

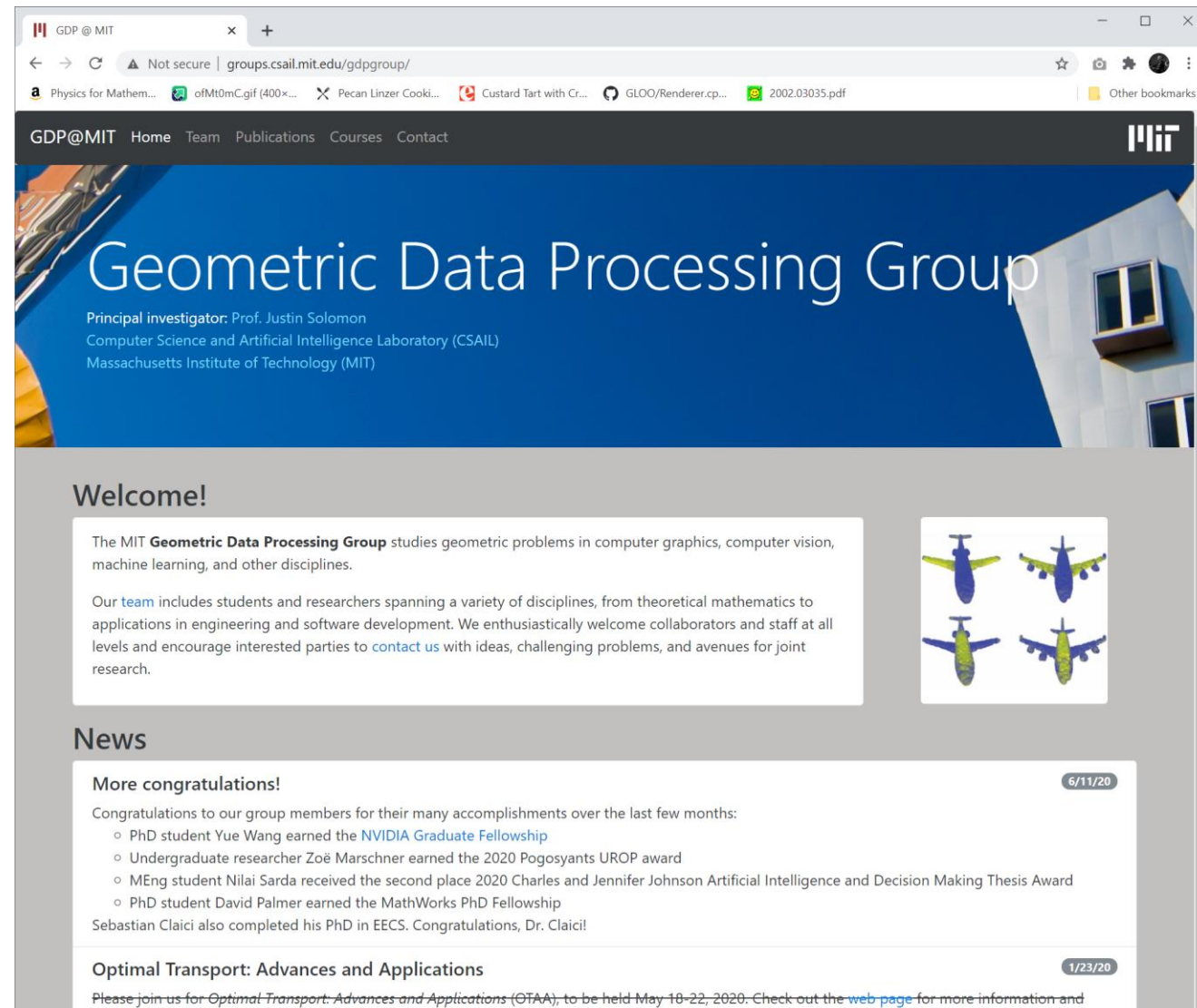
6.8410: Shape Analysis

Justin Solomon

Fall 2026



Course Instructor



Instructor: Justin Solomon
Email: jsolomon@mit.edu

Geometric Data Processing Group:
<http://gdp.csail.mit.edu>

Office hours:
Wednesdays 10am-12pm (32-D460)

TA



Madelyn Andersen, MSc

she/her

I'm a PhD student in Bayesian Statistics in [Tamara Broderick's](#) group within [LIDS](#) at [MIT](#), working on variational inference, robustness, and probabilistic modeling.

📍 Cambridge, MA, USA

🏛️ MIT

About me

I earned a B.S. in Theoretical Mathematics from [Harvey Mudd College](#), with a focus on theory, along with a dance concentration. I then completed an M.S. at [MIT](#) and am currently a PhD candidate at MIT in [LIDS](#), working in [Tamara Broderick's](#) group.

Current research interests

My research focuses on improving variational inference methods. I use techniques from machine learning, probabilistic programming, and statistical theory to bridge the gap between abstract theory and practical application.

I'm currently thinking about variational inference, normalizing flows, and simulation based inference.

TA: Madelyn Andersen
Email: madelyna@mit.edu

Office Hours:
Tuesdays/Thursdays
12pm-1pm (24-319)

Course Website

https://groups.csail.mit.edu/gdpgroup/68410_fall_2026.html

6.8410: Shape Analysis (Fall 2026)

Description:

Introduces mathematical, algorithmic, and statistical tools needed to analyze geometric data and to apply geometric techniques to data analysis, with applications to fields such as computer graphics, machine learning, computer vision, medical imaging, and architecture. Potential topics include applied introduction to differential geometry, discrete notions of curvature, metric embedding, geometric PDE via the finite element method (FEM) and discrete exterior calculus (DEC); computational spectral geometry and relationship to graph-based learning, correspondence and mapping, level set method, descriptor, shape collections, optimal transport, and vector field design. ([catalog listing](#))



Logistics:

Time: Monday/Wednesday, 2:30pm-4pm

Location: 32-124

Staff:

Instructor: [Justin Solomon](#), office hours Wednesdays 10am-12pm ([32-D460](#))

TA: [Madelyn Anderson](#), office hours Tuesdays/Thursdays 12pm-1pm ([24-319](#))

Online resources:

Prerequisites

- **Coding**

Matlab (or Python??)

- **Math**

Fluency in linear algebra and multivariable calculus

- **Not required (won't hurt):**

Graphics, differential geometry, numerical methods, ML

Philosophy

We want you to take this course!

Assignments intended to be interesting
(may be unintentionally easy/hard!).



Grading

- **Homeworks (30%)**

- 5 total
- 3 (24 hour) late days / semester; then 25% loss / day

Must complete everything to pass!
(or have excuse approved by instructor)

- **Quizzes (30%)**

- 5 total
- 30 minutes in class, written; Monday (or Tuesday) after HW due

- **Project (40%)**

- Open ended!
- Instructions already on course website

AI Policy

- **Homework:**

You may use AI tools as a tutor and to discuss general ideas and directions related to the homework. We expect you to formulate and write up your homework solutions yourself, without the use of AI tools.

- **Project:**

AI tools may be used on the course project. Projects that use AI tools will be judged on the human contributions to the project, which should be articulated clearly in the writeup.

- **Acknowledgment:**

In both cases, any use of AI tools must be acknowledged in the relevant writeup, with details on exactly how the tools were used.

Theme

1. *Geometric data analysis:* The analysis of geometric data

Modifier

Noun
2. *Geometric data analysis:* Data analysis using geometric techniques

Modifier

Noun

Applied Geometry

- I. Theoretical toolbox**
- II. Computational toolbox**
- III. Application areas**

Mostly a picture book!

Applied Geometry

I. Theoretical toolbox

II. Computational toolbox

III. Application areas

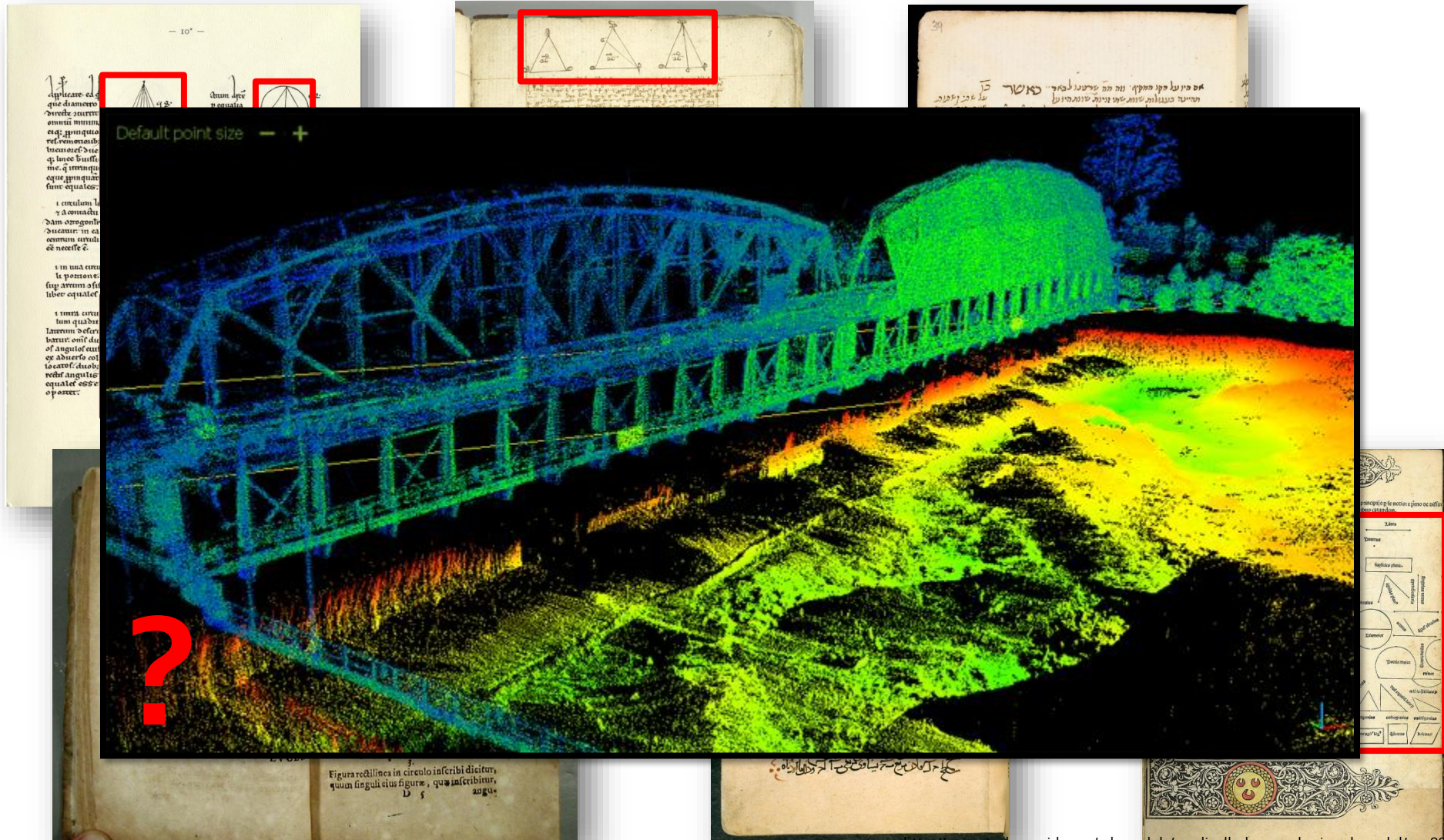
Euclidean Geometry



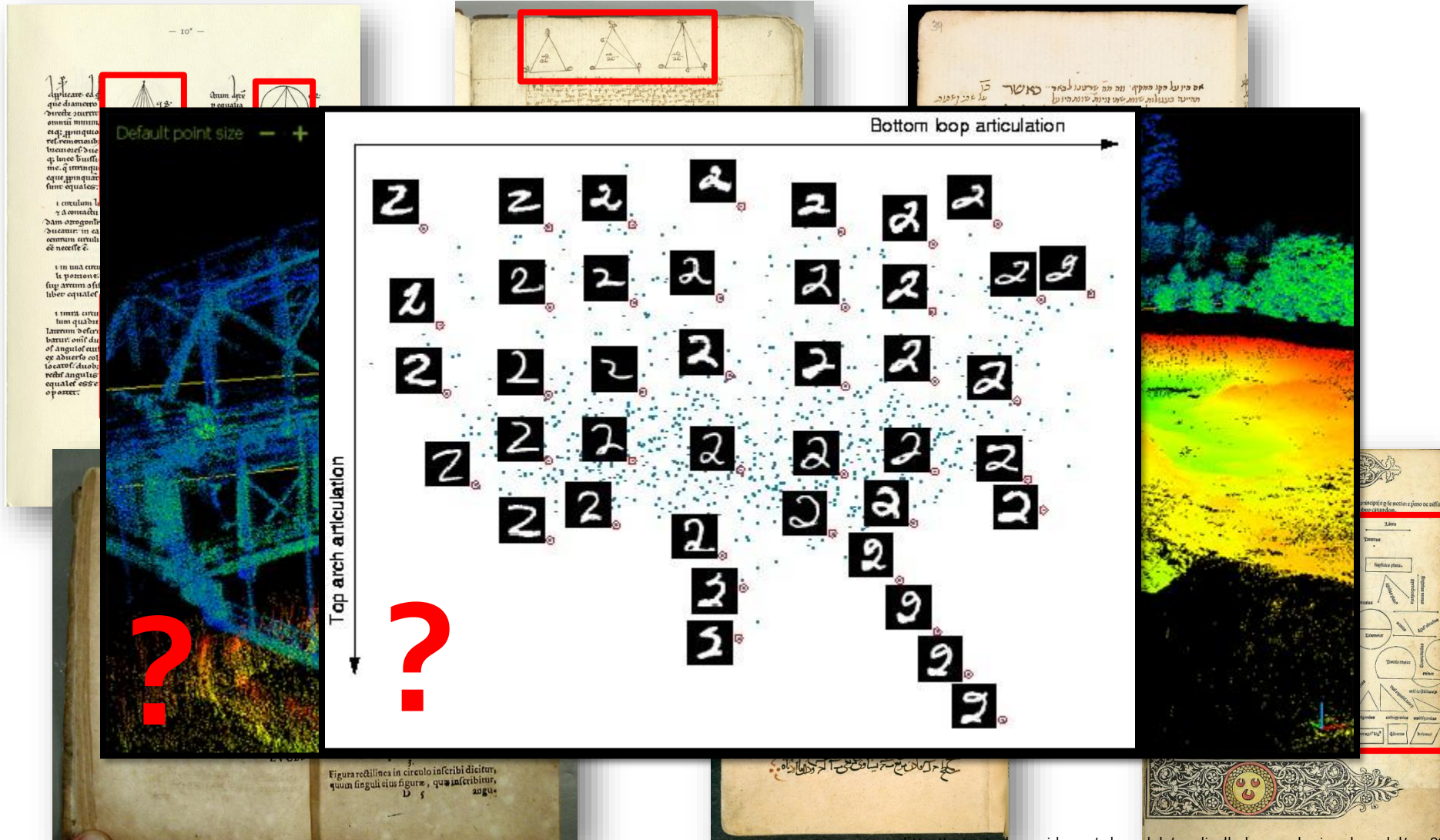
Euclidean Geometry



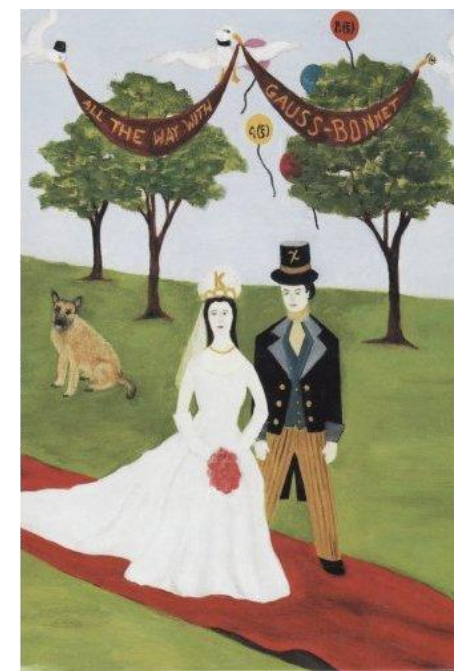
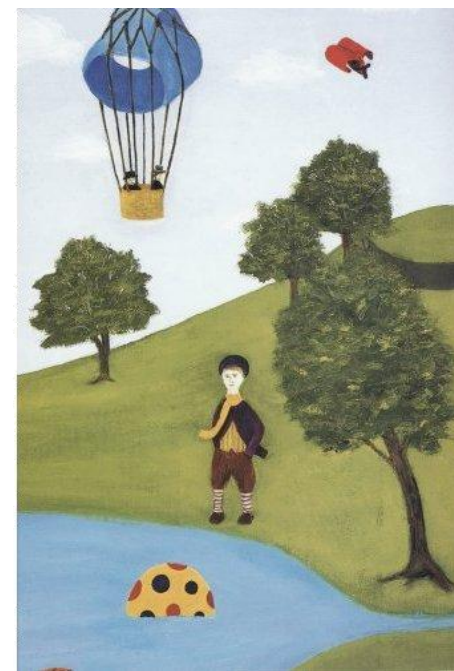
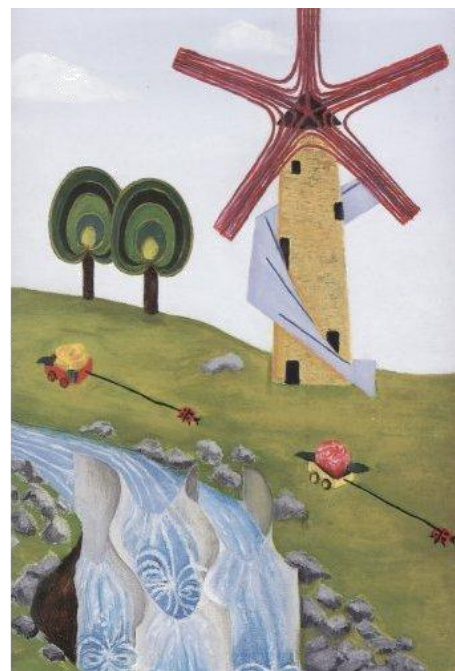
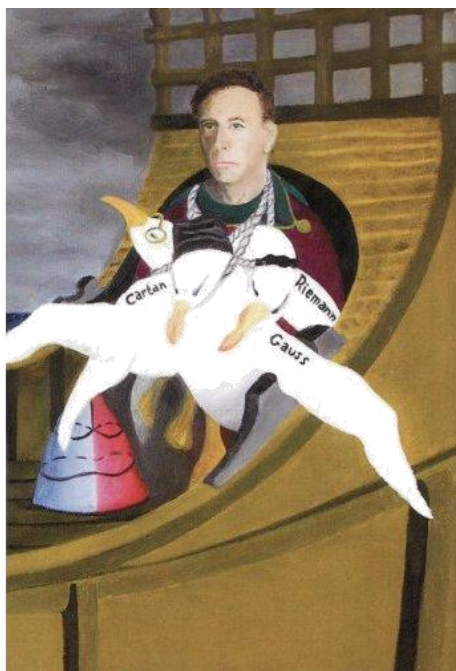
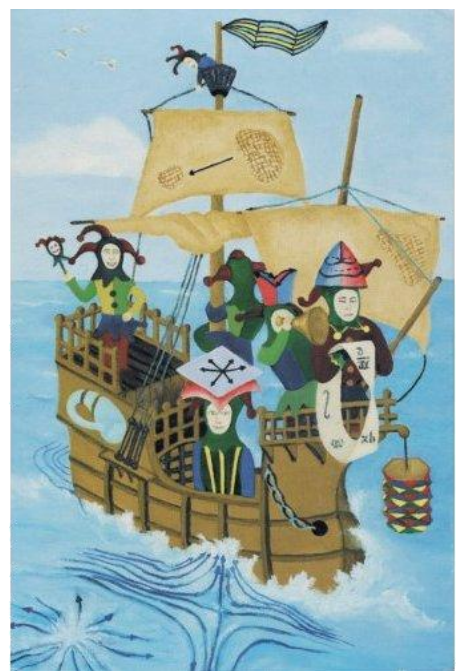
Euclidean Geometry



Euclidean Geometry

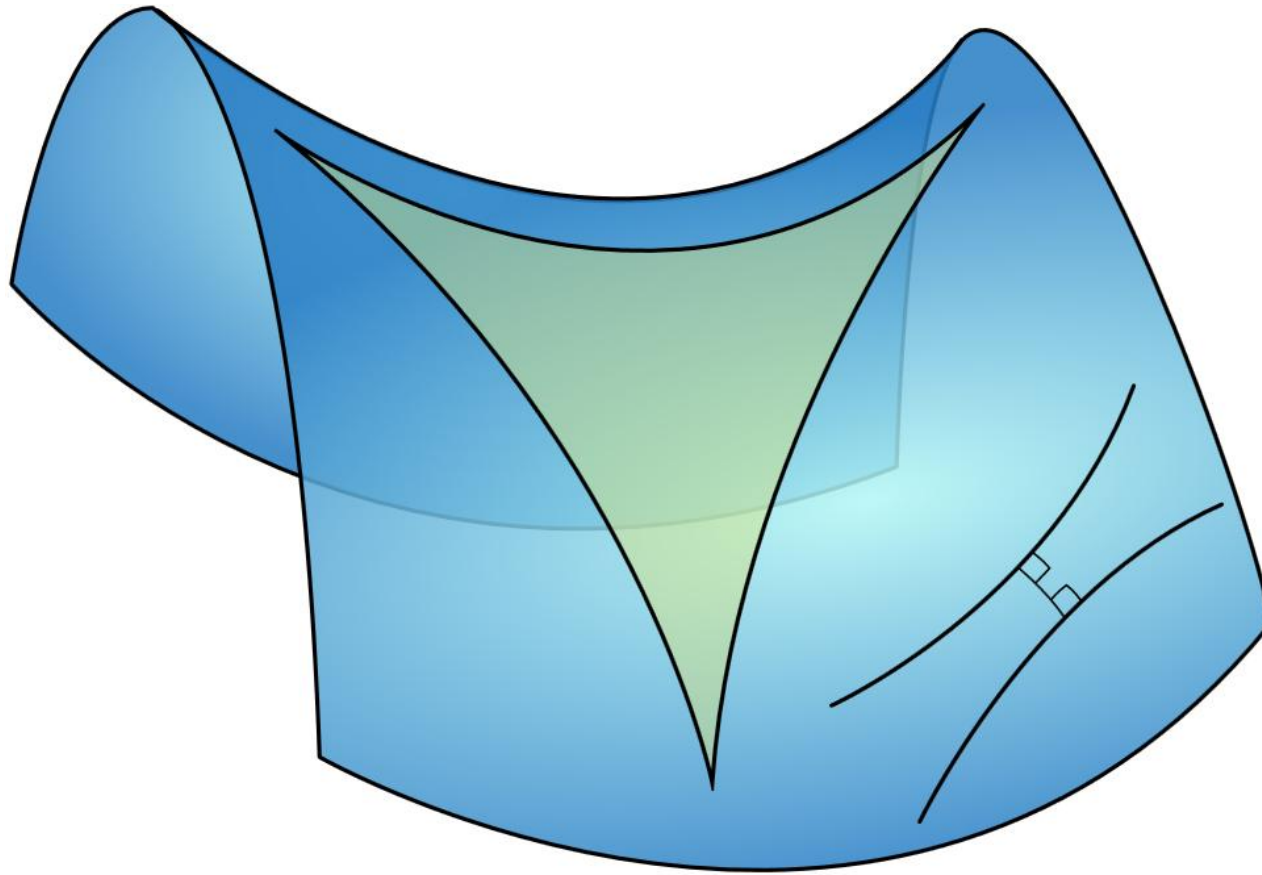


Differential Geometry



Spivak: *A Comprehensive Introduction to Differential Geometry*

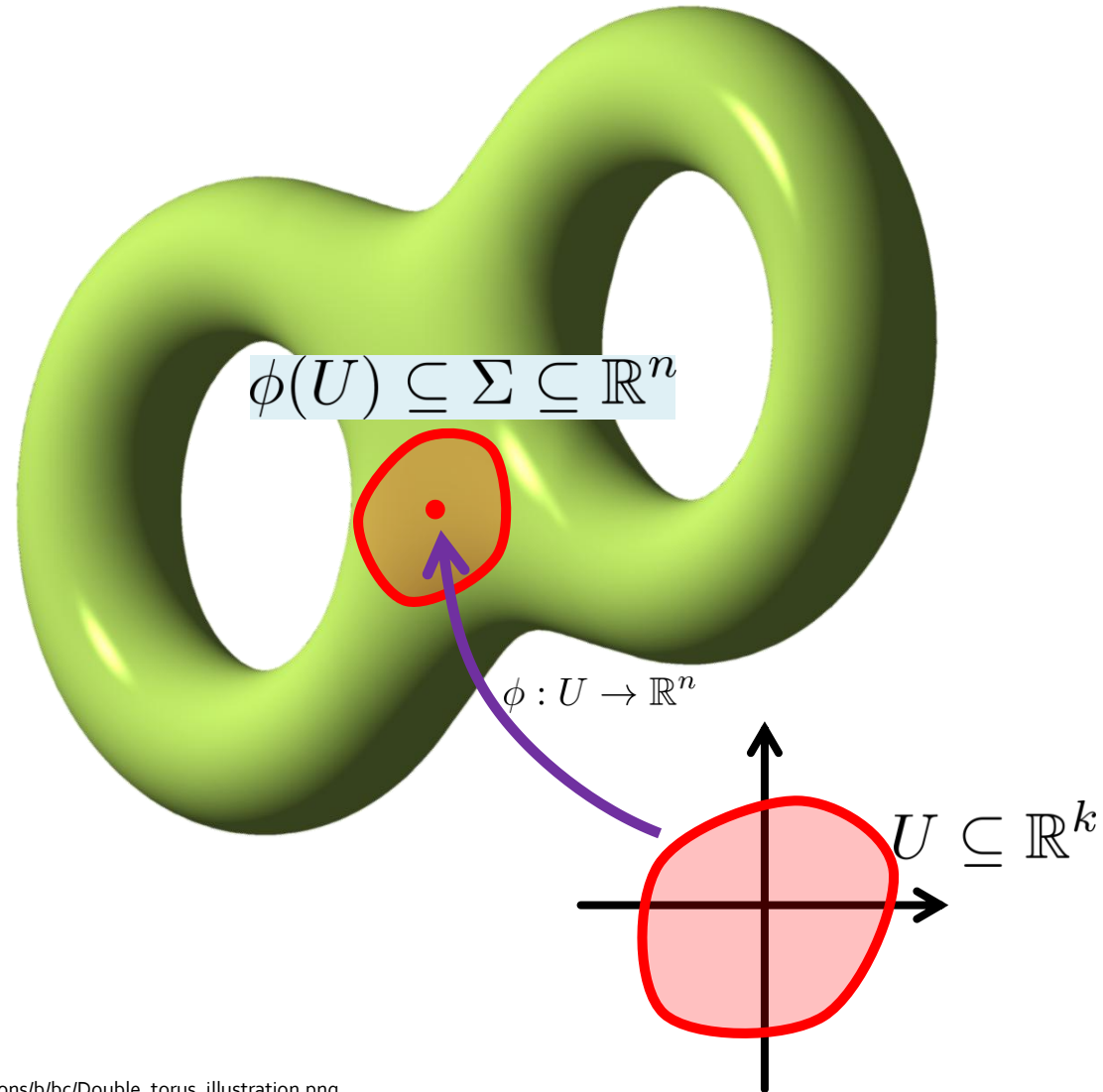
Differential Geometry



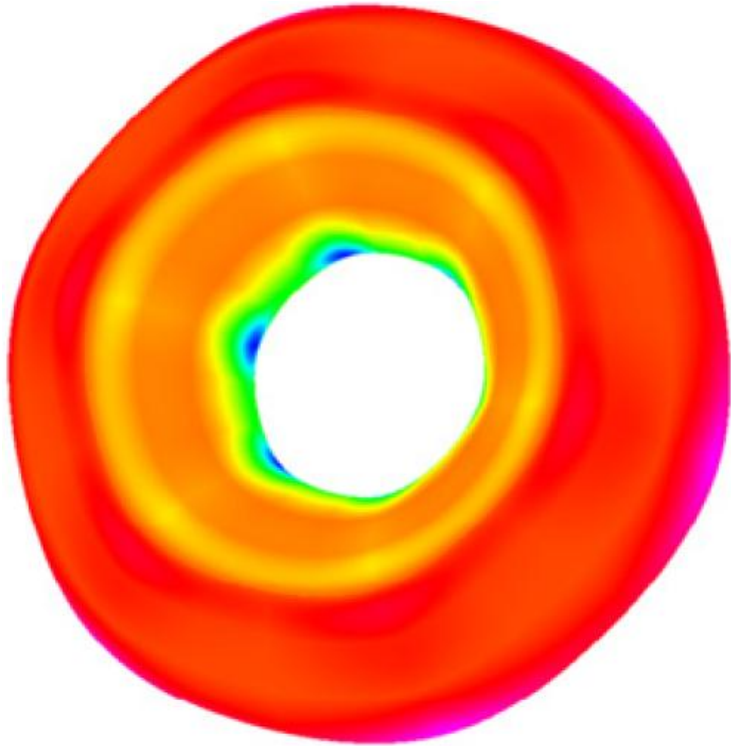
http://en.wikipedia.org/wiki/Differential_geometry

Study of smooth manifolds

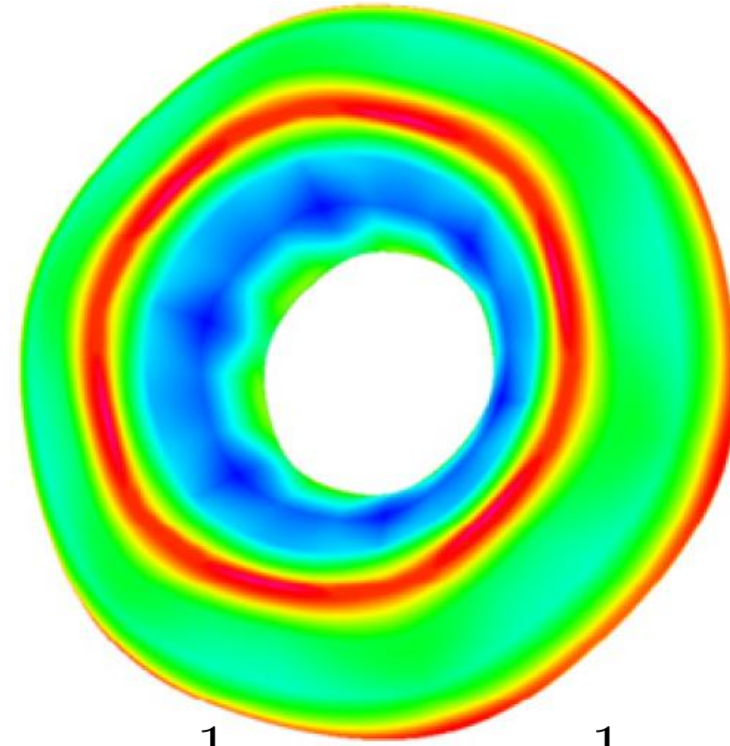
Manifold



Differential Geometry Toolbox



$$K := \kappa_1 \kappa_2 = \det \mathbb{I}$$

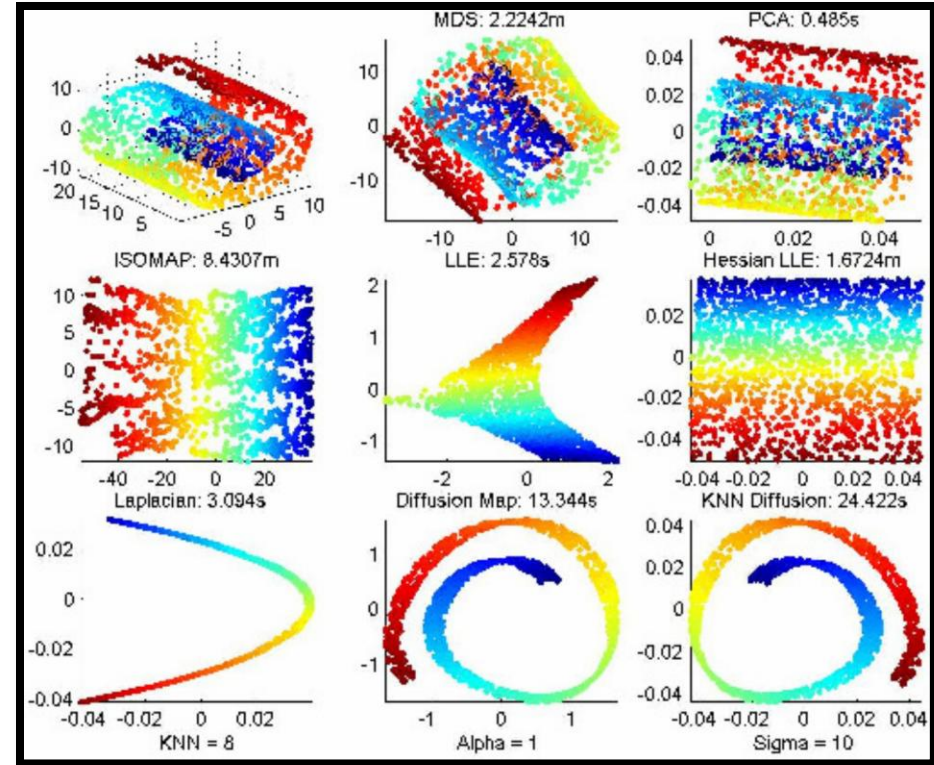
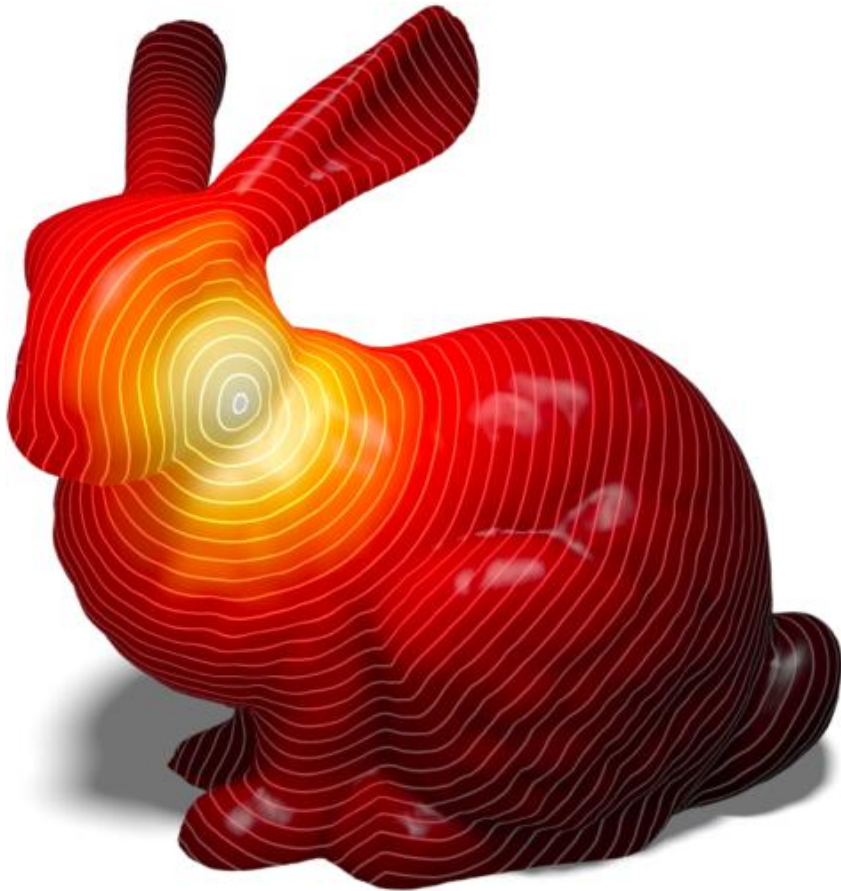


$$H := \frac{1}{2}(\kappa_1 + \kappa_2) = \frac{1}{2}\text{tr } \mathbb{I}$$

<http://www.sciencedirect.com/science/article/pii/S0010448510001983>

Curvature and shape properties

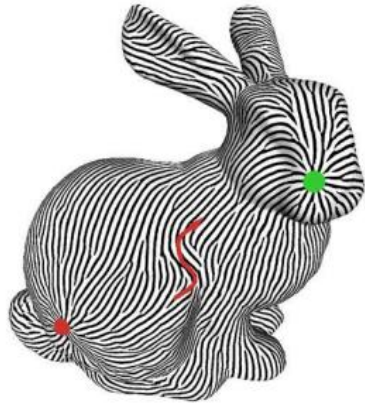
Differential Geometry Toolbox



Crane, Weischedel, Wardetzky. *Geodesics in heat*. TOG 2013.
Wittman. Manifold learning techniques.

Distances

Differential Geometry Toolbox



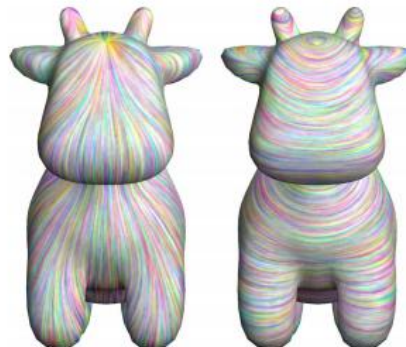
(a) [FSDH07]



(b) [BCBSG10]



(c) [KCPS13]



Symmetry

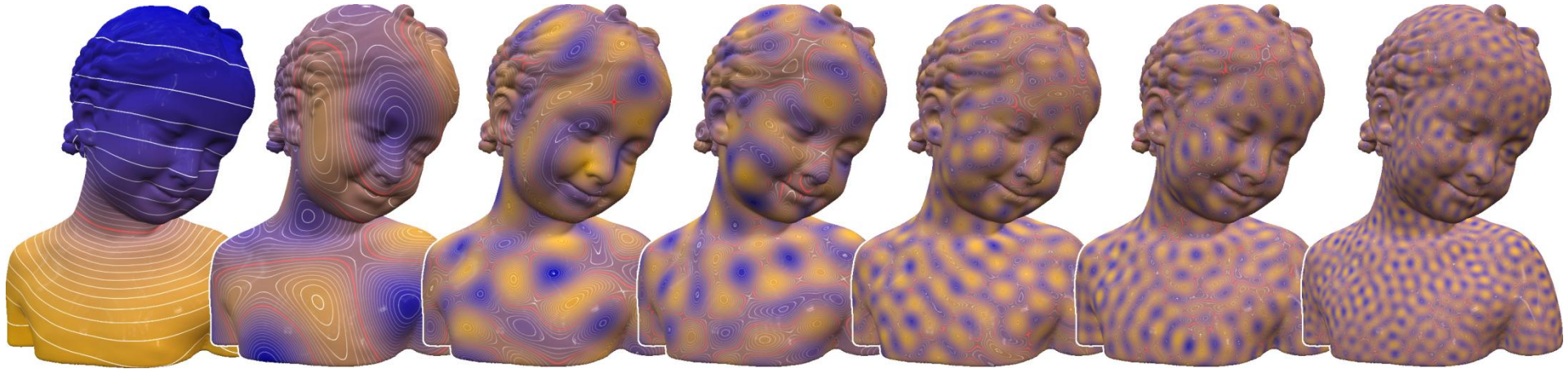
Anti-symmetry

(d) [ABCCO13]

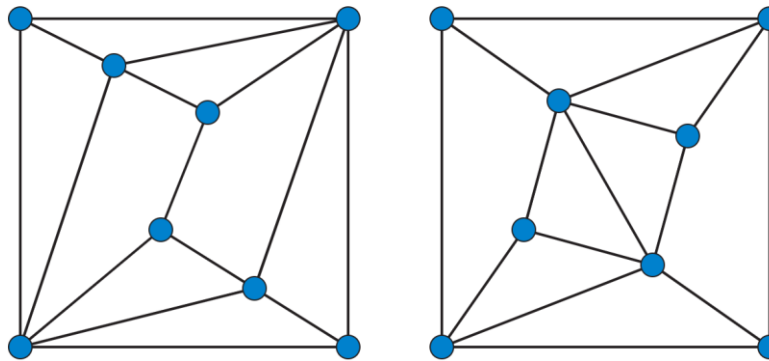
Vaxman et al.
Directional field synthesis, design, and processing.
EG STAR 2016.

Flows and vector fields

Differential Geometry Toolbox



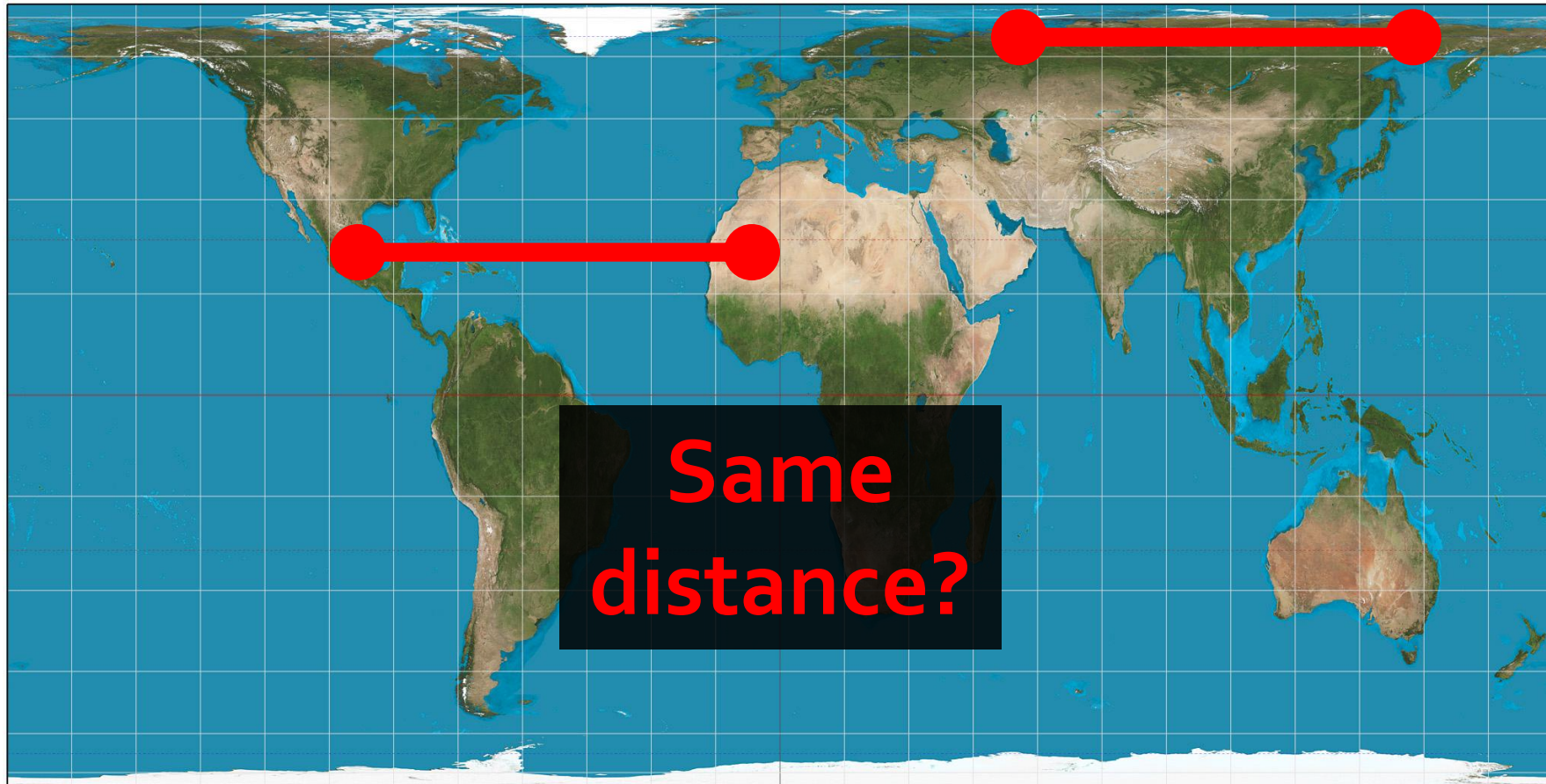
Vallet and Lévy. *Spectral Geometry Processing with Manifold Harmonics*. EG 2008



"Enneahedra"

Differential operators

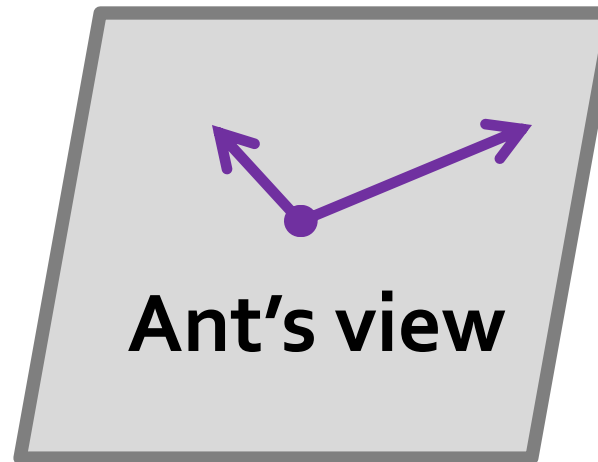
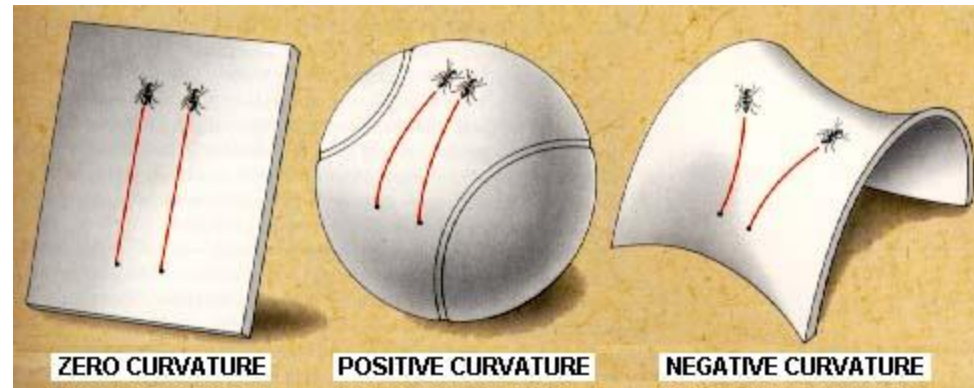
Riemannian Geometry



http://upload.wikimedia.org/wikipedia/commons/2/2c/Hobo%E2%80%93Dyer_projection_SW.jpg

Only need angles and distances

Riemannian Viewpoint

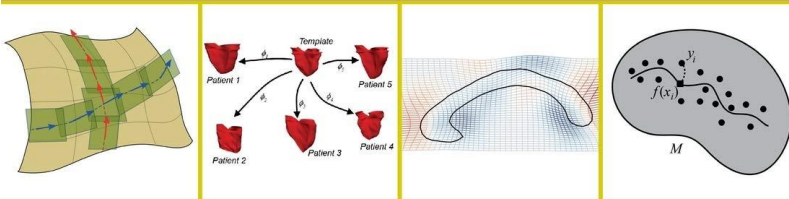


<http://www.phy.syr.edu/courses/modules/LIGHTCONE/pics/curved.jpg>

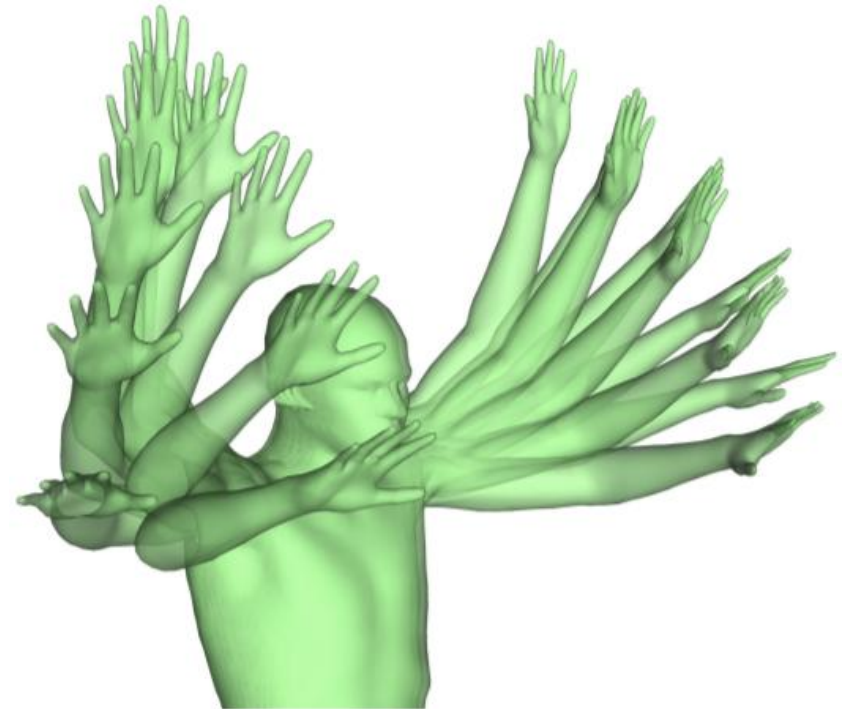
Only need angles and distances

High-Dimensional Geometry

RIEMANNIAN GEOMETRIC STATISTICS IN MEDICAL IMAGE ANALYSIS

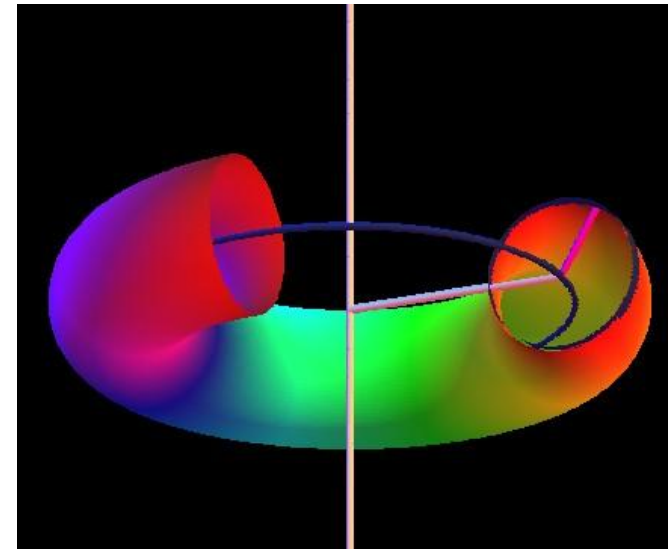
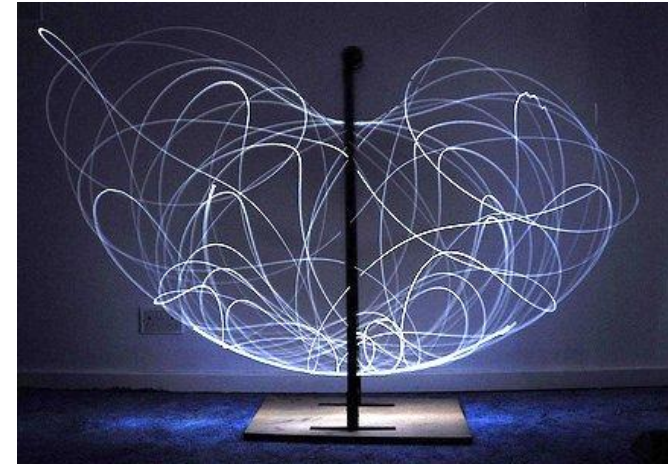
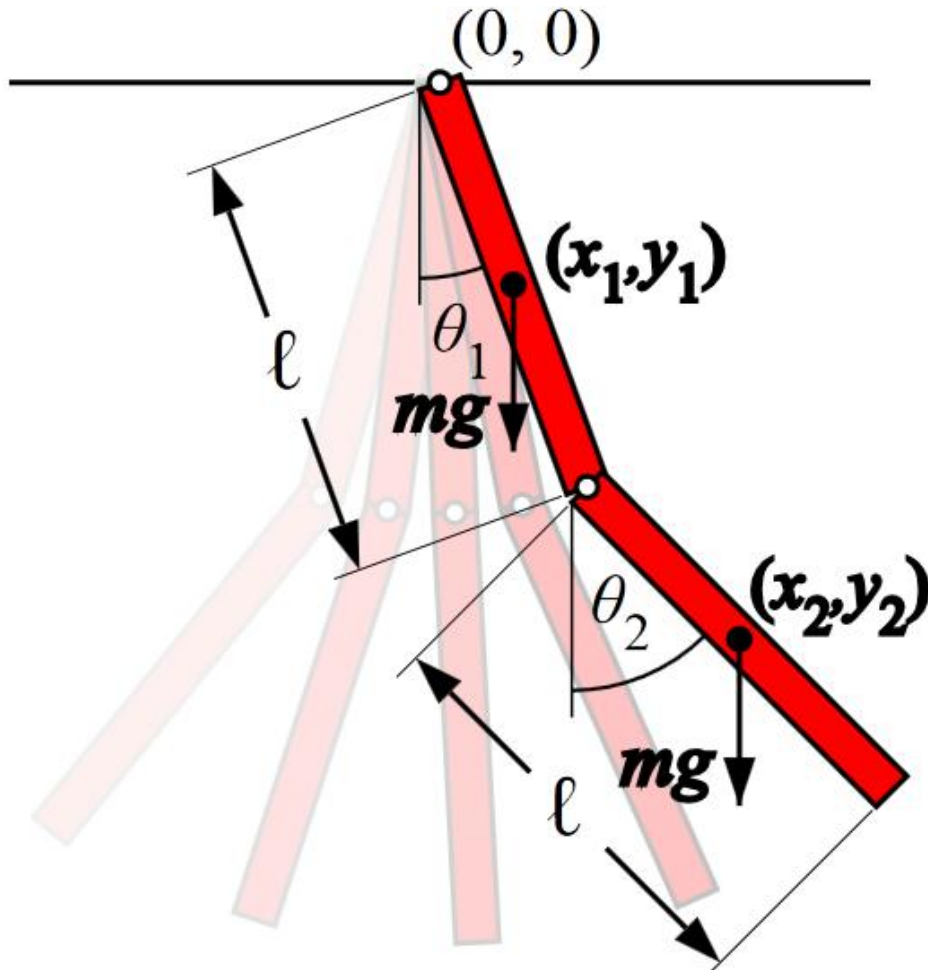


Edited by
Xavier Pennec,
Stefan Sommer, Tom Fletcher

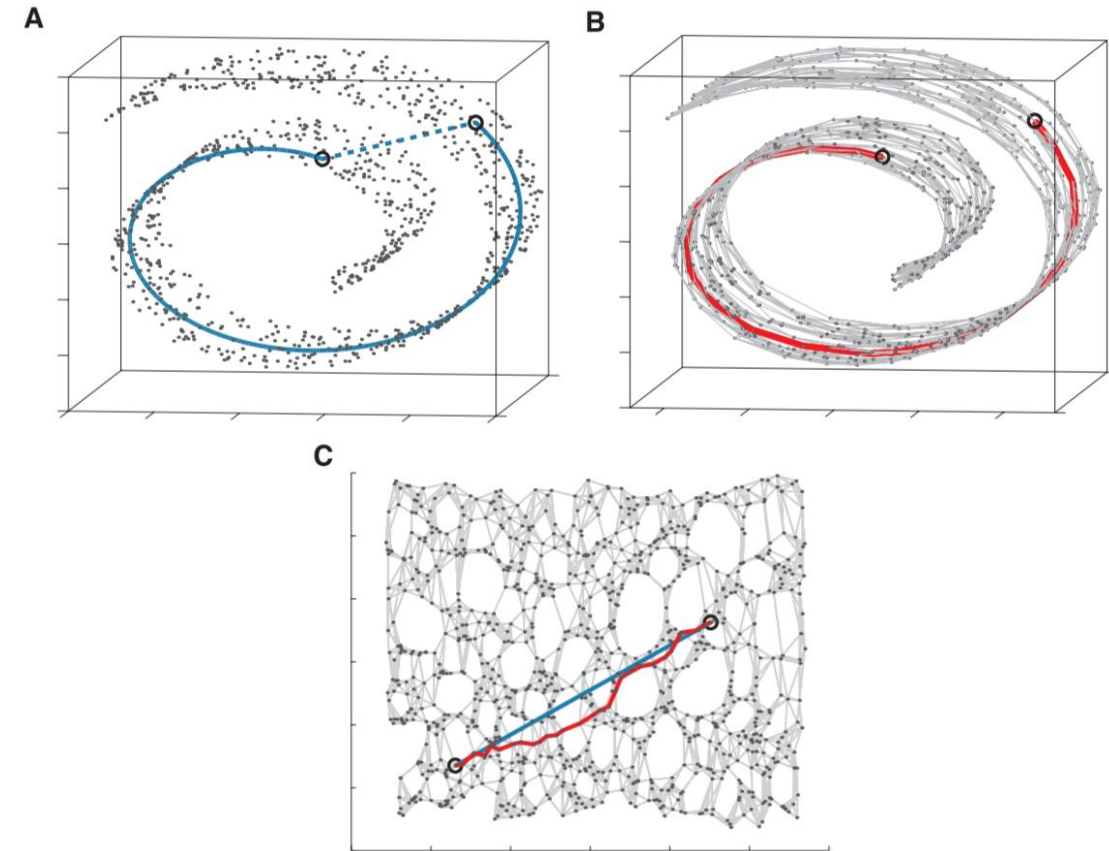


Heeren et al.
Splines in the Space of Shells.
SGP 2016.

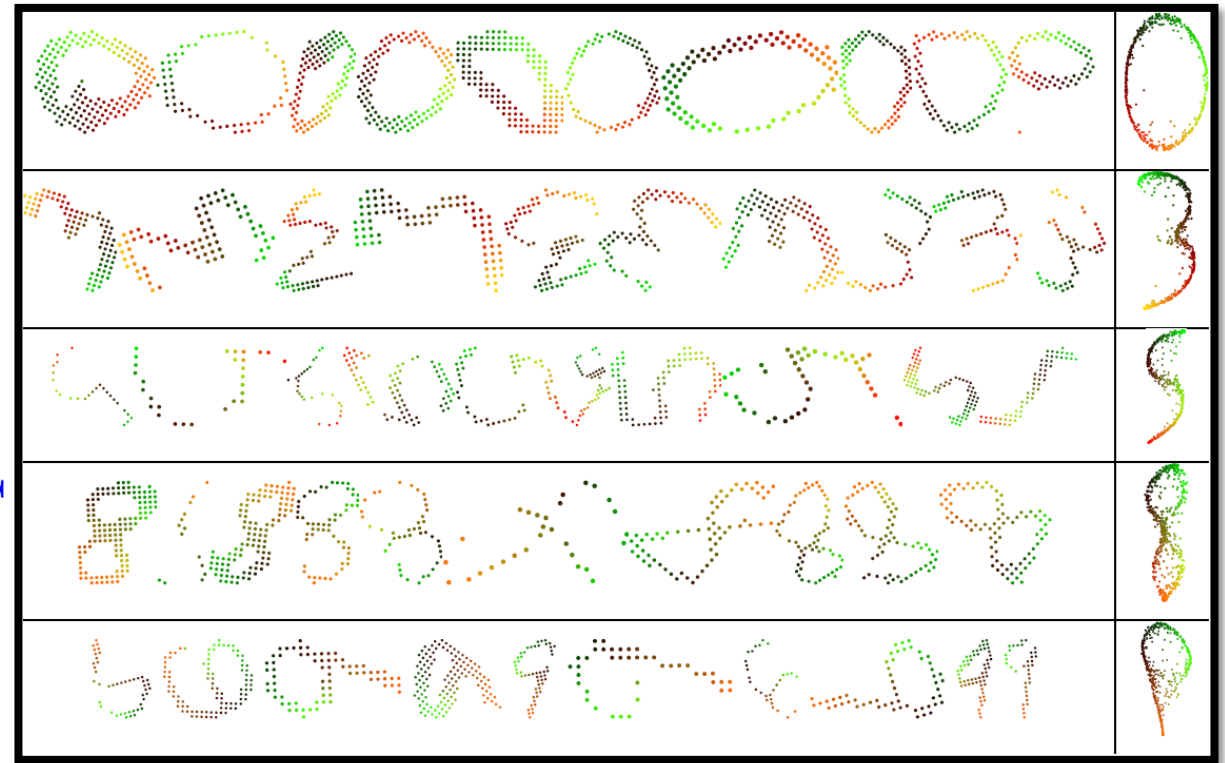
Geometric Mechanics and Lie Groups



Metric Geometry and Metric Embedding



Input data

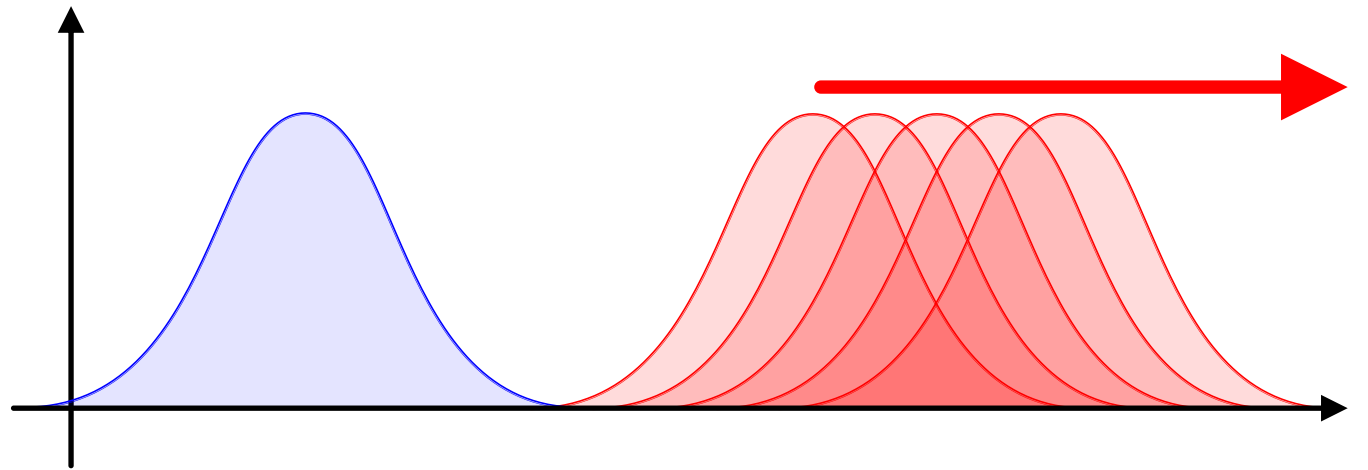
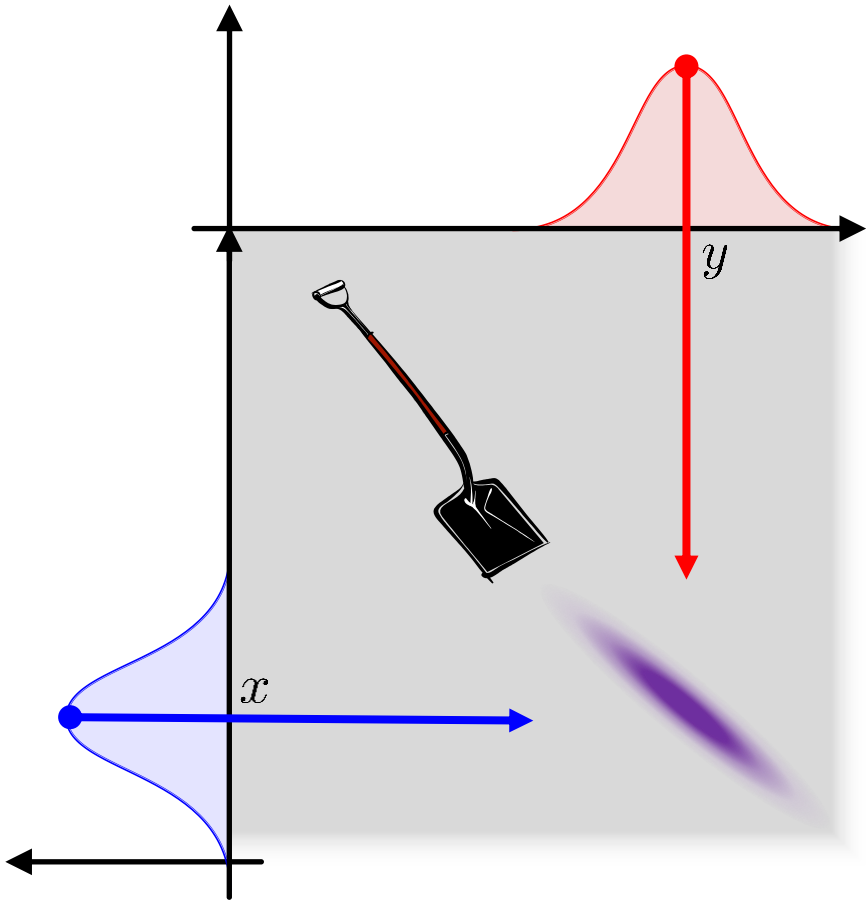


Barycenter (MDS)

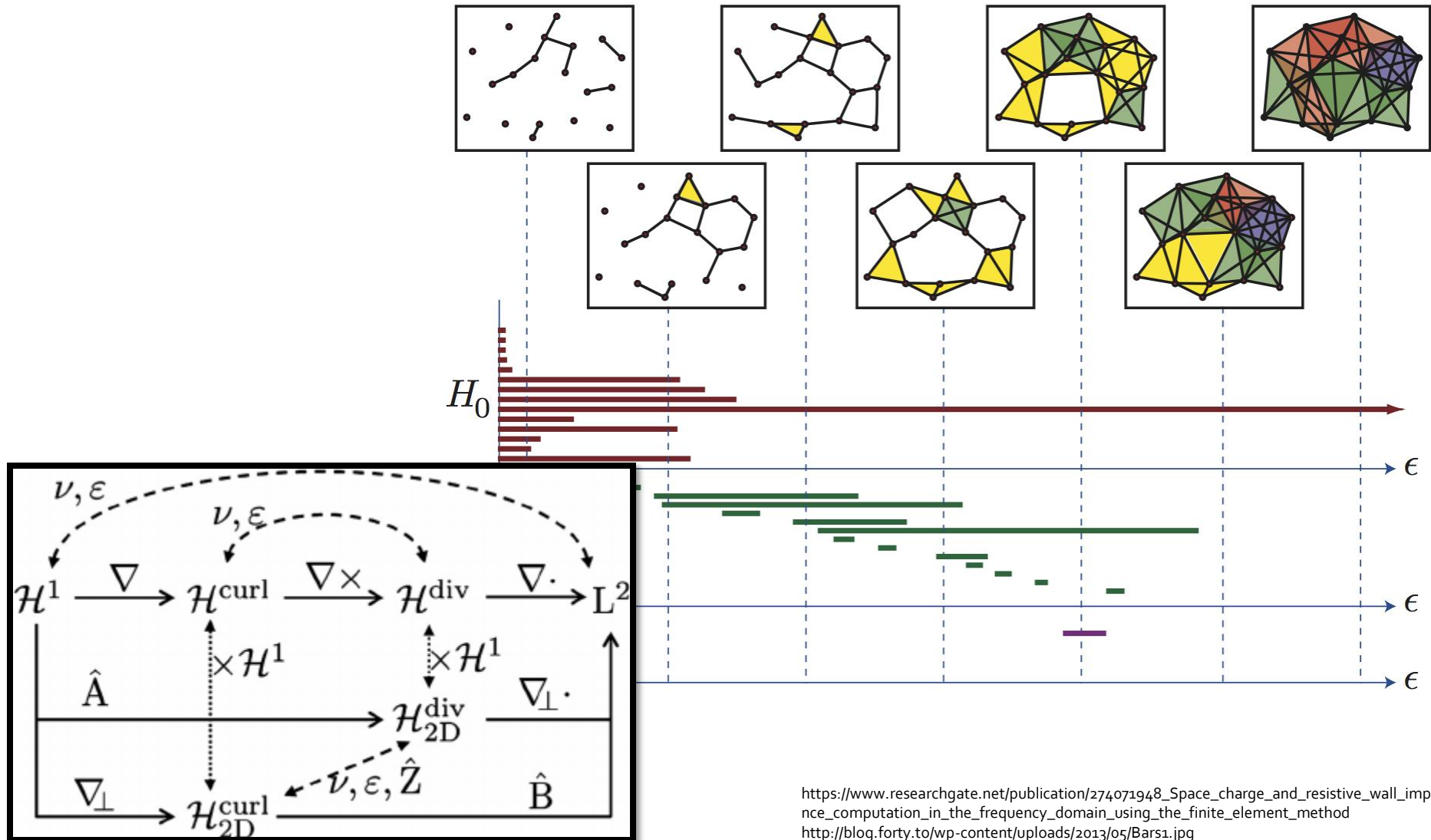
Tenenbaum et al.
A Global Geometric Framework for Nonlinear Dimensionality Reduction.
Science 2000.

Peyré, Cuturi, and Solomon.
Gromov-Wasserstein Averaging of Kernel and Distance Matrices.
ICML 2016.

Optimal Transport



{Differential/Morse/Persistent/...} Topology



Plan for Today

I. Theoretical toolbox

II. Computational toolbox

III. Application areas

Many Notions of Shape

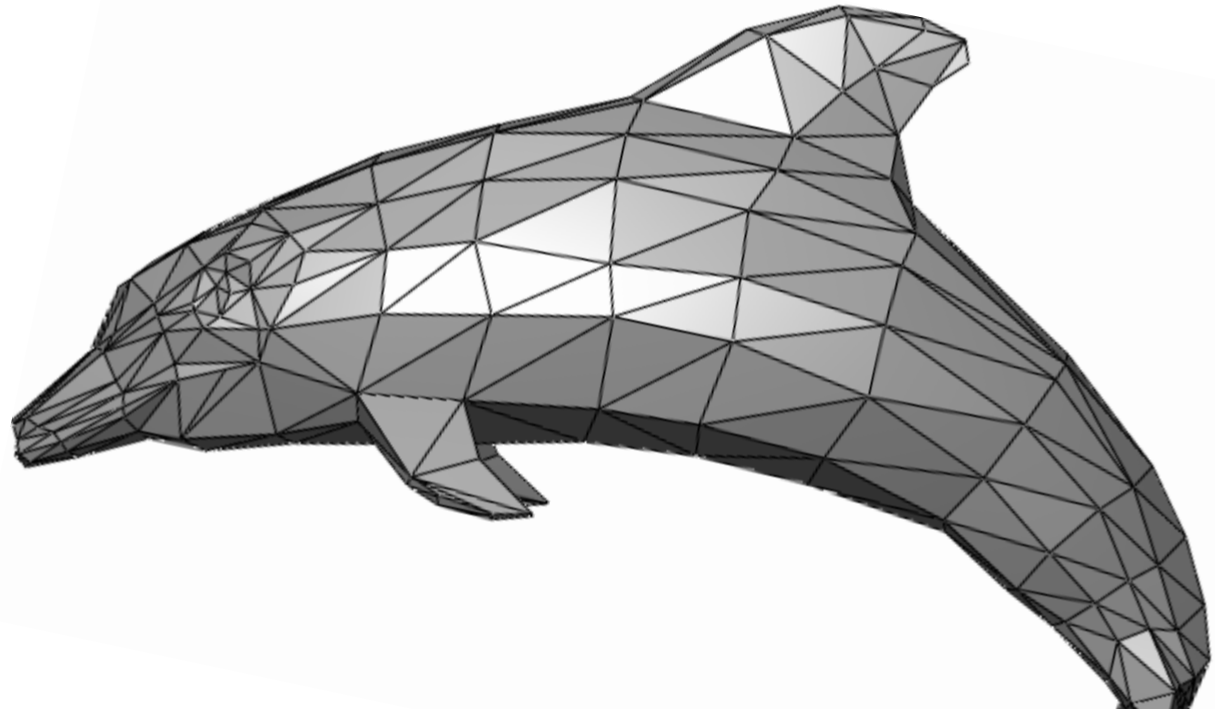
- Triangle mesh
- Triangle soup
 - Graph
- Point cloud
- Pairwise distance matrix
 - Dataset
 - Network

Nearly anything with a notion of
proximity/distance/curvature/...

Typical issue:

How to Interpret Geometric Data

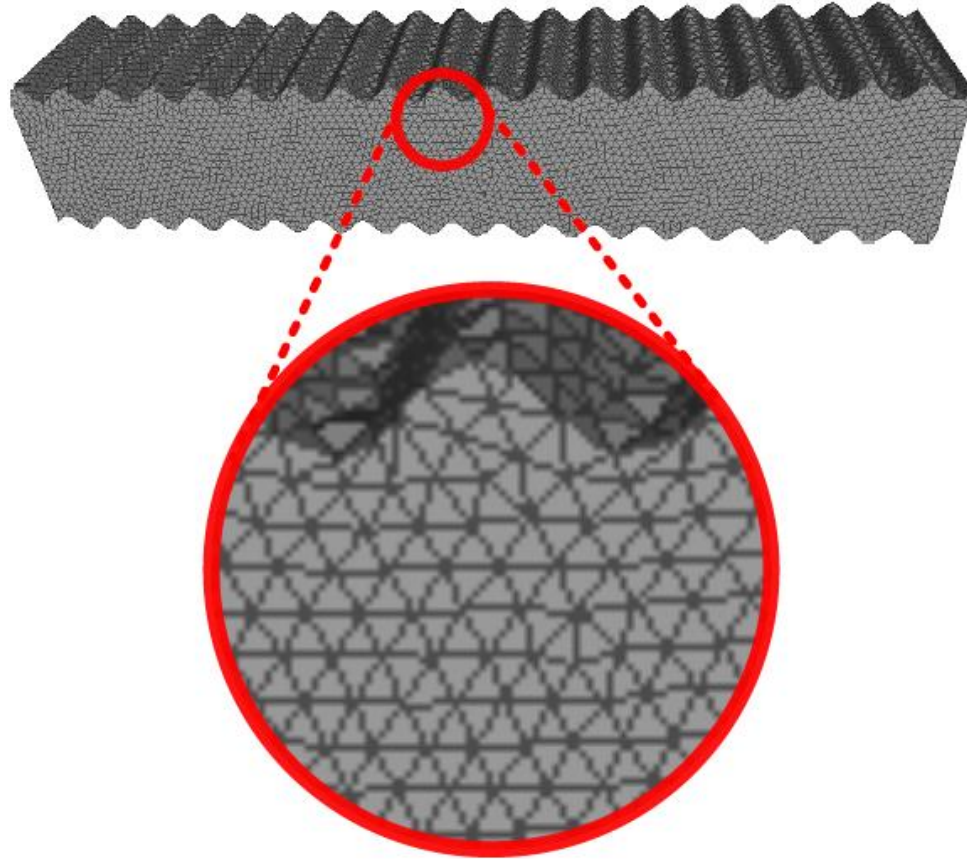
- Collection of **flat triangles**
- Approximates a **smooth surface**





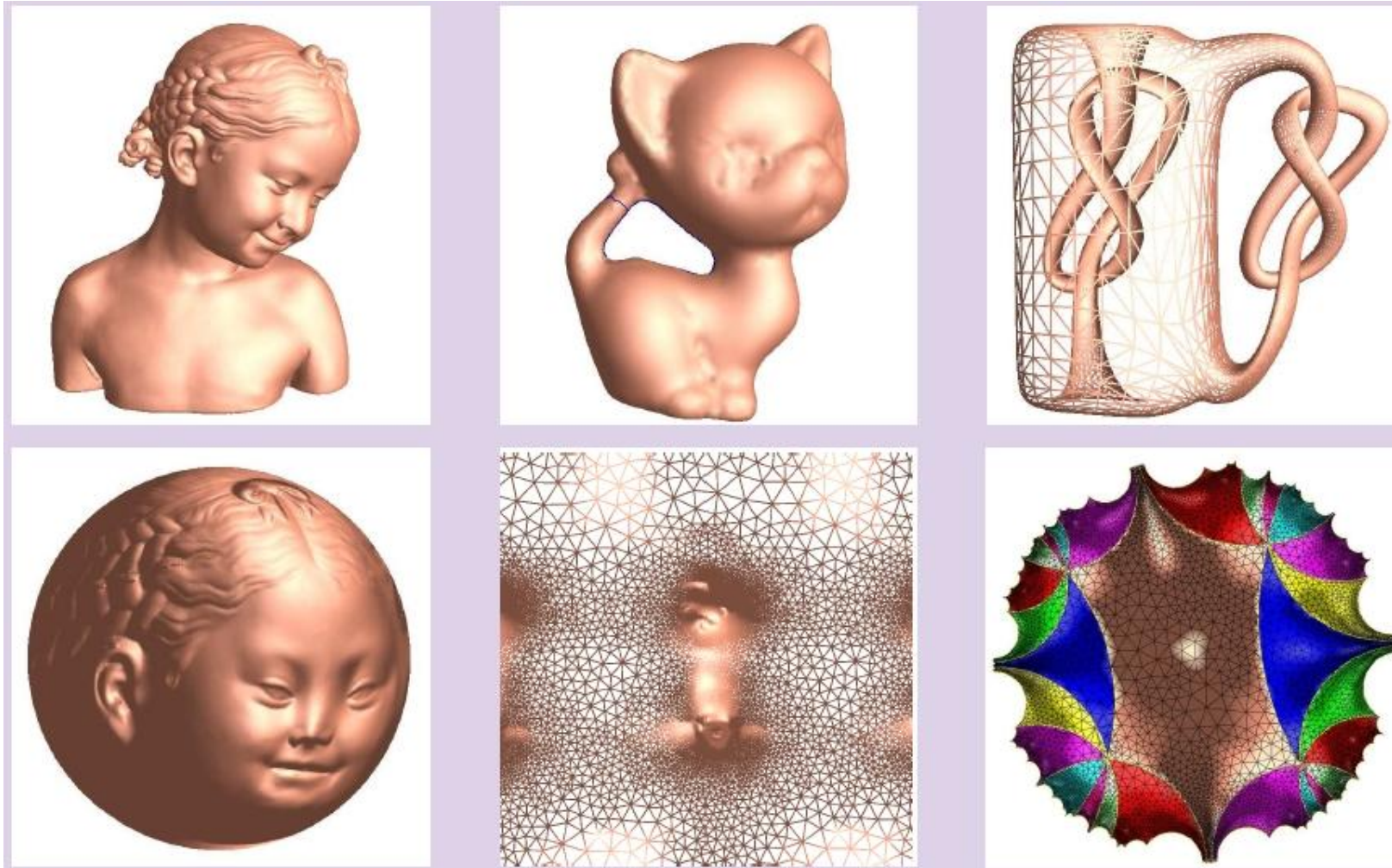
- Can a triangle mesh
- have **curvature**?

Jack of All Trades



Combine smooth and discrete

Example: Discrete Differential Geometry



Modern Approach

Discrete

vs.

Discretized

Discrete Differential Geometry

Discrete theory *paralleling*
differential geometry.

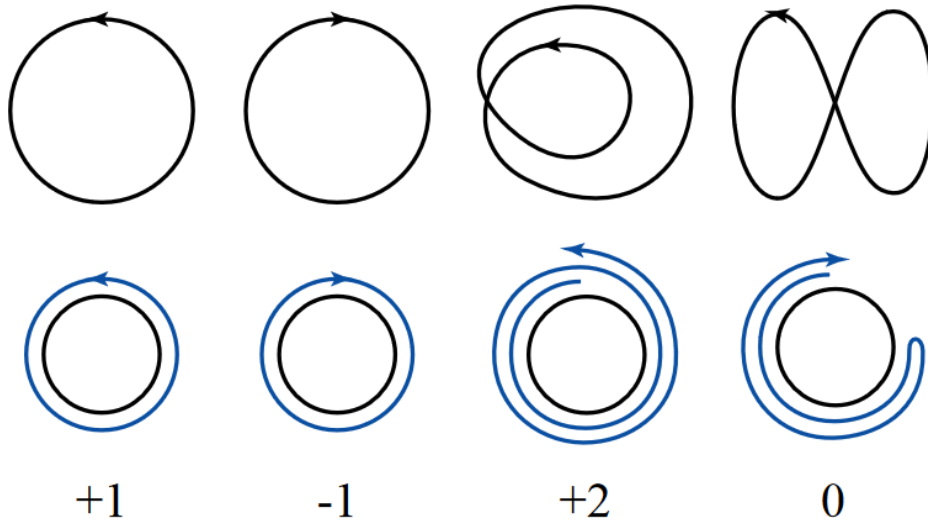
Structure preservation

[struhk-cher pre-zur-vey-shuh n]:

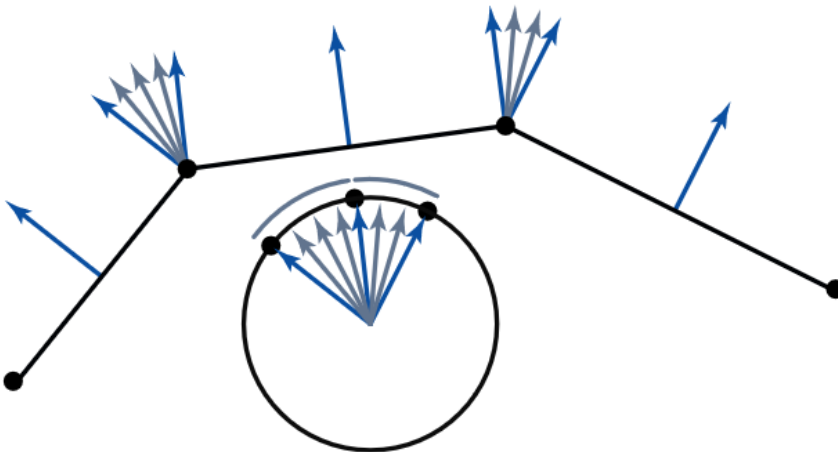
Keeping properties from the continuous abstraction exactly true in a discretization.



Example: Turning Numbers



$$\int_{\Omega} \kappa \, ds = 2\pi k$$



$$\sum_i \alpha_i = 2\pi k$$

Images from: Grinspun and Secord, "The Geometry of Plane Curves" (SIGGRAPH 2006)

Convergence

[*kuh* n-**vur**-*juh* ns]:

Increasing approximation
quality as a discretization is
refined.



Convergence *and* Structure

Can you have it all?



Disappointing Result

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

Max Wardetzky¹

Saurabh Mathur²

Felix Kälberer¹

Eitan Grinspun^{2 †}

¹Freie Universität Berlin, Germany

²Columbia University, USA

Abstract

Discrete Laplace operators are ubiquitous in applications spanning geometric modeling to simulation. For robustness and efficiency, many applications require discrete operators that retain key structural properties inherent to the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

In applications one often requires certain structural prop-

1.1. Properties of smooth Laplacians

Consider a smooth surface S , possibly with boundary, equipped with a Riemannian metric, *i.e.*, an intrinsic notion of distance. Let the intrinsic L^2 inner product of functions u and v on S be denoted by $(u, v)_{L^2} = \int_S uv \, dA$, and let $\Delta = -\operatorname{div} \operatorname{grad}$ denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

(NULL) $\Delta u = 0$ whenever u is constant

Disappointing Result

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

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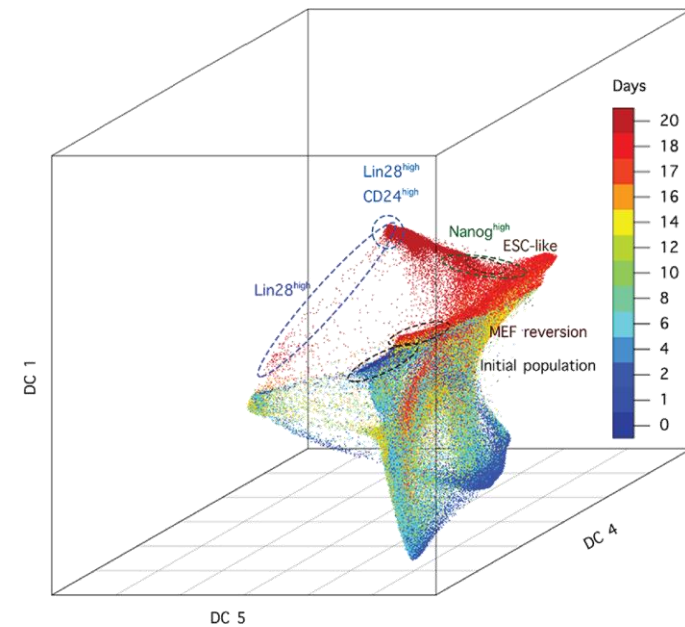
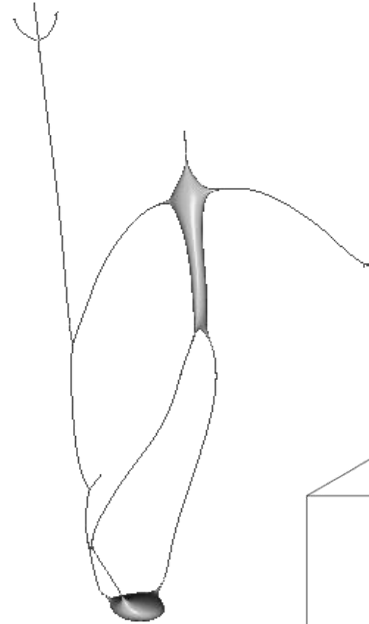
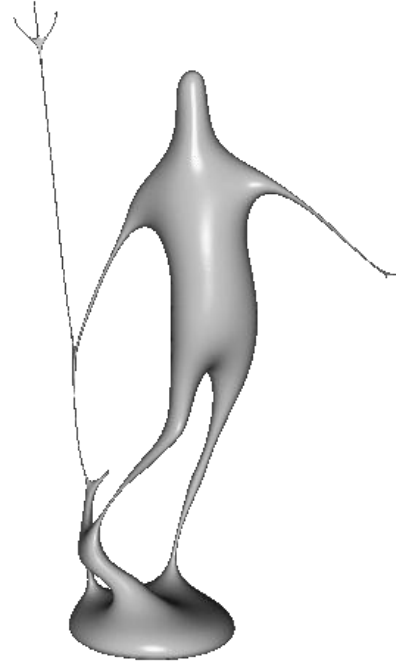
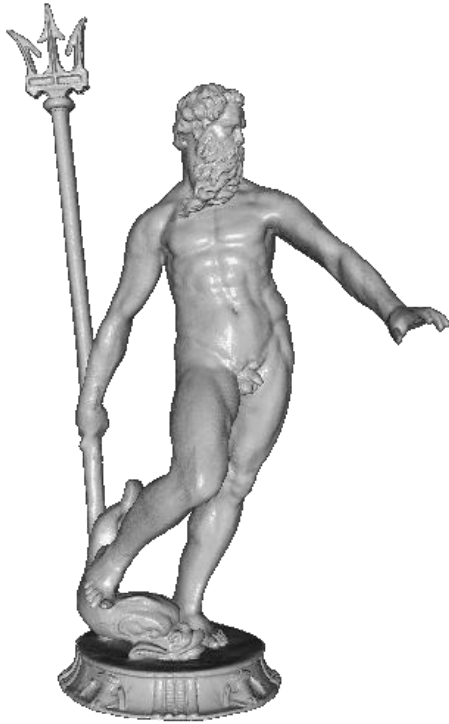
(NULL) $\Delta u = 0$ whenever u is constant

Theme

Pick and choose
which properties you need.

But there is a huge toolbox to draw from!

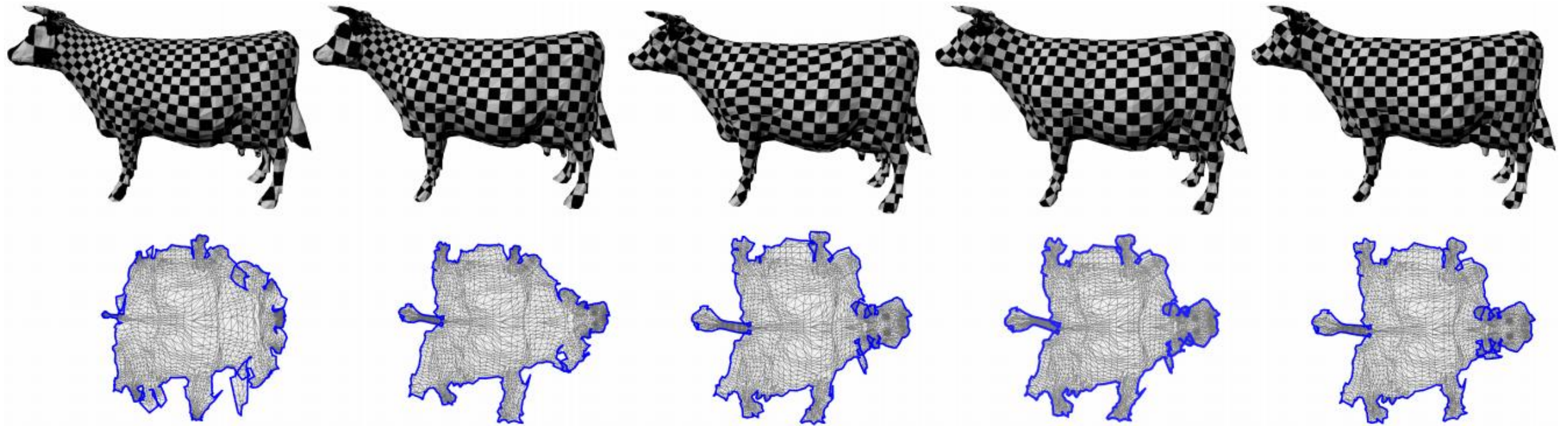
Numerical PDE



Chuang and Kazhdan. *Fast Mean-Curvature Flow via Finite-Elements Tracking*. CGF 2011.

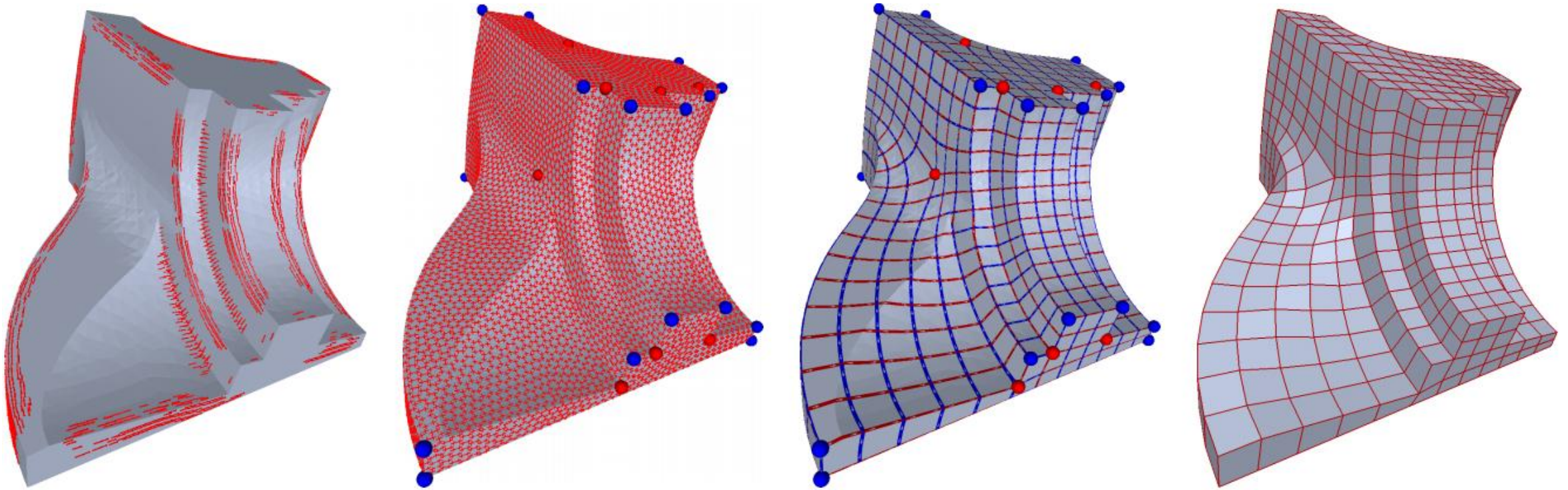
Coifman & Lafon. *Diffusion Maps*. ACHA 2006.

Large-Scale Smooth Optimization



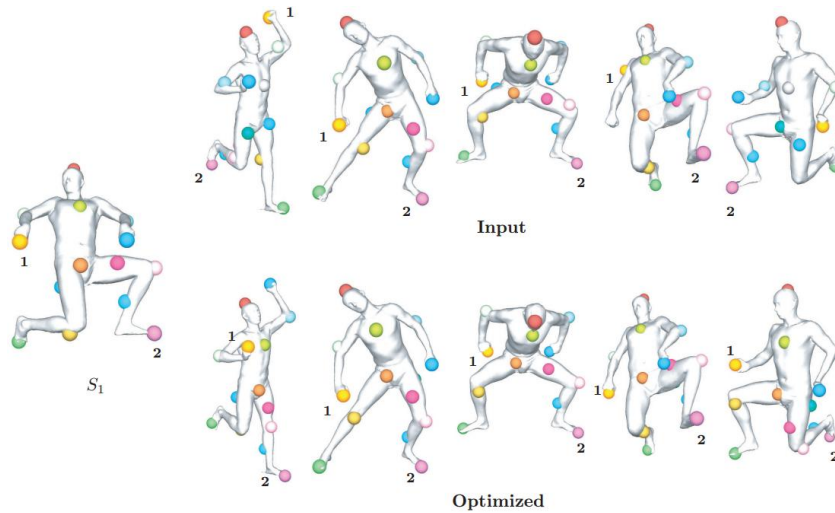
Smith and Schaefer. *Bijective parameterization with free boundaries*. SIGGRAPH 2015.

Discrete Optimization

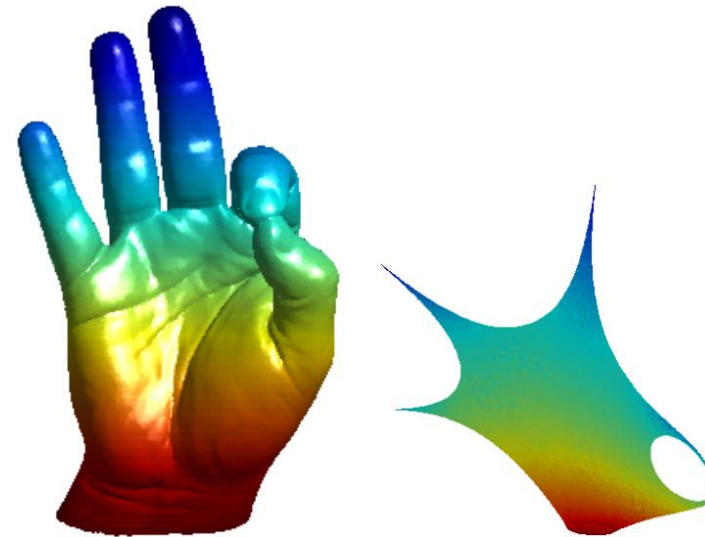


Bommes, Zimmer, Kobbelt. *Mixed-integer quadrangulation*. SIGGRAPH 2009.

Linear Algebra



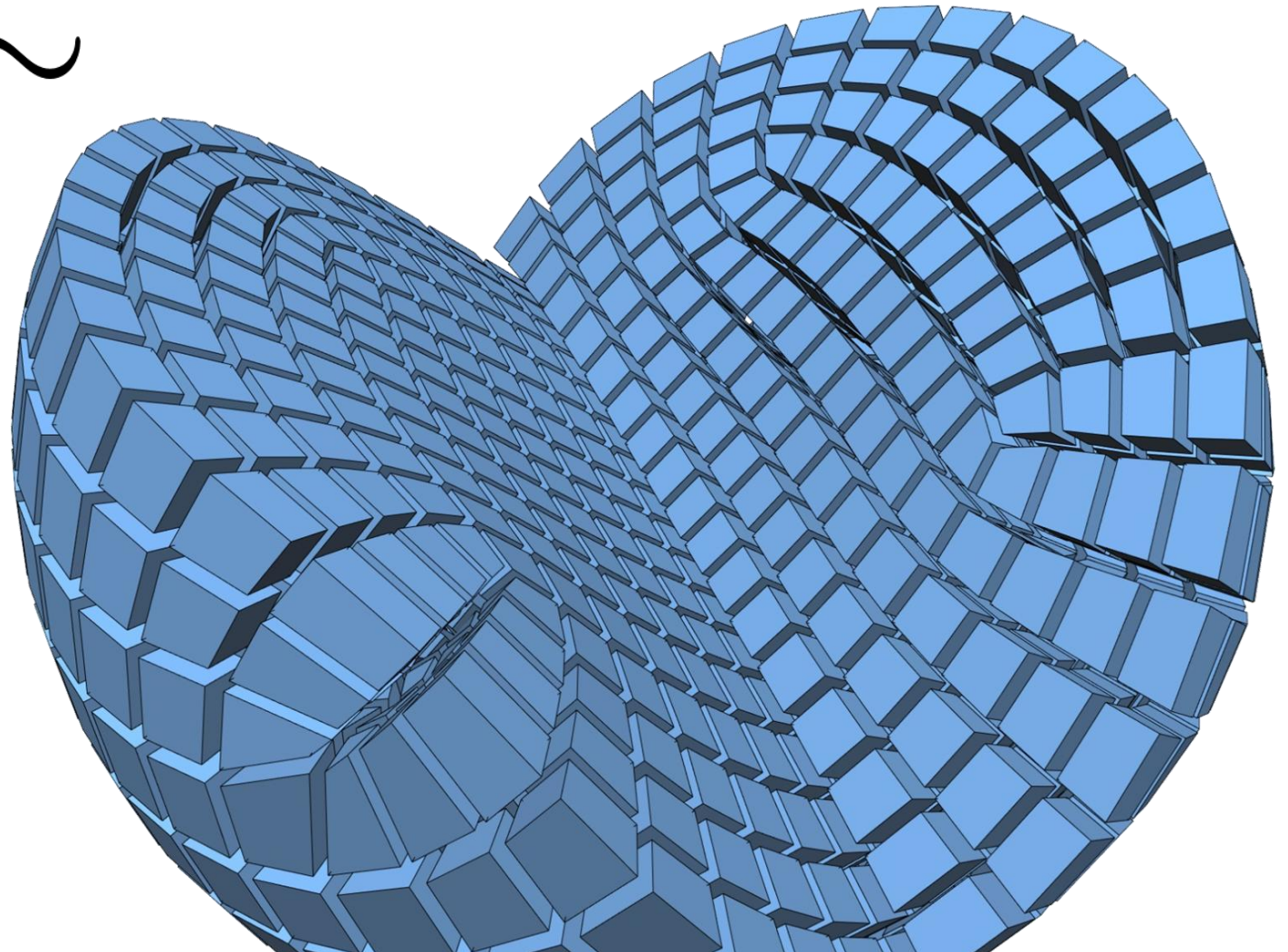
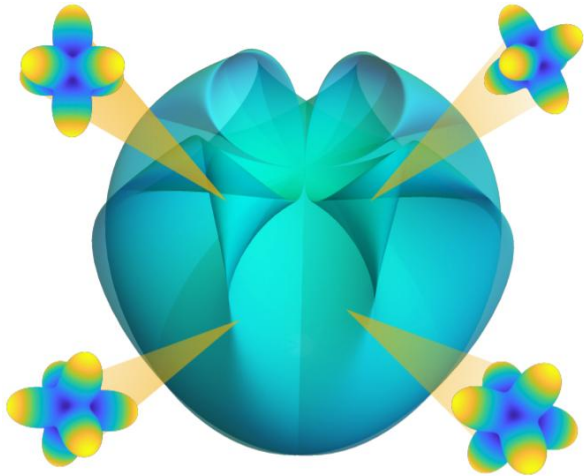
Huang, Guibas. *Consistent shape maps via semidefinite programming*. SGP 2013.



Krishnan, Fattal, Szeliski. *Efficient preconditioning of Laplacian matrices for computer graphics*. SIGGRAPH 2013.

Algebra & Representation Theory

$$SO(3) / \sim$$



Plan for Today

I. Theoretical toolbox

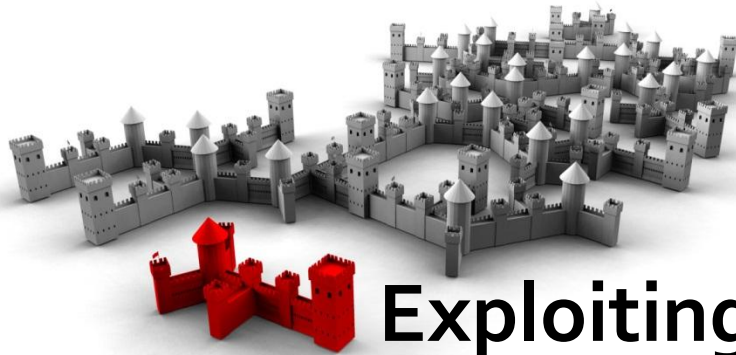
II. Computational toolbox

III. Application areas

Applications

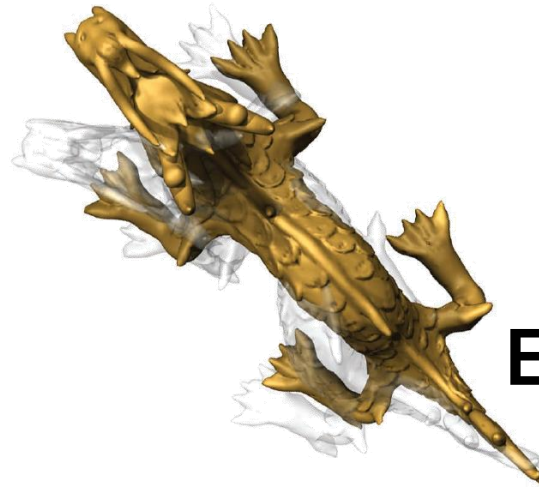
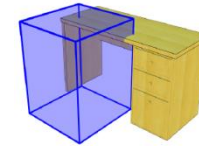


Transfer



Exploiting patterns

Retrieval



Editing

Mertens et al. "Texture Transfer Using Geometry Correlation."

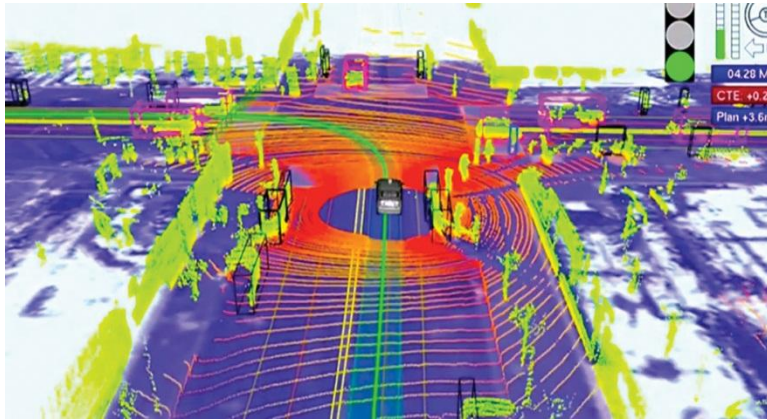
Fisher et al. "Context-Based Search for 3D Models."

Mitra et al. "Symmetrization."

Bokeloh et al. "A connection between partial symmetry and inverse procedural modeling."

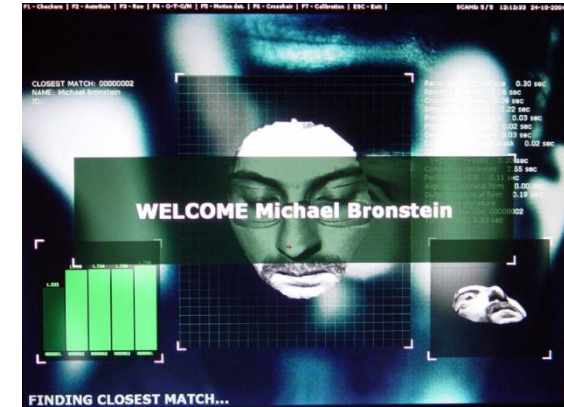
Graphics

Applications

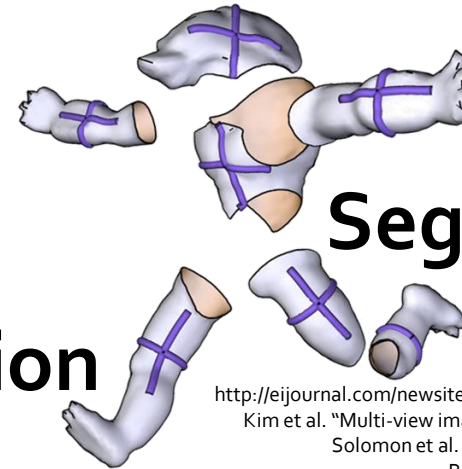


Recognition

Navigation



Reconstruction

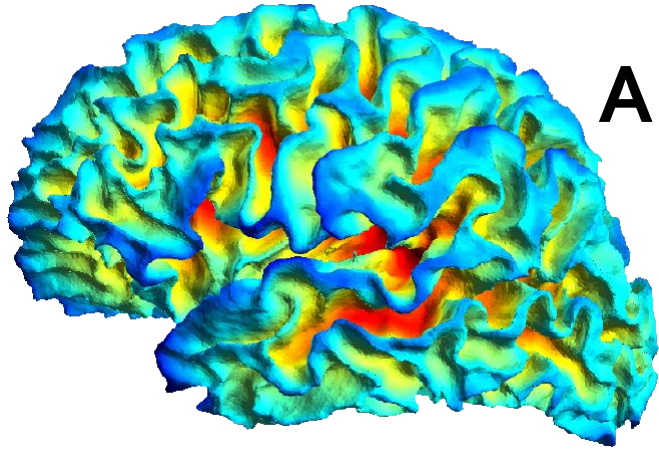


Segmentation

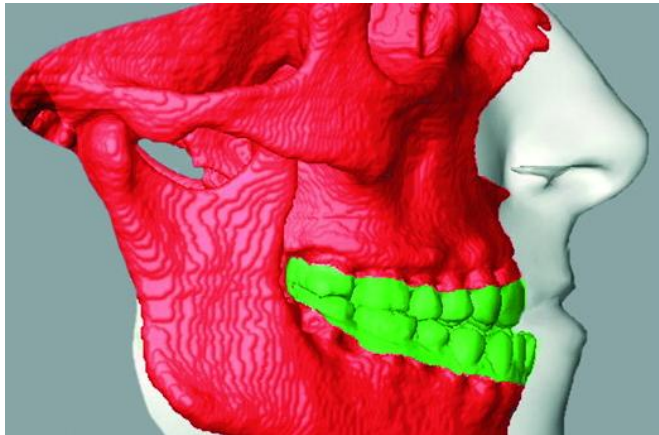
<http://eijournal.com/newsite/wp-content/uploads/2012/01/VELODYNE-IMAGE.jpg>
Kim et al. "Multi-view image and tof sensor fusion for dense 3d reconstruction."
Solomon et al. "Discovery of Intrinsic Primitives on Triangle Meshes."
Bronstein et al. "Three-Dimensional Face Recognition."

Vision

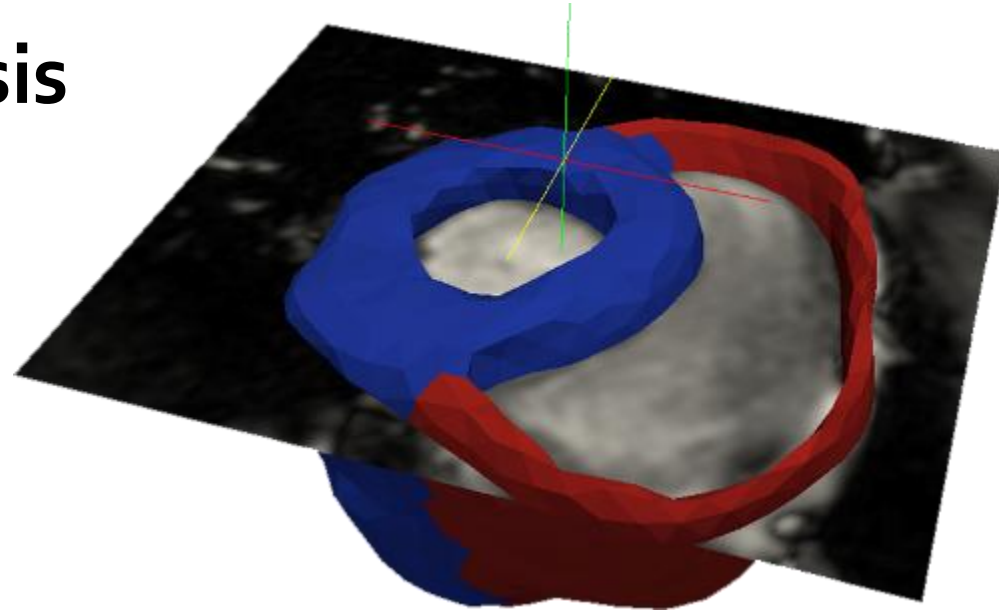
Applications



Analysis



Registration

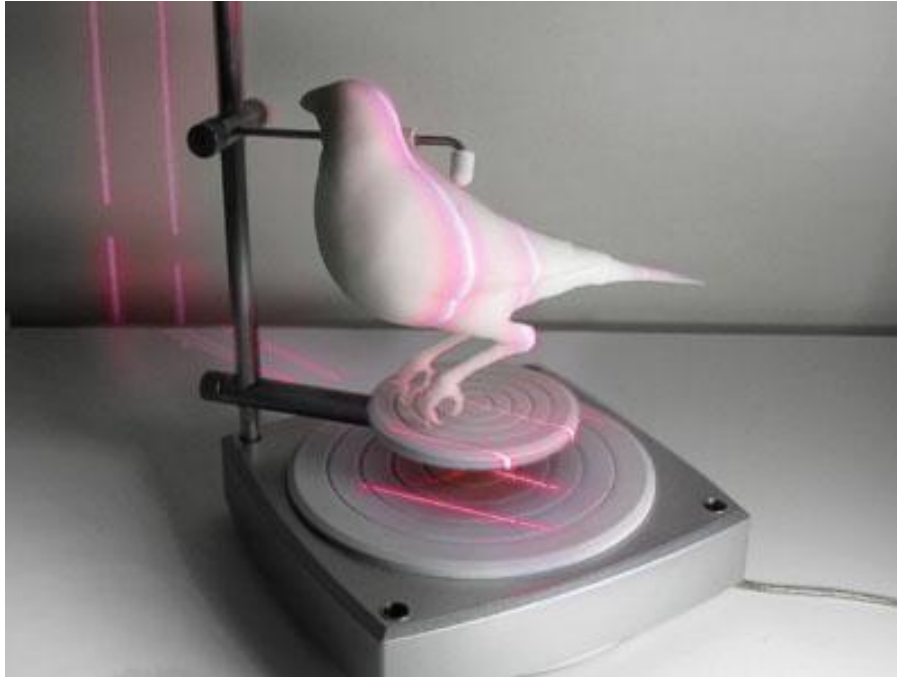


Segmentation

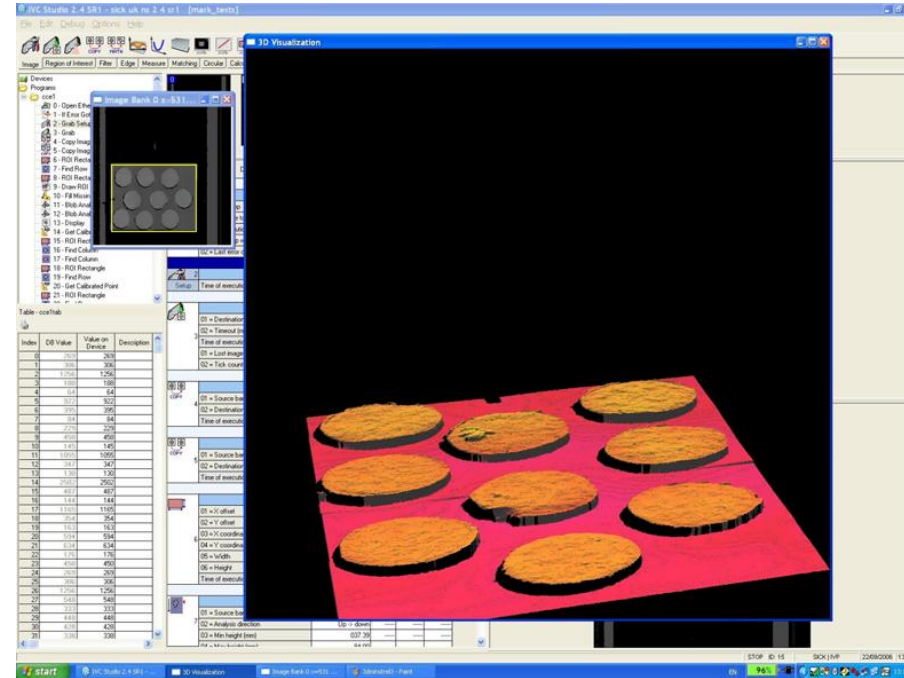
Medical Imaging

<http://dmfr.birjournals.org/content/33/4/226/F3.large.jpg>
<http://www-sop.inria.fr/asclepios/software/inriaviz4d/SphericalImTransp.png>
<http://www.creatis.insa-lyon.fr/site/sites/default/files/segm2.png>

Applications



Scanning

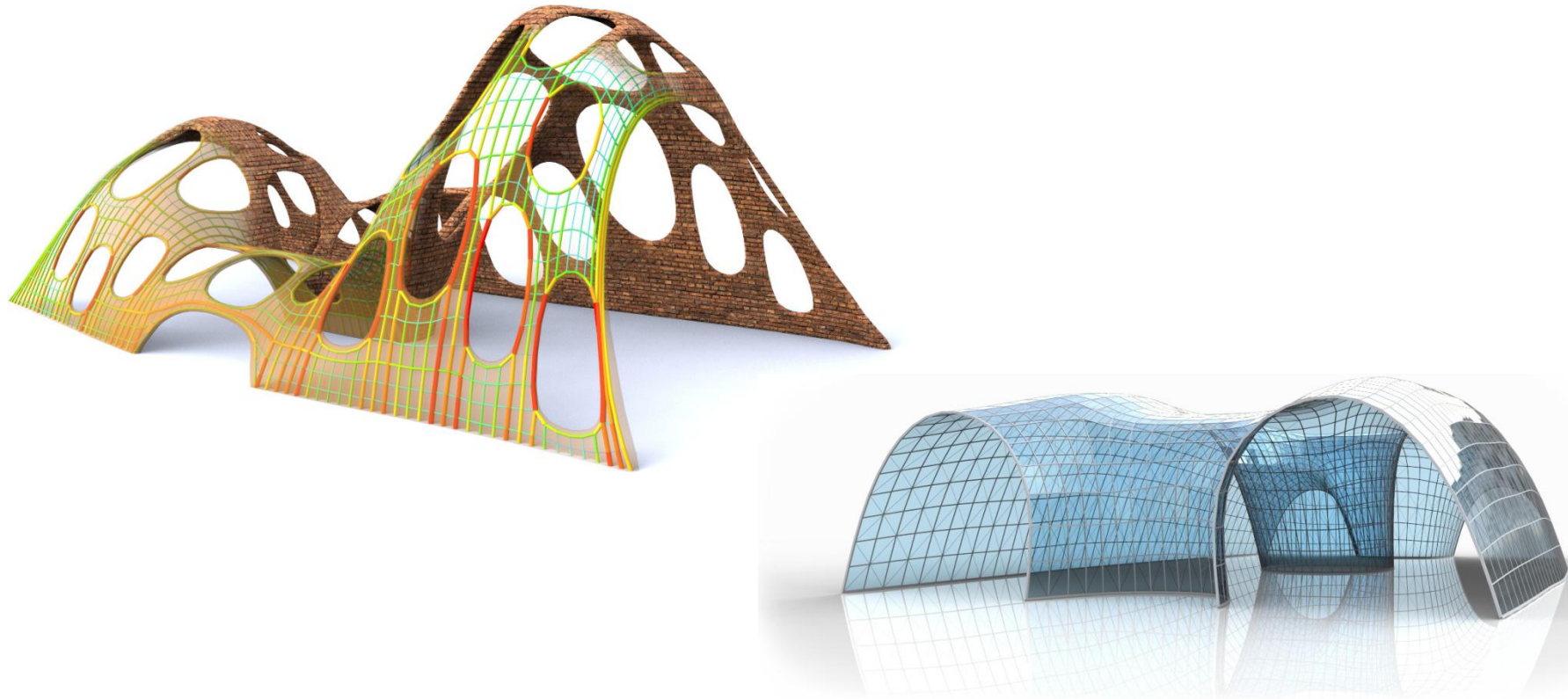


Defect detection

<http://www.conduitprojects.com/php/images/scan.jpg>
http://www.emeraldinsight.com/content_images/fig/0330290204005.png

Manufacturing and Fabrication

Applications

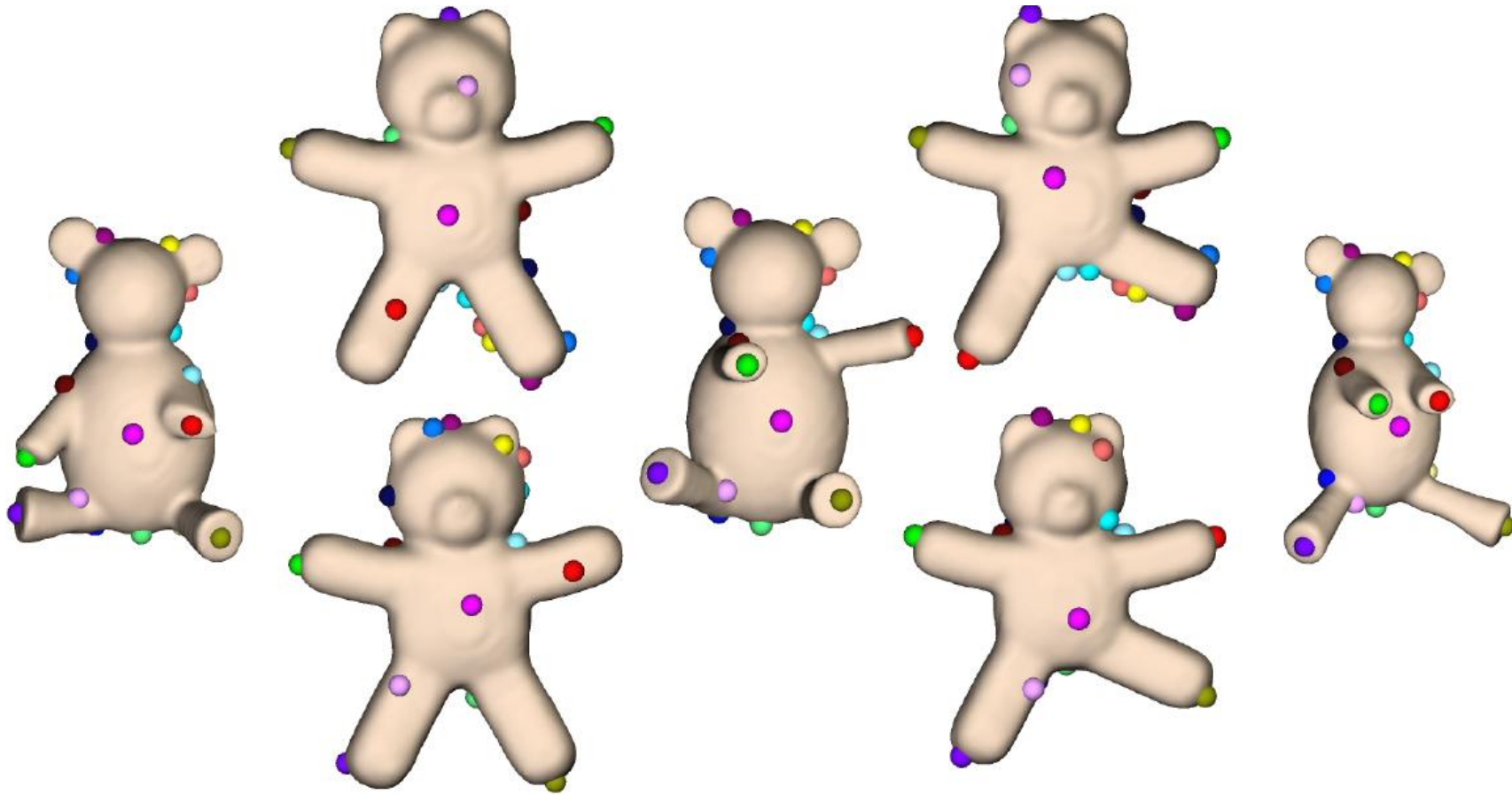


Design and analysis

Vouga et al. "Design of self-supporting surfaces."

Architecture

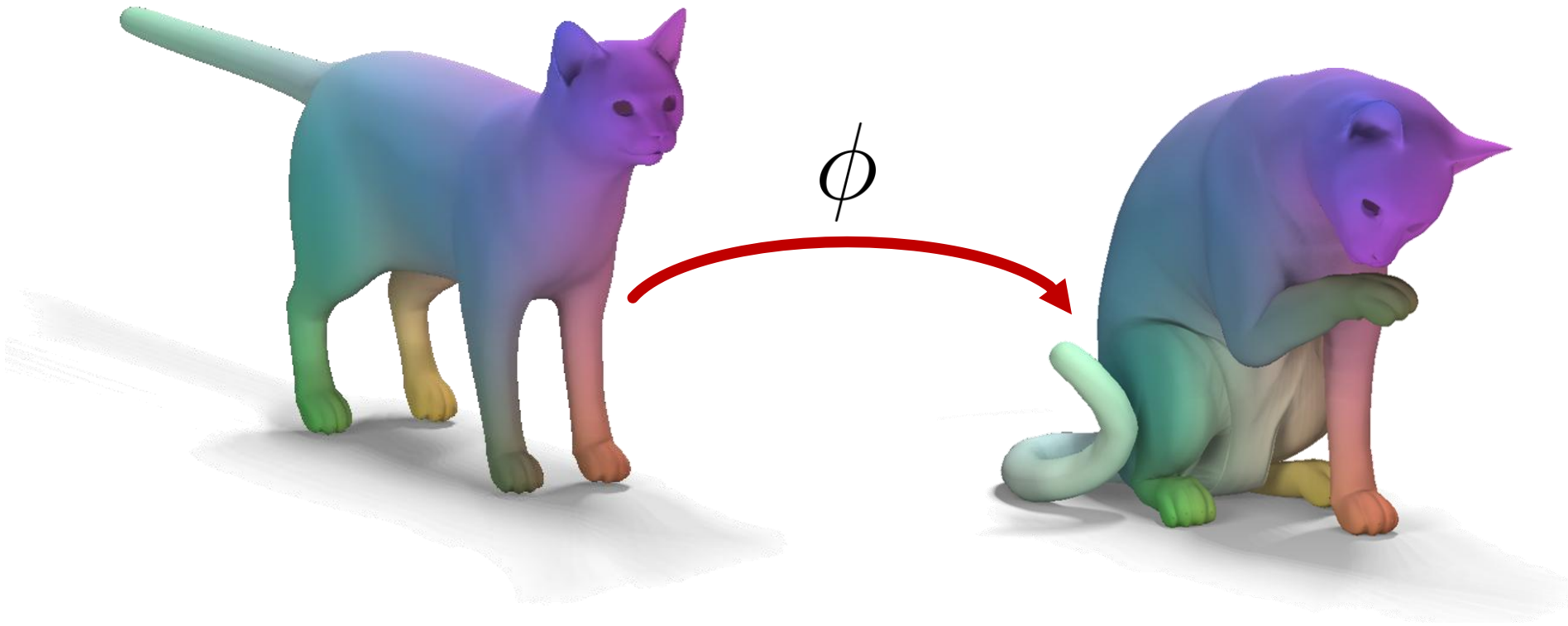
Applications



Huang et al. "Consistent shape maps via semidefinite programming."

Shape collections

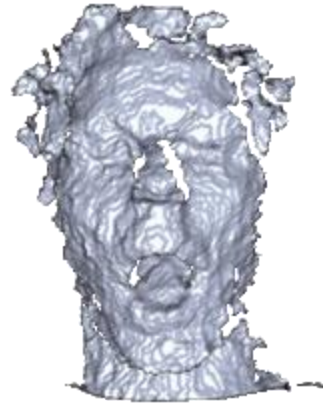
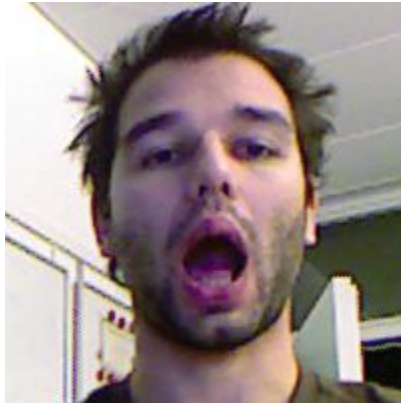
Applications



Ovsjanikov et al. "Functional maps."

Correspondence

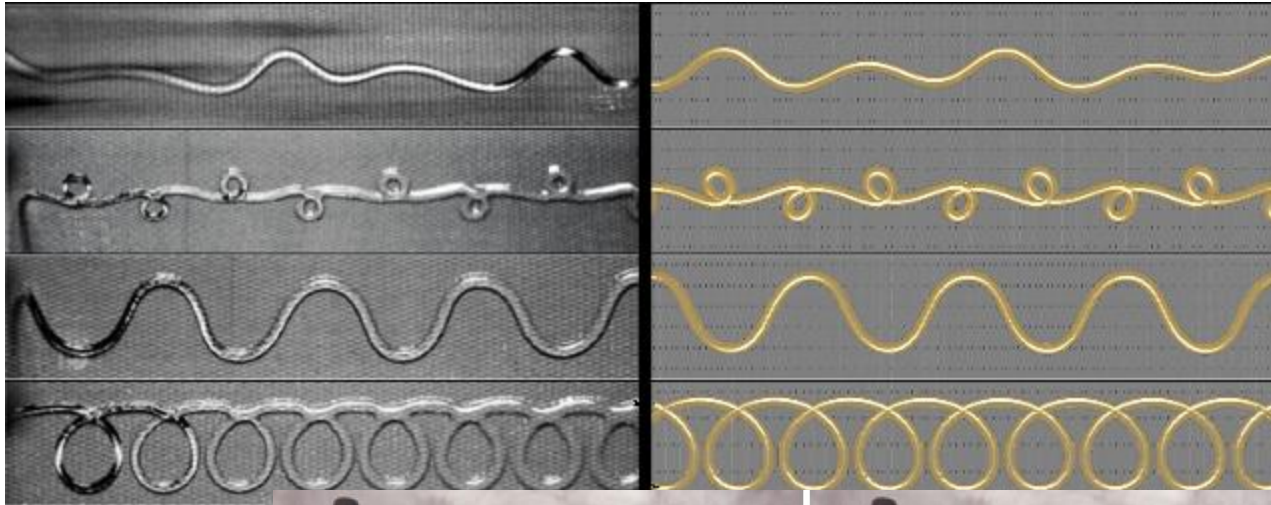
Applications



Weise et al. "Realtime performance-based facial animation."

Deformation transfer

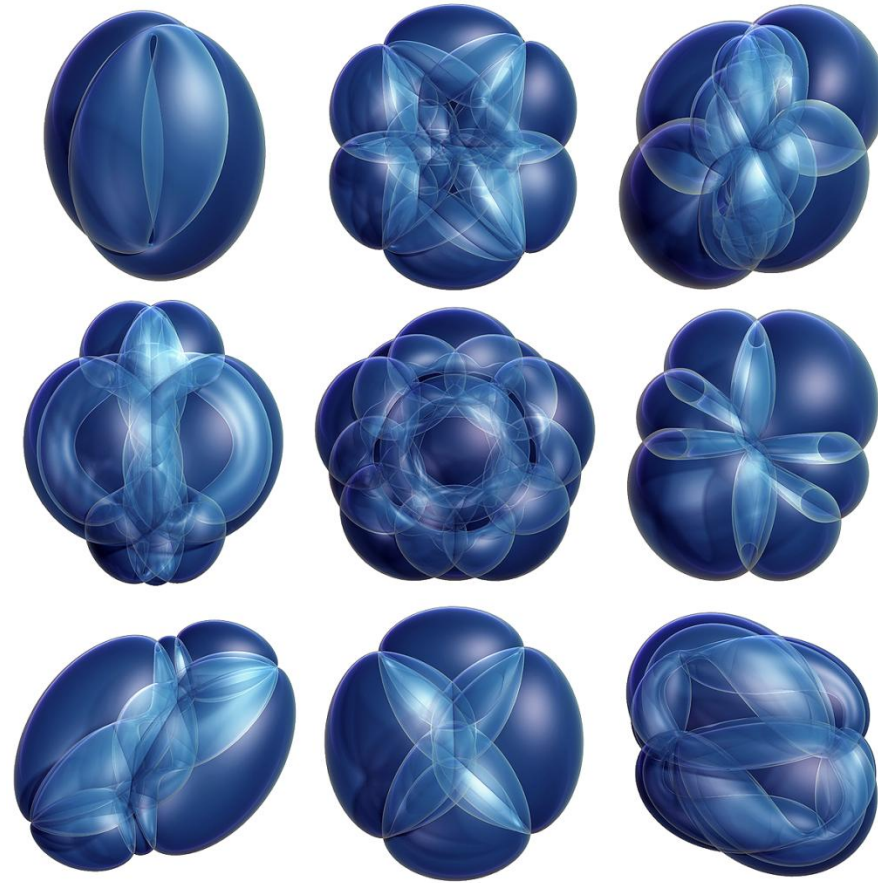
Applications



Bergou et al. "Discrete viscous threads."
Wardetzky et al. "Discrete quadratic curvature energies."

Simulation

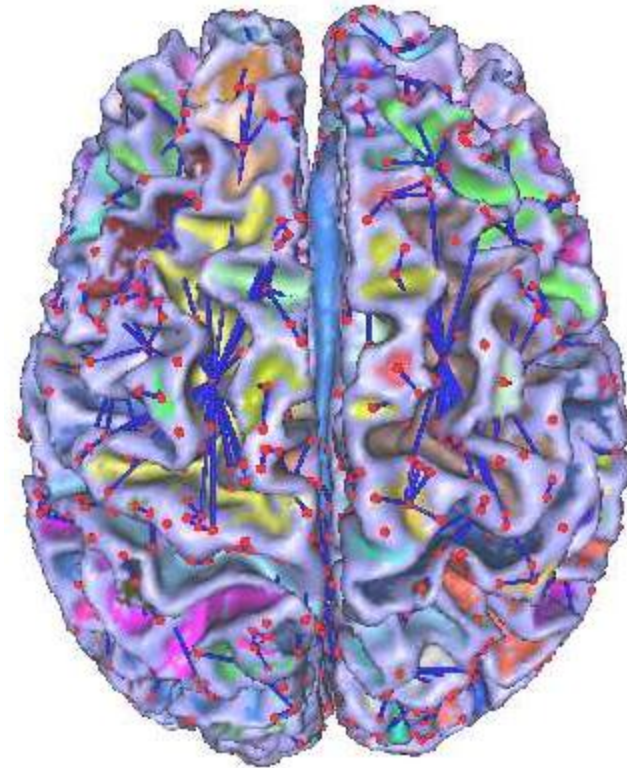
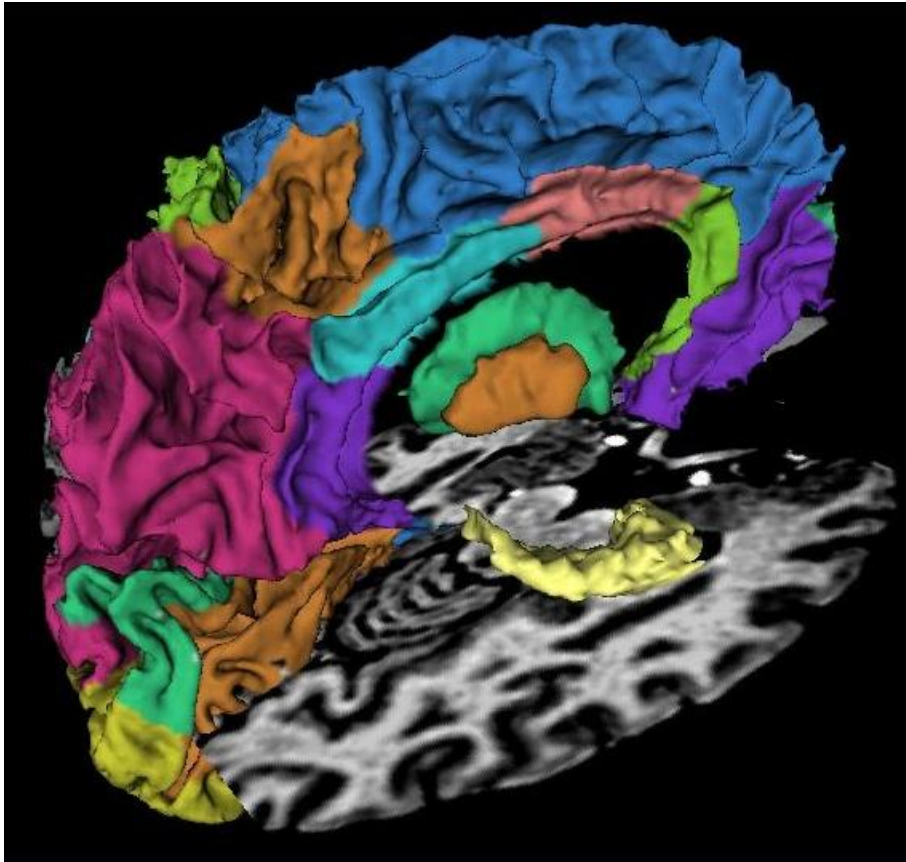
Applications



Crane et al. "Spin Transformations of Discrete Surfaces."

Scientific visualization

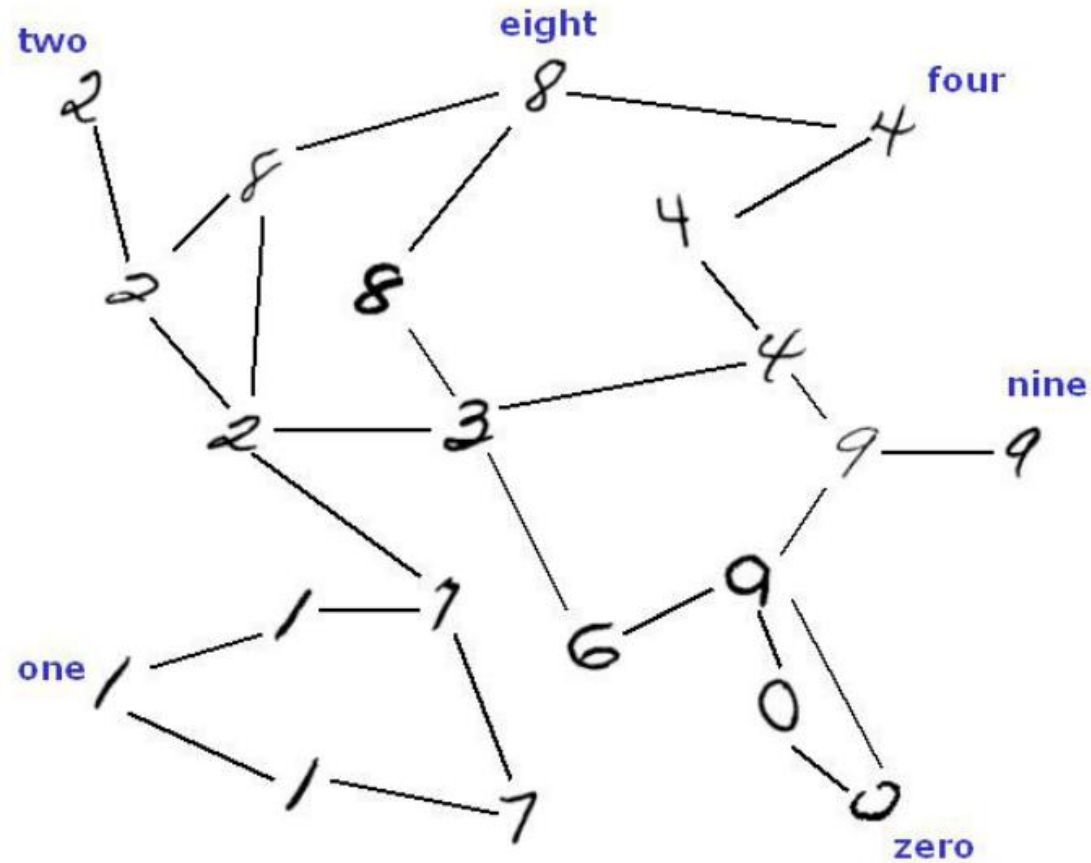
Applications



<http://www.bioinformaticslaboratory.nl/twiki/pub/EBioScience/News/freesurfer-3d.jpg>
http://hal.inria.fr/docs/00/40/21/30/IMG/vivodtzev_et_al-Dagstuhlo3.jpg

Segmentation

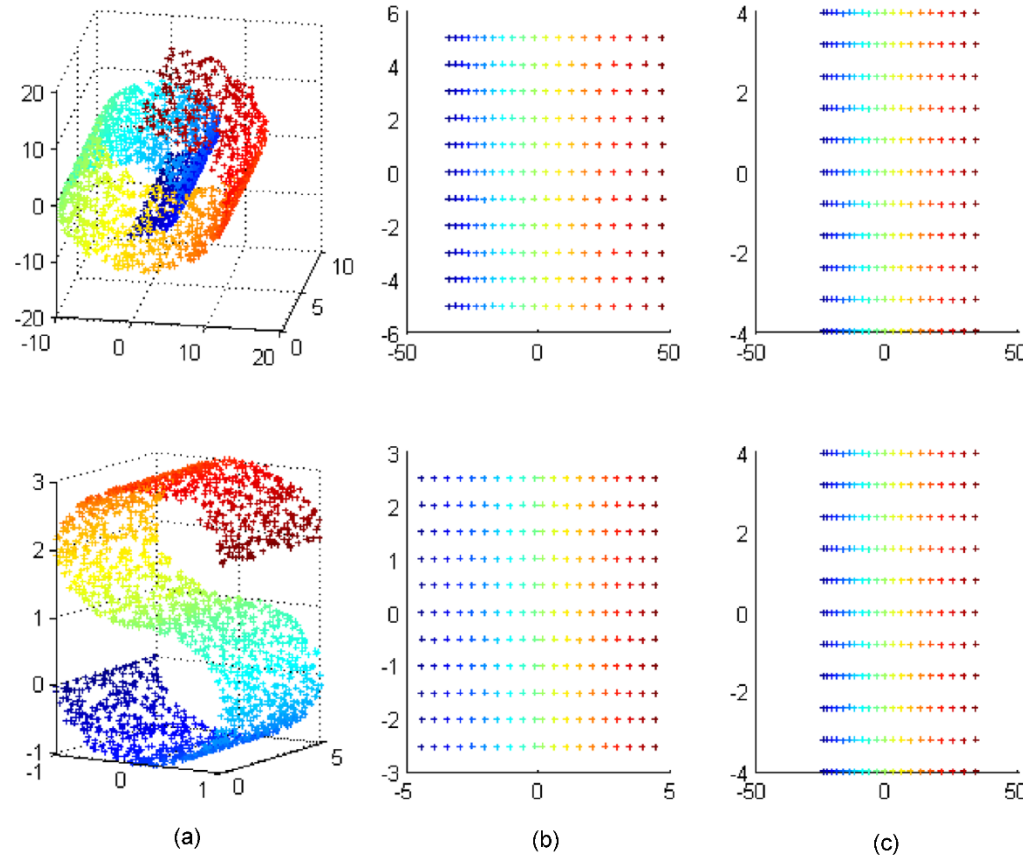
Applications



Zhu et al. *Semi-Supervised Learning Using Gaussian Fields and Harmonic Functions*. ICML 2003.

Machine learning

Applications



Hou et al. *Novel semisupervised high-dimensional correspondences learning method*. Opt. Eng. 2008.

Statistics

6.8410: Shape Analysis

Justin Solomon

Fall 2026

