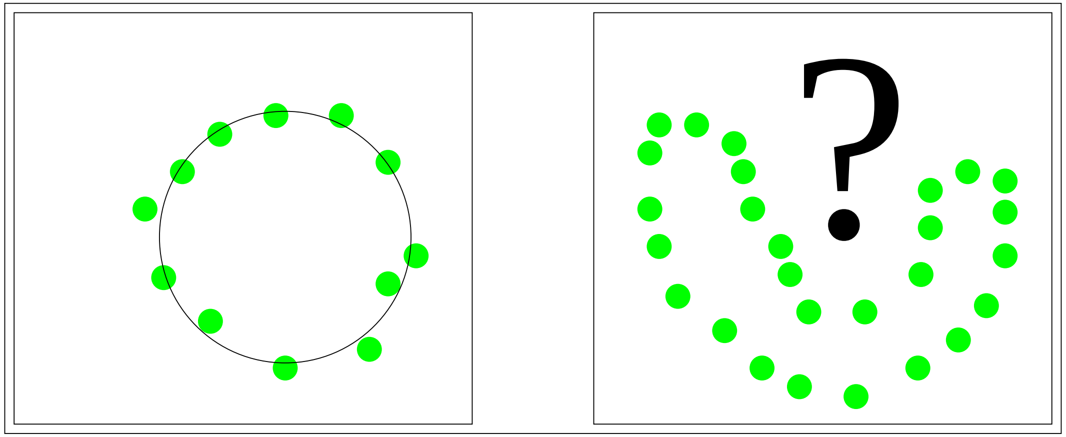
Technique	Assumptions
Fit parametric surface	

Fitting Parametric Surfaces



- Assume surface is from some known family (e.g. sphere, cylinder, plane, hyperboloid, etc)
- Find best parameters to fit data

Fitting Parametric Surfaces



- Fast, accurate for good data
- Useless when data is of unknown type

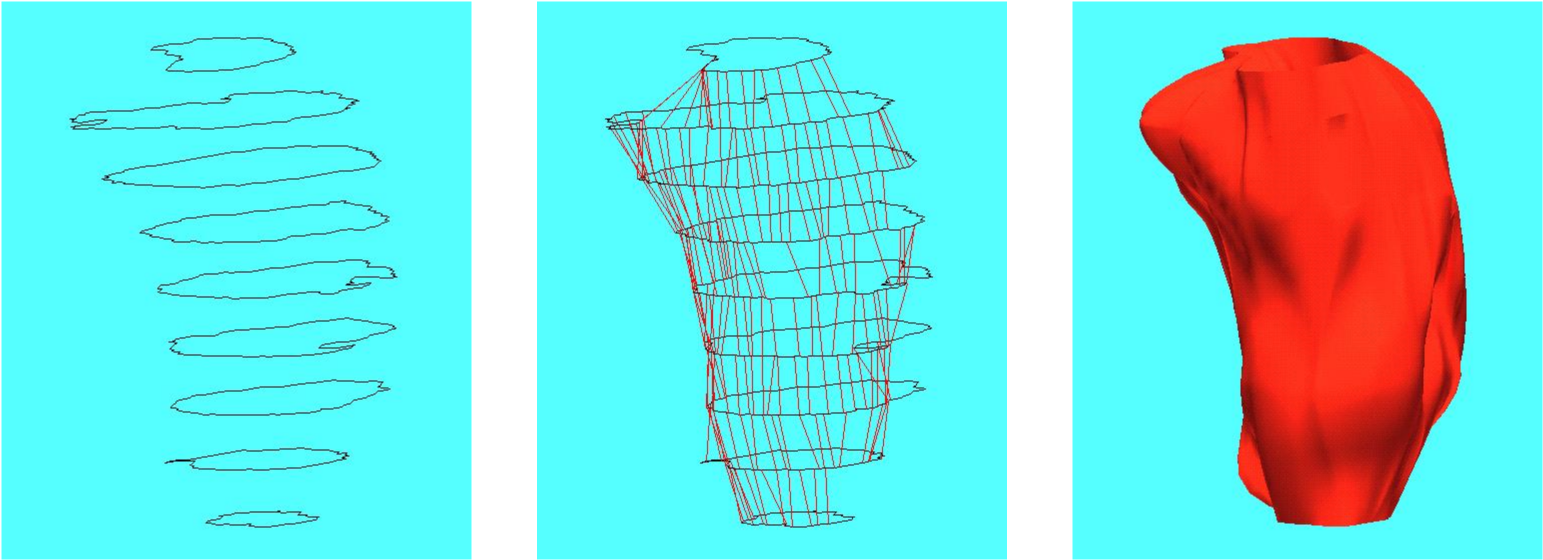
Techniques for Surface Reconstruction

Technique	Assumptions
Fit parametric surface	Data fits model

Techniques for Surface Reconstruction

Technique	Assumptions
Fit parametric surface	Data fits model
Piece together parallel contours	

Contour Data Reconstruction



- Piece together image from parallel slices
- Assumes data is “pre-structured”
- Applications: medical, topographic terrain maps

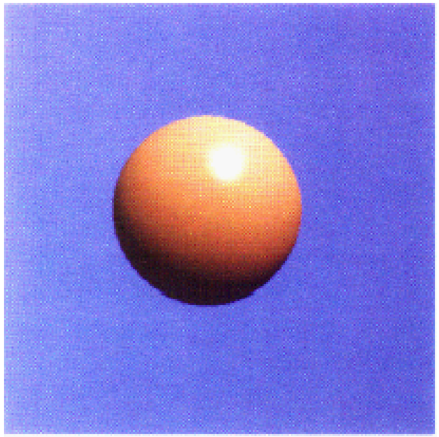
Techniques for Surface Reconstruction

Technique	Assumptions
Fit parametric surface	Data fits model
Piece together parallel contours	Data from known device

Techniques for Surface Reconstruction

Technique	Assumptions
Fit parametric surface	Data fits model
Piece together parallel contours	Data from known device
Fit Gaussian Kernels	

Fitting Gaussian balls



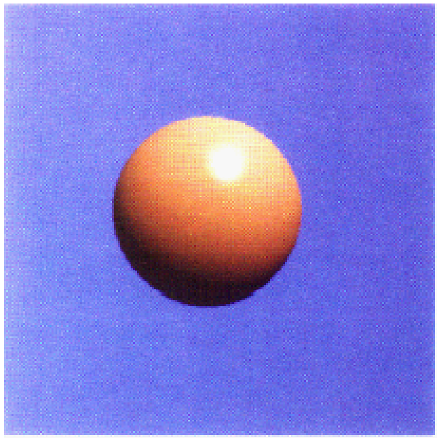
(a) $N = 1$

(d) $N = 35$

(f) $N = 243$

- Take linear combination of 3D Gaussians
  $f(\vec{x}) = \sum c_i \, e^{\left((\vec{x}-\vec{\mu})^\top K_i (\vec{x}-\vec{\mu})\right)}$
- Surface $S = \{\vec{s} \mid f(\vec{s}) = 0\}$
  (inside = positive, outside = negative)

Fitting Gaussian balls



(a) $N = 1$

(d) $N = 35$

(f) $N = 243$

Problems:

- Must know surface normal at each point
- Output always watertight, bubbly-shaped
- Useful for range scanner data

Techniques for Surface Reconstruction

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Piece together parallel contours	Data from known device
Fit Gaussian Kernels	Given normal at each point

Techniques for Surface Reconstruction

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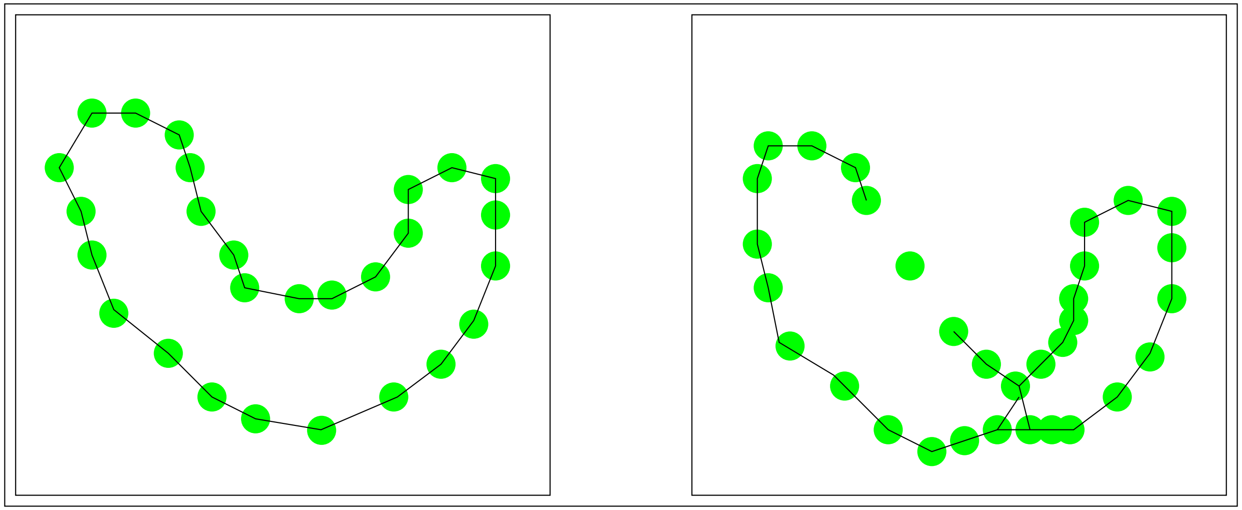
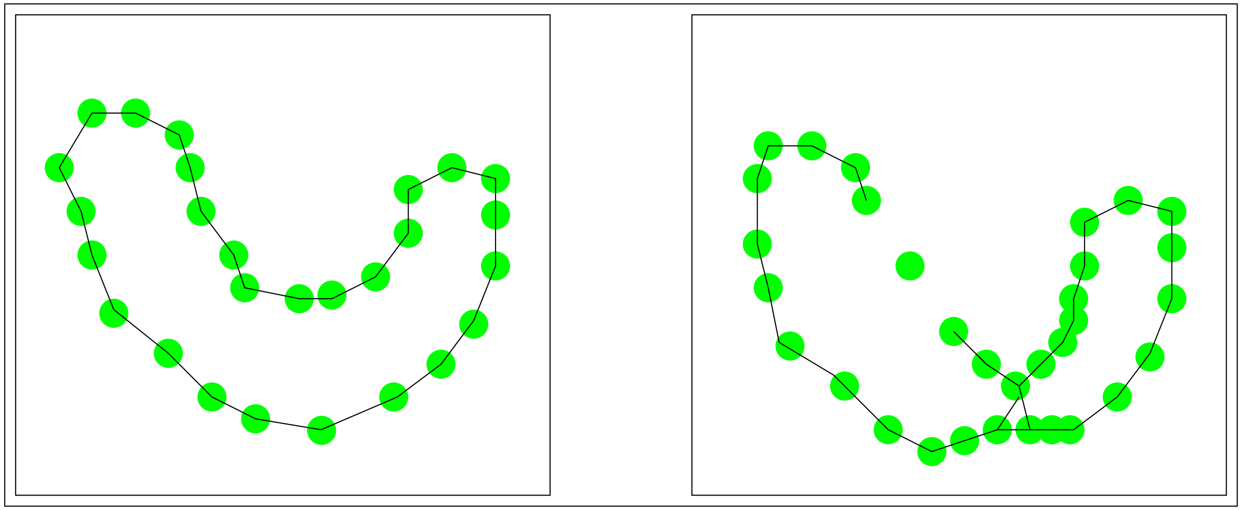
α-shape Triangulation	

α-shape triangulation



- Start with Delaunay triangulation
- Take subset of the edges “on” the surface S
  (In fact, just take shortest edges in graph)

α-shape triangulation



- Bad when samples unevenly spaced
  (can be fixed using weights on sample points)
- Works only for noise-free data

Techniques for Surface Reconstruction

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Piece together parallel contours	Data from known device
Fit Gaussian Kernels	Given normal at each point
α-shape Triangulation	Noise-free

Techniques for Surface Reconstruction

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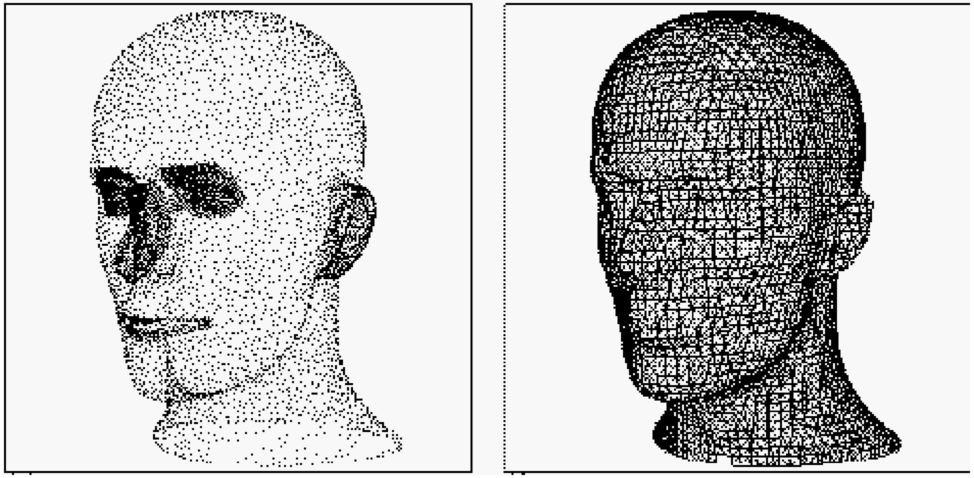
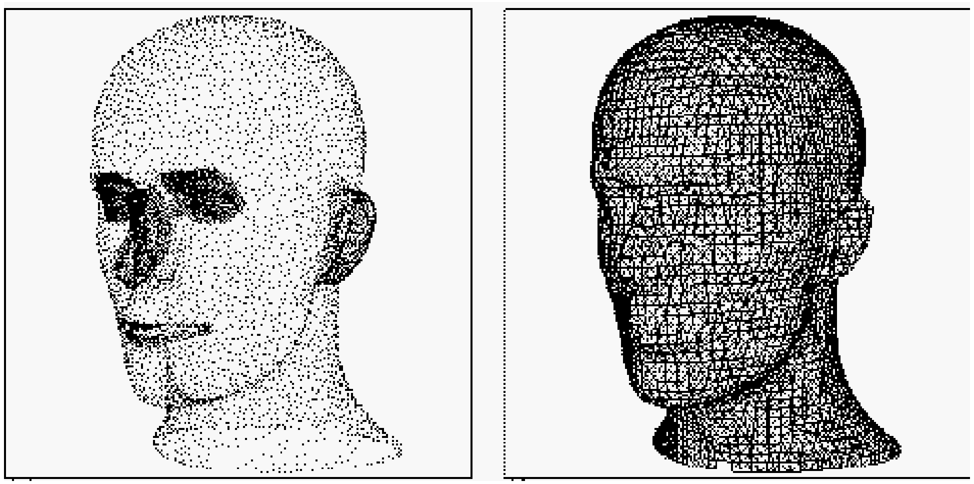
"Mesh" methods	

Mesh methods



- Exploit local information to find a mesh S approximating surface U
- Simplify mesh afterwards

Mesh methods



- Handles noisy data
- Assumes only sample dense near features (edges, bumps)
- Methods ad hoc; Rigorous analysis difficult

Techniques for Surface Reconstruction

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α-shape Triangulation	Noise-free
“Mesh” methods	Sample Dense near Features

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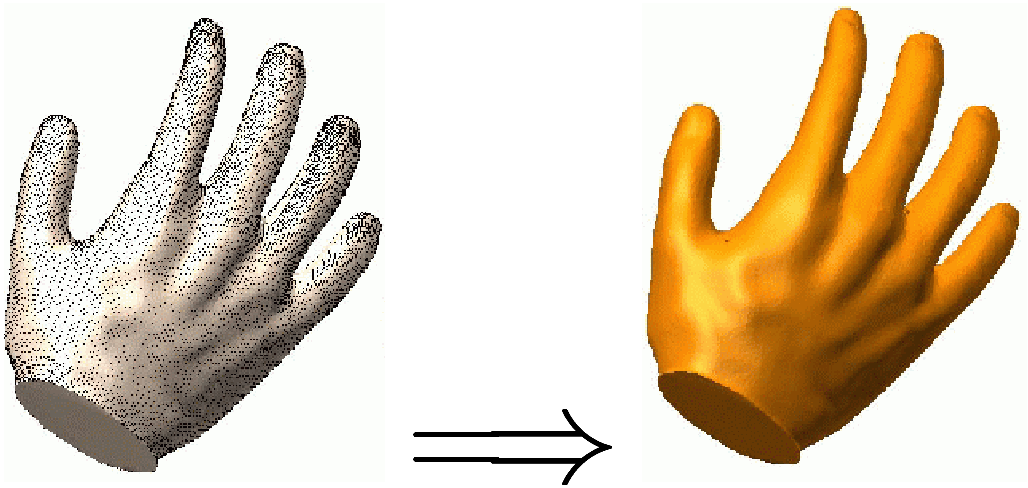
Crust Methods	

Crust methods



- Focus of this lecture
- Assume only dense sampling
- Provide other information on S: Volume, Skeletal Structure

Techniques for Surface Reconstruction

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“Mesh” methods	Sample Dense Near Features
Crust Methods	Sample Dense Near Features

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Weighted Distance and Power Diagrams

Weighted Distance and Power Diagrams

- Weighted distance
- Power Diagrams (= Weighted Voronoi)
- α-shapes

Unions of balls

Key concept:

> Solids can be roughly approximated (exact in the limit) as a union of balls (discs in 2D).

Given a set of points X, we can view X as *centers* of balls. How can we use this?

Unions of balls

Key concept:

> Solids can be roughly approximated (exact in the limit) as a union of balls (discs in 2D).

Given a set of points X, we can view X as *centers* of balls. How can we use this?

> Try to visualize “shape” of X.

Adding weights

Some points are "bigger" than others.

- X sampled from a surface $\Longrightarrow$ points where sampling is less dense are "bigger".
- X = centers of atoms in a molecule $\Longrightarrow$ heavier atoms are "bigger".
- In Power Crust (later), this will be crucial.

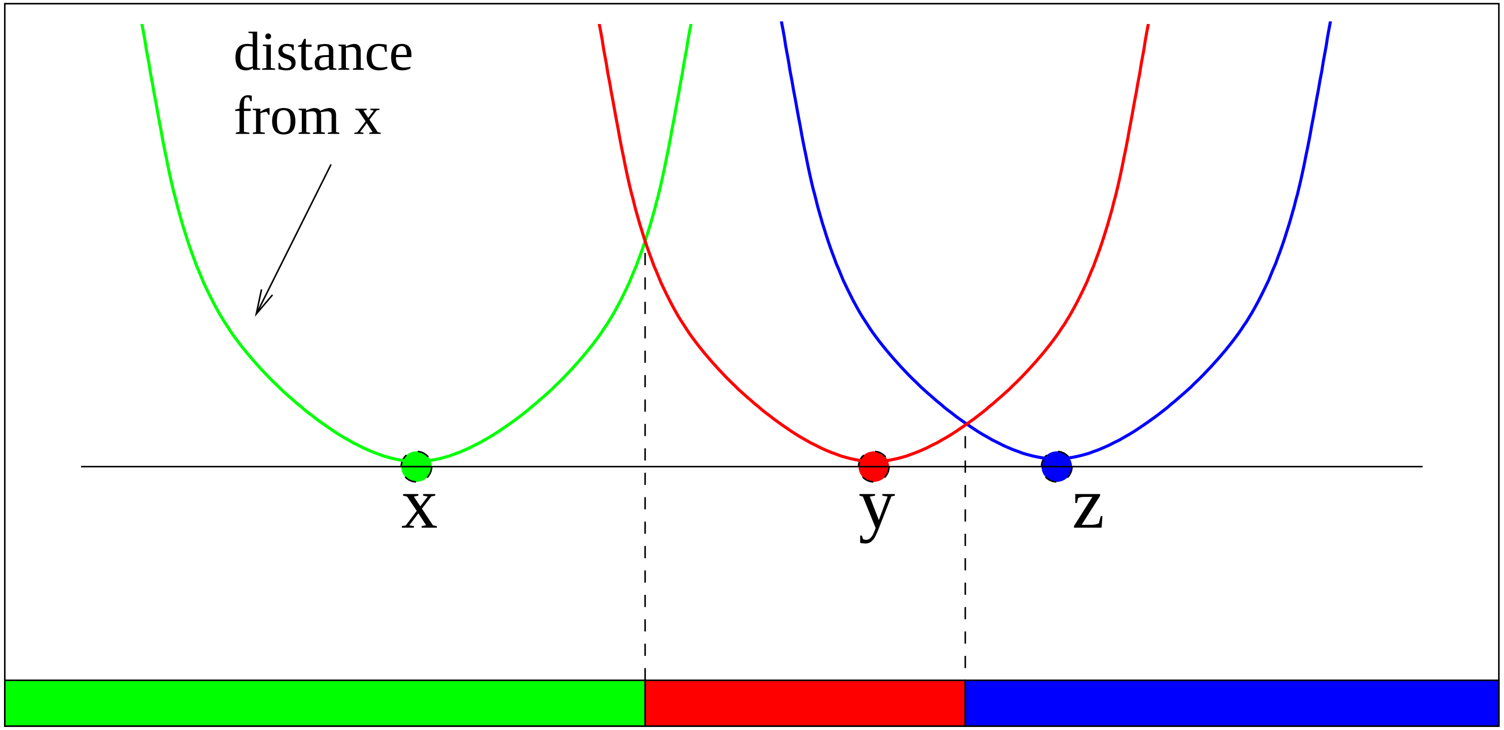
Each point $x \in X$ gets a weight r_x (its radius).

Weighted Distance

- Work with weighted distance. Distance from a point p to a ball (x, r_x):

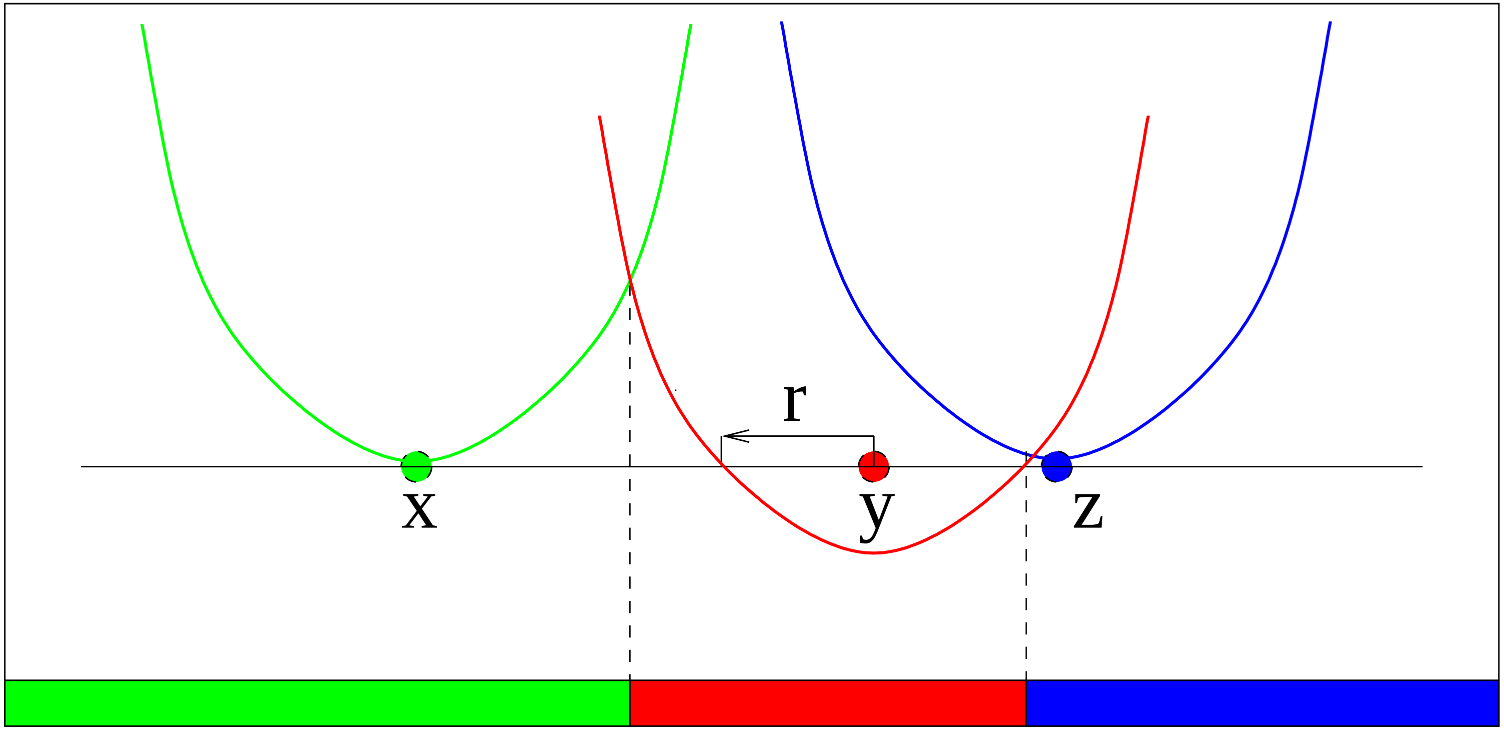
$$d_{r_x}(x, p) = d(x, p)^2 - {r_x}^2$$

Normal Distance: $(p - x)^2$



In 1D, the (normal) squared distance induced by each point x gives a parabola centered at x.

$d_r(y,p) = (p-y)^2 - r^2$



Point y has radius $r_y \Longrightarrow$ parabola gets lowered to intersect axis at distance r from y.

Power Diagram

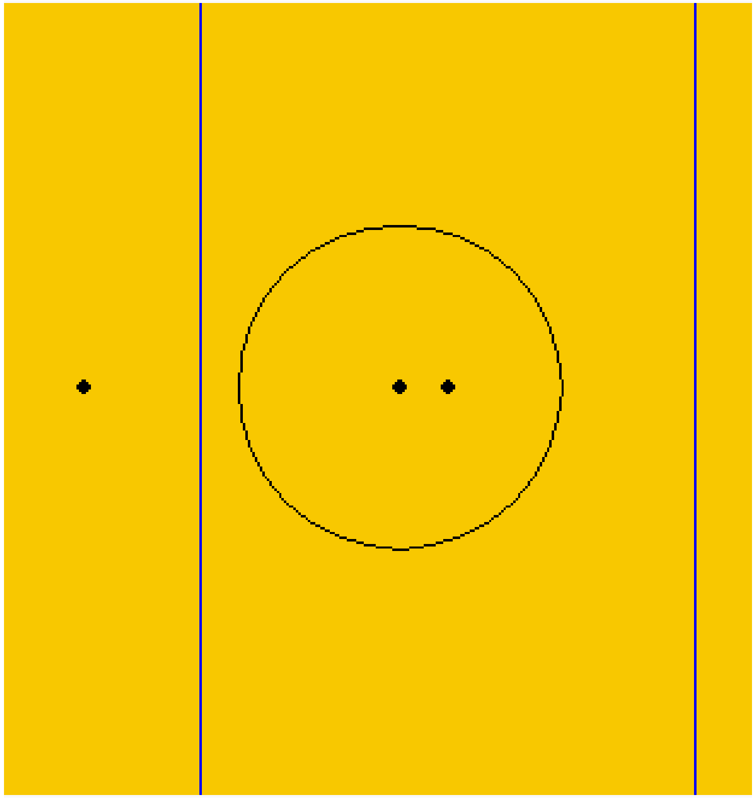
- Distance: $d_{r_x}(x, p) = d(x, p)^2 - {r_x}^2$
- Weighted Voronoi cell of (x, r_x) is set of points p that have smaller weighted distance to x than to any other point in X:

  $cell(x) = \{p \,|\, d_{r_x}(x, p) \leq d_{r_{x'}}(x', p)$ for all $x' \in X\}$

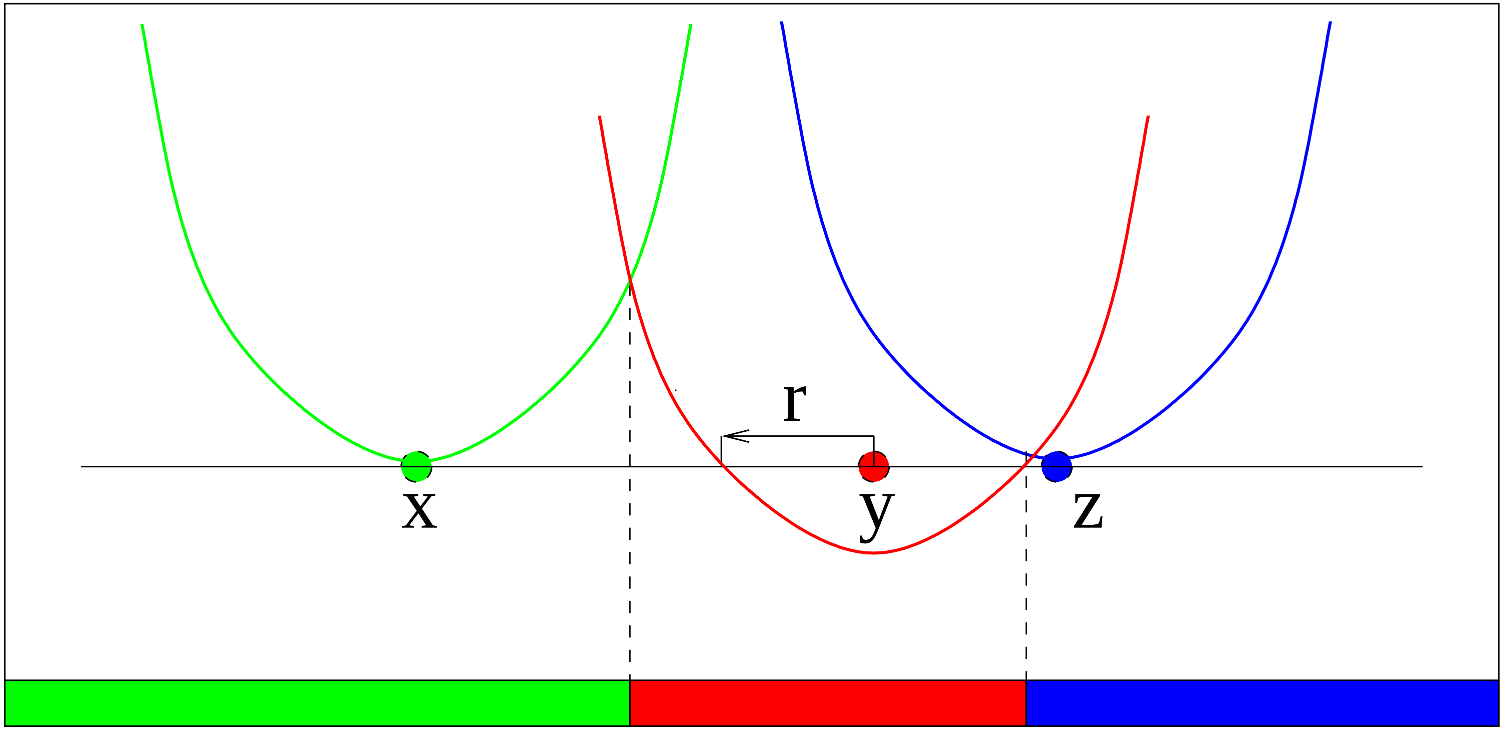
- When all weights are equal, get the usual Voronoi diagram.

Power Diagram Demo

Demo

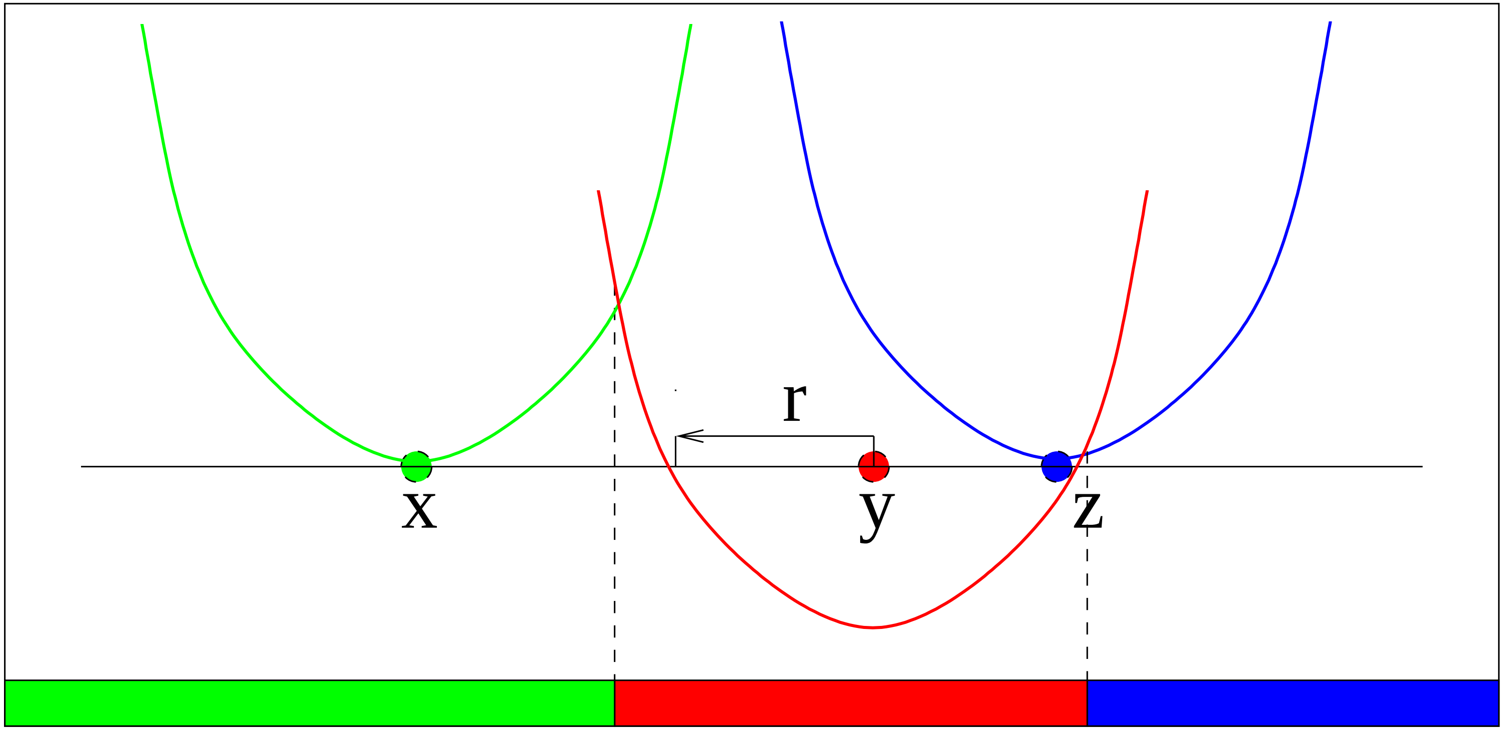


$d_r(y, p) = (p - y)^2 - r^2$



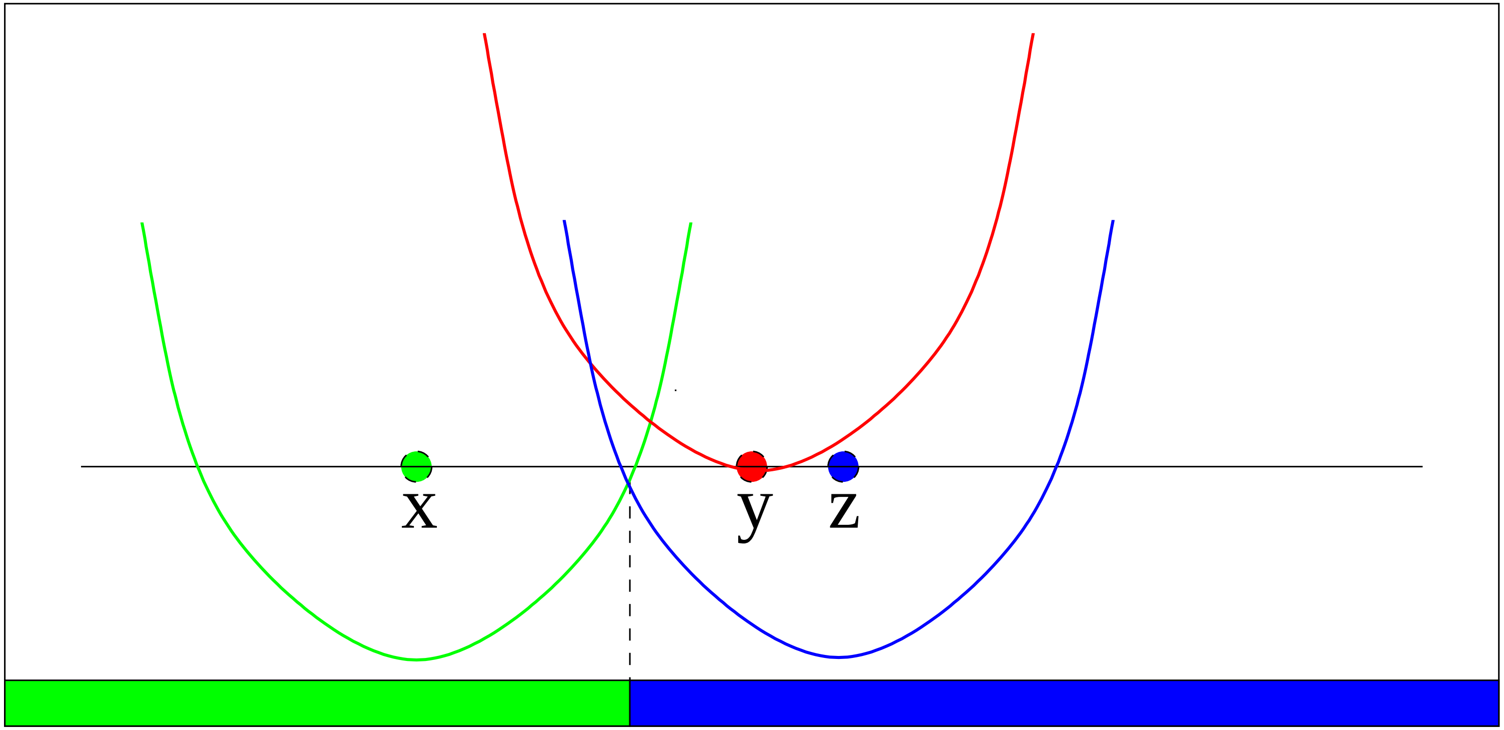
Point y has radius $r_y \implies$ parabola gets lowered to intersect axis at distance r from y.

$d_r(x, y) = (x - y)^2 - r^2$



Weighted Voronoi cell for z doesn't necessarily contain z.

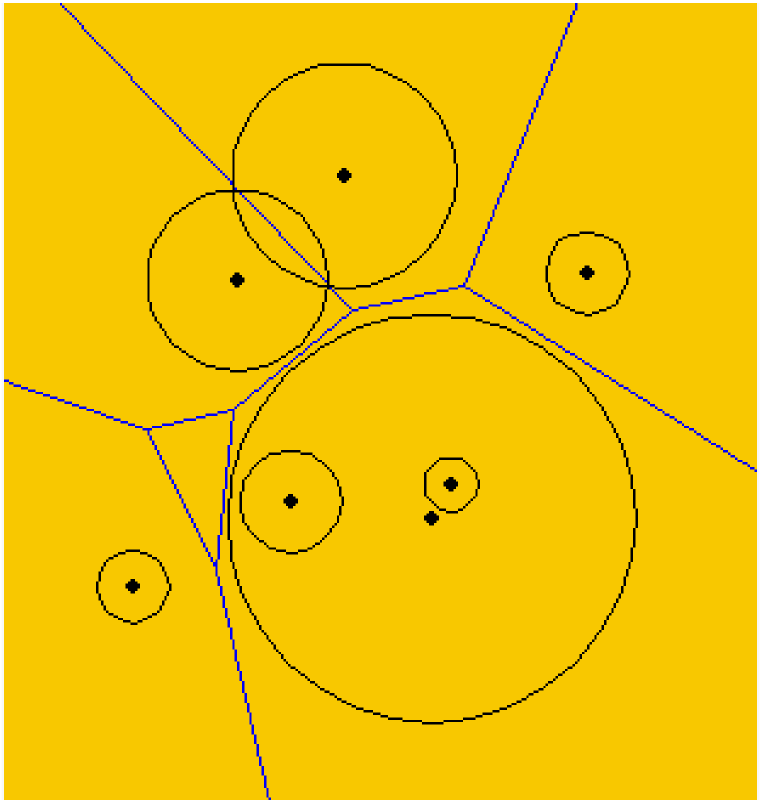
$d_r(x, y) = (x - y)^2 - r^2$



Some Voronoi cells may be empty!

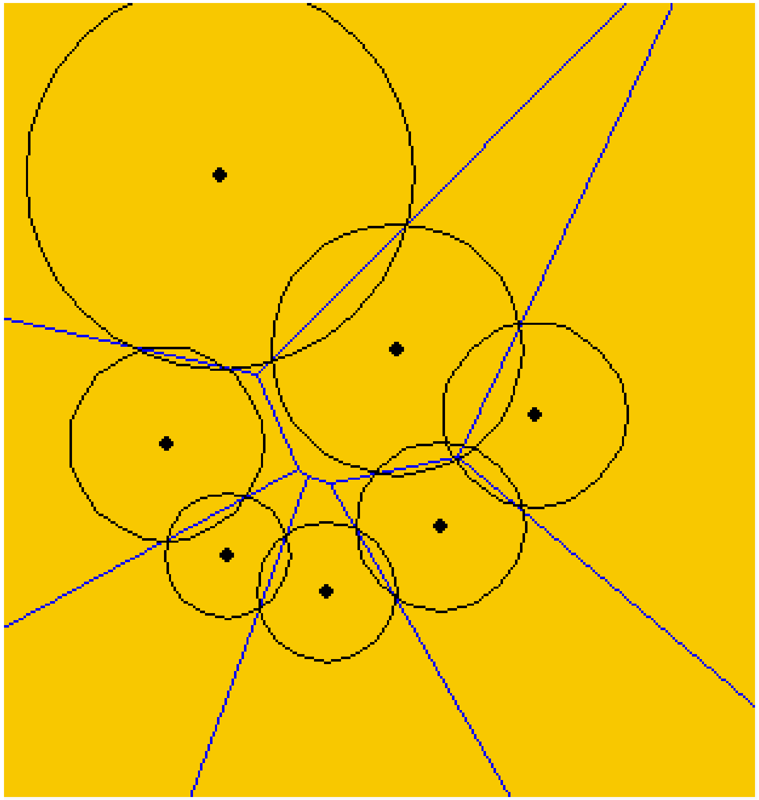
Power Diagram Demo

More Demo



Intersections

Power diagram edges always go through the intersections of circles

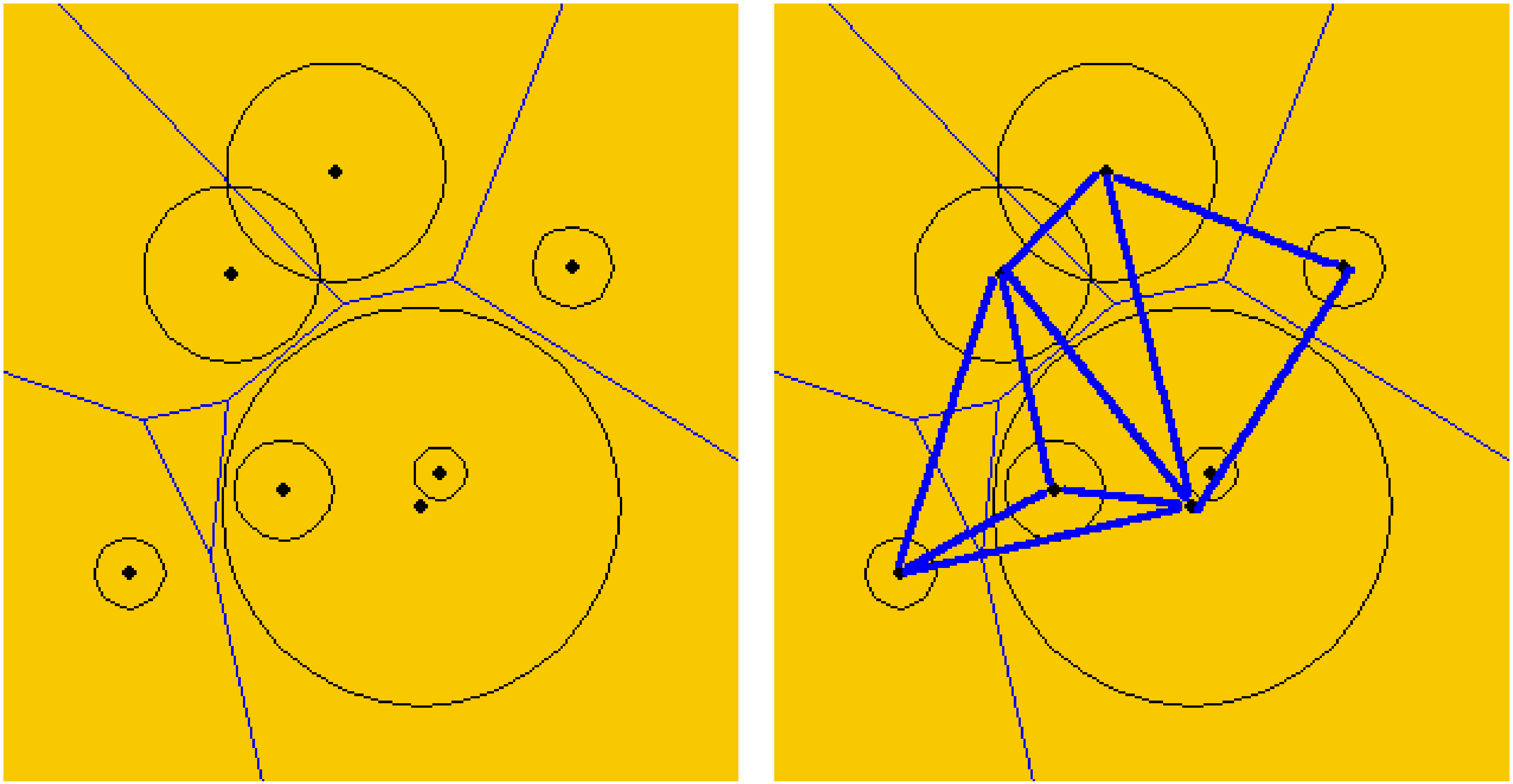


Weighted Delaunay Complex

Weighted Delaunay Complex
= Dual of Power Diagram

- Add an edge $\{x, y\}$if cells of x, y interesect
- Add a triangle $\{x, y, z\}$ if cells of x, y, z intersect
- Add a tetrahedron...

Weighted Delaunay Complex

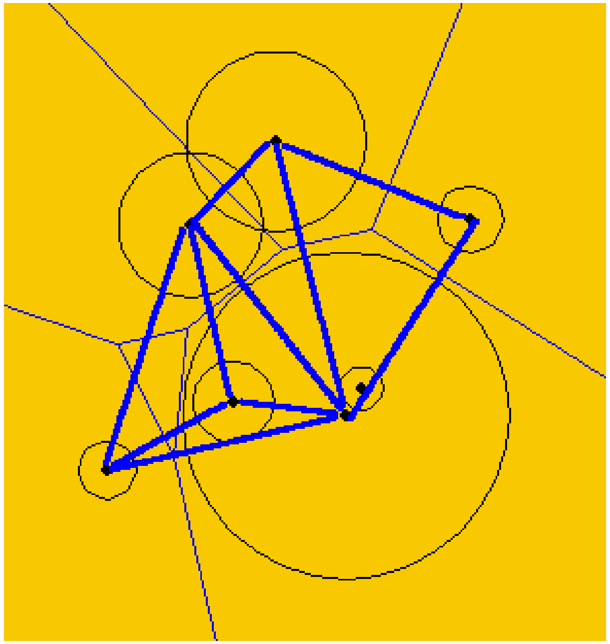


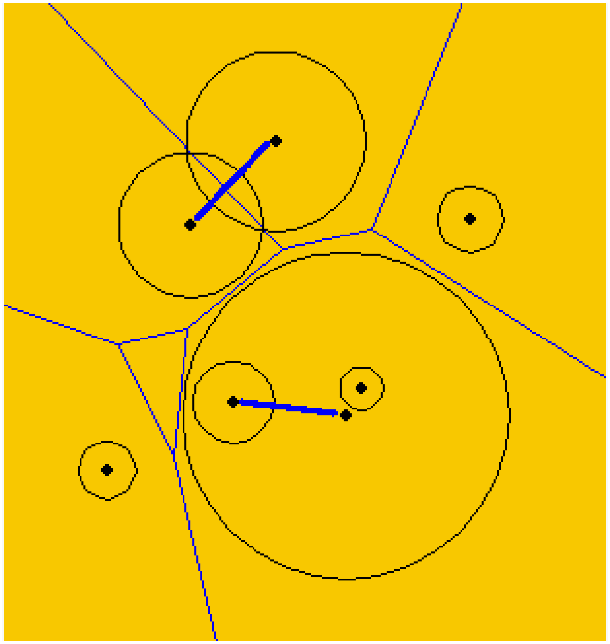
Weighted Distance and Power Diagram

- Weighted distance
- Power Diagrams (= Weighted Voronoi)
- α-shapes

Dual Complex

- Subset of weighted Delaunay graph
- Only keep edge (x, y) if balls at x, y intersect: $d(x, y) \leq r_x + r_y$.



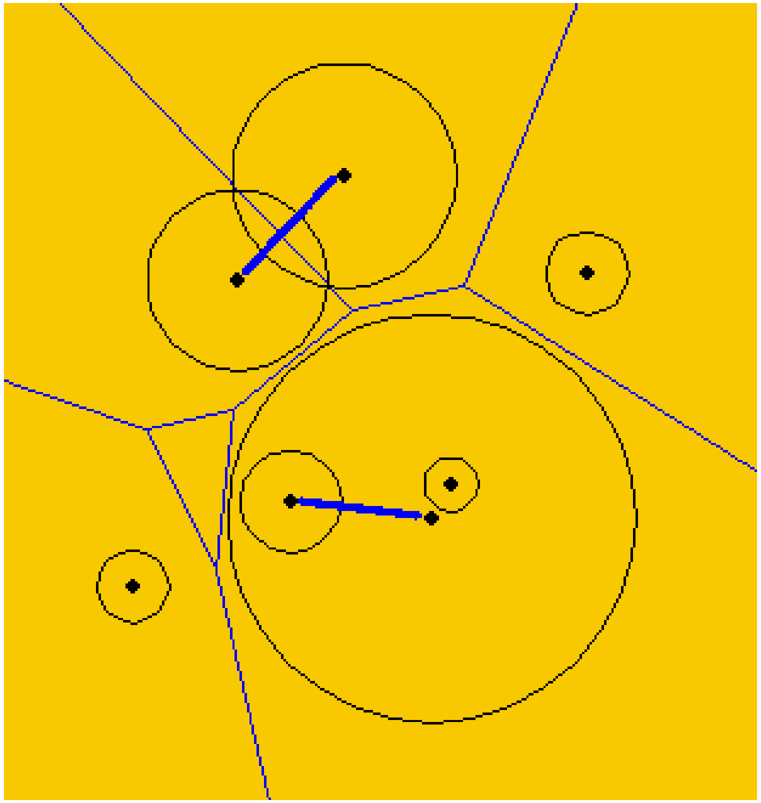


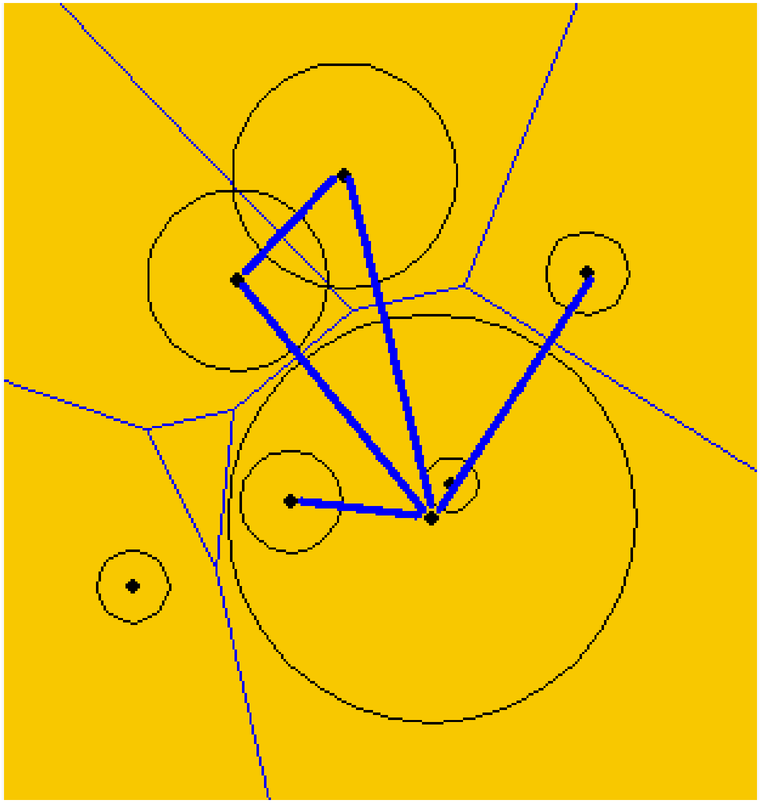
Changing the radii

- Fix a parameter $\alpha^2 \in (-\infty, \infty)$ (i.e. $\alpha \in \mathbb{C}$)
- Consider new radii $r'_x = \sqrt{r_x^2 + \alpha^2}$
- Power diagram stays the same since $d_{r'}(x, p) = d_r(x, p) - \alpha^2$.
- Weighted Delaunay graph stays the same.
- Dual Complex
  - grows if $\alpha^2 > 0$
  - shrinks if $\alpha^2 < 0$

α-shapes

As α^2 grows from $-\infty$ to ∞, progress from empty graph to full weighted Delaunay graph:





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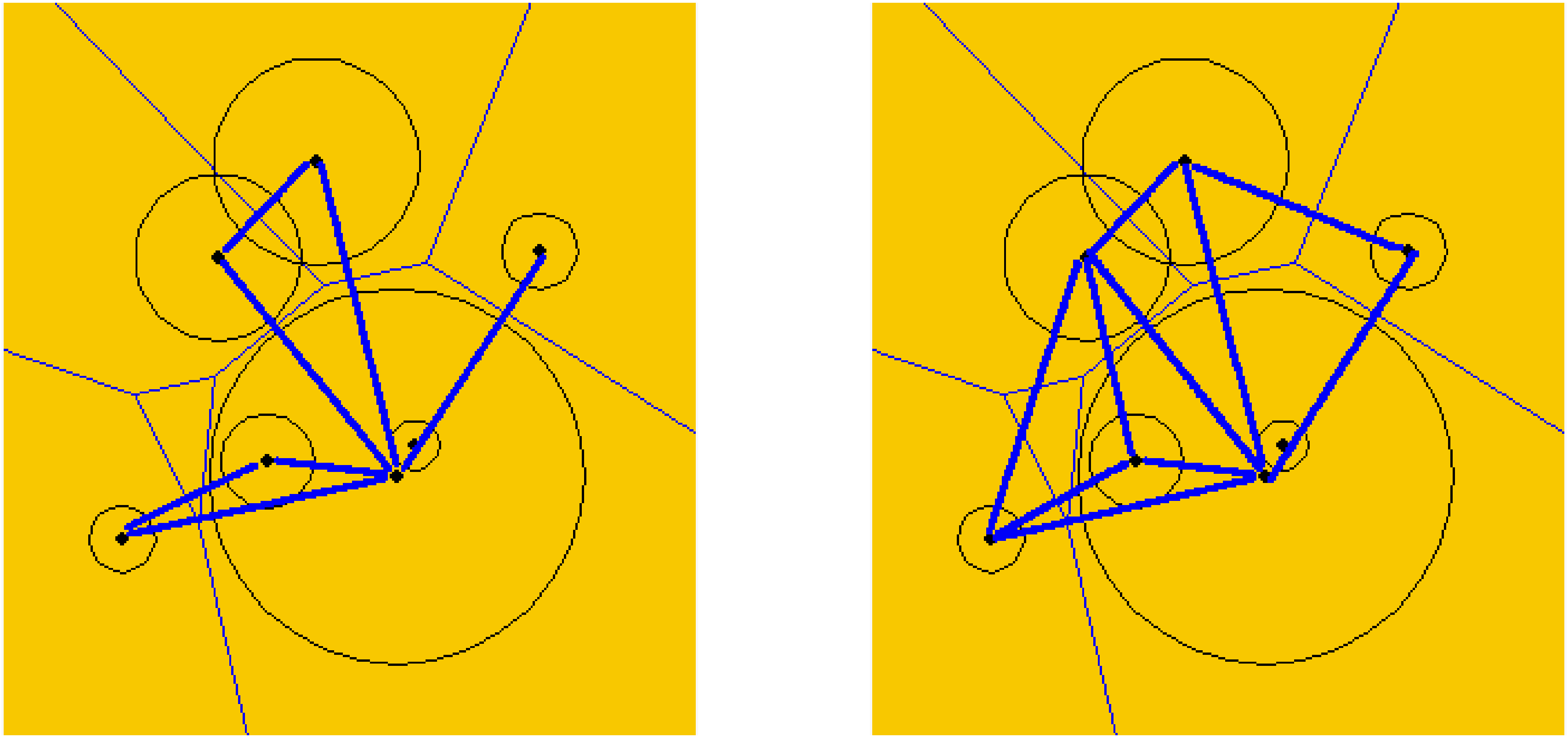

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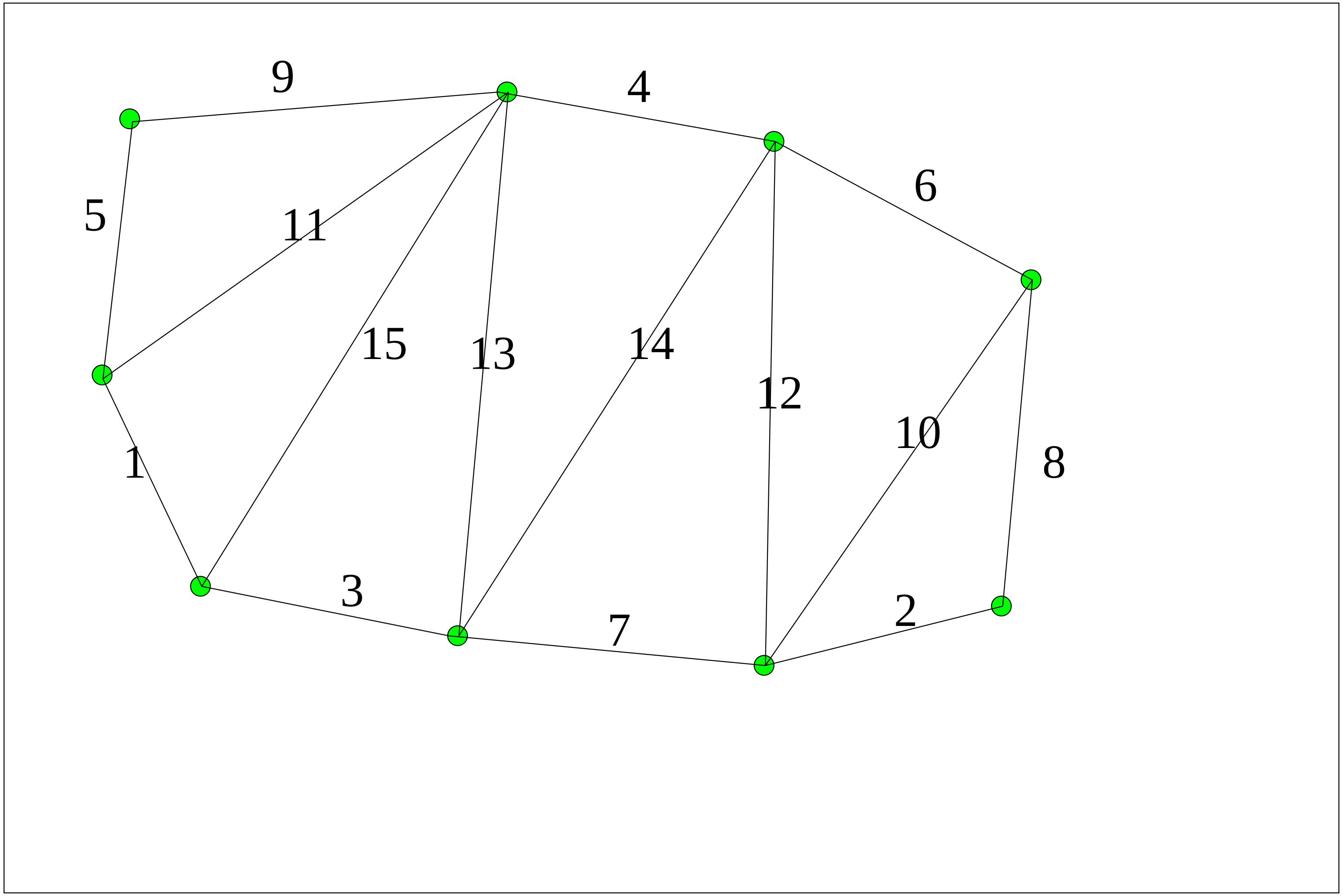

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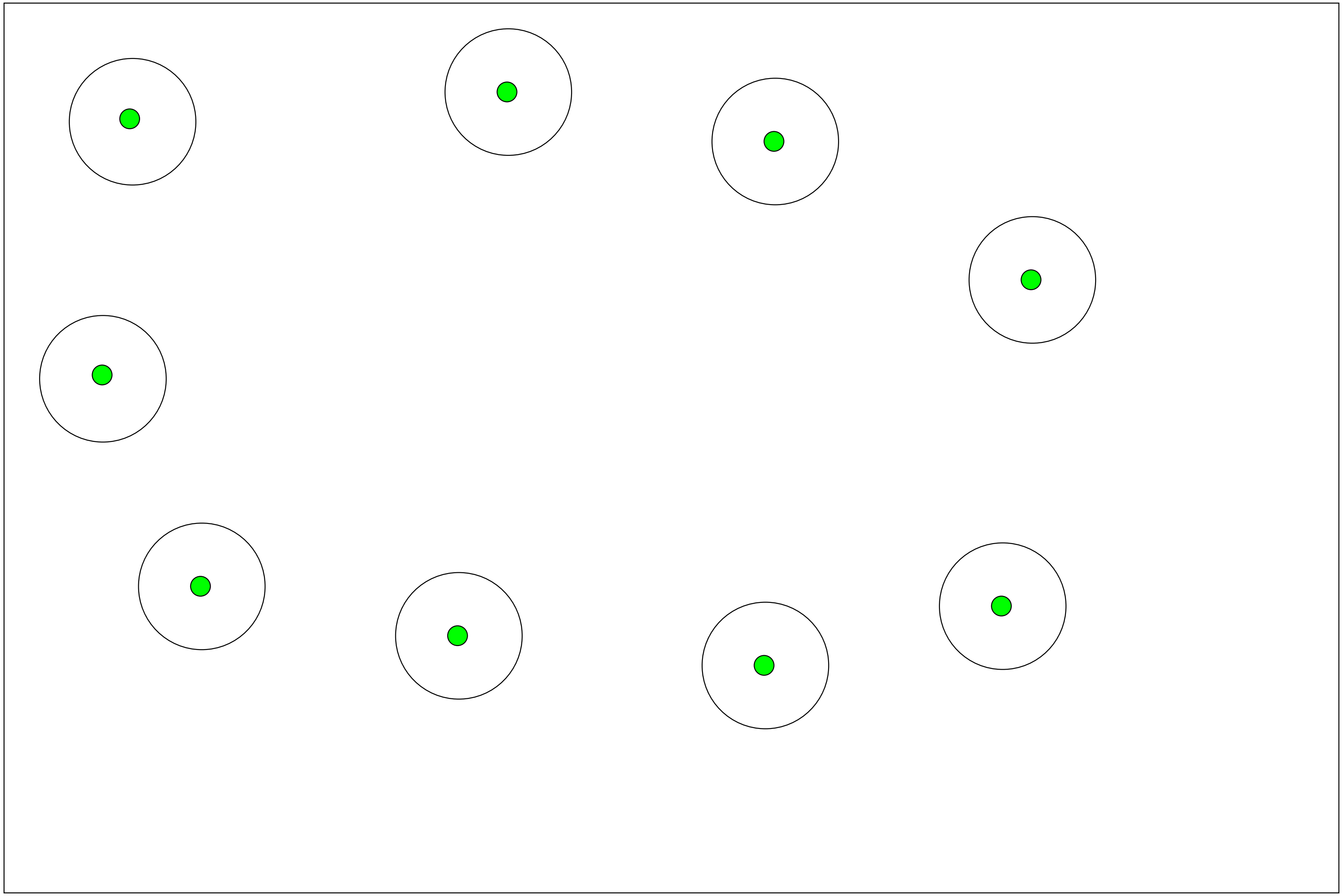

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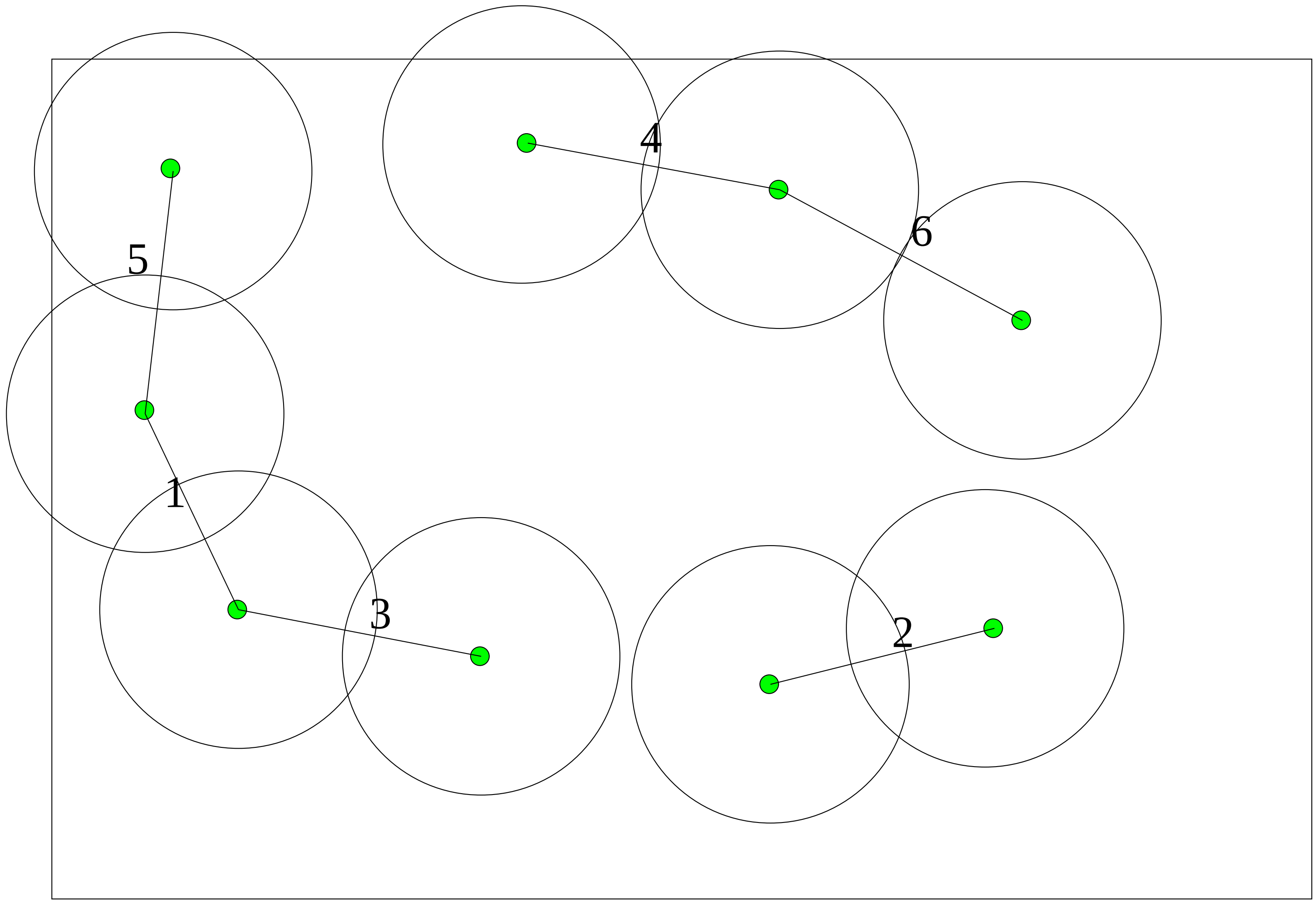

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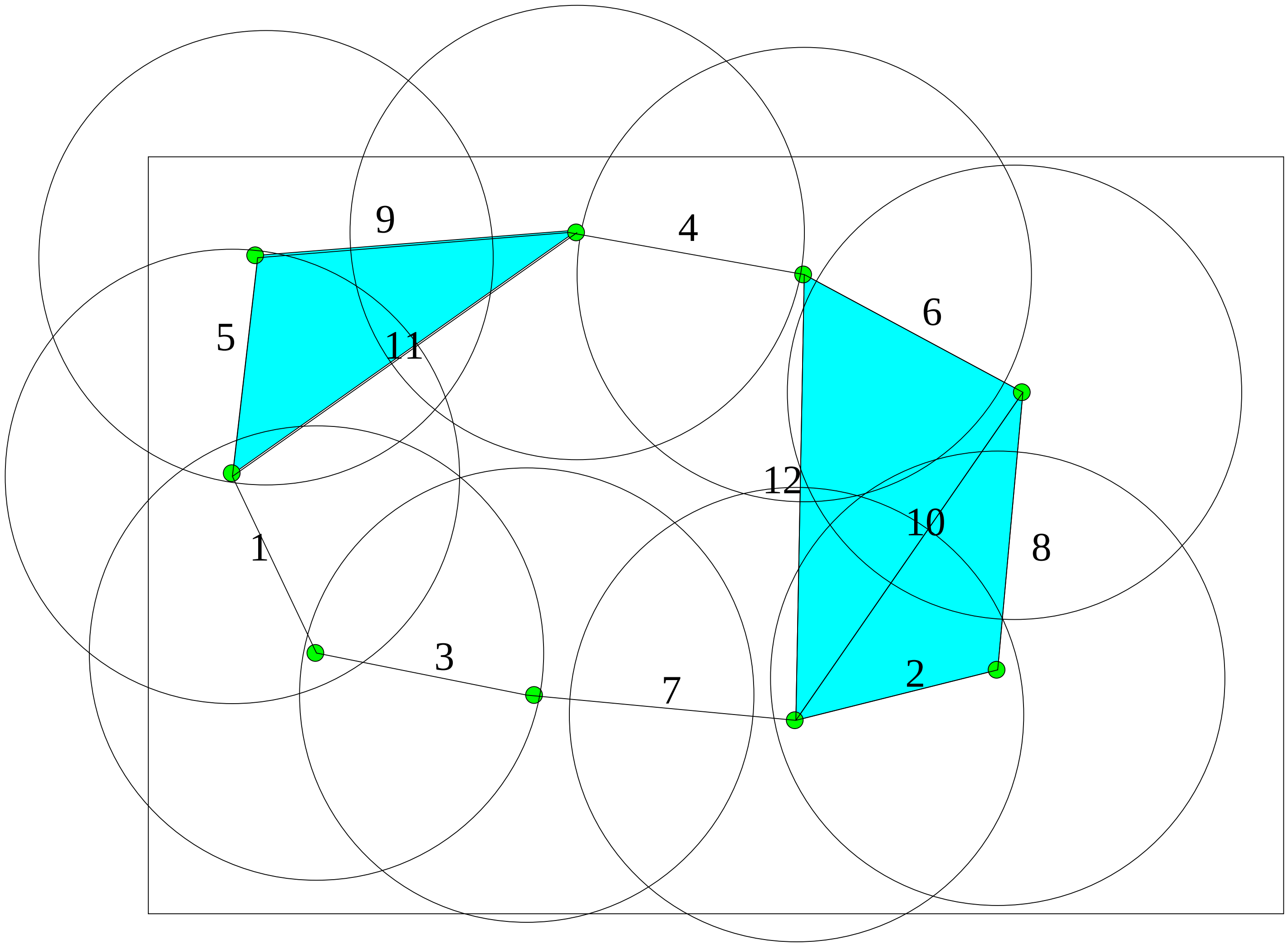
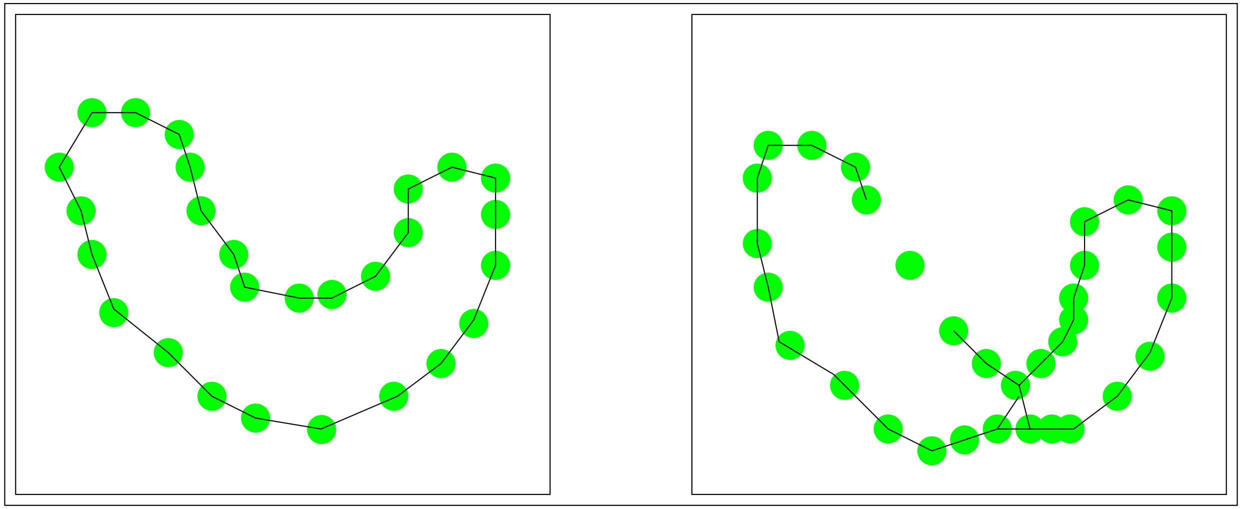


Who cares?

This ordering is useful for visualizing structure of the point set.
Example: Simple surface reconstruction



Other apps: chemical modeling, visualization

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Selected References

- Hughes Hoppe. *Surface reconstruction from unorganized points*. Ph.D. Thesis, University of Washington, June 1994.
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