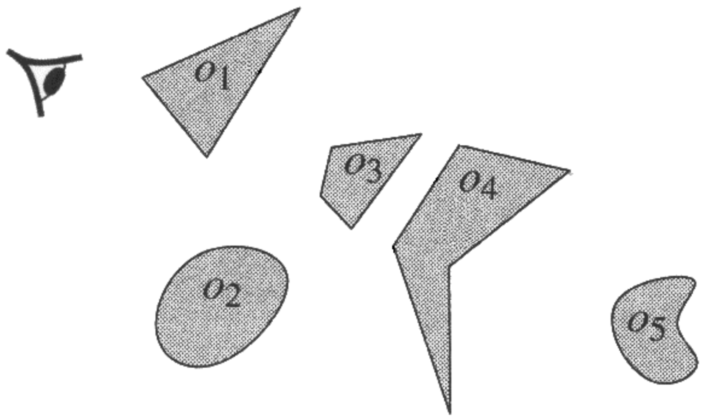
Motivation for BSP Trees: The Visibility Problem

We have a set of objects (either 2d or 3d) in space.

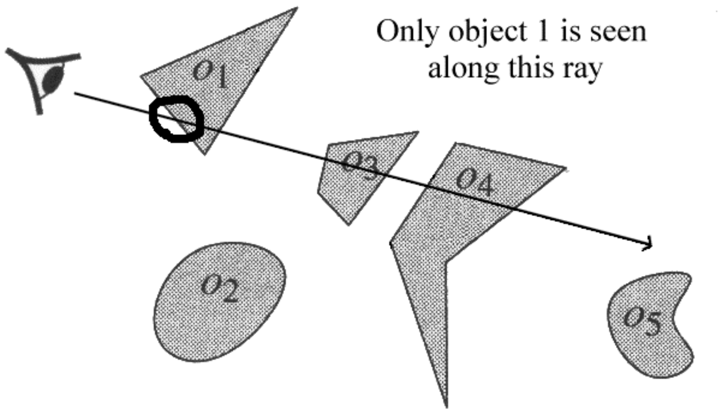
We have an "eye" at some point in this space, looking at the objects from a particular direction.



Drawing the Visible Objects

We want to generate the image that the eye would see, given the objects in our space.

How do we draw the correct object at each pixel, given that some objects may obscure others in the scene?



A Simple Solution: The Z-buffer

- Keep a buffer that holds the z-depth of the pixel currently at each point on screen
- Draw each polygon: for each pixel, test its depth versus current screen depth to decide if we draw it or not

Drawbacks to Z-buffering

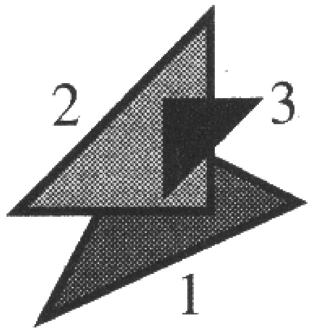
This used to be a very expensive solution!

- Requires memory for the z-buffer – extra hardware cost was prohibitive
- Requires extra z-test for every pixel

So, a software solution was developed …

The Painter's Algorithm

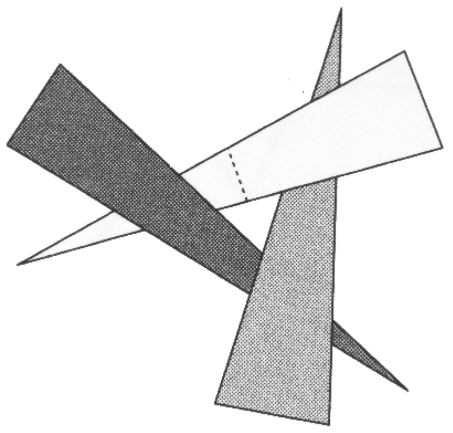
Avoid extra z-test & space costs by scan converting polygons in back-to-front order



Is there always a correct back-to-front order?

How Do We Deal With Cycles?

In 3 dimensions, polygons can overlap, creating cycles in which no depth ordering would draw correctly.



How do we deal with these cases?

BSP Trees

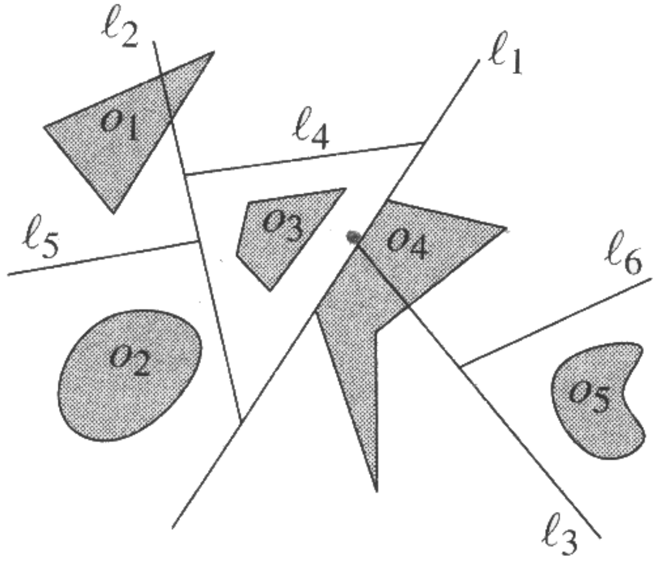
Having a pre-built BSP tree will allow us to get a correct depth order of polygons in our scene for any point in space.

We will build a data structure based on the polygons in our scene, that can be queried with any point input to return an ordering of those polygons.

The Big Picture

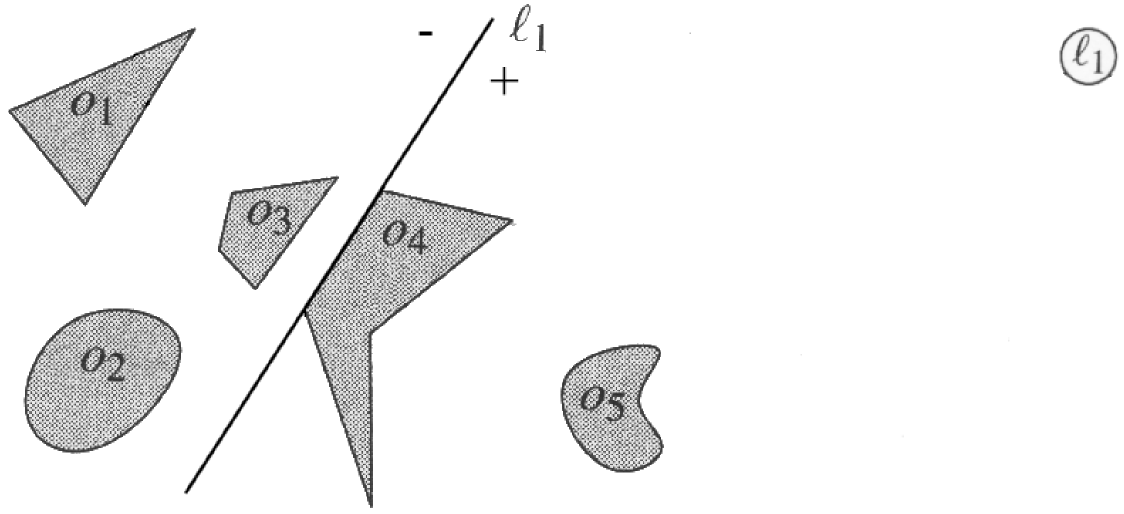
Assume that no objects in our space overlap.

Use planes to recursively split our object space, keeping a tree structure of these recursive splits.



Choose a Splitting Line

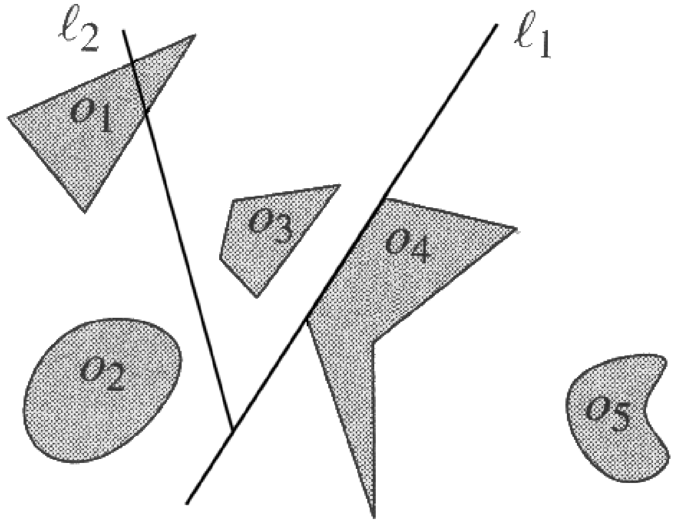
Choose a splitting plane, dividing our objects into three sets – those on each side of the plane, and those fully contained on the plane.

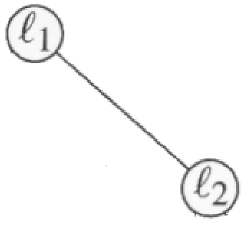


Choose More Splitting Lines

What do we do when an object (like object 1) is divided by a splitting plane?

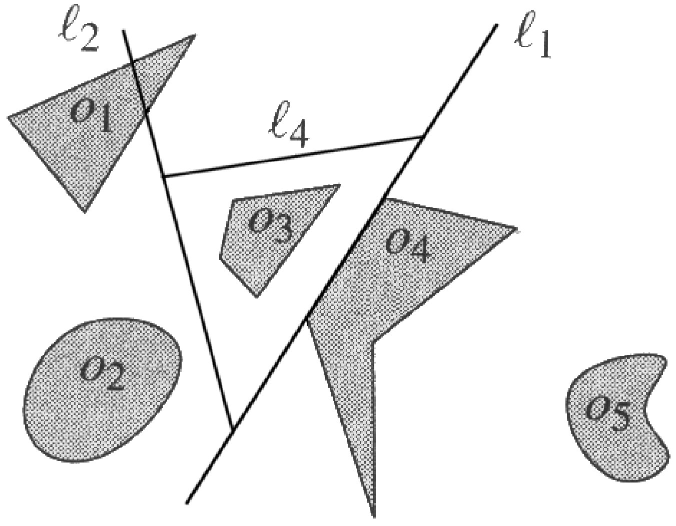
It is divided into two objects, one on each side of the plane.

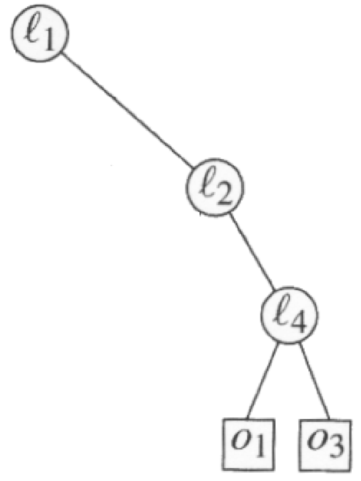




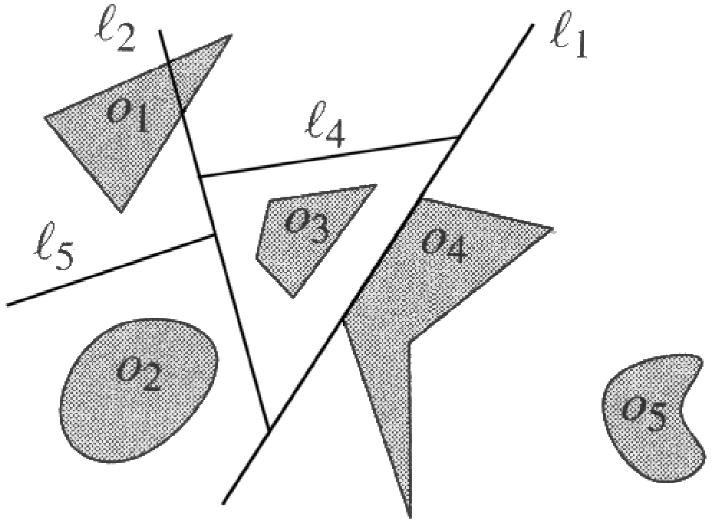
Split Recursively Until Done

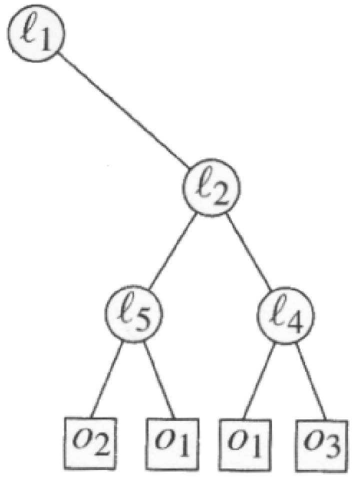
When we reach a convex space containing exactly zero or one objects, that is a leaf node.



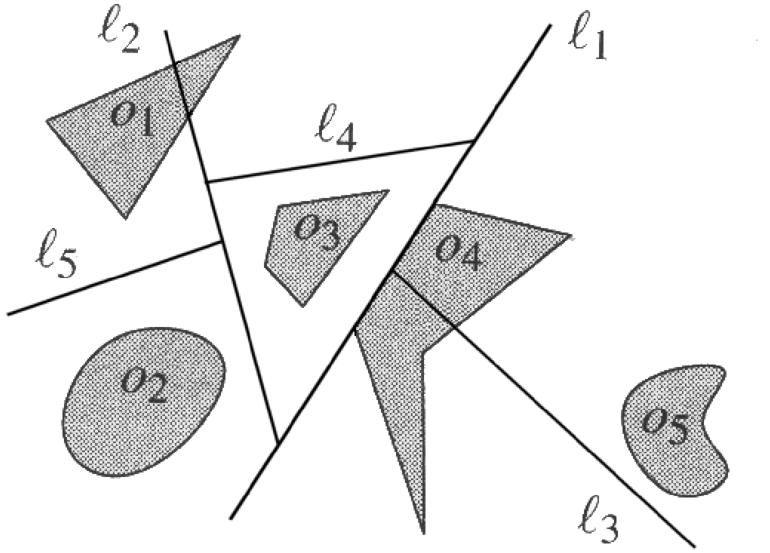


Continue

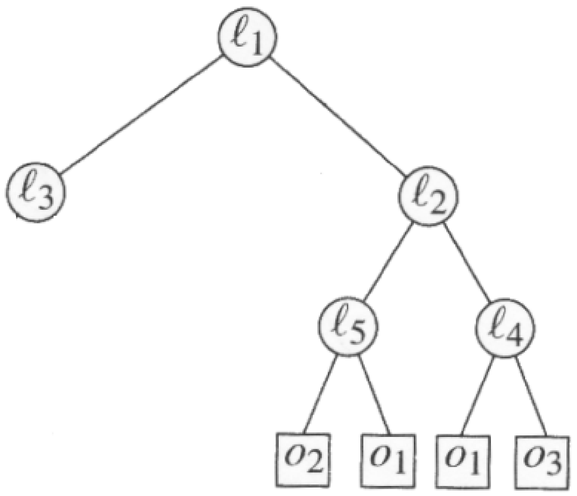

ℓ_2
ℓ_1
o_1
ℓ_4
o_3
o_4
ℓ_5
o_2
o_5


ℓ_1
ℓ_2
ℓ_5
ℓ_4
o_2
o_1
o_1
o_3

Continue



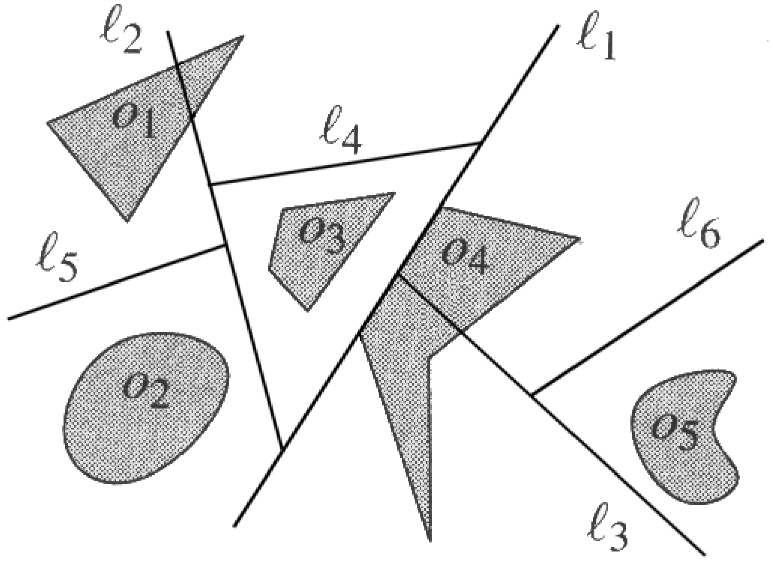
ℓ_2
ℓ_1
o_1
ℓ_4
o_3
o_4
ℓ_5
o_2
o_5
ℓ_3

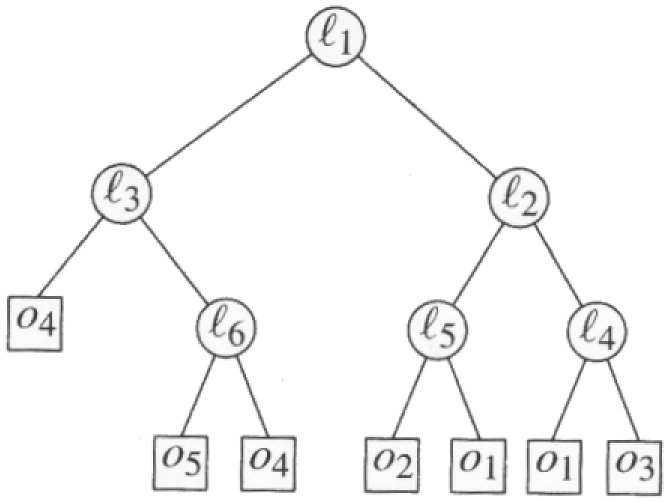


ℓ_1
ℓ_3
ℓ_2
ℓ_5
ℓ_4
o_2
o_1
o_1
o_3

Finished

Once the tree is constructed, every root-to-leaf path describes a single convex subspace.



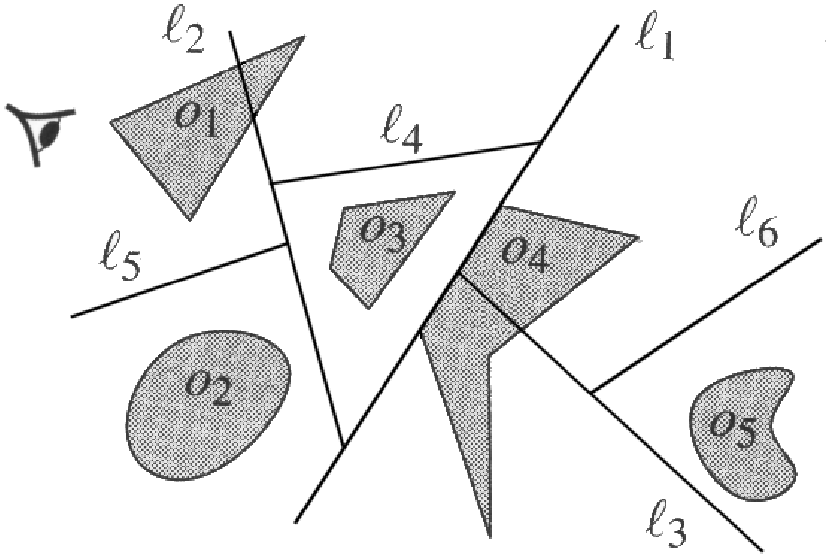


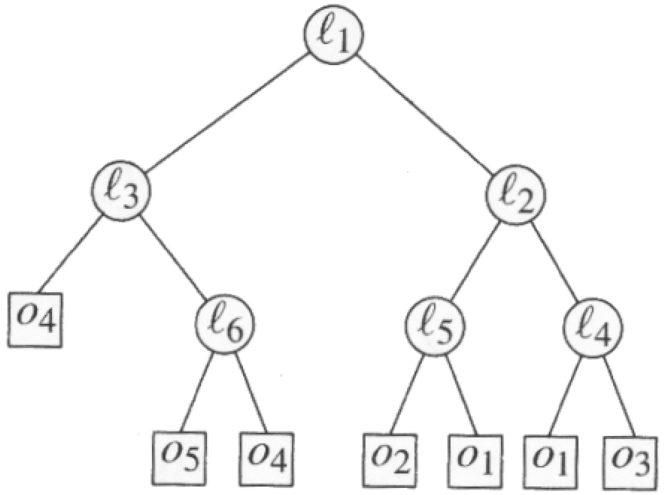
Querying the Tree

If a point is in the positive half-space of a plane, then everything in the negative half-space is farther away -- so draw it first, using this algorithm recursively.

Then draw objects on the splitting plane, and recurse into the positive half-space.

What Order Is Generated From This Eye Point?





How much time does it take to query the BSP tree, asymptotically?

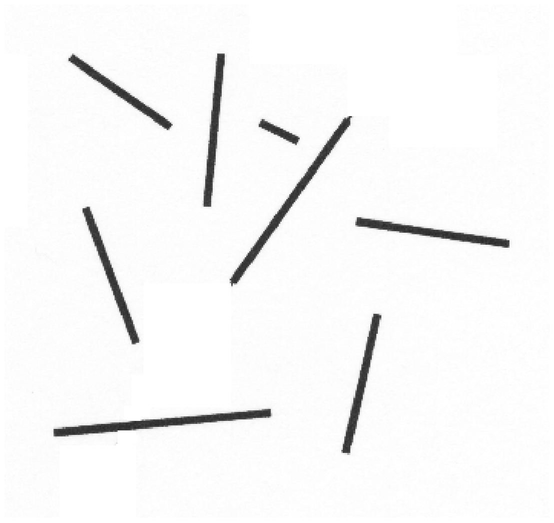
Structure of a BSP Tree

- Each internal node has a +half space, a -half space, and a list of objects contained entirely within that plane (if any exist).
- Each leaf has a list of zero or one objects inside it, and no subtrees.
- The *size* of a BSP tree is the total number of objects stored in the leaves & nodes of the tree.
- This can be larger than the number of objects in our scene because of splitting.

Building a BSP Tree From Line Segments in the Plane

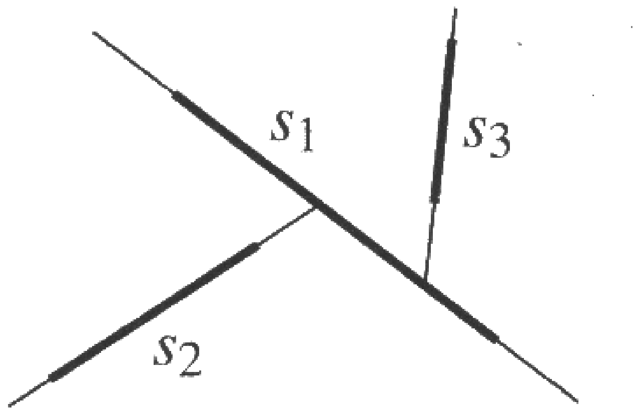
We’ll now deal with a formal algorithm for building a BSP tree for line segments in the 2D plane.

This will generalize for building trees of D-1 dimensional objects within D-dimensional spaces.



Auto-Partitioning

Auto-partitioning: splitting only along planes coincident on objects in our space

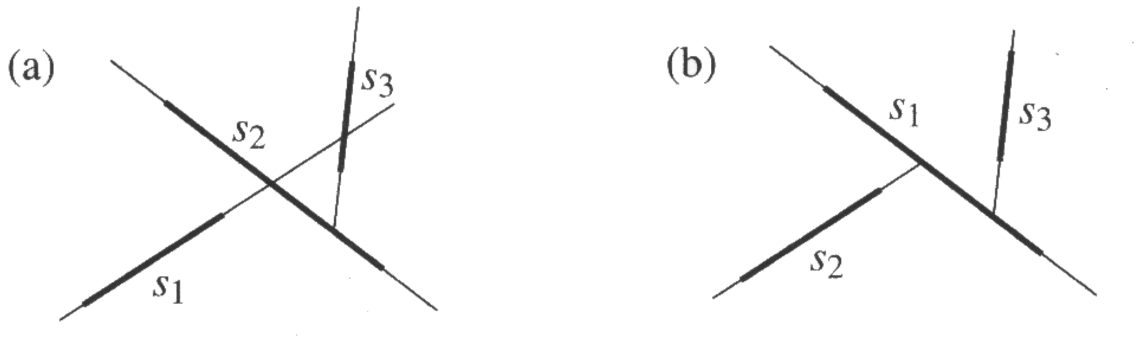


Algorithm for the 2d Case

- If we only have one segment 's', return a leaf node containing s.
- Otherwise, choose a line segment 's' along which to split.
- For all other line segments, one of four cases will be true:
    1) The segment is in the +half space of s
    2) The segment is in the -half space of s
    3) The segment crosses the line through s
    4) The segment is entirely contained within the line through s.
- Split all segments who cross 's' into two new segments -- one in the +half space, and one in the -half space.
- Create a new BSP tree from the set of segments in the +half space of s, and another on the set of segments in the -half space.
- Return a new node whose children are these +/- half space BSP’s, and which contains the list of segments entirely contained along the line through s.

How Small Is the BSP From This Algorithm?

- Different orderings result in different trees



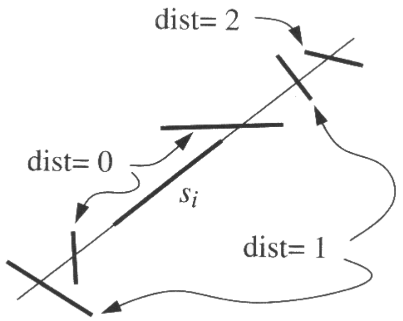
- Greedy approach doesn't always work -- sometimes it does very badly, and it is costly to find

Random Approach Works Well

If we randomly order segments before building the tree, then we do well in the average case.

Expected Number of Fragments: O(n Log n)

Let $\text{Dist}_{si}(sj)$ = The # of segments intersecting line(si) between si and sj, if line(si) intersects sj, or +infinity otherwise



$$\Pr[\ell(s_i) \text{ cuts } s_j] \leqslant \frac{1}{\text{dist}_{s_i}(s_j)+2}$$

$$\begin{aligned} \text{E[number of cuts generated by } s_i] &\leqslant \sum_{j\neq i} \frac{1}{\text{dist}_{s_i}(s_j)+2} \\ &\leqslant 2\sum_{k=0}^{n-2} \frac{1}{k+2} \\ &\leqslant 2\ln n. \end{aligned}$$

Thus, the expected number of fragments = O(n + n log n).

Expected Running Time: $O(n^2 \log n)$

The time taken at any particular node is linear in the number of fragments in its input set.

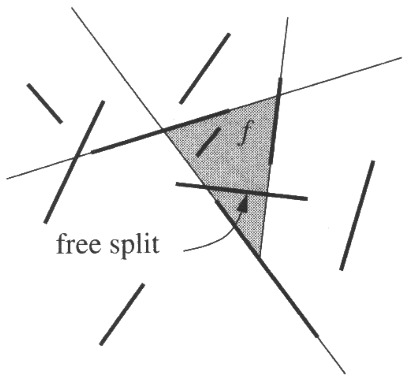
Each call to the algorithm thus takes $O(n)$ time, and the number of recursive calls is bounded by $O(n \log n)$.

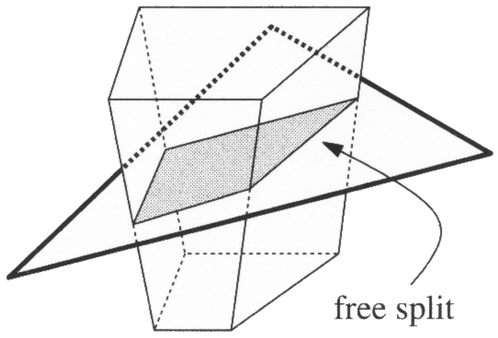
Our total expected running time is then

$O(n^2 \log n)$.

Optimization: Free Splits

- Sometimes a segment will entirely cross a convex region described by some interior node -- if we choose that segment to split on next, we get a "free split" -- no splitting needs to occur on the other segments since it crosses the region.



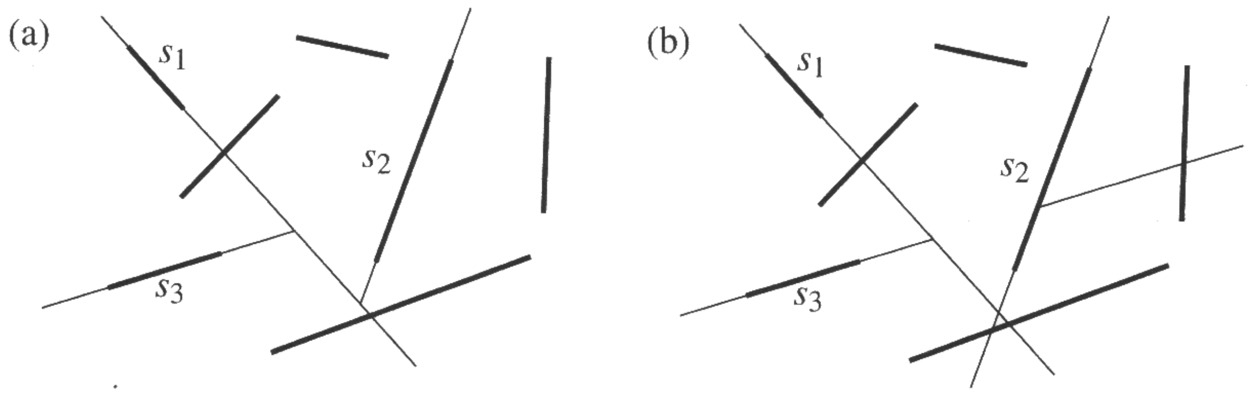


Our Building Algorithm Extends to 3D!

The same procedure we used to build a tree from two-dimensional objects can be used in the 3D case – we merely use polygonal faces as splitting planes, rather than line segments.

A Slight Modification, for the Sake of Analysis:

- Split *both* the + and - half space of a node with the same plane, when we choose to do a split.
- An exception: When a split does not subdivide some cell, it isn't necessary to include an extra node for it.



Expected number of fragments: $O(n^2)$

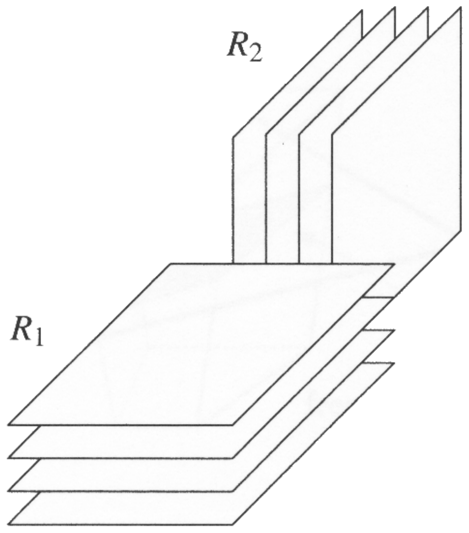
With that modification, we can show that the expected number of subdivisions for each polygon in our input is O(k), where k is its position in the random sequence.

The summation of these subdivisions over k=1 to n will be $O(n^2)$. (Proof is on p.261 of the book if you are interested in details.)

More analysis: How good are auto-partitions?

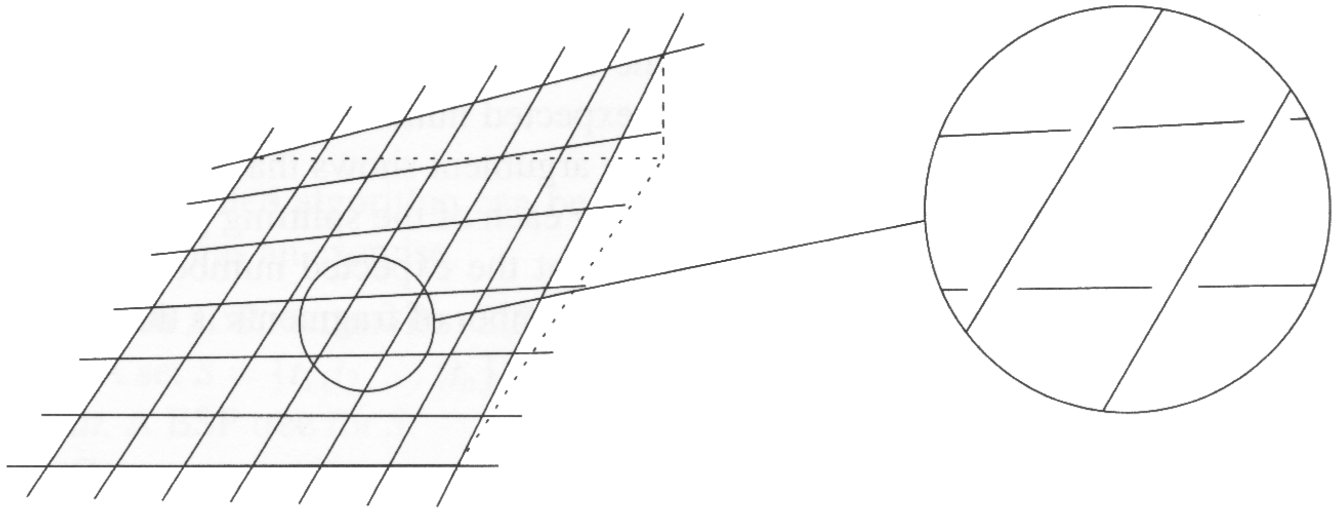
- There are sets of triangles in 3-space for which autopartitions guarantee $\Omega(n^2)$ fragments.

- Can we do better by not using autopartitions?



Sometimes, No

There are actually sets of triangles for which *any* BSP will have $\Omega(n^2)$ fragments!



More applications

- Querying a point to find out what convex space it exists within
- Cell visibility -- put extra data into your BSP leaves, such as links to adjacent convex subspaces