Lecture 10: Tuesday, 12 October 1999

Reminders:
- Asst 4 Exhibition – Inventor Scenes
- Asst 5 (`ivscan`) due Friday 5pm
- Asst 6A (Ray Casting) out today

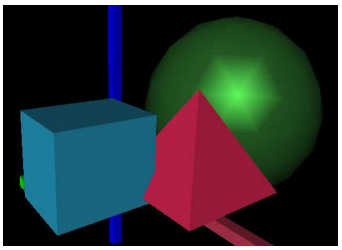
Today
- Ray Casting (Prelude to Ray Tracing)

Thursday
- Recursive Ray Tracing
- Asst 6B (Ray Tracing) out
  - This is the last assignment !

Next Week:
- Tuesday: Final Project Brainstorming
  - Start thinking about project ideas, teams !
- Thursday: Special Guest Lecture
  - Solid Modeling

Scan Conversion and Visibility

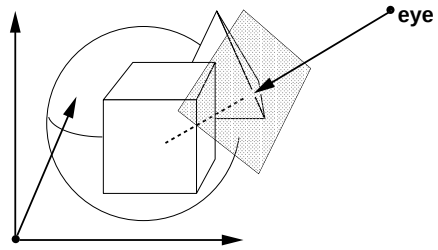
Hidden surfaces eliminated at each pixel
- scanline algorithms
- z buffering
- Shading computed many times at each pixel
- Each pixel stores only constant state



Limitations:
- Restricted to scan-convertable primitives
  - Requires polygonalization of objects
  - Causes faceting, shading artifacts
- Effective resolution h/w dependent (aliasing)
- No handling of shadows, reflection, transparency

Ray Casting for Visibility

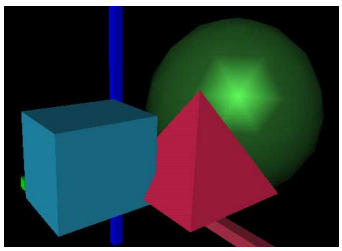
```
Object Cast ( Point R, Ray D ) {
  find minimum t>0 such that R + t D hits object
  if ( object hit )
    return object
  else
    return background object
} // Cast
```

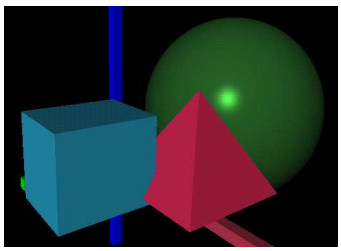



```
FrameBuffer Render ( frustum, viewport ) {
  For each raster y
    For each pixel x
      E = ray from eye through pixel x, y
      obj = Cast ( eye, E );
      FB[x][y] = obj->color
  return FB
} // Render by Ray Casting
```


Ray Casting for Visibility

Advantages?
- Smooth variation of normal, silhouettes





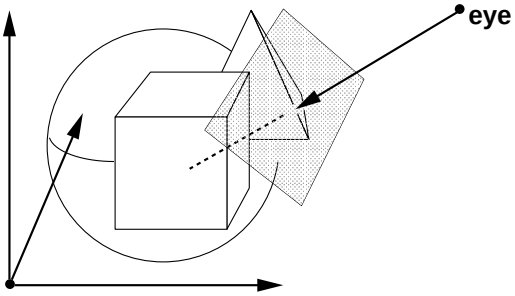
- Generality: can render anything with
  - which a ray can be intersected !
- Compactness of representation

Disadvantages?
- Time complexity (N objects, R pixels)
- Wasted work at "background" pixels

Why aren't ray casters usually found in h/w?

Ray – Scene Intersection

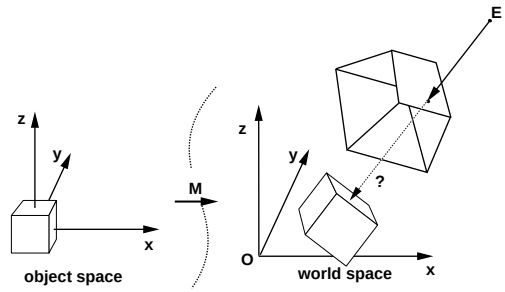
Fundamental operation in ray tracing:
  Given: a ray, and a scene of transformed primitives
  Determine:
    Closest ray-primitive intersection **P**, if any
    Primitive's surface normal **N** at point **P**



Typically do this by computing t for each intersection, and retaining t_{min}, the least t

Ray – Primitive Intersection

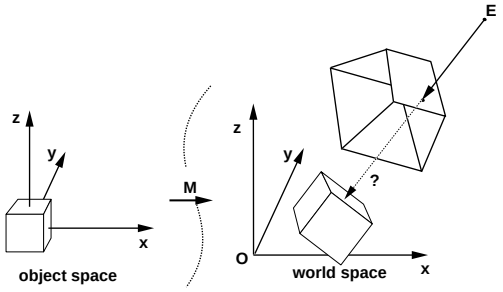
First concentrate on **individual** primitives
  Ray is typically generated in world space
  Primitives defined in object space, then transformed



What are our options?

Intersection in World Space

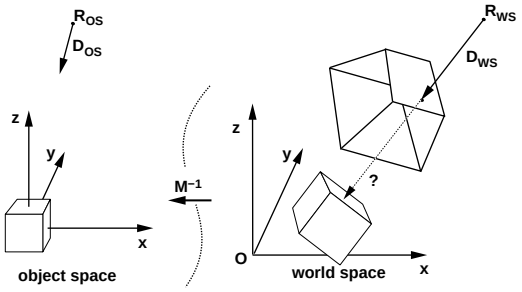
Option 1: transform *primitive* into *world* space
  Reexpress it there, then perform intersection



Polygon: vertices as $\mathbf{Mp}$, plane as $\mathbf{HM}^{-1}$
  Polyhedra: collections of polygons
Quadrics: transform as $\mathbf{MQM}^{-1}$
  But: quadric class is not invariant !
Torii, spline surfaces, other primitives: harder.
  Each requires a different transformation rule

Intersection in Object Space

Option 2: transform *ray* into *object* space!
  Intersections straightforward, even trivial there

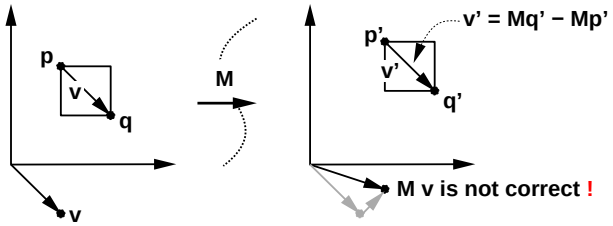


How do we transform the ray's origin and direction?
Careful: must deal with isotropic, anisotropic scaling

Ray Transformation

Know how points transform: $\mathbf{p}' = \mathbf{Mp}$

Thus, transforming ray *origin* is easy



But how to transform the ray *direction* ?

Clearly not as a point [see example, T(1,1,0)]

Direction vector transformation

What's going on?

"Endpoints" of ray have equal translations

Equivalently: ray is a pure *direction*

So: transform both ends, subtract results

Equivalent to ignoring translation part of $\mathbf{M}$

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & T_x \\ M_{21} & M_{22} & M_{23} & T_y \\ M_{31} & M_{32} & M_{33} & T_z \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

Instead, use

$$\mathbf{M}^* = \begin{pmatrix} M_{11} & M_{12} & M_{13} & \mathbf{0} \\ M_{21} & M_{22} & M_{23} & \mathbf{0} \\ M_{31} & M_{32} & M_{33} & \mathbf{0} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

Again: can treat direction (a, b, c) as homogeneous $(a, b, c, 0)$

[Inventor supplies this as M.multDirMatrix $(\mathbf{v}, \mathbf{v}')$]

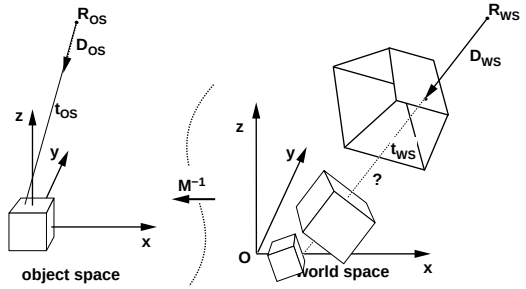
Ray Scaling – Transforming t

Problem: ray might be scaled *anisotropically* by $\mathbf{M}$

Ok; can still compute intersections in object space

(Unit length of $\mathbf{N}$ often comes in handy...)

How to get t_{WS} from t_{OS}?



Two options:

Don't normalize $\mathbf{D_{OS}}$; compute t_{OS}

Then [ ] !

Must handle unnormalized vectors, but xform easy

Normalize $\mathbf{D_{OS}}$; compute t_{OS}

Then [ ] !

Need handle only normalized vecs, but xform harder

Primitive Intersection

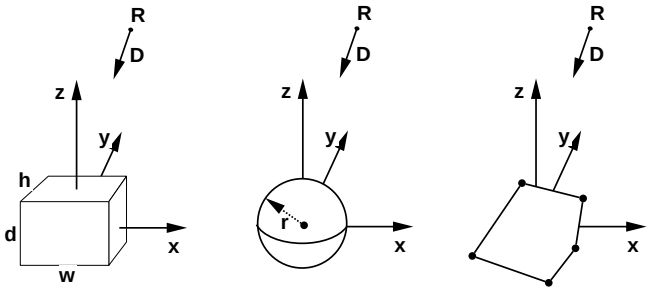
Now we can work in object space, then

revert to world space to return values

Wish to compute closest intersection point,

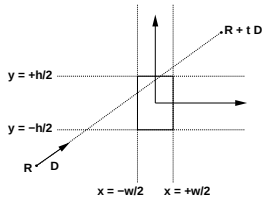
and normal at that point (if any intersection)

Consider ray-primitive intersection, with primitives



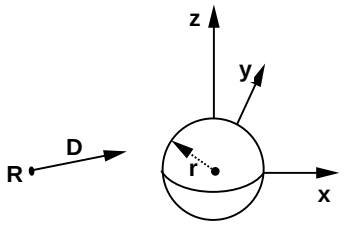
Axial parallelepiped; sphere of radius r; polygon

Ray-Parallelepiped Intersection




```
static Vector hats[3] = { xhat, yhat, zhat };
// ray is of form R + t D; assign min t as thit; normal N
float t1, t2, tmin = 0, tmax = HUGE; // from <math.h>
Vector extent = Vector ( width / 2, height / 2, depth / 2 );
// intersect ray with x, y, z ‘‘slabs’’ (k = 0, 1, 2)
for ( int k = 0; k < 3; k++ ) {
  if ( D[k] != 0. ) {
    t1 = (-extent[k] - R[k]) / D[k];    // plane x_k = -dx_k
    t2 = ( extent[k] - R[k]) / D[k];    // plane x_k = +dx_k
    tmin = fmax ( tmin, fmin (t1, t2)); // intersect [tmin..
    tmax = fmin ( tmax, fmax (t1, t2)); //  tmax],  [t1..t2]
    if ( tmax <= tmin ) return FALSE;   // no intersection
    else if ( tmin == t1 ) N = -hats[k]; // hit x_k = -dx_k
    else if ( tmin == t2 ) N =  hats[k]; // hit x_k = +dx_k
    } // if
  } // k
thit = tmin; // return parameter of closest intersection
return TRUE;
```

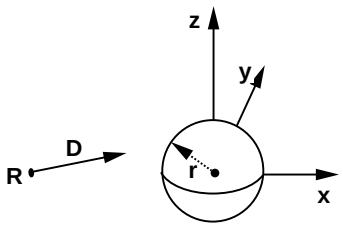

Ray-Sphere Intersection



Ray equation (explicit): $\mathbf{P}(t) = \mathbf{R} + t\mathbf{D}$, with $|\mathbf{D}| = 1$
Sphere equation (implicit): $\mathbf{P} \cdot \mathbf{P} = r^2$
Intersection means both are satisfied, so

$$\begin{aligned} 0 &= \mathbf{P} \cdot \mathbf{P} - r^2 \\ &= (\mathbf{R} + t\mathbf{D}) \cdot (\mathbf{R} + t\mathbf{D}) - r^2 \\ &= \mathbf{R} \cdot \mathbf{R} + 2t\mathbf{D} \cdot \mathbf{R} + t^2\mathbf{D}^2 - r^2 \\ &= t^2 + 2t\mathbf{D} \cdot \mathbf{R} + \mathbf{R} \cdot \mathbf{R} - r^2 \end{aligned}$$

Ray-Sphere Intersection



This is just a quadratic $at^2 + bt + c = 0$, where

$$\begin{aligned} a &= 1 \\ b &= 2\mathbf{D} \cdot \mathbf{R} \\ c &= \mathbf{R} \cdot \mathbf{R} - r^2 \end{aligned}$$

With discriminant

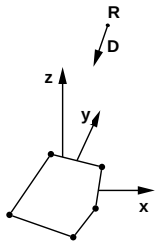
$$d = \sqrt{b^2 - 4ac}$$

and solutions

$$t_{\pm} = \frac{-b \pm d}{2a}$$

Three cases, depending on sign of $b^2 - 4ac$
Which root (t_+ or t_-) should you choose?

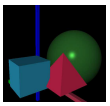
Ray-Polygon Intersection



Options:
Project polygon to shadow on principal plane
  Choose principal plane
  Project 3D edges to this plane
  Solve 2D inclusion problem
Intersect ray with support plane
  Compute intersection point
  Express 3D edges as plane constraints
  Solve 3D inclusion problem
Tradeoffs?

Aliasing

So far, both rasterization and raycasting compute one color per pixel:

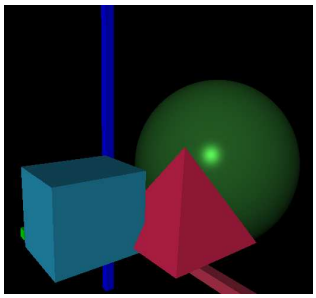


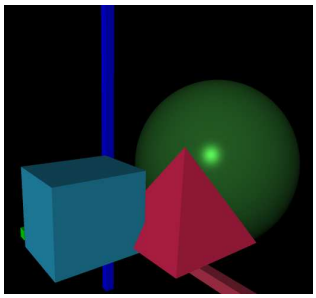
Look closely at:

Notice "jaggies" at object boundaries
Why do they happen ?
Can we do better? How?

Super Sampling

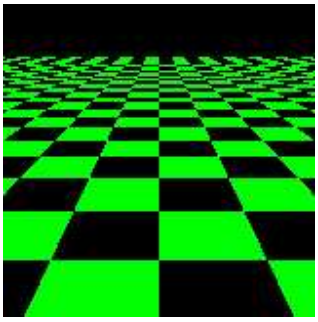
We can *average* multiple samples per pixel:

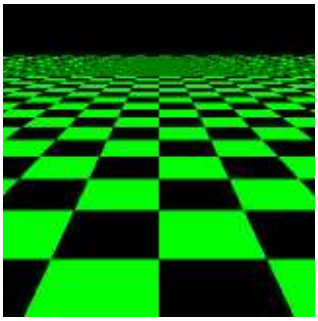




More Antialiasing

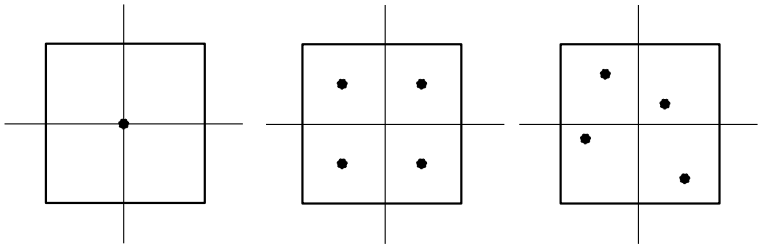
Important when rendering high-frequency textures:



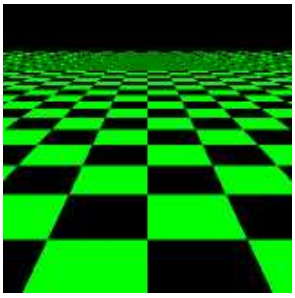


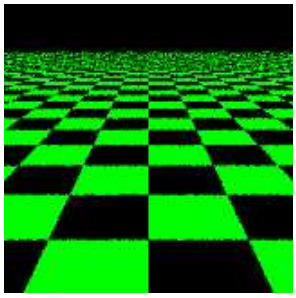
More about this in L15 (28 Oct, Justin Legakis)

Placing Samples



Regular, Jittered Supersampling





Demos: Asst 4 Exhibition; `ivray` (Damian)