Lecture 14: Tuesday, 26 October 1999

Final project topics!
- Inspiration – past projects
- Email feedback all week
- Team formation (use web page)
- Proposals by Friday
  - Put on web (plaintext, simple HTML)
  - Send staff email, with proposal URL
    - Make sure to `Cc:` all team members !
    - We'll create aliases of form
      - `teamXX@graphics.lcs.mit.edu`

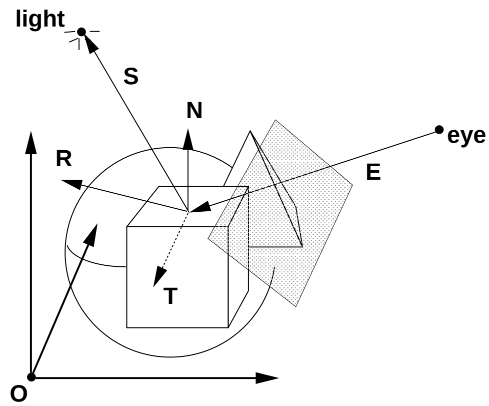
Today:
- Radiosity (a Global Illumination algorithm)

Thursday:
- Texture Mapping (Legakis)

Next Tuesday
- Embodied Conversational Agents (Cassell)

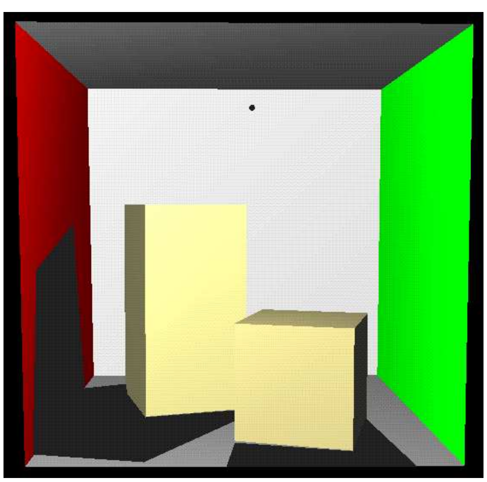
Radiosity Solvers

One possibility: modify ray-tracer to sample area light sources, hemispherical irradiance:



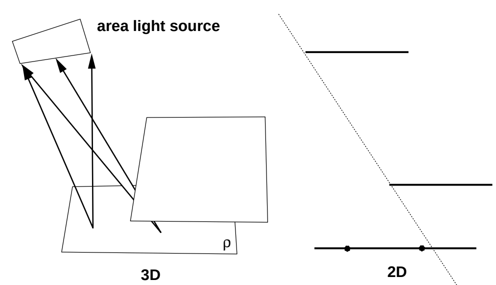
this requires lots of CPU for ray casting
any other disadvantages ?

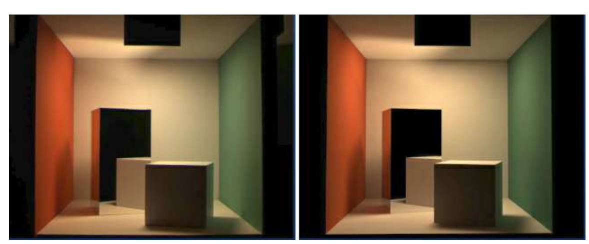
Hard shadows



Abrupt changes in illumination arise in ray tracing
- "Point" light source is non-physical; it goes abruptly from visible to invisible

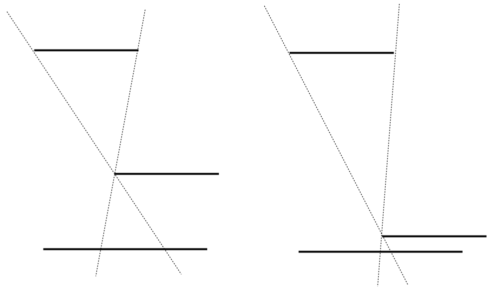
Soft shadows





Illumination Gradients

What if occluder is relatively far from receiver?
What if occluder is relatively close to receiver?



Inter-Reflections

Zeroth-order (direct illumination)
First-order (reflectors of direct illumination)
Second-order (reflectors of reflected illum., etc.)

So ... we want an algorithm that handles
  Primary, secondary, tertiary emitters, etc.
And handles soft shadows. Radiosity !

Radiosity Methods

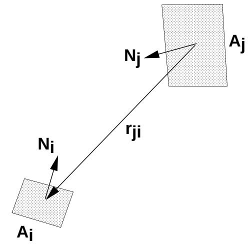
Arose from thermal radiation literature
  (Analyzing heat thrown from blast furnaces)
Find **equilibrium** distribution of radiant energy
  Equilibrium means ...
Suitable for ideal *diffuse* environments
  Why ?

Radiosity Equation - Intuitive Form

Radiosity equation:

$$\begin{aligned} \text{Exitant Power} &= \text{Emitted Power} + \text{Reflected Power} \\ B_i A_i &= E_i A_i + \rho_i (\text{Total Incident Power}) \\ &= E_i A_i + \rho_i \sum_j (\text{Incident Power from } j) \\ &= E_i A_i + \rho_i \sum_j B_j A_j F_{ji} \end{aligned}$$

Where F_{ji} is the **fraction of radiant power leaving j that arrives at i**



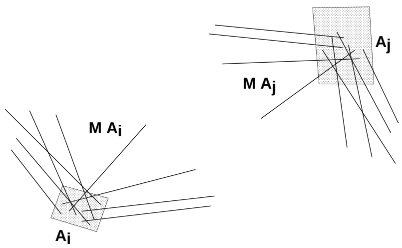
(Or, fraction of j's hemisphere occupied by i, integrated over all points on j)
F_{ji} called the "Form Factor" between j and i
  Note order, interpretation of subscripts!

Radiosity Equation: Computational Form

Digression: Line dualities and measures
Geometric fact:

$$A_i F_{ij} = A_j F_{ji}$$

Proof: lines can be counted under some measure



\# of lines through A_i is MA_i
\# of lines through A_j is MA_j
Why? What is M?
So # of lines leaving A_i arriving at A_j is MA_iF_{ij},
and # of lines leaving A_j arriving at A_i is MA_jF_{ji}
But both quantities describe the same set of lines!

$$MA_iF_{ij} = MA_jF_{ji}$$

Radiosity Equation: Computational Form

Relate total power as follows

$$\begin{aligned} B_iA_i &= E_iA_i + \rho_i \sum_j B_jA_jF_{ji} \\ &= E_iA_i + \rho_i \sum_j B_jF_{ij}A_i \end{aligned}$$

This is the "power equation" for radiosity

Dividing power equation through by A_i yields

$$B_i = E_i + \rho_i \sum_j B_jF_{ij}$$

This is the "radiosity equation"
More efficient, but more confusing
Be careful to get subscripts right!

Solving the Radiosity Equation

First of all ... what does "solving it" mean ?
Can't be done in closed form (in general)
so use *iterative numerical methods*
Set up initial conditions (for all i):
Assign E_i:
$E_i > 0$ (light sources)
$E_i = 0$ (all other surfaces)
Assign B_i, initial guess for i^{th} surface's radiosity
What is a good heuristic for initial guess?
Determine **geometric** coupling:
Compute F_{ij} for all i, j
(How long does this take?)

Numerical Iteration

So, get initial guesses somehow, then ...
Iterate system until convergence
Improve "guess" for each B_i based on the B_j, $j \neq i$
In practice, can't capture fine-scale illumination
effects unless original surfaces are "meshed"
Terminology (somewhat arbitrary):
Surfaces: boundaries of scene objects
Patches: surface portion acting as illumination source
Elements: patch portion receiving illumination

Radiosity Solvers

Numerical iteration of radiosity values

At equilibrium:

$$B_i = E_i + \rho_i \sum_j B_j F_{ij}$$

So, given estimates at step t, produce estimates at step $t+1$ as:

$$B_i^{t+1} = E_i + \rho_i \sum_j B_j^t F_{ij}$$

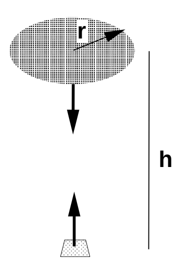
What is the intuition behind this ?
When do we stop iterating ?

Computing Form Factors

What is the form factor? (Intuition.)
- If patches are back-facing?
- If patches are front-facing, and fully intervisible?
  - What is a good approximation?
- If patches are front-facing, but occluded?

Computing VISIBLE Form Factors

Approximating front-facing, intervisible F_{ij}



For i differential, j a parallel disk of radius r
Then form factor F_{ij} is

$$\frac{r^2}{h^2 + r^2}$$

If non-parallel, incorporate cosine term
Closed-form expressions exist for a few other cases

Computing PARTIAL Form Factors

Front-facing, but only **partially** intervisible?
- Very complex; no known closed-form sol'n
- What to do?

How to compute form factors ?
- Up front, or ...
- Progressively, as computation proceeds !