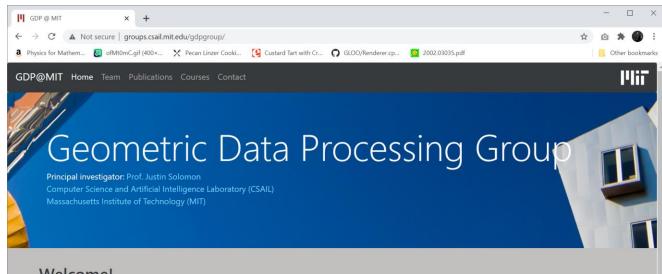
6.838: Shape Analysis

Justin Solomon Spring 2021



Course Instructor



Welcome!

The MIT Geometric Data Processing Group studies geometric problems in computer graphics, computer vision, machine learning, and other disciplines.

Our team includes students and researchers spanning a variety of disciplines, from theoretical mathematics to applications in engineering and software development. We enthusiastically welcome collaborators and staff at all levels and encourage interested parties to contact us with ideas, challenging problems, and avenues for joint research.



6/11/20

1/23/20

News

More congratulations!

Congratulations to our group members for their many accomplishments over the last few months:

- PhD student Yue Wang earned the NVIDIA Graduate Fellowship
- MEng student Nilai Sarda received the second place 2020 Charles and Jennifer Johnson Artificial Intelligence and Decision Making Thesis Award
- PhD student David Palmer earned the MathWorks PhD Fellowship

Sebastian Claici also completed his PhD in EECS. Congratulations, Dr. Claici!

Optimal Transport: Advances and Applications

Please join us for Optimal Transport: Advances and Applications (OTAA), to be held May 18-22, 2020. Check out the web page for more information

Instructor: Justin Solomon Email: jsolomon@mit.edu

Geometric Data Processing Group: http://gdp.csail.mit.edu

Will cover administrative details over Zoom.

Prerequisites

Coding Julia, Python, or Matlab preferred

Math

Fluency in linear algebra and multivariable calculus

Not required (won't hurt):

Graphics, differential geometry, numerics, ML

Philosophy

We <u>want</u> you to take this course!

Assignments intended to be interesting

(may be unintentionally easy/hard!).



Image from postermywall.com

Theme

1. *Geometric data analysis:* The analysis of geometric data Modifier Noun

2. *Geometric data analysis:* Data analysis using geometric techniques Modifier Noun

Applied Geometry

I. Theoretical toolbox

II. Computational toolbox III. Application areas

Mostly a picture book!

Applied Geometry

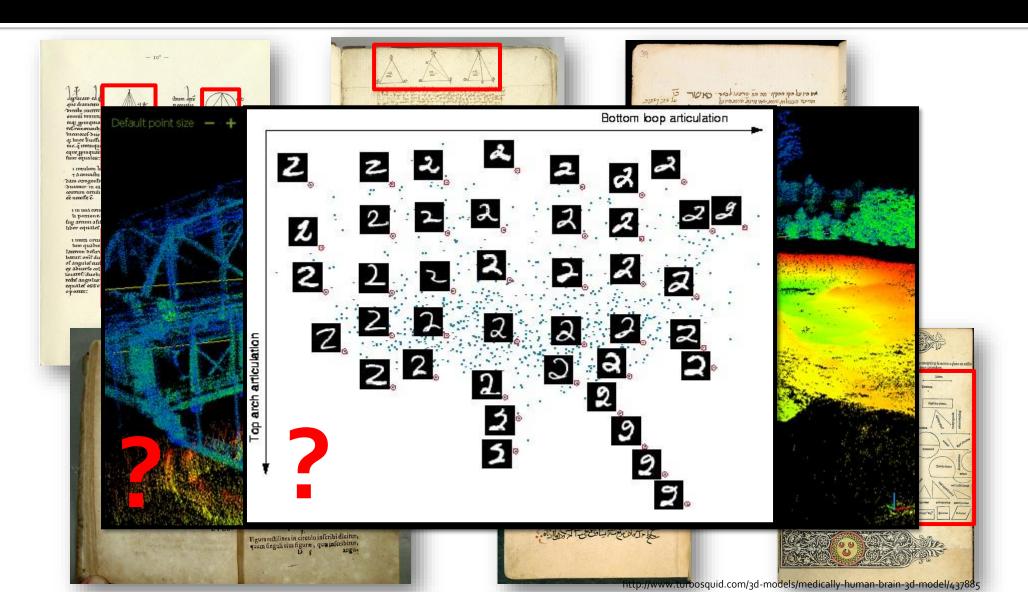
I. Theoretical toolbox

II. Computational toolbox III. Application areas

- 10* -10 01 Applicare ed f que diamero Dirette suirir אם היועל הקו החקיף נוה מה שרטנו לבארי כאטור 12 Anum dere pequatia fecare non dif 21081 202 12 תהיינה בענולות שוות שתי ונינת שונת היועל 48 שווכ הנה הוויות המרכזאו היו על המקיף המשל כופי שתי עבולות אבל רהז mus dans eritar "po de ver dar best dan buie the ver ha da da de the the on the "Aven an energy of the but vert rich" to the reliving a causian of verta vert rich" to שוות ושתי קשתנת כל הו שוות על הם שתי זויות בעל החו omnui mmma. שוות ושתי קשתות כל זה שוות עלימש שתי קויות ולם נשל etq: prinquito ret.remononib; bacanozef.due i redulinent angulut m ferniar auto fup aveum ofutar. reduité 5 i m portione femiciruito maioze. המרכואוט המקיף הנה אום בי הם שתיהם שונת אי אפשר כיחם זה שאם היה אפשר נאמר שתהיה זוית בטג יותר q: unee buille me. q uminque הענה אלויה החז וטנמיד recto minaz. ע נקורתה מקו מה eque punquar, funt equales. fi un minoze? 109. reeto matoz. זוית החצ כמו זוית. בעוב הנה השת פג 1 enculum linea resta co 1 aralue Y a contactu in entulum linea q כמוקשת הכאבו Inea reda dam orogonticomingar. a gradu uo in arculum quedam fe קשת כן ככר היה to a grant state instance genes and with the state and the state of th כמו קשת הי הנה קשו centrum artuli cant curailum recha linea pier an trun ducanne quofaunq: angu הו כמו הצ הגדולה כמו הקשנה והשקר אם בואין בשנ בלתי שוה לורתה הואס lof faur. duof duob; angulis. at 1 In und circu כן היא עום לה ושתי הנויות אשר תחיינה על הקשת באן הרו in alternant circule fup areum co NY 1-1 חע זוית בטו החו השוות העה אטבן שוותי וזה מה שרעינו, le pomone. future pomonit: funt equales. fup areum sfiftant: angulof quot כן למארי המיהריה השווים אשר בענוות משוות כרילו unu danun קאמת שיים שוות הוצולה לוצולה והקיים בי הו המשל בובי שתי עגולות אבל רהן שוות ושט מיתיי בל הו שוות הנדולה לנדולה והקטטה לקטטה corenhum 1 tint's circu Dare reche 68. lum quadat Laterum deferi הטשוים הנה אומר ני linee. que שנה אנלם יבדילו קשתות שונה אנלם קשת בו כמו קשת הו יואולם wamerro mi barut omf du nime maiaz evilar. equof angulof euf ex aduerfo col lo carof. duob; . 0.0. קשת באג בקשת כרו lineam figu reda Angulis then all a deal march והיושני המרכזי כועא eft ins uent equaler esse that. ונוצאטב ועג וחה 01. ens. LIBER IV. SE EVCLID. ELEM. GEOM. EVCLIDIS er EVCLID. ELEM. GEOM. verð tangatt quad fuð tota fecante, kette rius interpunktum & contexam periphe. riam affunpta comprehenditur; redangu. lumæqualerri er, quad å tangente dafeti. ELEMENTVM Firstnared catas parto dri. dl. Linea erd logitudo time latminducenti de sindé cy-remitates fie no pulcta. Climet recar è ab von policito ad luiti servillinare, que ho i cerremitatos fina o vertias cos rece in con Cemptone di logitudo el luitino maine tur ib scont termi quale li luitino. Incha eft mine peno eft. E Línea eft tombus catandom QVARTVM. 2.iees bitur, quadrato. DEFINITIONES. "Countrad Theoremage. Propo-The second secon fitio 37. عام دست مراسی می مرا المار موسط محال مر عام دست مارسی می مرابط مد المحرف المحرف من حرج الدفان مدین مرافع مد محرف المحد محرف مرابط المان مداور مطلح مر مداری است محرف مرابع مارسی مطلح مر مدین محصر مع foofisies obers. Figura rectilinea in figura rectilinea in-feribi dicitur, cum fingu-li eius figura; que inferi-bitur, angoli, fingula late-bitur, angoli, fingula late-Si extra circulum fumatur punctum all. quod, ab coque puncto in circulum cadant duz refiz linez, quarum altera circulum NY. fecet, altera in cum incidat; fit autem, quod Ferenjeter in eum incrudatificauten, quod fuib tota fecante, & exte-nis inter pandtum , & conuesam peripheriaria allampis, comprehendi-turreckangulum, #qua-leci, quod ab incidente ra cius, in quainfcribitur, مر معالی مانی وی لونها مانی مانی جر مرد الذی مر بر مرد الم مانی مرا cangunt. Similiter & figura eireum figuram defcribit We set that the set of the set o dicitur, quum fingula eius, que circum feri-bitur, lax describitur quadrato;intera fin-لكونها على حد eidensipfa circulum tanget. 34 gulos c-The set Oxigonias enbogrania FINIS ELEMENTI III. ra anguesta ett organis regentaren angerezenagale, gila ett untym: gote ett ogalatera: ied regentaren non et مزيع رت بدادى ح 1 للمادى فسنها ديدا ذالة تعرف في ح كو مر دادي محسم بدادة فات في المر دوله los teti gerint, circum quam illa describitur. circum quam un act. Figura recilines in circulo inferibi dicitor, quum finguli cius figure, qua inferibitor, quum finguli cius figure, qua inferibitor, guum finguli cius figure, qua inferibitor, D 5 EVCLL U





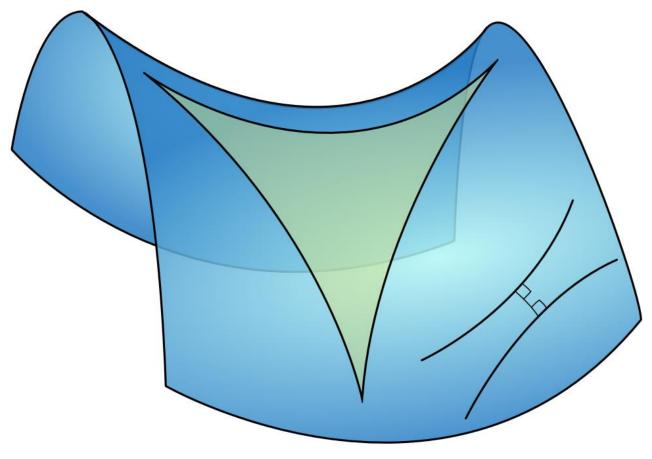


Differential Geometry



Spivak: A Comprehensive Introduction to Differential Geometry

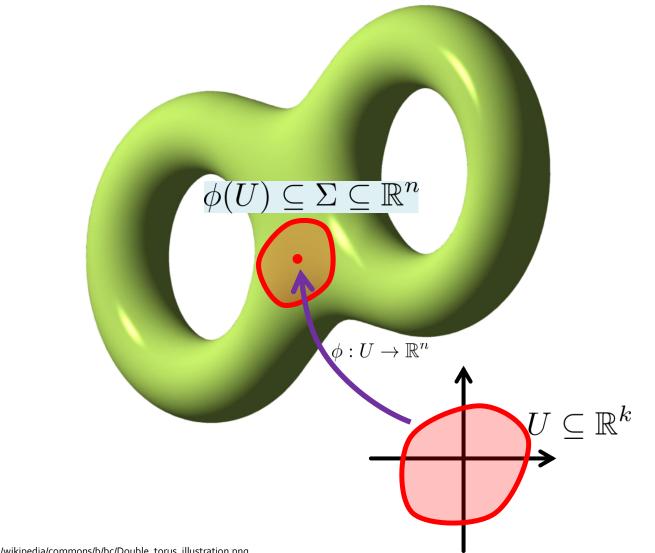
Differential Geometry



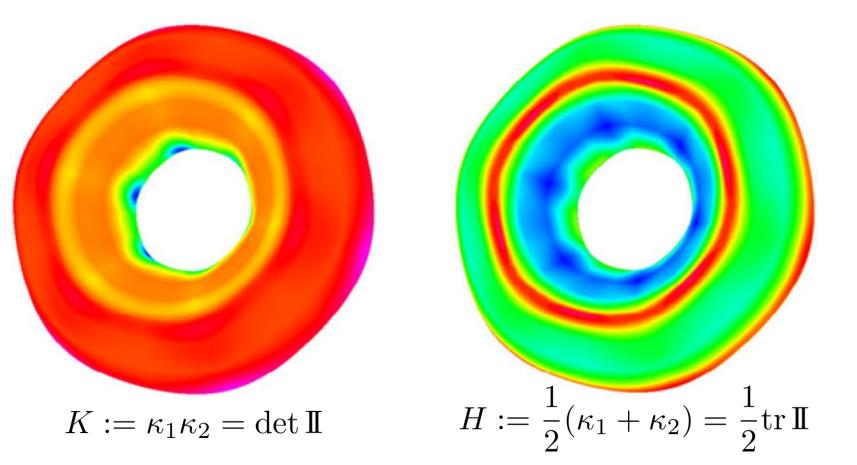
http://en.wikipedia.org/wiki/Differential_geometry

Study of smooth manifolds

Manifold

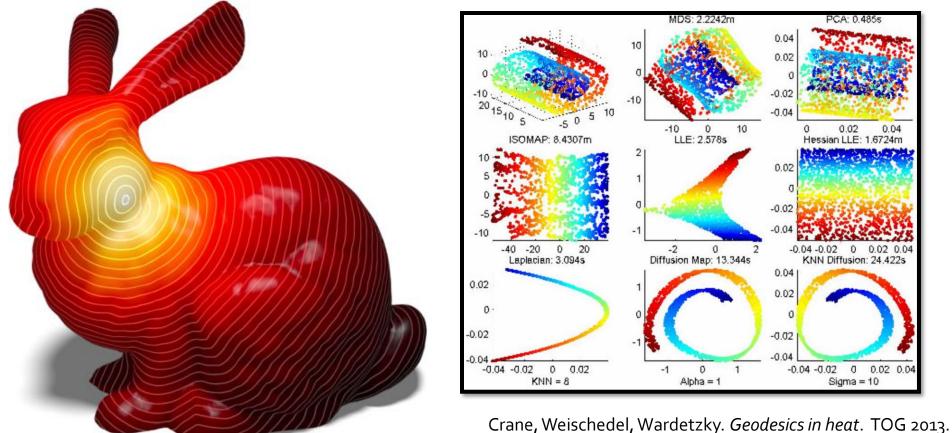


http://upload.wikimedia.org/wikipedia/commons/b/bc/Double_torus_illustration.png



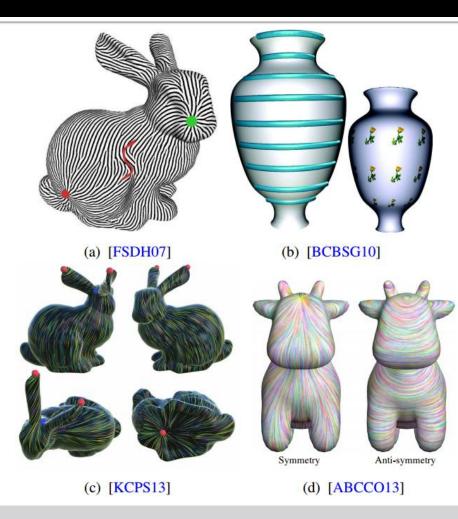
http://www.sciencedirect.com/science/article/pii/S0010448510001983

Curvature and shape properties



Wittman. Manifold learning techniques.



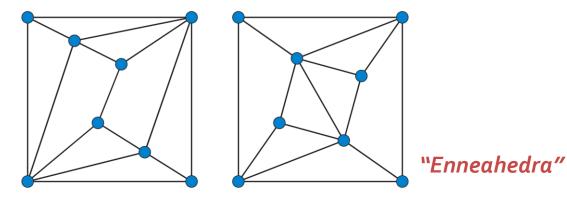


Vaxman et al. Directional field synthesis, design, and processing. EG STAR 2016.

Flows and vector fields

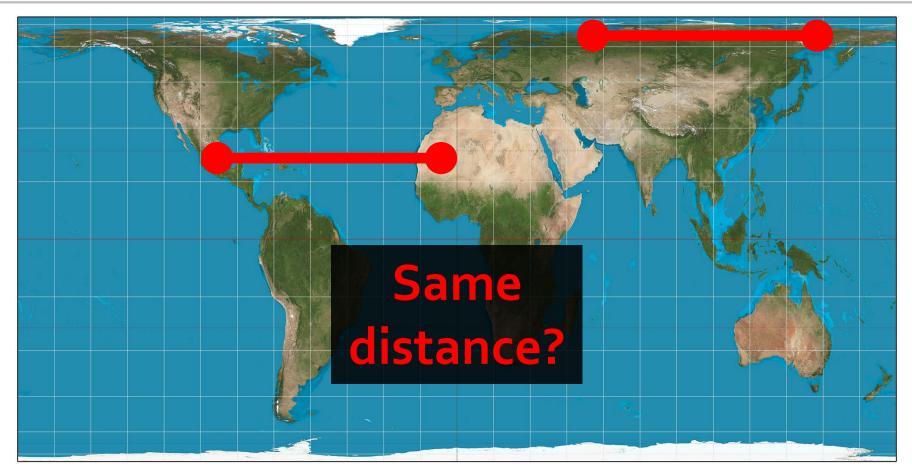


Vallet and Lévy. Spectral Geometry Processing with Manifold Harmonics. EG 2008



Differential operators

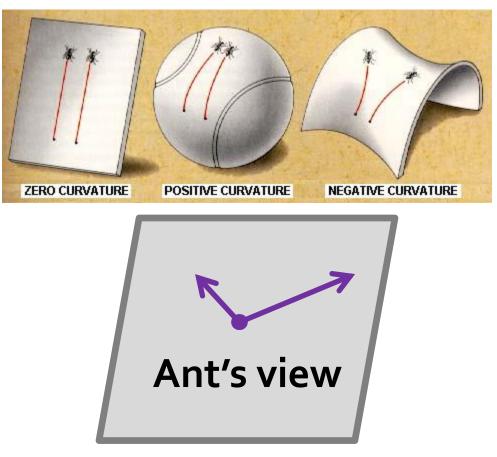
Riemannian Geometry



http://upload.wikimedia.org/wikipedia/commons/2/2c/Hobo%E2%8o%93Dyer_projection_SW.jpg

Only need angles and distances

Riemannian Viewpoint

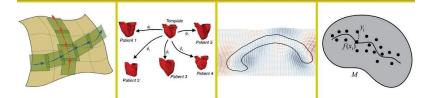


http://www.phy.syr.edu/courses/modules/LIGHTCONE/pics/curved.jpg

Only need angles and distances

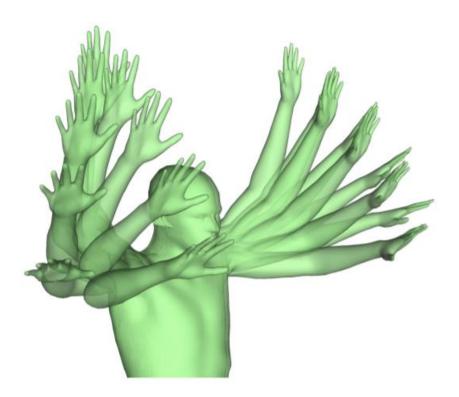
High-Dimensional Geometry

RIEMANNIAN GEOMETRIC STATISTICS IN MEDICAL IMAGE ANALYSIS



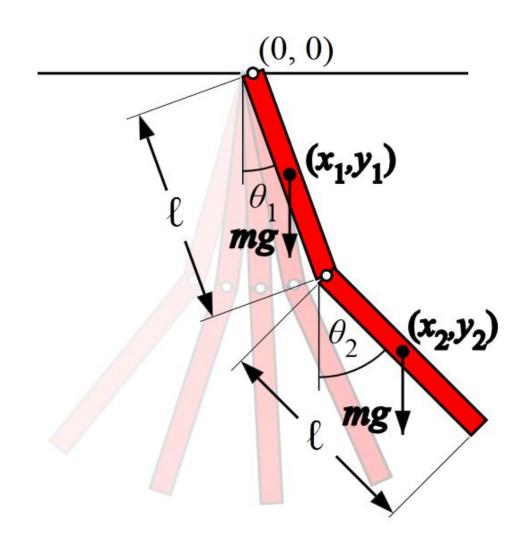
Edited by Xavier Pennec, Stefan Sommer, Tom Fletcher

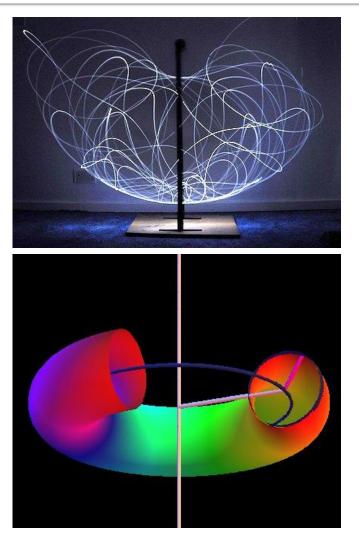




Heeren et al. Splines in the Space of Shells. SGP 2016.

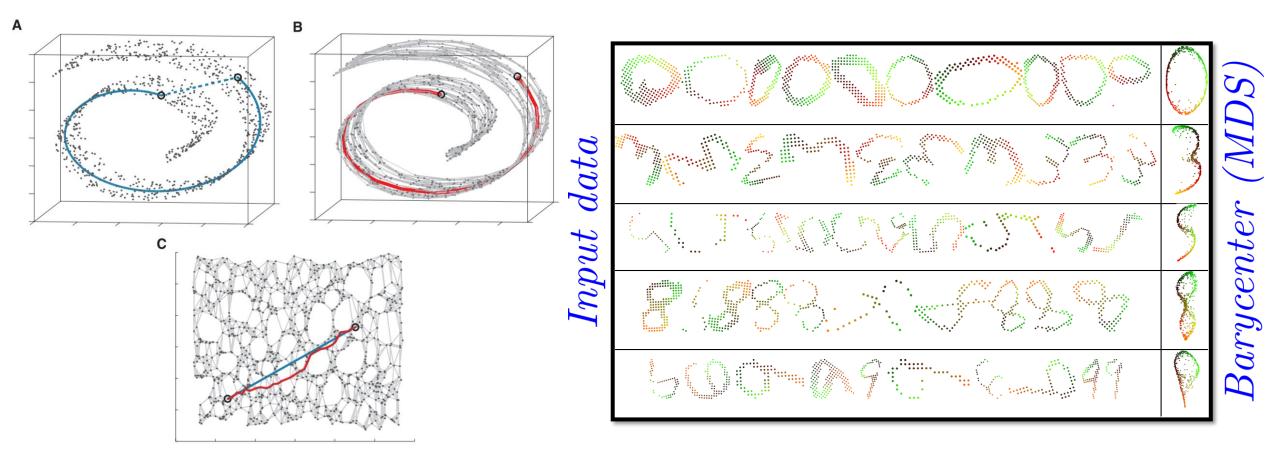
Geometric Mechanics and Lie Groups





http://en.wikipedia.org/wiki/Double_pendulum http://www.ualberta.ca/dept/math/gauss/fcm/Bscldeas/SpcDmnsn/pndlm2.htm

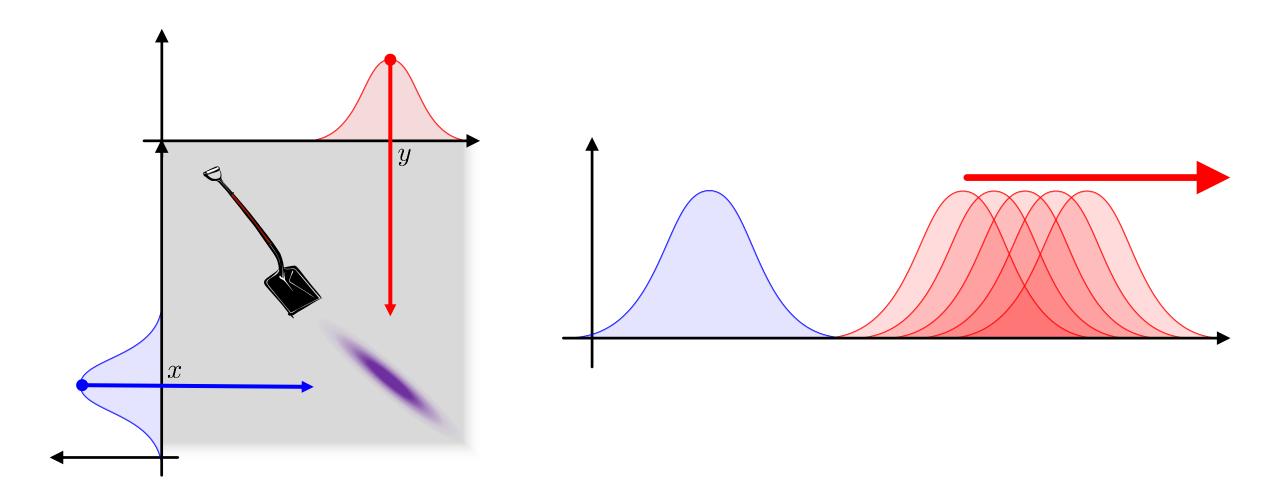
Metric Geometry and Metric Embedding



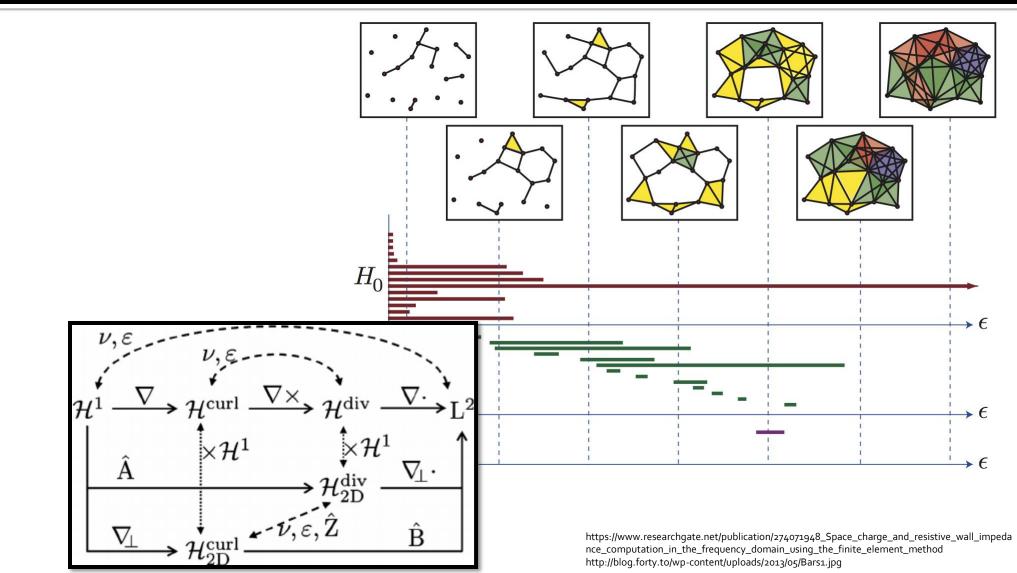
Tenenbaum et al. *A Global Geometric Framework for Nonlinear Dimensionality Reduction*. Science 2000.

Peyré, Cuturi, and Solomon. Gromov-Wasserstein Averaging of Kernel and Distance Matrices. ICML 2016.

Optimal Transport



{Differential/Morse/Persistent/...} Topology



Plan for Today

I. Theoretical toolbox

II. Computational toolbox III. Application areas

Many Notions of Shape

Triangle mesh
Triangle soup

Graph
Point cloud

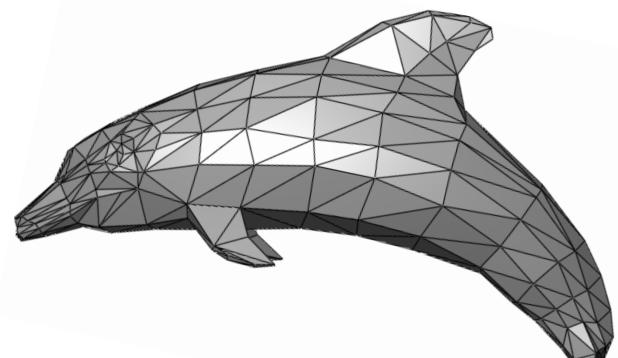
Pairwise distance matrix

Dataset
Network

Nearly anything with a notion of proximity/distance/curvature/...

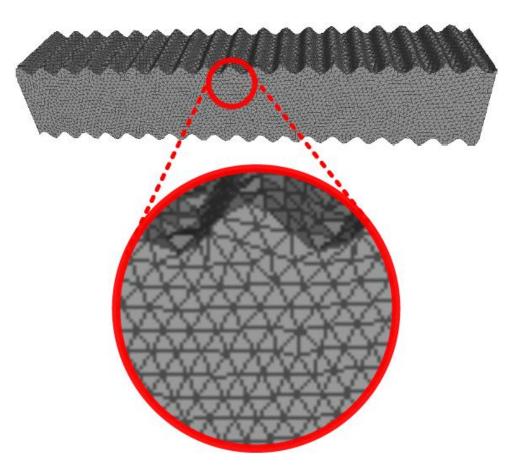
Typical issue: How to Interpret Geometric Data

- Collection of flat triangles
- Approximates a smooth surface



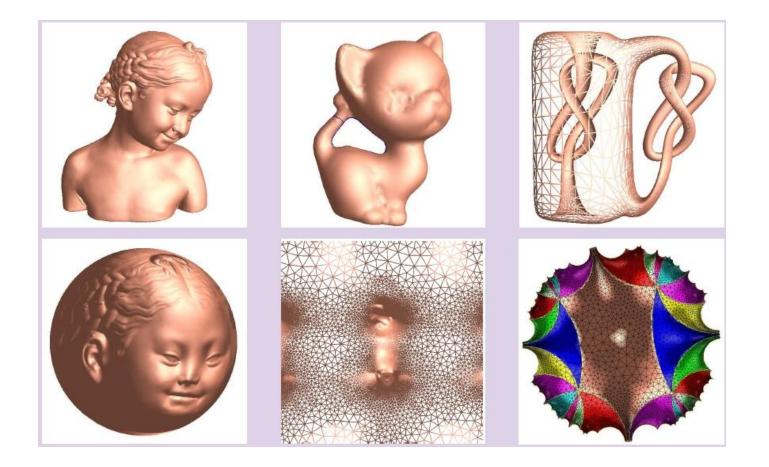
Can a triangle mesh have curvature?

Jack of All Trades



Combine smooth and discrete

Example: Discrete Differential Geometry



Modern Approach

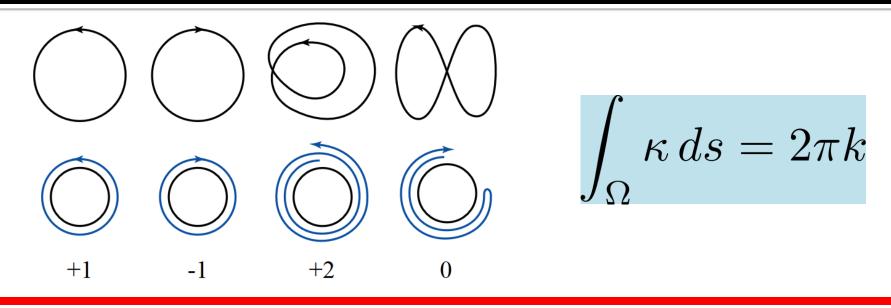
Discrete VS. Discretized

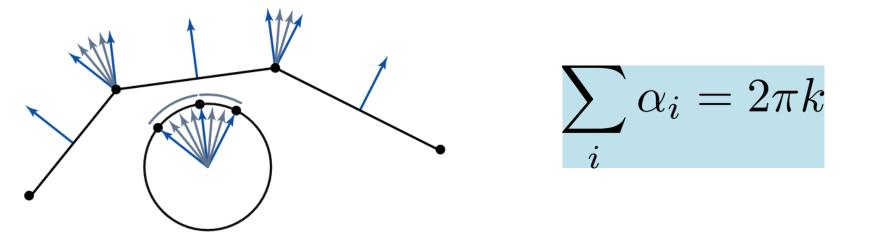
Discrete Differential Geometry

Discrete theory *paralleling* differential geometry.

Structure preservation [struhk-cher pre-zur-vey-shuh n]: Keeping properties from the continuous abstraction exactly true in a discretization.

Example: Turning Numbers





Images from: Grinspun and Secord, "The Geometry of Plane Curves" (SIGGRAPH 2006)

Convergence

[k*uh* n-**vur**-j*uh* ns]**:**

Increasing approximation quality as a discretization is refined.

Convergence and Structure

Can you have it all?



Fey et al. *30 Rock* (2006-2013).

Disappointing Result

surographics Symposium on Geometry Processing (2007)	
Alexander Belyaev, Michael Garland (Editors)	

Discrete Laplace operators: No free lunch

Max Wardetzky¹

Saurabh Mathur²

Felix Kälberer¹ Eitan Grinspun²[†]

¹Freie Universität Berlin, Germany

²Columbia University, USA

Abstract

Discrete Laplace operators are ubiquitous in applications spanning geometric modeling to simulation. For robustness and efficiency, many applications require discrete operators that retain key structural properties inherent to the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

In applications one often requires certain structural prop-

1.1. Properties of smooth Laplacians

Consider a smooth surface *S*, possibly with boundary, equipped with a Riemannian metric, *i.e.*, an intrinsic notion of distance. Let the intrinsic L^2 inner product of functions *u* and *v* on *S* be denoted by $(u, v)_{L^2} = \int_S uv \, dA$, and let $\Delta = -\text{div}$ grad denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

(NIII I) $\Delta u = 0$ whenever u is constant

Disappointing Result

Eurographics Symposium on Geometry P Alexander Belyaev, Michael Garland (Edi				
Discrete Laplace operators: No free lunch				
Max Wardetzky ¹	Saurabh Mathur ²	Felix Kälberer ¹	Eitan Grinspun ² [†]	
¹ Freie	Universität Berlin, Germany	² Columbia Universi	ty, USA	

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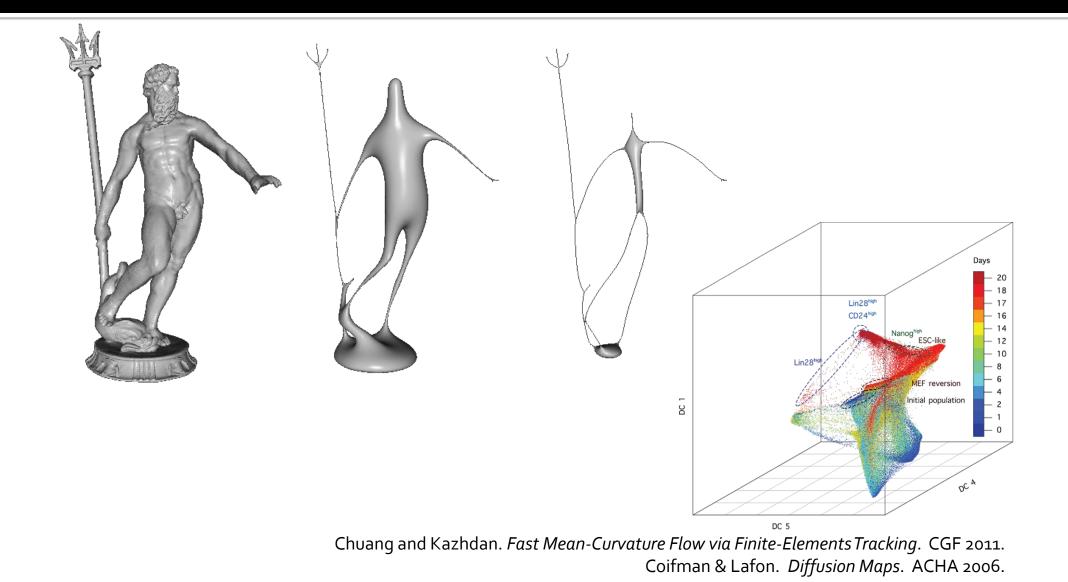
(NIII I) $\Delta u = 0$ whenever u is constant



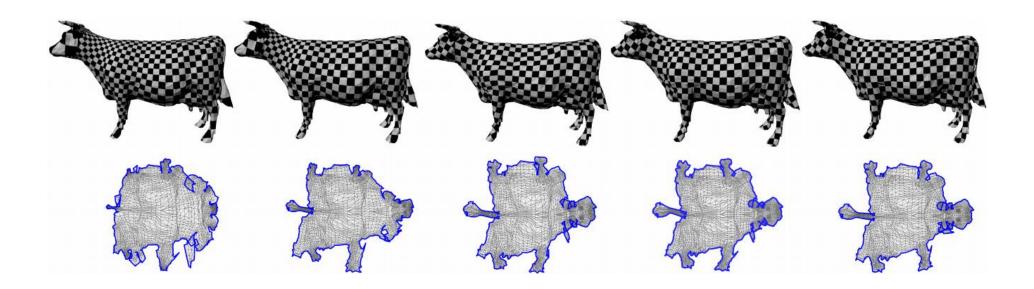
Pick and choose which properties you need.

But there is a huge toolbox of algorithms to draw from!

Numerical PDE

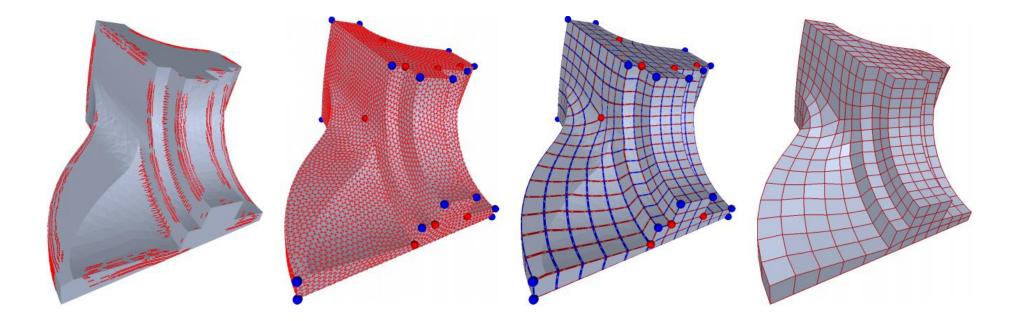


Large-Scale Smooth Optimization



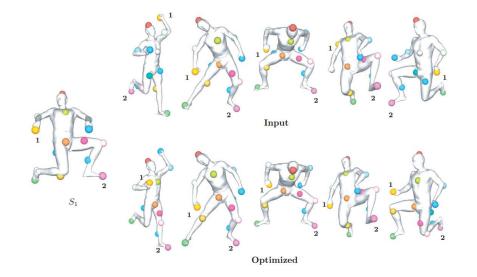
Smith and Schaefer. *Bijective parameterization with free boundaries*. SIGGRAPH 2015.

Discrete Optimization

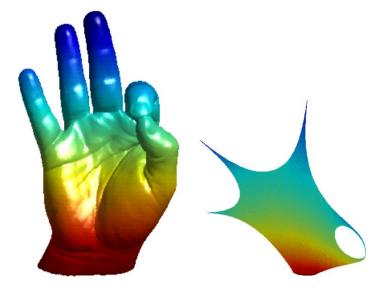


Bommes, Zimmer, Kobbelt. *Mixed-integer quadrangulation*. SIGGRAPH 2009.

Linear Algebra



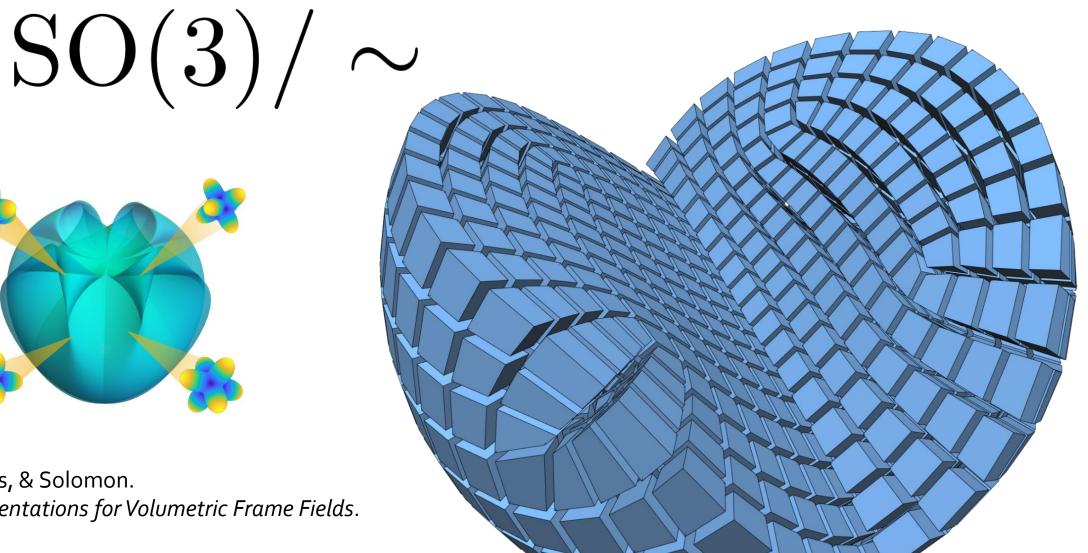
Huang, Guibas. *Consistent shape maps via semidefinite programming*. SGP 2013.



Krishnan, Fattal, Szeliski. Efficient preconditioning of Laplacian matrices for computer graphics. SIGGRAPH 2013.

Algebra & Representation Theory

Palmer, Bommes, & Solomon. Algebraic Representations for Volumetric Frame Fields. TOG 2020.

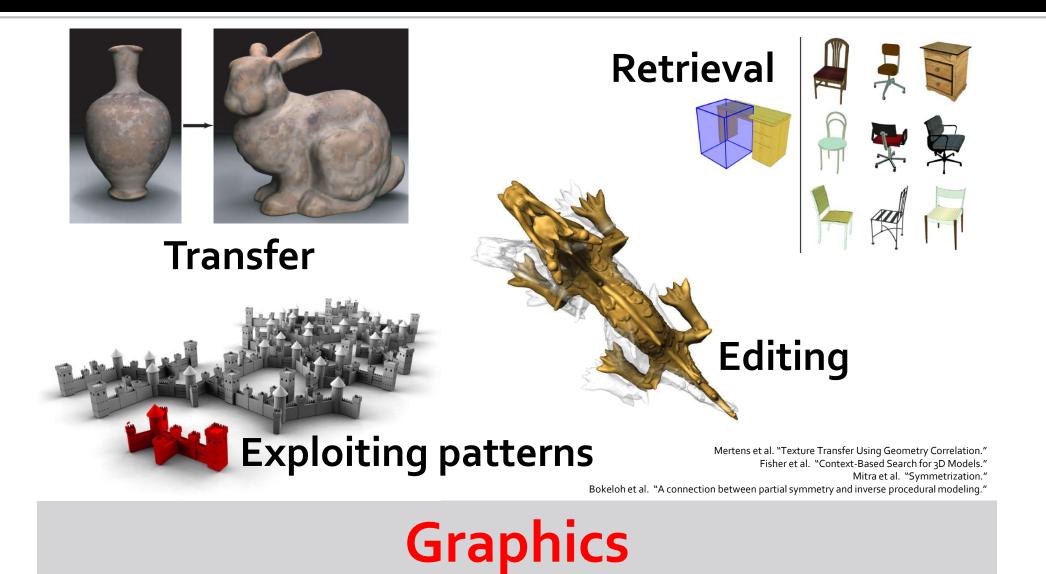


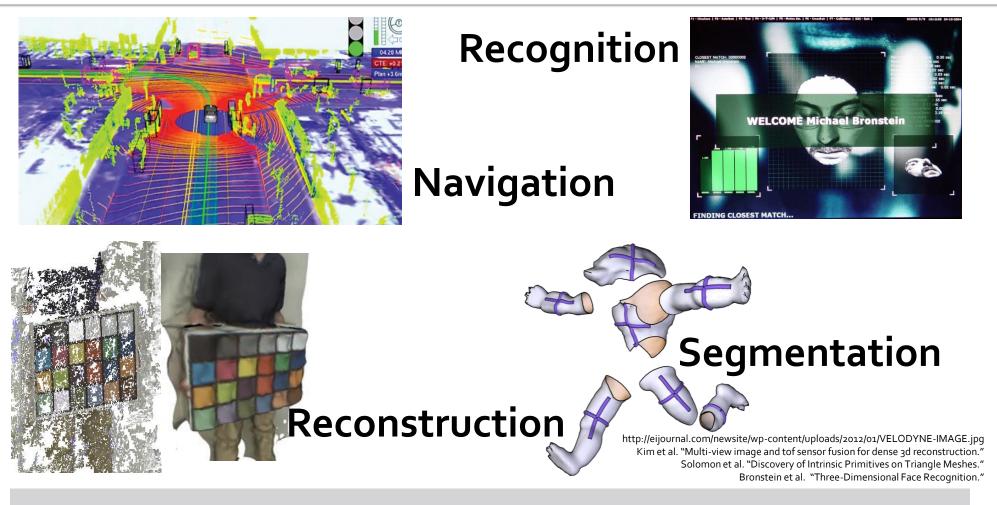
Plan for Today

I. Theoretical toolbox

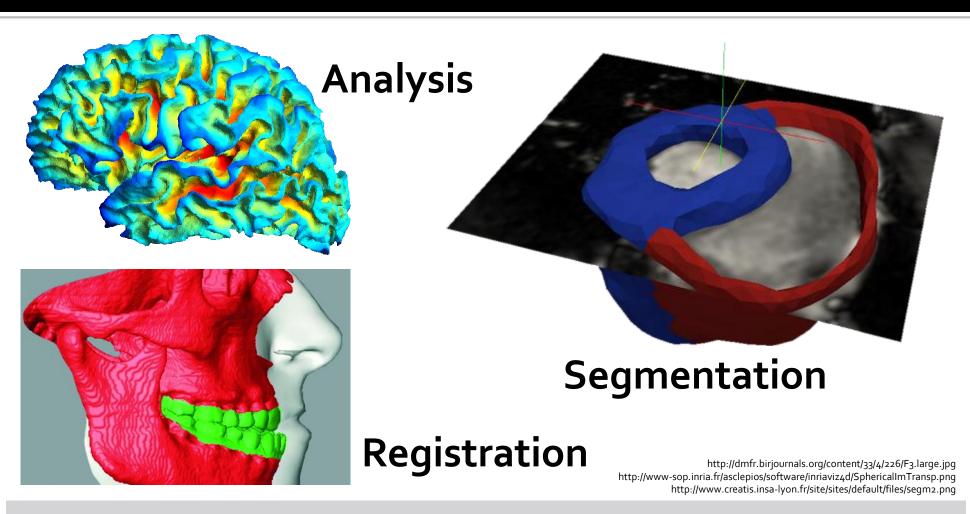
II. Computational toolbox

III. Application areas

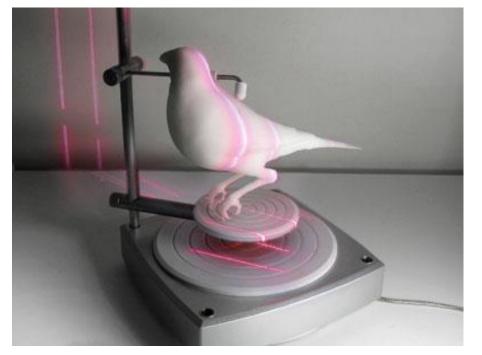


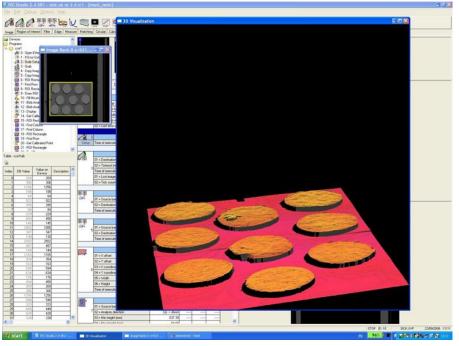


Vision



Medical Imaging



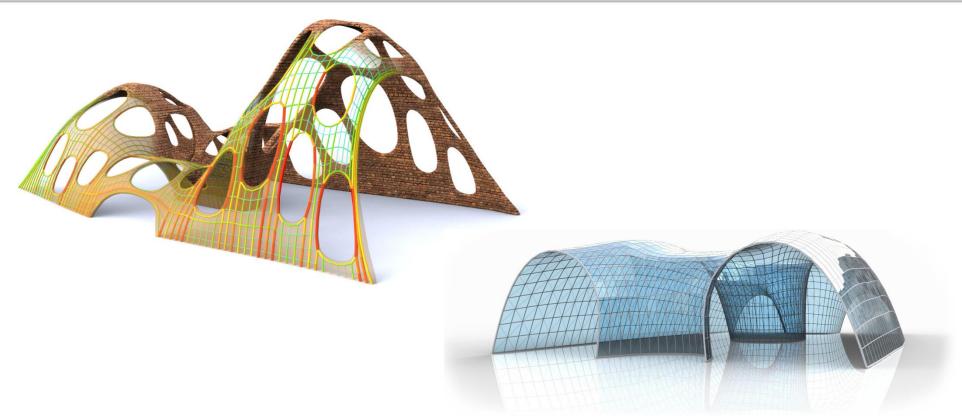


Scanning

Defect detection

http://www.conduitprojects.com/php/images/scan.jpg http://www.emeraldinsight.com/content_images/fig/0330290204005.png

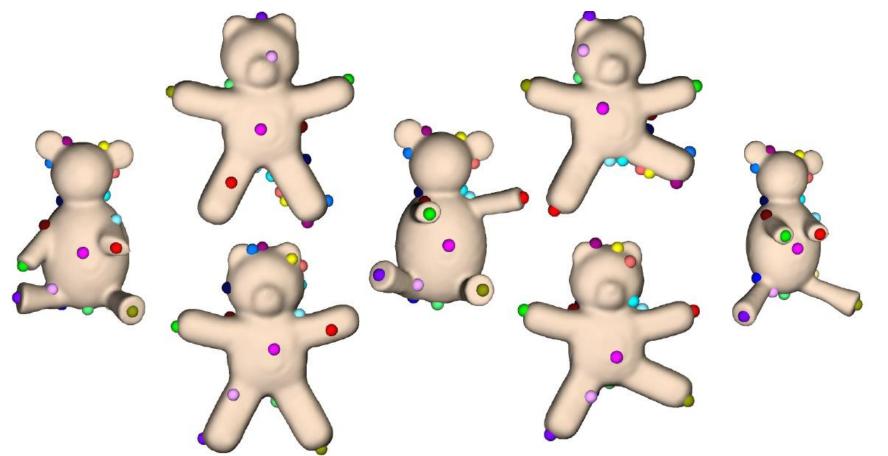
Manufacturing and Fabrication



Design and analysis

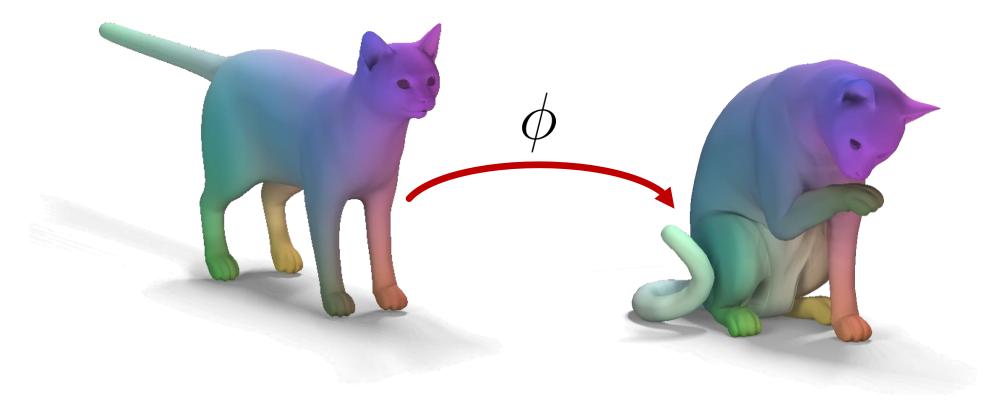
Vouga et al. "Design of self-supporting surfaces."





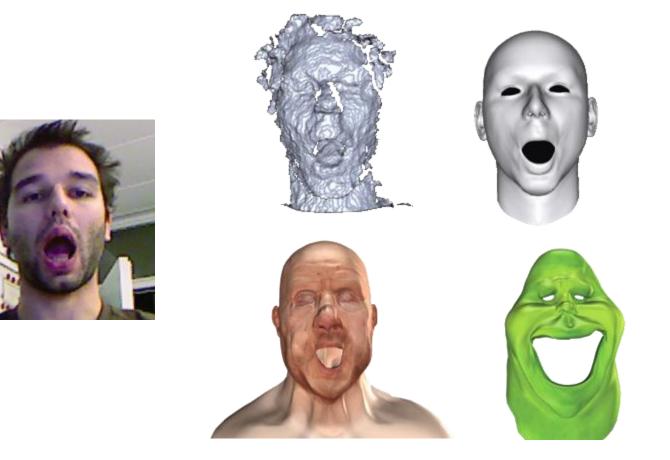
Huang et al. "Consistent shape maps via semidefinite programming."

Shape collections



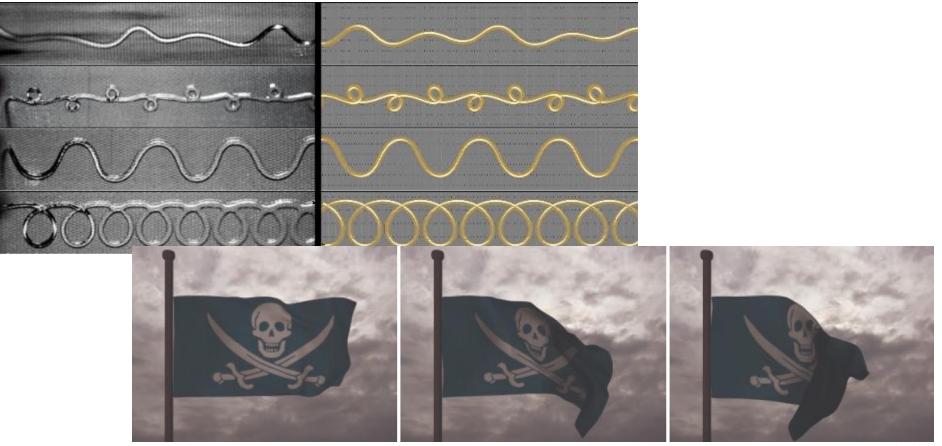
Ovsjanikov et al. "Functional maps."

Correspondence



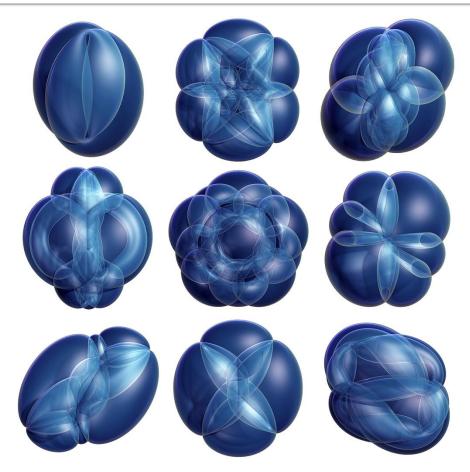
Weise et al. "Realtime performance-based facial animation."

Deformation transfer



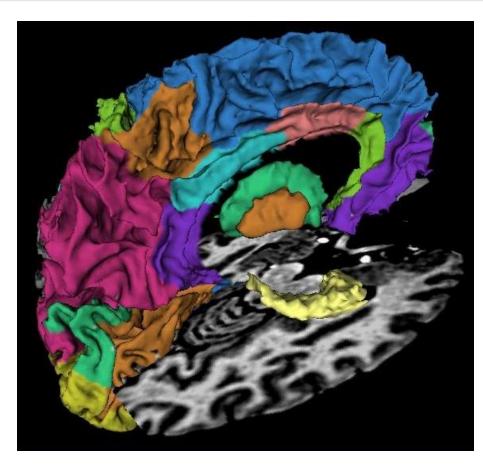
Bergou et al. "Discrete viscous threads." Wardetzky et al. "Discrete quadratic curvature energies."

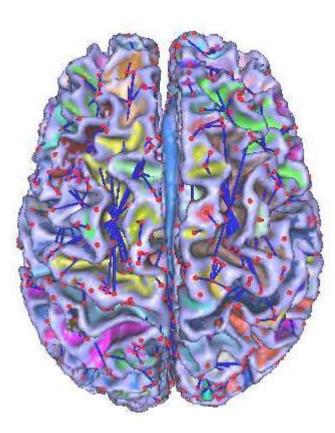
Simulation



Crane et al. "Spin Transformations of Discrete Surfaces."

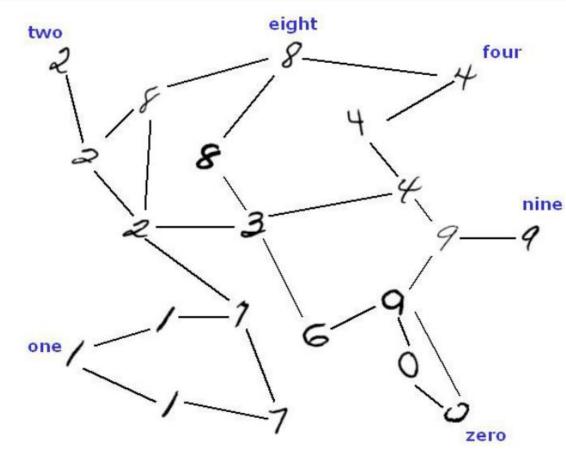
Scientific visualization





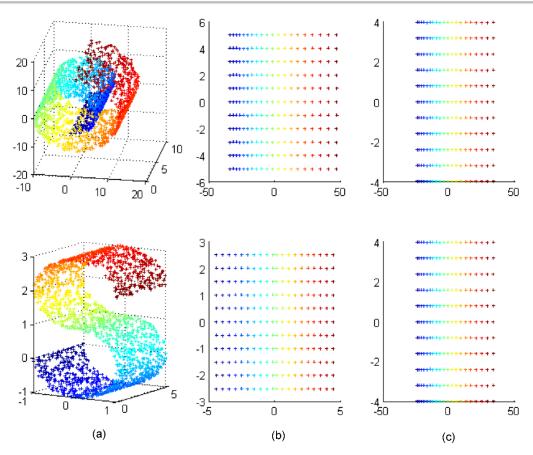
http://www.bioinformaticslaboratory.nl/twiki/pub/EBioScience/News/freesurfer-3d.jpg http://hal.inria.fr/docs/oo/40/21/30/IMG/vivodtzev_et_al-Dagstuhlo3.jpg

Segmentation



Zhu et al. Semi-Supervised Learning Using Gaussian Fields and Harmonic Functions. ICML 2003.

Machine learning



Hou et al. Novel semisupervised high-dimensional correspondences learning method. Opt. Eng. 2008.



6.838: Shape Analysis

Justin Solomon Spring 2021

