Consistent Correspondence

Justin Solomon

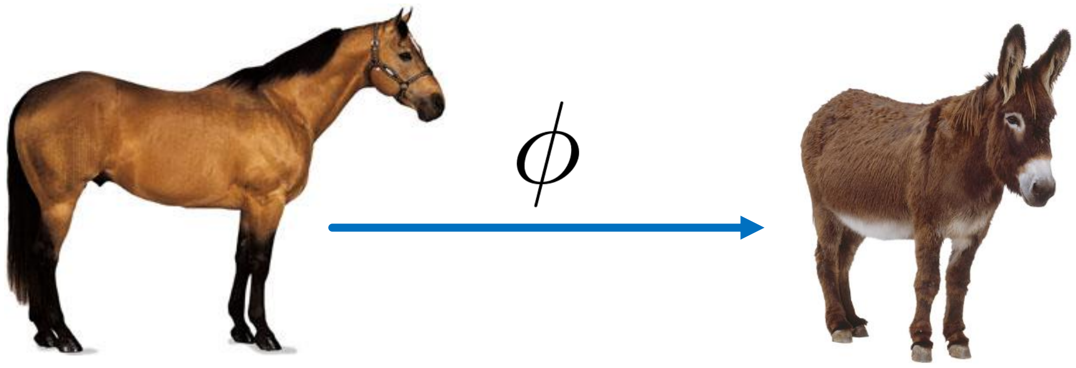
6.838: Shape Analysis

Spring 2021

MIT EECS

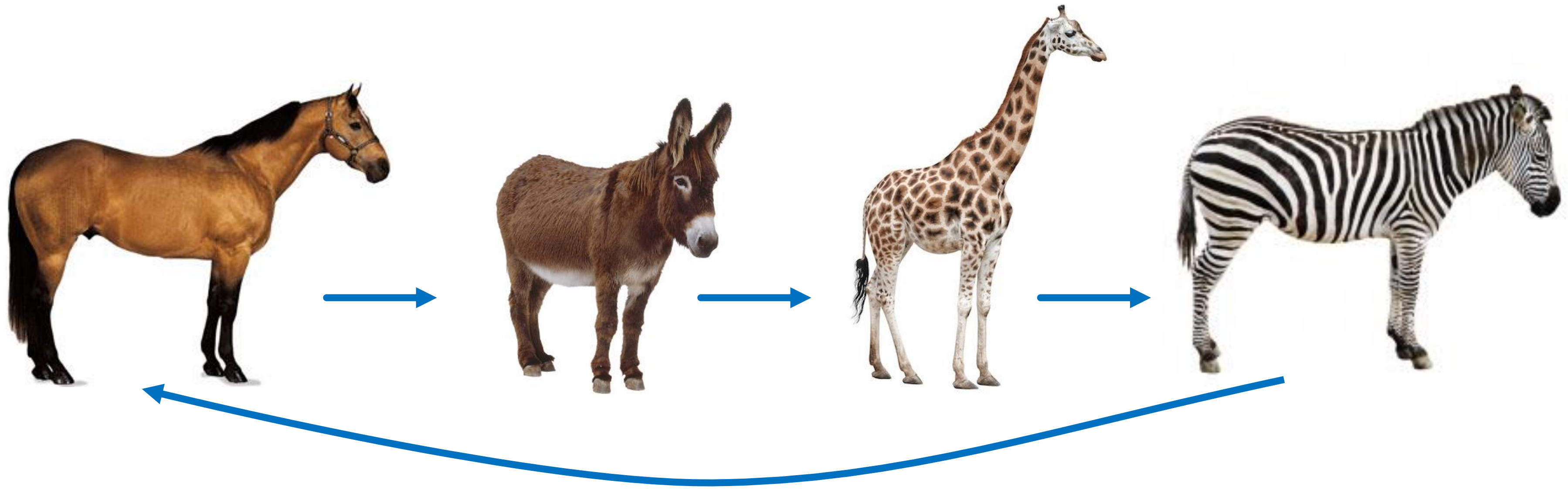
Previously

Map between two shapes.



Question

What happens if you compose these maps?



Q:

What do you **expect** if you compose around a cycle?

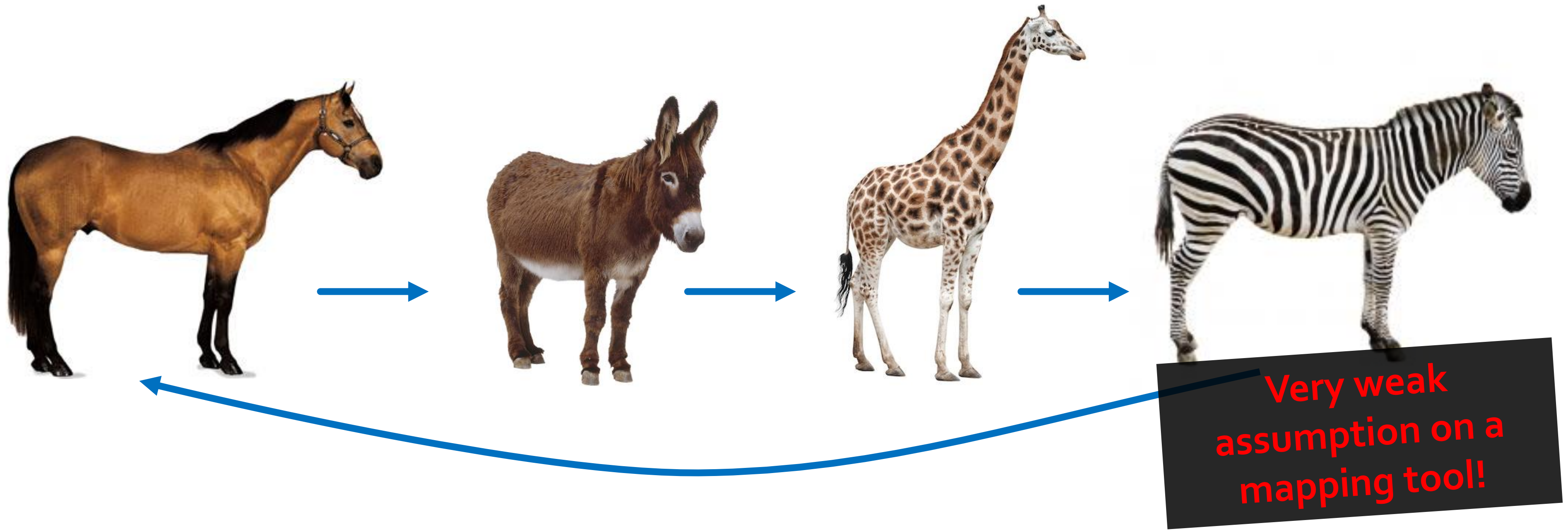
Cycle consistency

[**sahy**-k*uh* l k*uh* n-**sis**-t*uh* n-see]:

Composing maps in a cycle yields the identity

Philosophical Point

You should have a good reason if your correspondences are **inconsistent.**



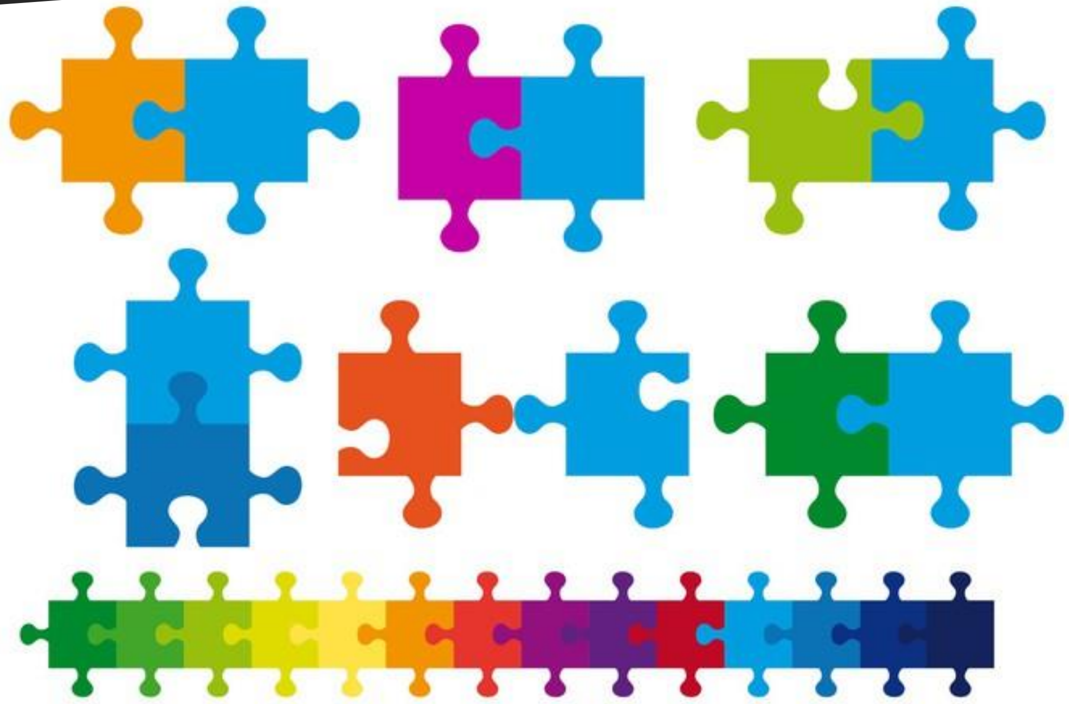
An Unpleasant Constraint

$$\phi_1(\phi_2(\phi_3(x))) = \mathrm{Id}$$

Cycle consistency

Contrasting Viewpoint

Many possible pairwise matches!



https://s3.pixers.pics/pixers/700/FO/39/51/09/46/700_FO39510946_cd54b90a83d46f5dbd96440271eadfec.jpg

Additional data should <u>help</u>!

Today

Sampling of methods for consistent correspondence.

- Spanning tree
- Inconsistent cycle detection
- Convex optimization

Holy Grail

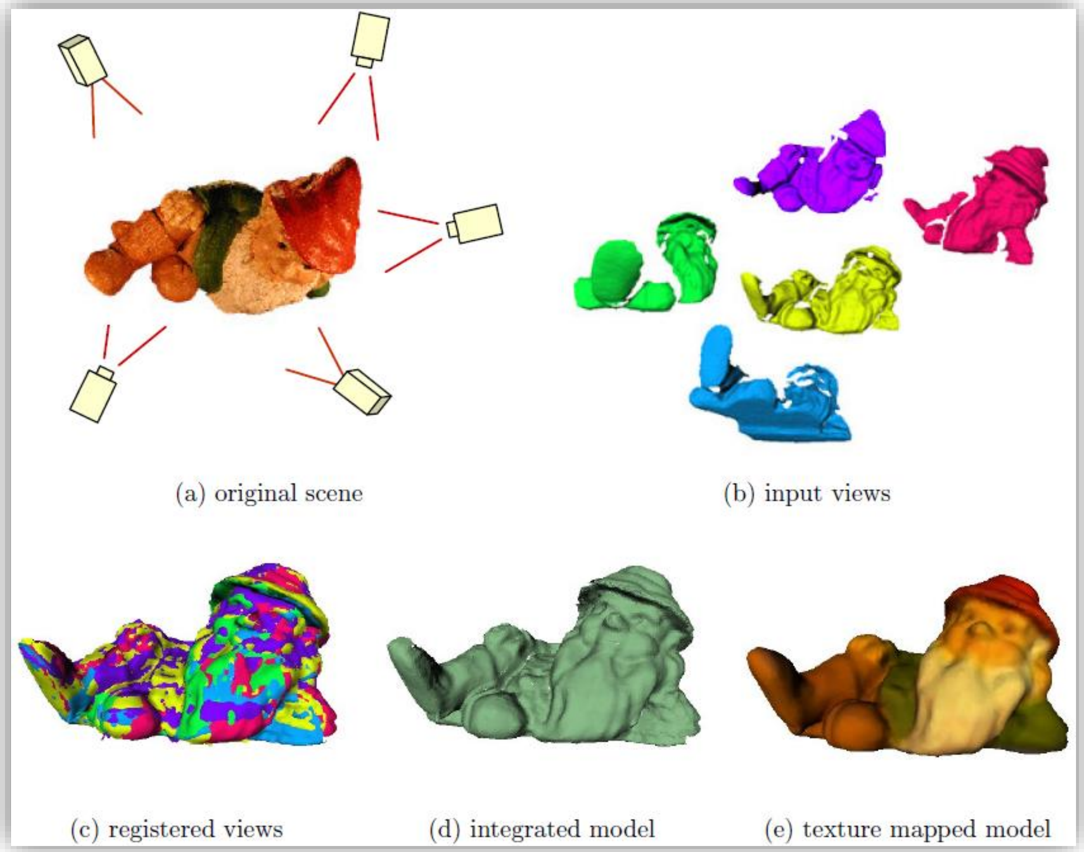
Simultaneously optimize all maps in a collection.

Open problem!

Joint Matching: Simplest Formulation

- Input
  - N shapes
  - N^2 maps (see last lecture)
- Output
  - Cycle-consistent approximation

Spanning Tree: Original Context

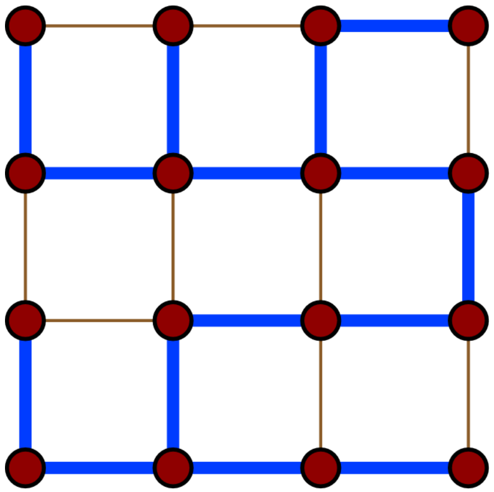


"Automatic Three-Dimensional Modeling from Reality" (Huber, 2002)

Multi-view registration

Unsurprisingly…

Given: Model graph $G = (S, E)$
Find: Largest consistent spanning tree

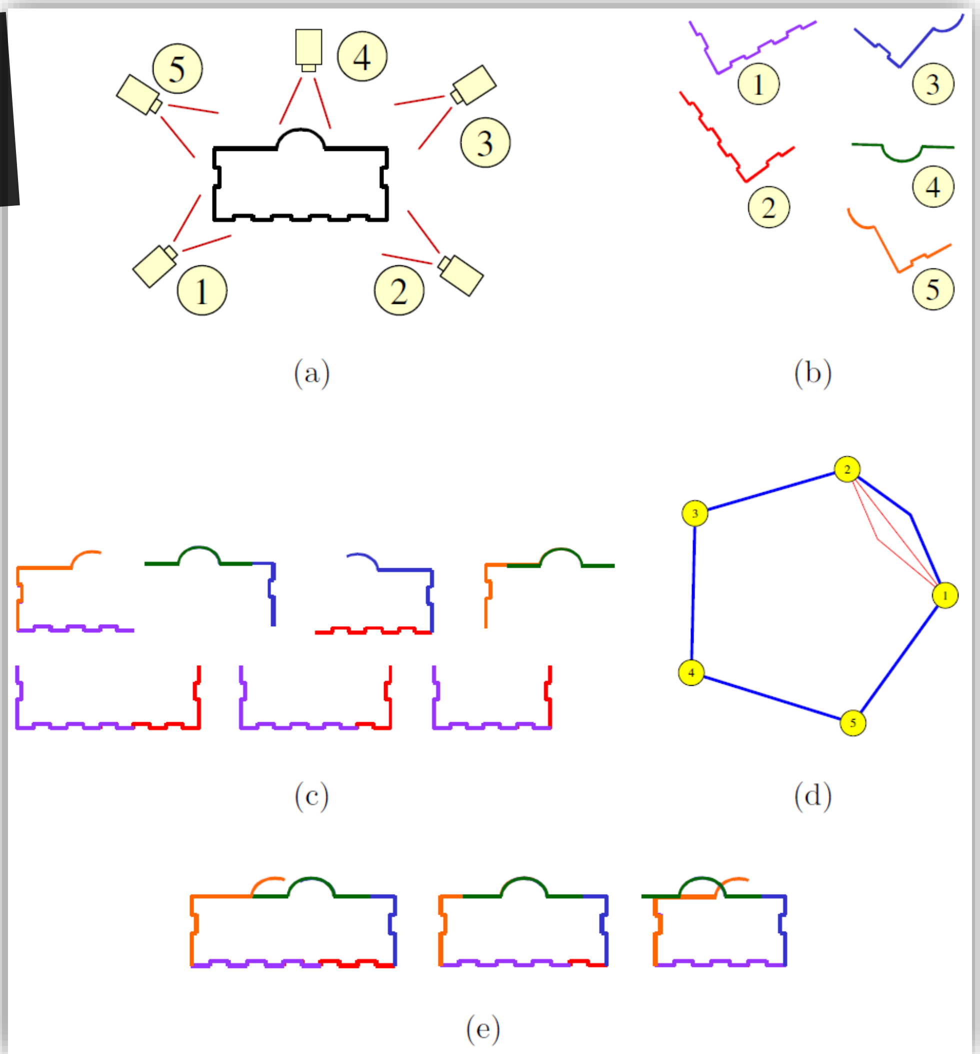


"Automatic Three-Dimensional Modeling from Reality" (Huber, 2002)

NP-hard

Heuristic Algorithm

Extract consistent spanning tree in model graph



Issues

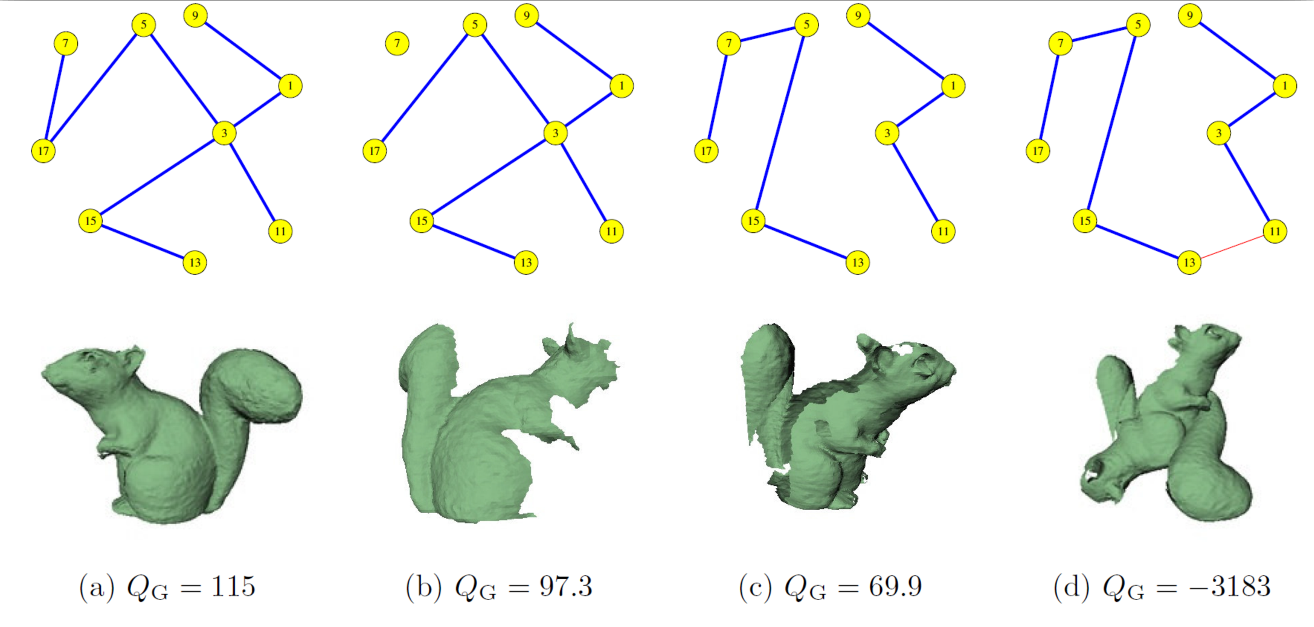
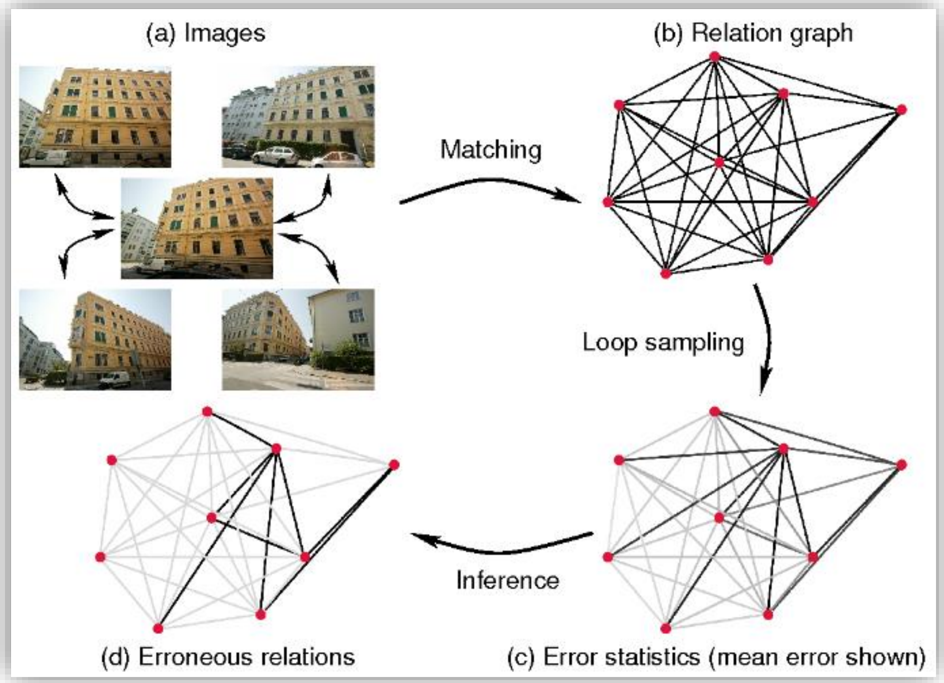


Figure 3.13: Global quality values for several versions of the squirrel model. The model hypothesis is shown in the top row with the corresponding 3D visualization in the bottom row. a) Correct model. b) Correct model with a single view detached; c) Correct model split into equally sized two parts (only one part shown in 3D). d) Model with one error.

- Many spanning trees
- Single incorrect match can destroy the maps

Inconsistent Loop Detection

Used to deal with repeating structures like windows!



Large for inconsistent cycles

$$\begin{aligned} \max \quad & \textstyle\sum_L \rho_L x_L \\ \text{s.t.} \quad & x_L \geq x_e \ \forall e \in L \\ & x_L \leq \textstyle\sum_{e \in L} x_e \\ & x_L, x_e \in [0, 1] \end{aligned}$$

x_e = 1 for false positive edge

x_L = max of x_e over loop

"Disambiguating Visual Relations Using Loop Constraints" (Zach et al., CVPR 2010)

Relationship: Consistency vs. Accuracy

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Mario Botsch and Scott Schaefer
(Guest Editors)

Volume 30 (2011), Number 5

An Optimization Approach to Improving Collections of Shape Maps

Andy Nguyen[1] Mirela Ben-Chen[1] Katarzyna Welnicka[2] Yinyu Ye[1] Leonidas Guibas[1]

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Abstract

Finding an informative, structure-preserving map between two shapes has been a long-standing problem ... ometry processing, involving a variety of solution approaches and applications. However, in many cases, ... given not only two related shapes, but a collection of them, and considering each pairwise map independent... not take full advantage of all existing information. For example, a notorious problem with computing shape maps is the ambiguity i... there exist two m... them based on the... sensitivity to how... Given the context... map consistency, ... chosen in the netw... help us replace a... sense interpolate... mization problem... and individually a... shapes, as long as... for improving map...

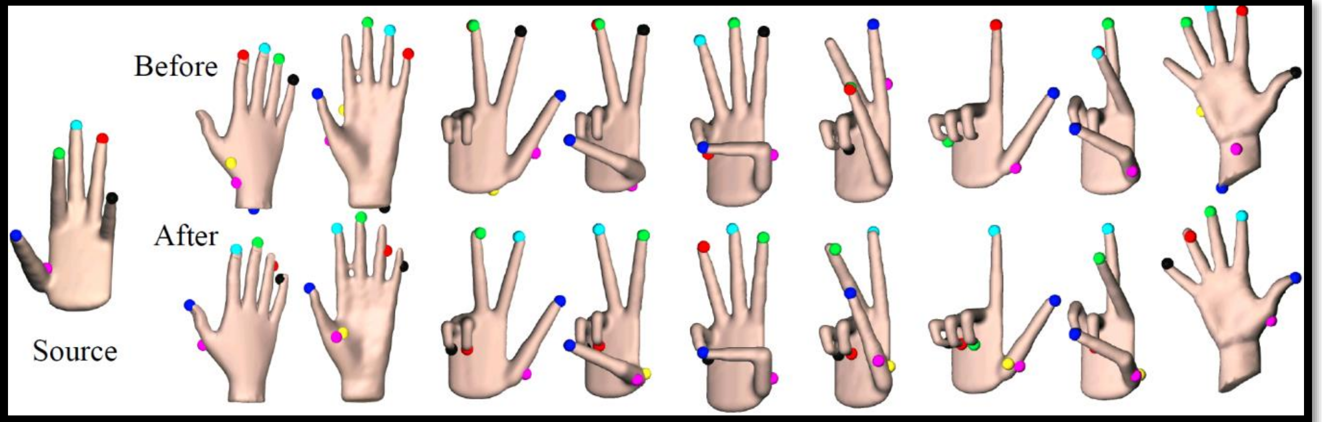
Categories and Su...

1. Introduction

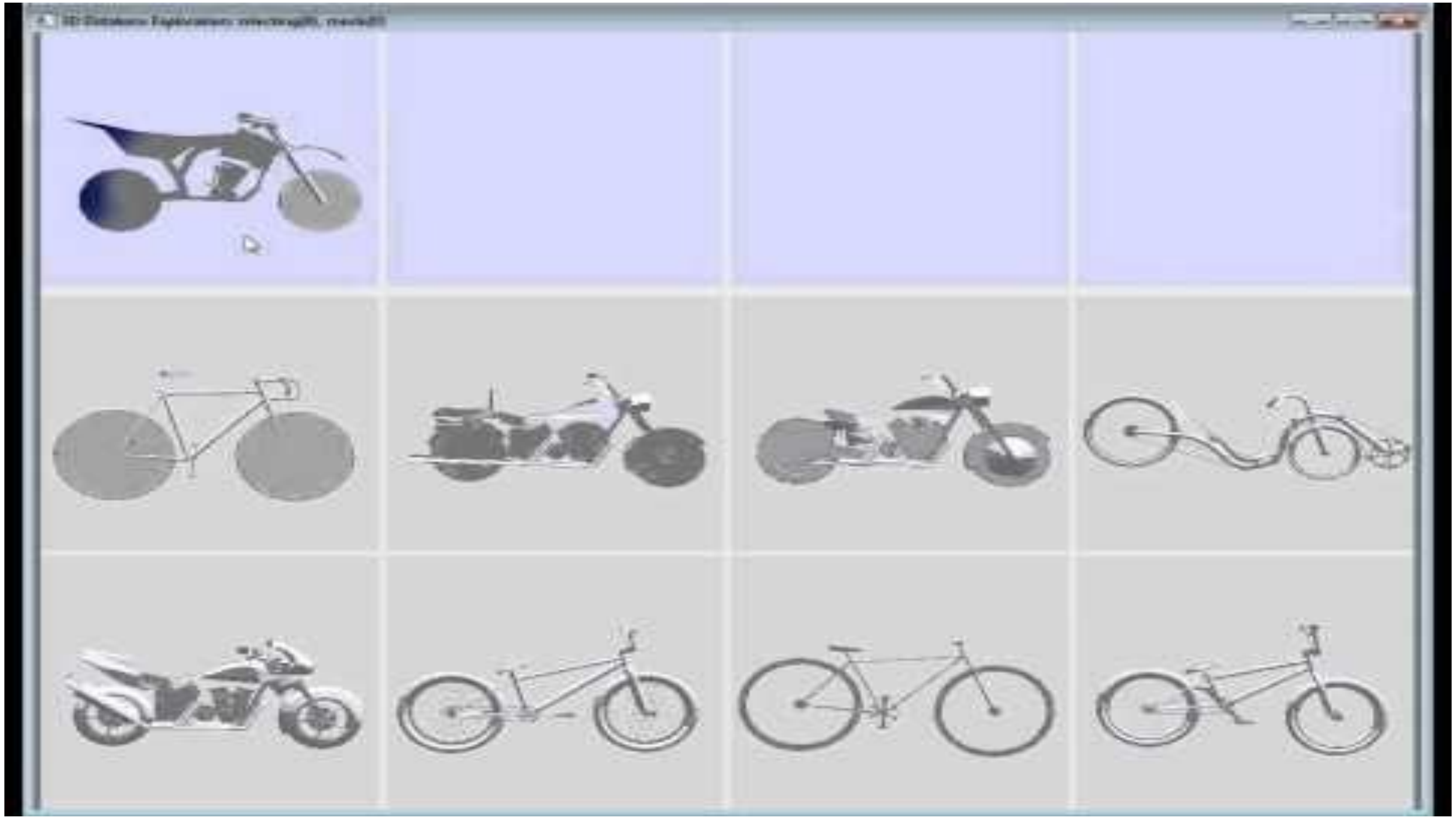
a common measure of distortion is the difference between

Iteratively fix triplets and reweight

Definition 3 Given a collection of maps $\mathcal{M}$, let $\mathcal{B}(\mathcal{M}) = \{m_{i,j} \in \mathcal{M} \mid E_{acc}(m_{i,j}) > 0\}$ — the collection of inaccurate maps. Then we say that $\mathcal{M}$ is *almost accurate*, if there do not exist two maps $m_1, m_2 \in \mathcal{B}(\mathcal{M})$, which both belong to the same 3-cycle in $G_{\mathcal{M}}$. We call such maps *isolated*.



Fuzzy Correspondences

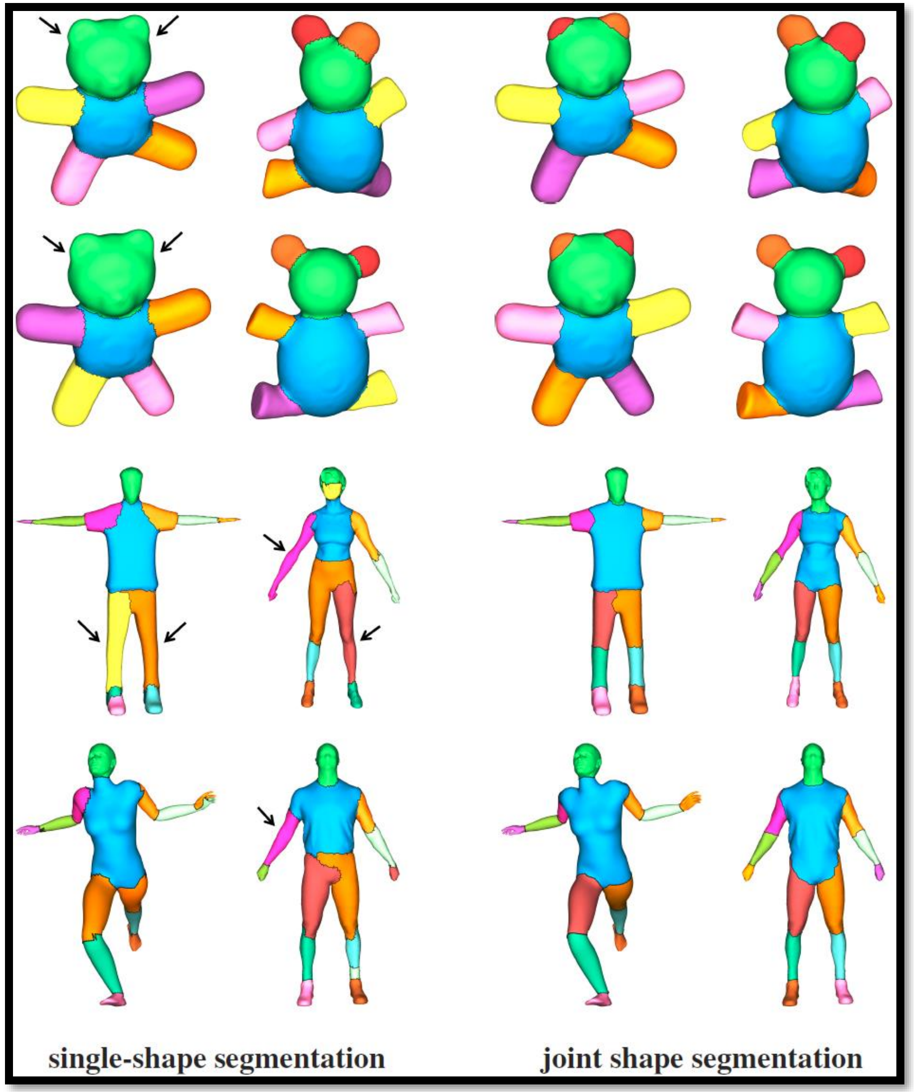


Exploring Collections of 3D Models using Fuzzy Correspondences (Kim et al., SIGGRAPH 2012)

Fuzzy Correspondences: Idea

- Compute Nk x Nk similarity matrix
  - Same number of samples per surface
  - Align similar shapes
- Compute spectral embedding
- Use as descriptor: Display $e^{-|d_i - d_j|^2}$

Consistent Segmentation



Global optimization to choose among many possible segmentations

"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011)

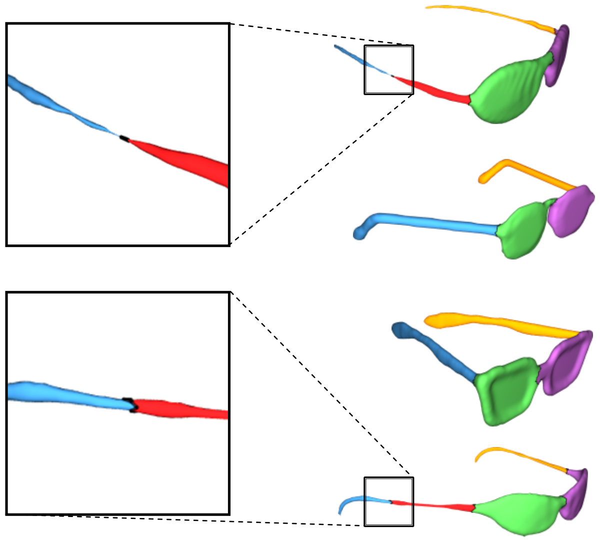
Joint Segmentation: Motivation

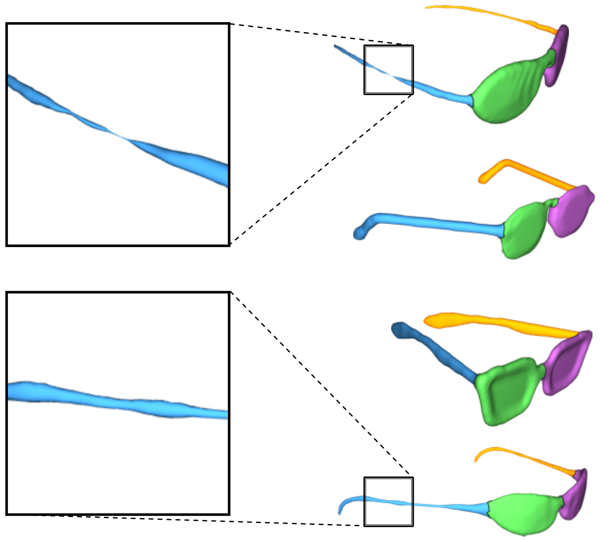
Structural similarity of segmentations

- Extraneous geometric clues

Single shape segmentation [Chen et al. 09]

Joint shape segmentation [Huang et al. 11]





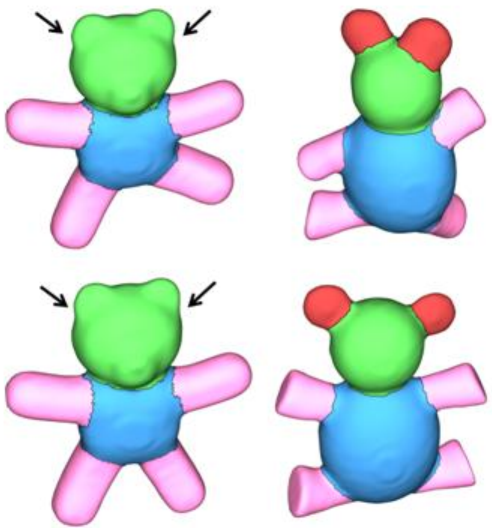
"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011; slides provided by authors)

Joint Segmentation: Motivation

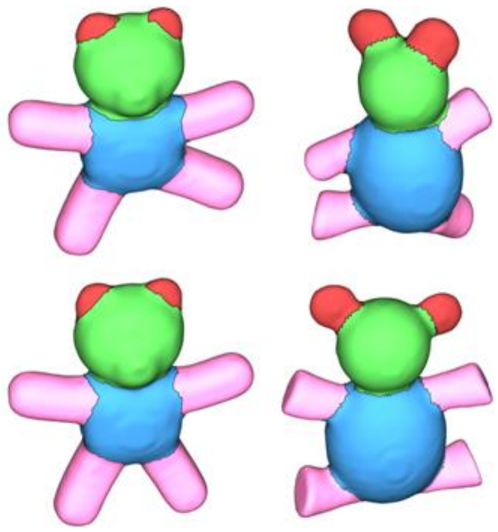
Structural similarity of segmentations

- Low saliency

Single shape segmentation
[Chen et al. 09]



Joint shape segmentation
[Huang et al. 11]



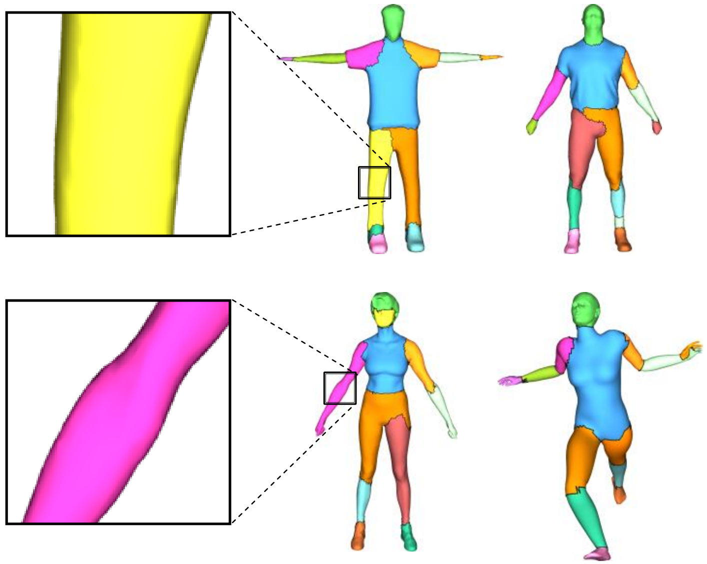
"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011; slides provided by authors)

Joint Segmentation: Motivation

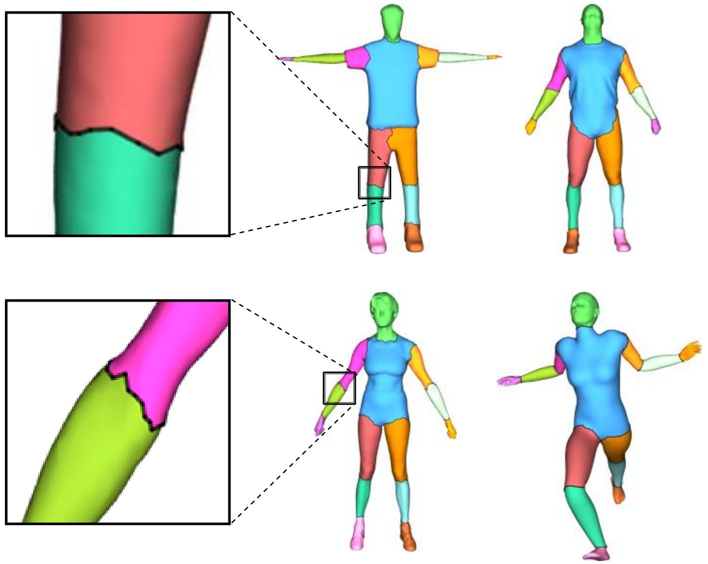
(Rigid) invariance of segments

- Articulated structures

Single shape segmentation
[Chen et al. 09]



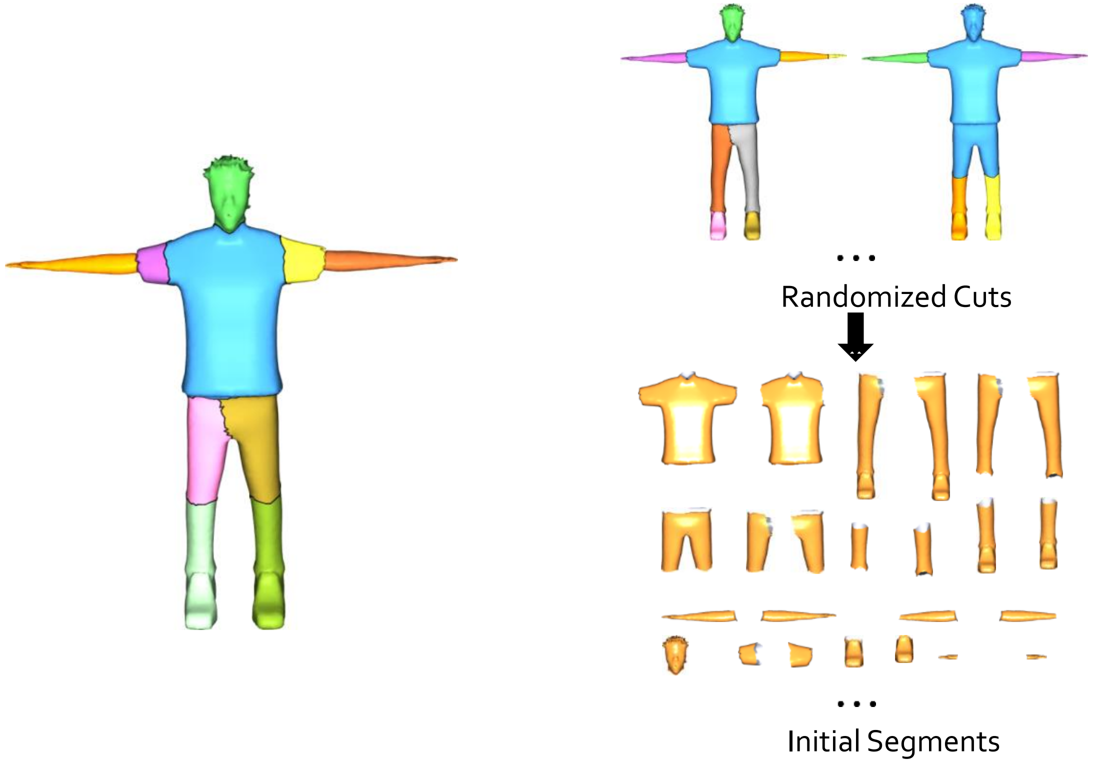
Joint shape segmentation
[Huang et al. 11]



"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011; slides provided by authors)

Parameterization

Initial subsets of randomized segmentations



"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011; slides provided by authors)

Segmentation Constraint/Score

- Each point covered by one segment

$$|\mathrm{cover}(p)| = 1 \ \forall p \in W$$

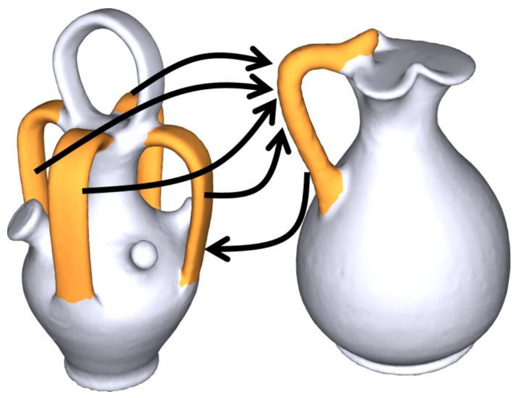
- Avoid tiny segments

$$\mathrm{score}(S) = \sum_{s \in S} \mathrm{area}(s) \cdot \mathrm{repetitions}_s$$

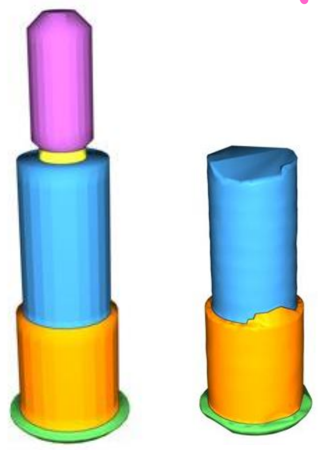
"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011; slides provided by authors)

Consistency Term

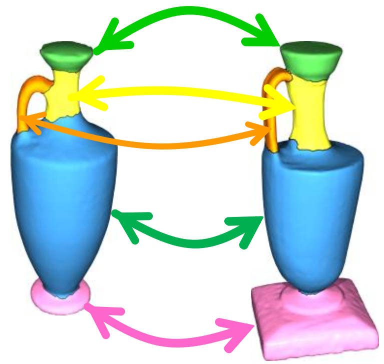
- Defined in terms of mappings
  - Oriented
  - Partial



Many-to-one correspondences



Partial similarity

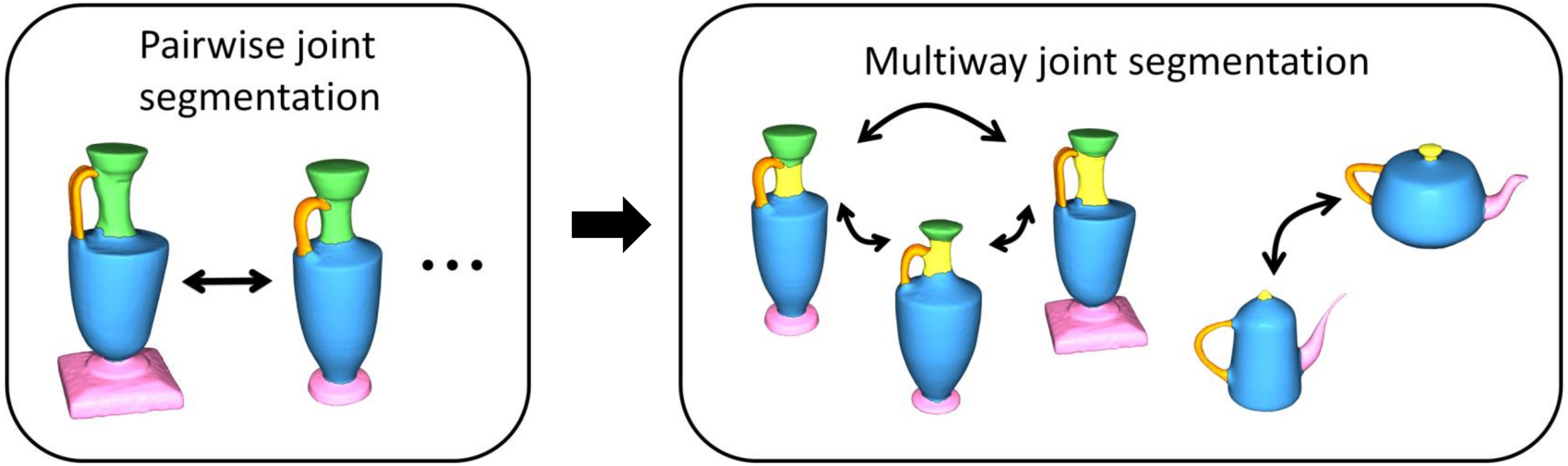


"Joint Shape Segmentation with Linear Programming"
(Huang, Koltun, Guibas; SIGGRAPH Asia 2011; slides provided by authors)

Multi-Way Joint Segmentation

- Objective function

$$\sum_{i=1}^{n} \text{score}(S_i) + \sum_{(S_i,S_j)\in\mathcal{E}} \text{consistency}(S_i, S_j)$$



See paper: Linear program relaxation

Q:

Can you extract consistent maps in a globally optimal way?

Basic Setup

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & \cdots & 0 \end{pmatrix}$$

Map as a permutation matrix

Q:

What is the inverse of a permutation matrix?

Discrete Relaxation

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & \cdots & 0 \end{pmatrix}$$

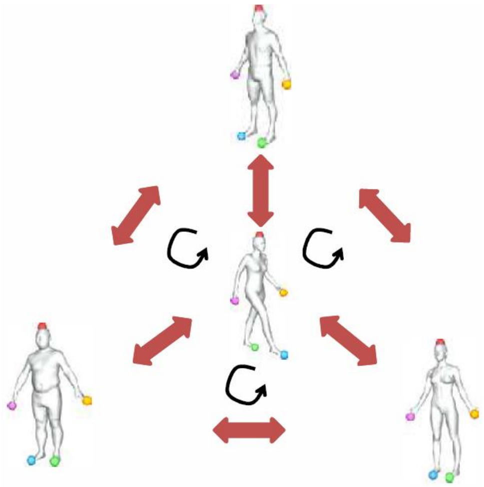
Sums to 1

Sums to 1

Map as a doubly-stochastic matrix

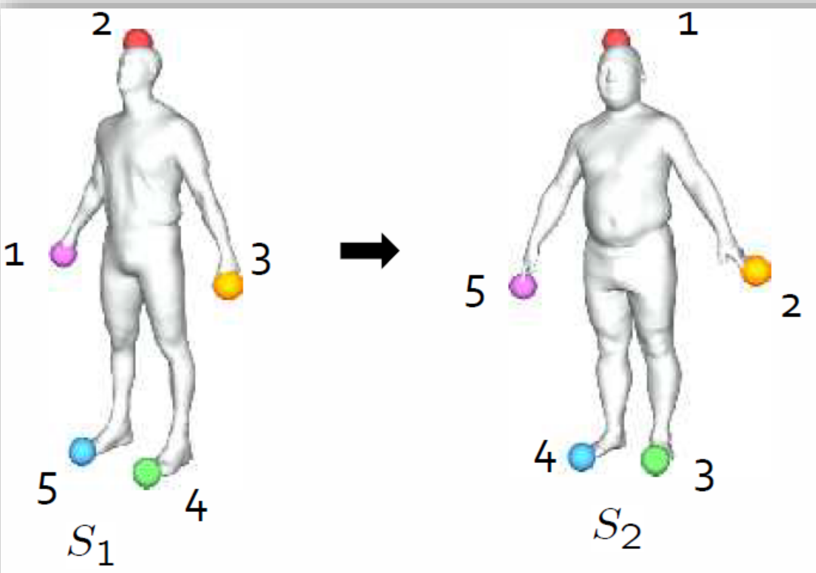
Basic Setting

- Given n objects
- Each object sampled with m points



“Consistent Shape Maps via Semidefinite Programming” (Huang & Guibas, SGP 2013)

Map Collection: Matrix Representation



$$X_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{12}^T & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{(n-1),n} \\ X_{1n}^T & \vdots & X_{(n-1),n}^T & I_m \end{bmatrix}$$

- Diagonal blocks are identity matrices
- Off diagonal blocks are permutation matrices
- Symmetric

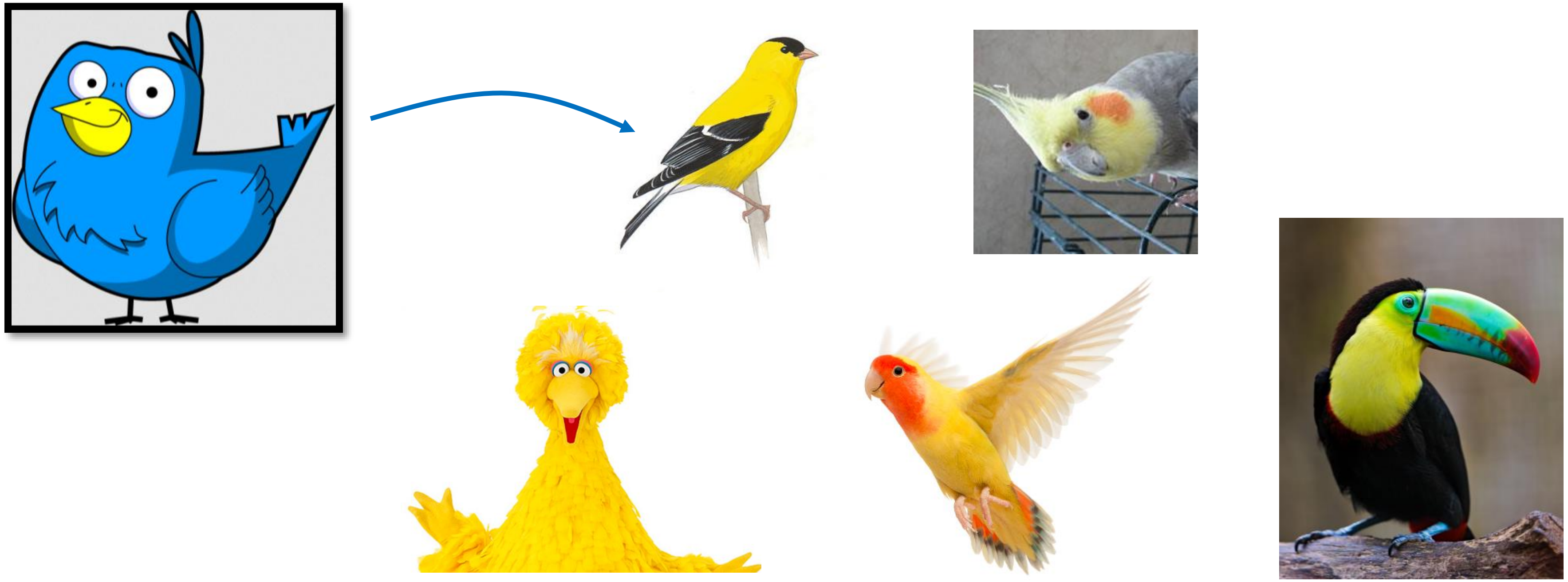
Q:

What is the rank of a consistent map collection matrix?

Hint: "Urshape" Factorization

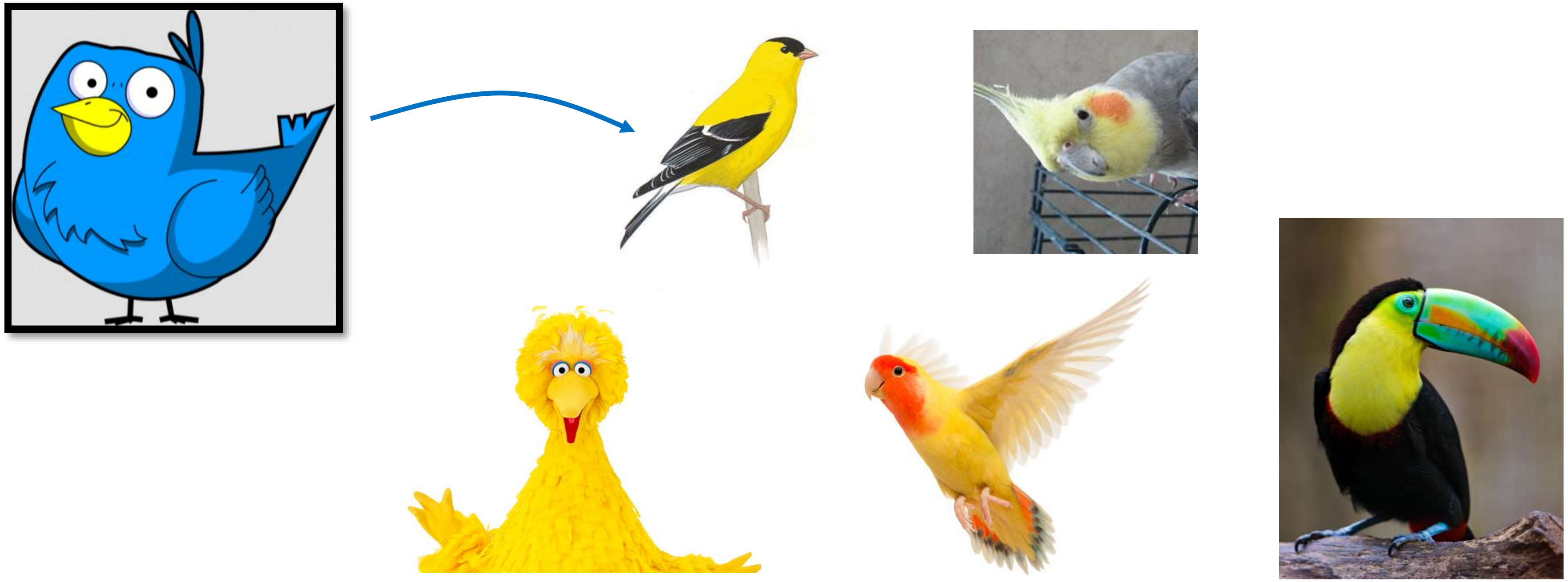
$$X = \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{12}^T & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{(n-1),n} \\ X_{1n}^T & \vdots & X_{(n-1),n}^T & I_m \end{bmatrix}$$

- Diagonal blocks are identity matrices
- Off diagonal blocks are permutation matrices
- Symmetric



Rank *m*, Number of Samples

$$X_{ij} = X_{j1}^\top X_{i1} \iff X = \begin{pmatrix} I_m \\ \vdots \\ X_{n1}^\top \end{pmatrix} \begin{pmatrix} I_m & \cdots & X_{n1} \end{pmatrix}$$



Many Equivalent Conditions

Definition 2.1 Given a shape collection $\mathcal{S} = \{S_1, \cdots, S_n\}$ of n shapes where each shape consists of the same number of samples, we say a map collection $\Phi = \{\phi_{ij} : S_i \to S_j | 1 \leq i, j \leq n\}$ of maps between all pairs of shapes is cycle consistent if and only if the following equalities are satisfied:

$$
\begin{aligned}
\phi_{ii} &= id_{S_i}, & 1 \leq i \leq n, & \quad \text{(1-cycle)} \\
\phi_{ji} \circ \phi_{ij} &= id_{S_i}, & 1 \leq i < j \leq n, & \quad \text{(2-cycle)} \\
\phi_{ki} \circ \phi_{jk} \circ \phi_{ij} &= id_{S_i}, & 1 \leq i < j < k \leq n, & \quad \text{(3-cycle)} \quad (1)
\end{aligned}
$$

where id_{S_i} denotes the identity self-map on S_i.

Equivalence for binary map matrix Φ:

1. Φ is cycle-consistent

2. $X = Y_i^\top Y_i$, where $Y_i = (X_{i1}, \ldots, X_{in})$

3. $X \succeq 0$

Equivalence for binary map matrix Φ:

1. Φ is cycle-consistent
2. $X = Y_i^\top Y_i$, where $Y_i = (X_{i1}, \ldots, X_{in})$
3. $X \succeq 0$

Approximation by Consistent Maps

$$
\begin{aligned}
\max_X \quad & \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\
\text{s.t.} \quad & X \in \{0,1\}^{nm \times nm} \\
& X \succeq 0 \\
& X_{ii} = I_m \\
& X_{ij}\mathbf{1} = \mathbf{1} \\
& X_{ij}^\top \mathbf{1} = \mathbf{1}
\end{aligned}
$$

Approximation by Consistent Maps

$$\max_X \quad \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle$$

$$\text{s.t.} \quad X \in \{0,1\}^{nm \times nm}$$

$$X \succeq 0$$

$$X_{ii} = I_m$$

$$X_{ij}\mathbf{1} = \mathbf{1}$$

$$X_{ij}^\top \mathbf{1} = \mathbf{1}$$

Maximize number of preserved matches

Approximation by Consistent Maps

$$\begin{aligned} \max_X \quad & \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\ \text{s.t.} \quad & X \in \{0,1\}^{nm \times nm} \\ & X \succeq 0 \\ & X_{ii} = I_m \\ & X_{ij}\mathbf{1} = \mathbf{1} \\ & X_{ij}^{\top}\mathbf{1} = \mathbf{1} \end{aligned}$$

Binary matrix

Approximation by Consistent Maps

$$\begin{aligned} \max_X \quad & \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\ \text{s.t.} \quad & X \in \{0,1\}^{nm \times nm} \\ & X \succeq 0 \\ & X_{ii} = I_m \\ & X_{ij}\mathbf{1} = \mathbf{1} \\ & X_{ij}^{\top}\mathbf{1} = \mathbf{1} \end{aligned}$$

Every block is a permutation

Approximation by Consistent Maps

$$
\begin{aligned}
\max_X \quad & \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\
\text{s.t.} \quad & X \in \{0,1\}^{nm \times nm} \\
& X \succeq 0 \\
& X_{ii} = I_m \\
& X_{ij} \mathbf{1} = \mathbf{1} \\
& X_{ij}^\top \mathbf{1} = \mathbf{1}
\end{aligned}
$$

Self maps are identity

Approximation by Consistent Maps

$$\begin{aligned} \max_X \quad & \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\ \text{s.t.} \quad & X \in \{0,1\}^{nm \times nm} \\ & X \succeq 0 \\ & X_{ii} = I_m \\ & X_{ij}\mathbf{1} = \mathbf{1} \\ & X_{ij}^\top \mathbf{1} = \mathbf{1} \end{aligned}$$

Already showed:
Equivalent to low-rank

Approximation by Consistent Maps

$$
\begin{aligned}
\max_X \quad & \sum_{ij\in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\
\text{s.t.} \quad & X \in \{0,1\}^{nm\times nm} \\
& X \succeq 0 \\
& X_{ii} = I_m \\
& X_{ij}\mathbf{1} = \mathbf{1} \\
& X_{ij}^\top \mathbf{1} = \mathbf{1}
\end{aligned}
$$

Nonconvex!

Convex Relaxation

$$
\begin{aligned}
\max_X \quad & \sum_{ij \in E} \langle X_{ij}^{\text{in}}, X_{ij} \rangle \\
\text{s.t.} \quad & X \geq 0 \\
& X \succeq 0 \\
& X_{ii} = I_m \\
& X_{ij} \mathbf{1} = \mathbf{1} \\
& X_{ij}^\top \mathbf{1} = \mathbf{1}
\end{aligned}
$$

Rounding Procedure

Guaranteed to give permutation

$$\begin{aligned} \max_X \quad & \langle X, X_0 \rangle \\ \text{s.t.} \quad & X \geq 0 \\ & X\mathbf{1} = \mathbf{1} \\ & X^\top \mathbf{1} = \mathbf{1} \end{aligned}$$

Linear assignment problem

Recovery Theorem

Can tolerate $\lambda_2/4(n-1)$ incorrect correspondences from each sample on one shape.

λ_2 is algebraic connectivity; bounded above by two times maximum degree

\omit{proof}

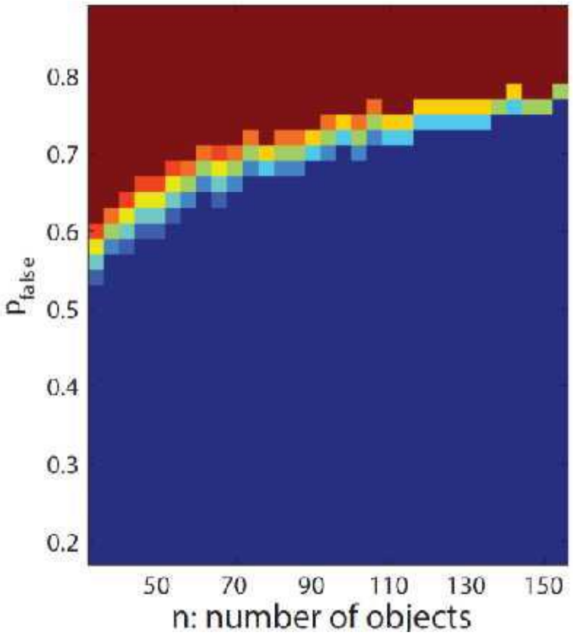
Recovery Theorem: Complete Graph

Can tolerate 25% incorrect correspondences from each sample on one shape.

λ_2 is algebraic connectivity; bounded above by two times maximum degree

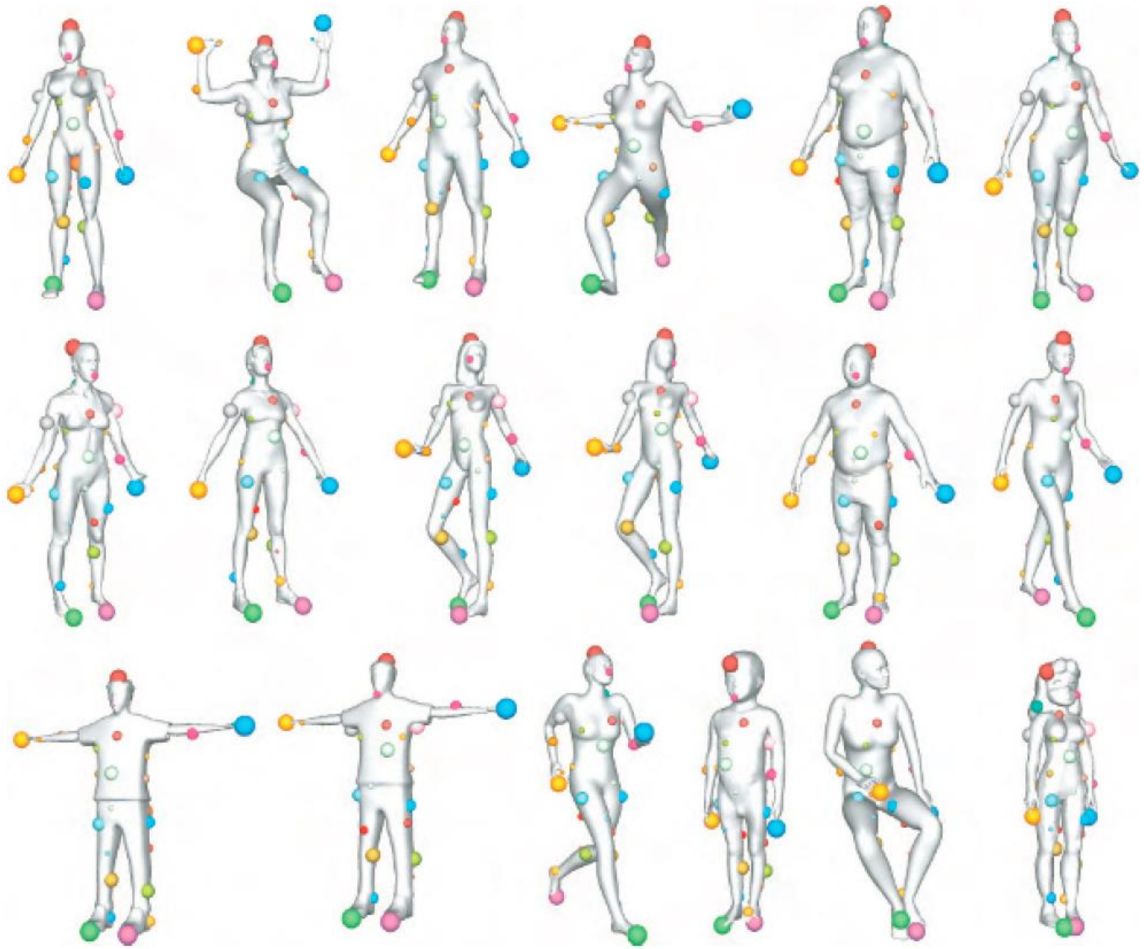
`\omit{proof}`

Phase Transition

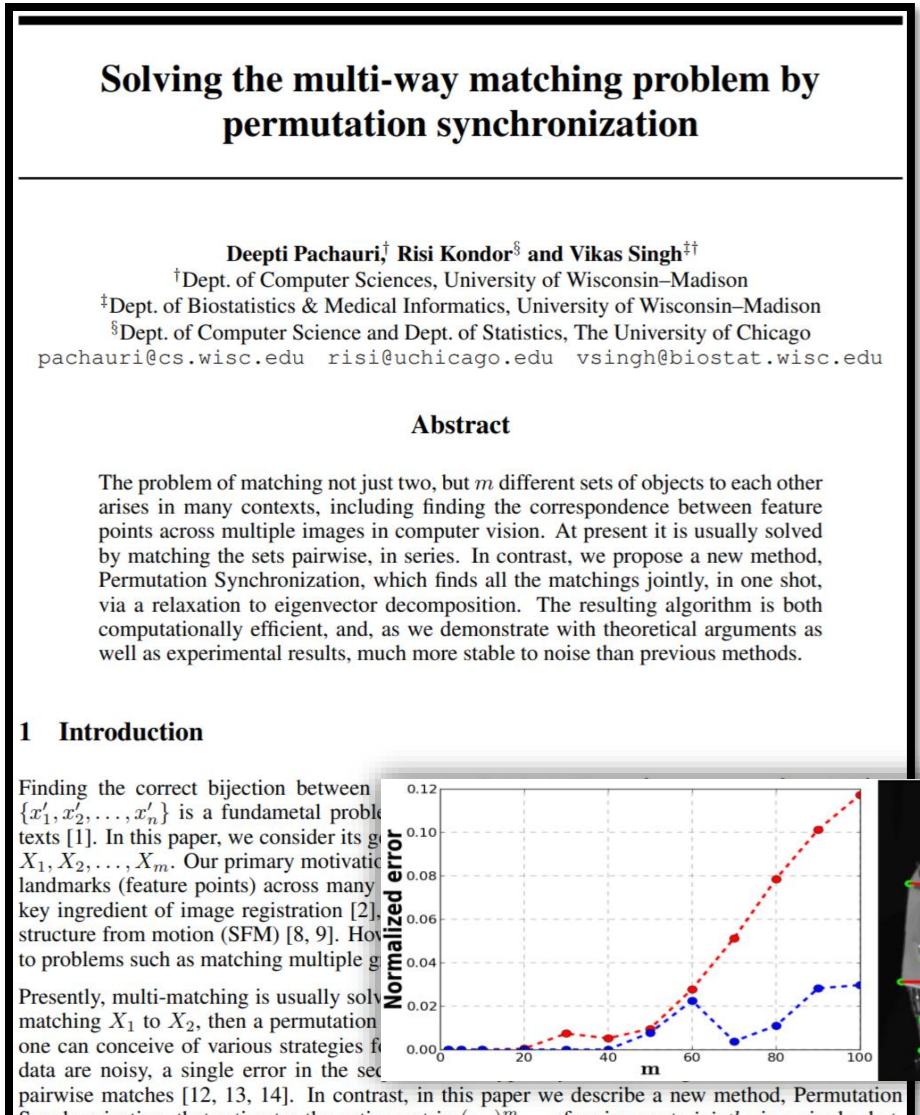


Always recovers / Never recovers

Example Result



Weaker Relaxation



Solving the multi-way matching problem by permutation synchronization

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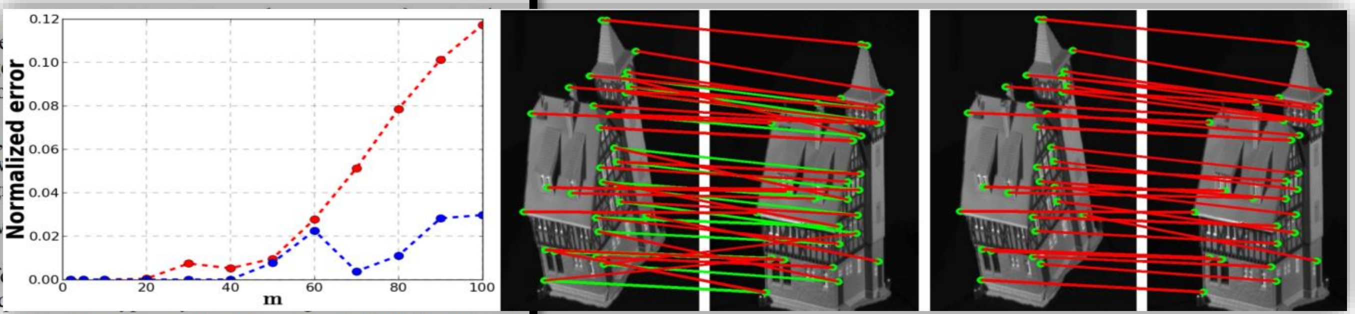
Abstract

The problem of matching not just two, but m different sets of objects to each other arises in many contexts, including finding the correspondence between feature points across multiple images in computer vision. At present it is usually solved by matching the sets pairwise, in series. In contrast, we propose a new method, Permutation Synchronization, which finds all the matchings jointly, in one shot, via a relaxation to eigenvector decomposition. The resulting algorithm is both computationally efficient, and, as we demonstrate with theoretical arguments as well as experimental results, much more stable to noise than previous methods.

1 Introduction

Finding the correct bijection between … $\{x'_1, x'_2, \ldots, x'_n\}$ is a fundametal proble… texts [1]. In this paper, we consider its g… $X_1, X_2, \ldots, X_m$. Our primary motivatio… landmarks (feature points) across many … key ingredient of image registration [2], … structure from motion (SFM) [8, 9]. Ho… to problems such as matching multiple g…

Presently, multi-matching is usually solv… matching X_1 to X_2, then a permutation … one can conceive of various strategies f… data are noisy, a single error in the se… pairwise matches [12, 13, 14]. In contrast, in this paper we describe a new method, Permutation



Eigenvector relaxation of the same problem

Q:

Where do the pairwise **input maps** come from?

Possible Extension with Guarantees

Eurographics Symposium on Geometry Processing 2015
Mirela Ben-Chen and Ligang Liu
(Guest Editors)

Volume 34 (2015), Number 5

Tight Relaxation of Quadratic Matching

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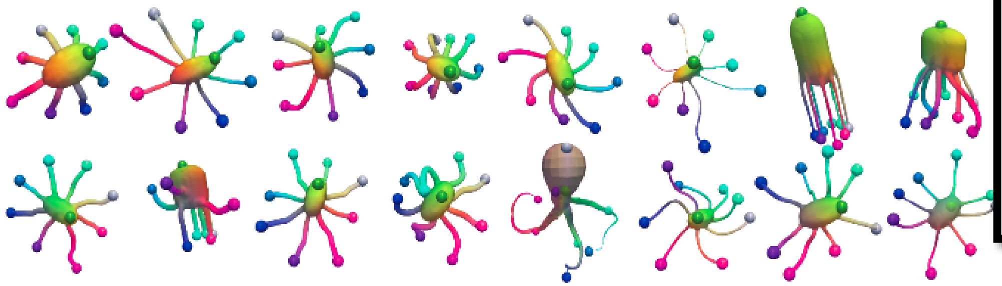

Figure 1: Consistent Collection Matching. Results of the proposed one-stage procedure for finding consistent correspondence between shapes in a collection showing strong variability and non-rigid deformations.

Abstract

Establishing point correspondences between shapes is extremely challenging as it involves both finding sets of semantically persistent feature points, as well as their combinatorial matching. We focus on the latter and consider the Quadratic Assignment Matching (QAM) model. We suggest a novel convex relaxation for this NP-hard problem that builds upon a rank-one reformulation of the problem in a higher dimension, followed by relaxation into a semidefinite program (SDP). Our method is shown to be a certain hybrid of the popular spectral and doubly-stochastic relaxations of QAM and in particular we prove that it is tighter than both.

Experimental evaluation shows that the proposed relaxation is extremely tight: in the majority of our experiments it achieved the certified global optimum solution for the problem, while other relaxations tend to produce sub-optimal solutions. This, however, comes at the price of solving an SDP in a higher dimension.

Our approach is further generalized to the problem of Consistent Collection Matching (CCM), where we solve

Heavy optimization problem!

$$
\begin{aligned}
\max_{Y} \quad & \operatorname{tr}(WY) && \text{(7a)}\\
\text{s.t.} \quad & Y \succeq [X][X]^T && \text{(7b)}\\
& X \in \operatorname{conv}\Pi_n^k && \text{(7c)}\\
& \operatorname{tr} Y = k && \text{(7d)}\\
& Y \geq 0 && \text{(7e)}\\
& \sum_{qrst} Y_{qrst} = k^2 && \text{(7f)}\\
& Y_{qrst} \leq \begin{cases} 0, & \text{if } q = s,\ r \neq t \\ 0, & \text{if } r = t,\ q \neq s \\ \min\{X_{qr}, X_{st}\}, & \text{otherwise} \end{cases} && \text{(7g)}
\end{aligned}
$$

$$
\begin{aligned}
\max_{\mathbf{X},\mathbf{Y}} \quad & \sum_{i,j} \operatorname{tr}\left(W^{ij}Y^{ij}\right) && && \text{(10a)}\\
\text{s.t.} \quad & \left(X^{ij}, Y^{ij}\right) \in \mathcal{C}^k && \forall i < j && \text{(10b)}\\
& X^{ii} \in \mathcal{D} \cap \operatorname{conv}\Pi_n^k && \forall i && \text{(10c)}\\
& \mathbf{X} \succeq 0 && && \text{(10d)}
\end{aligned}
$$

Approximate Methods

Consistent Partial Matching of Shape Collections via Sparse Modeling

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[1]University of Venice, Italy [2]TU Munich, Germany [3]Ohio State University, U.S.

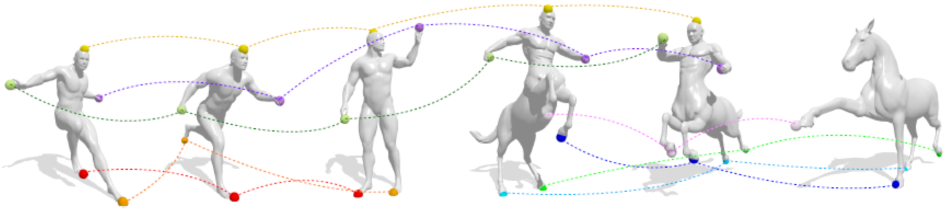


Figure 1: *A partial multi-way correspondence obtained with our approach on a heterogeneous collection of shapes. Our method does not require initial pairwise maps as input, as it actively seeks a reliable correspondе space of joint, cycle-consistent matches. Partially-similar as well as outlier shapes are au for by adopting a sparse model for the joint correspondence. A subset of all matches is show*

Abstract

Recent efforts in the area of joint object matching approach the problem by taking as i which are then jointly optimized across the whole collection so that certain accuracy satisfied. One natural requirement is cycle-consistency – namely the fact that map same result regardless of the path taken in the shape collection. In this paper, we in obtain consistent matches without requiring initial pairwise solutions to be given as i a joint measure of metric distortion directly over the space of cycle-consistent maps; i similar and extra-class shapes, we formulate the problem as a series of quadratic prog constraints, making our technique a natural candidate for analyzing collections with The particular form of the problem allows us to leverage results and tools from the theory. This enables a highly efficient optimization procedure which assures accur solutions in a matter of minutes in collections with hundreds of shapes.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics and Object Modeling—Shape Analysis

1. Introduction

Finding matches among multiple objects is a research topic

this end, a natural and widely accepted criterion is *cycle-consistency* [ZKP10], namely that composition of maps

Sequence of quadratic programs; based on metric distortion and WKS descriptor match

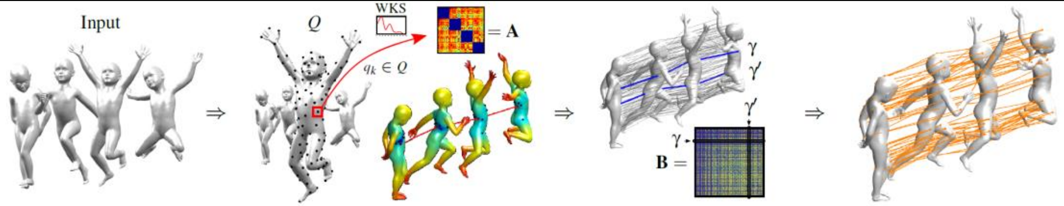
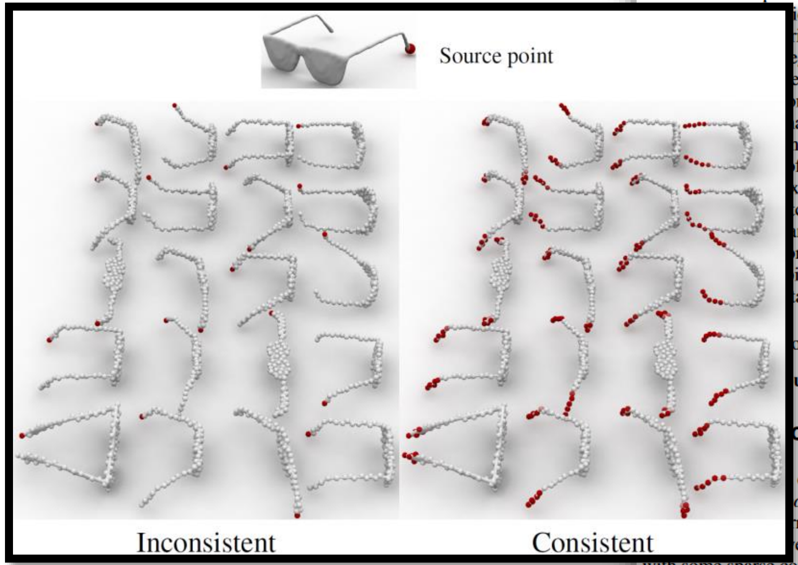


Figure 6: *Our matching pipeline. First sub-problem (from left): Given a collection of shapes as input, a set Q of queries are generated (e.g., by farthest point sampling in the joint WKS space); we then compute distance maps (shown here as heat maps over the shapes) in descriptor space from each shape point to each query $q_k \in Q$, and keep the vertices having distance smaller than a threshold; finally, a single multi-way match is extracted by solving problem (11). Second sub-problem: The multi-way matches extracted by iterating the previous step are compared using a measure of metric distortion; the final solution (in orange) is obtained by solving problem (13) over the reduced feasible set.*

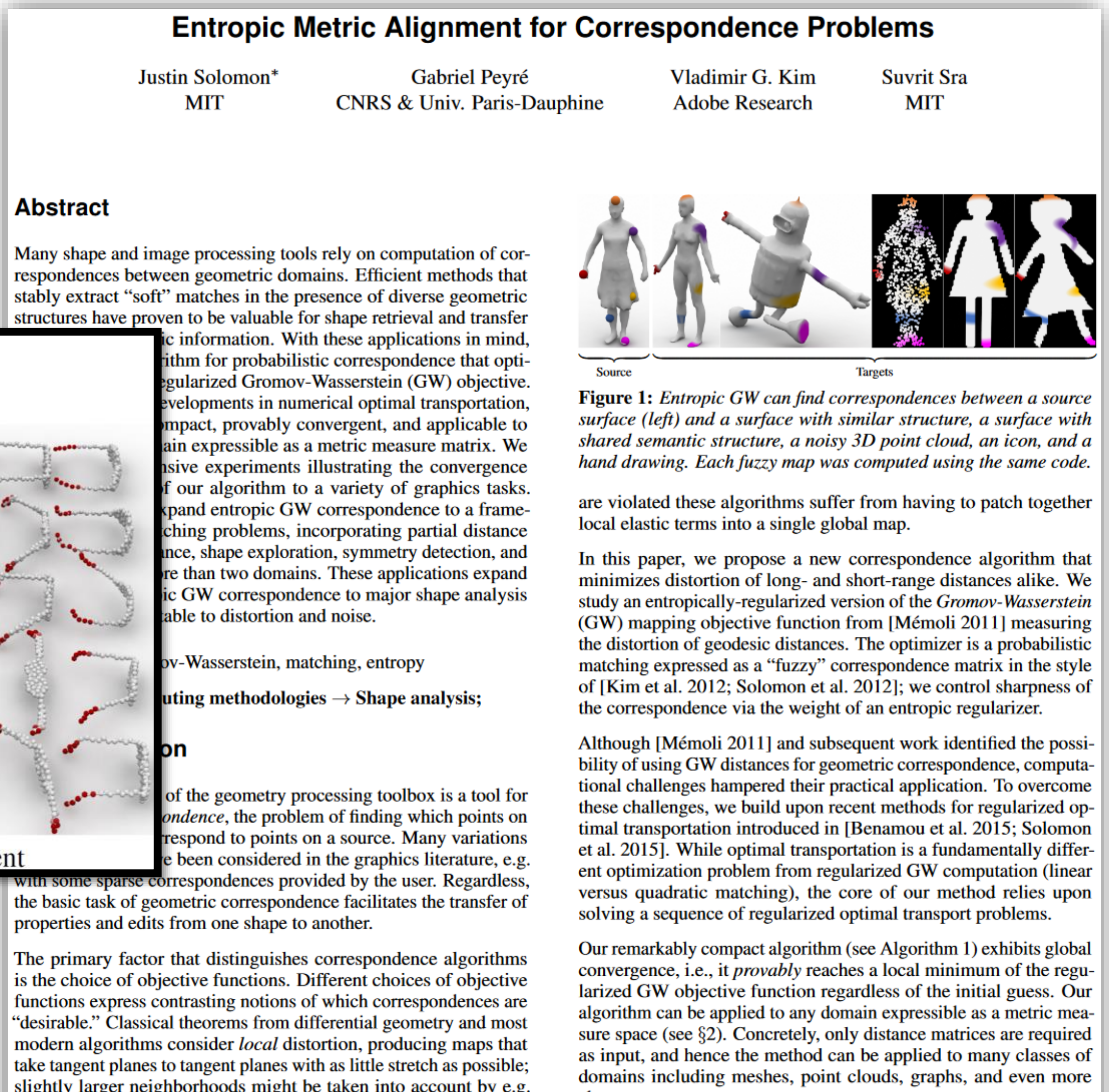
CGF 2017

Approximate Methods

Multiplicative updates for nonconvex nonnegative matrix factorization



$$\min_{A} \mathrm{KL}(G|AA^\top)$$



Entropic Metric Alignment for Correspondence Problems

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Gabriel Peyré — CNRS & Univ. Paris-Dauphine
Vladimir G. Kim — Adobe Research
Suvrit Sra — MIT

Abstract

Many shape and image processing tools rely on computation of correspondences between geometric domains. Efficient methods that stably extract "soft" matches in the presence of diverse geometric structures have proven to be valuable for shape retrieval and transfer ...ic information. With these applications in mind, ...rithm for probabilistic correspondence that opti... ...gularized Gromov-Wasserstein (GW) objective. ...evelopments in numerical optimal transportation, ...mpact, provably convergent, and applicable to ...ain expressible as a metric measure matrix. We ...nsive experiments illustrating the convergence ...f our algorithm to a variety of graphics tasks. ...xpand entropic GW correspondence to a frame... ...ching problems, incorporating partial distance ...nce, shape exploration, symmetry detection, and ...re than two domains. These applications expand ...ic GW correspondence to major shape analysis ...able to distortion and noise.

...ov-Wasserstein, matching, entropy

...uting methodologies → **Shape analysis;**

...on

...of the geometry processing toolbox is a tool for ...*ondence*, the problem of finding which points on ...respond to points on a source. Many variations ...e been considered in the graphics literature, e.g. with some sparse correspondences provided by the user. Regardless, the basic task of geometric correspondence facilitates the transfer of properties and edits from one shape to another.

The primary factor that distinguishes correspondence algorithms is the choice of objective functions. Different choices of objective functions express contrasting notions of which correspondences are "desirable." Classical theorems from differential geometry and most modern algorithms consider *local* distortion, producing maps that take tangent planes to tangent planes with as little stretch as possible; slightly larger neighborhoods might be taken into account by e.g.

Figure 1: *Entropic GW can find correspondences between a source surface (left) and a surface with similar structure, a surface with shared semantic structure, a noisy 3D point cloud, an icon, and a hand drawing. Each fuzzy map was computed using the same code.*

are violated these algorithms suffer from having to patch together local elastic terms into a single global map.

In this paper, we propose a new correspondence algorithm that minimizes distortion of long- and short-range distances alike. We study an entropically-regularized version of the *Gromov-Wasserstein* (GW) mapping objective function from [Mémoli 2011] measuring the distortion of geodesic distances. The optimizer is a probabilistic matching expressed as a "fuzzy" correspondence matrix in the style of [Kim et al. 2012; Solomon et al. 2012]; we control sharpness of the correspondence via the weight of an entropic regularizer.

Although [Mémoli 2011] and subsequent work identified the possibility of using GW distances for geometric correspondence, computational challenges hampered their practical application. To overcome these challenges, we build upon recent methods for regularized optimal transportation introduced in [Benamou et al. 2015; Solomon et al. 2015]. While optimal transportation is a fundamentally different optimization problem from regularized GW computation (linear versus quadratic matching), the core of our method relies upon solving a sequence of regularized optimal transport problems.

Our remarkably compact algorithm (see Algorithm 1) exhibits global convergence, i.e., it *provably* reaches a local minimum of the regularized GW objective function regardless of the initial guess. Our algorithm can be applied to any domain expressible as a metric measure space (see §2). Concretely, only distance matrices are required as input, and hence the method can be applied to many classes of domains including meshes, point clouds, graphs, and even more

Computer Vision Perspective

Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks

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Berkeley AI Research (BAIR) laboratory, UC Berkeley

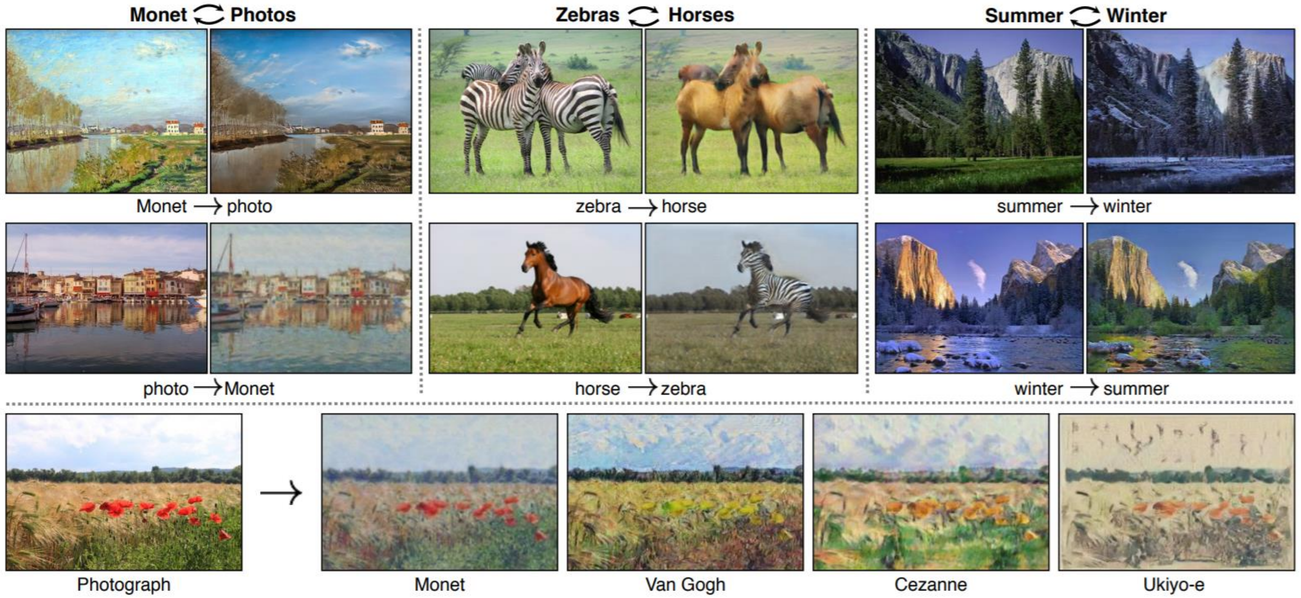


Figure 1: Given any two unordered image collections X and Y, our algorithm learns to automatically "translate" an image from one into the other and vice versa: *(left)* Monet paintings and landscape photos from Flickr; *(center)* zebras and horses from ImageNet; *(right)* summer and winter Yosemite photos from Flickr. Example application *(bottom)*: using a collection of paintings of famous artists, our method learns to render natural photographs into the respective styles.

Abstract

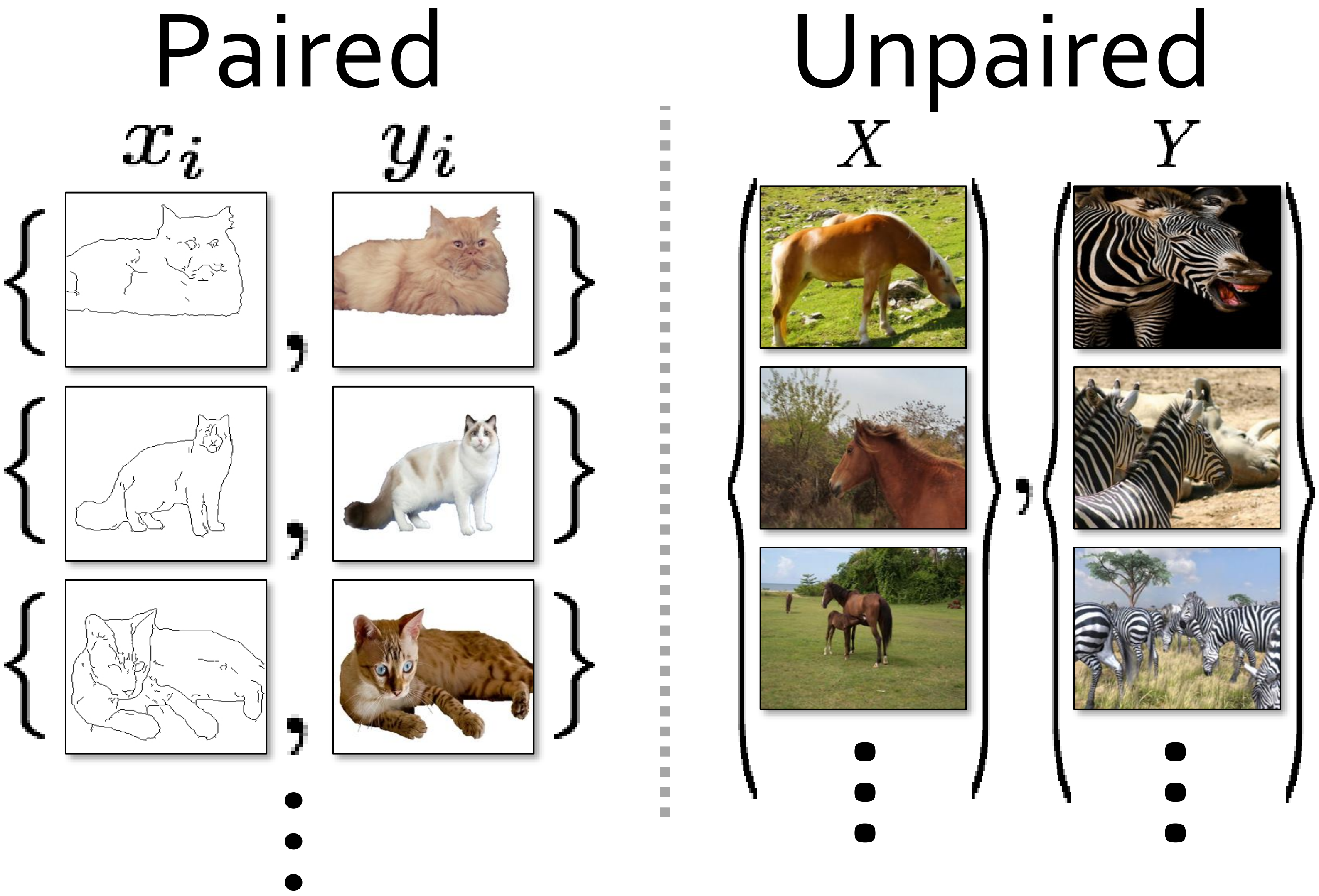
Image-to-image translation is a class of vision and graphics problems where the goal is to learn the mapping

1. Introduction

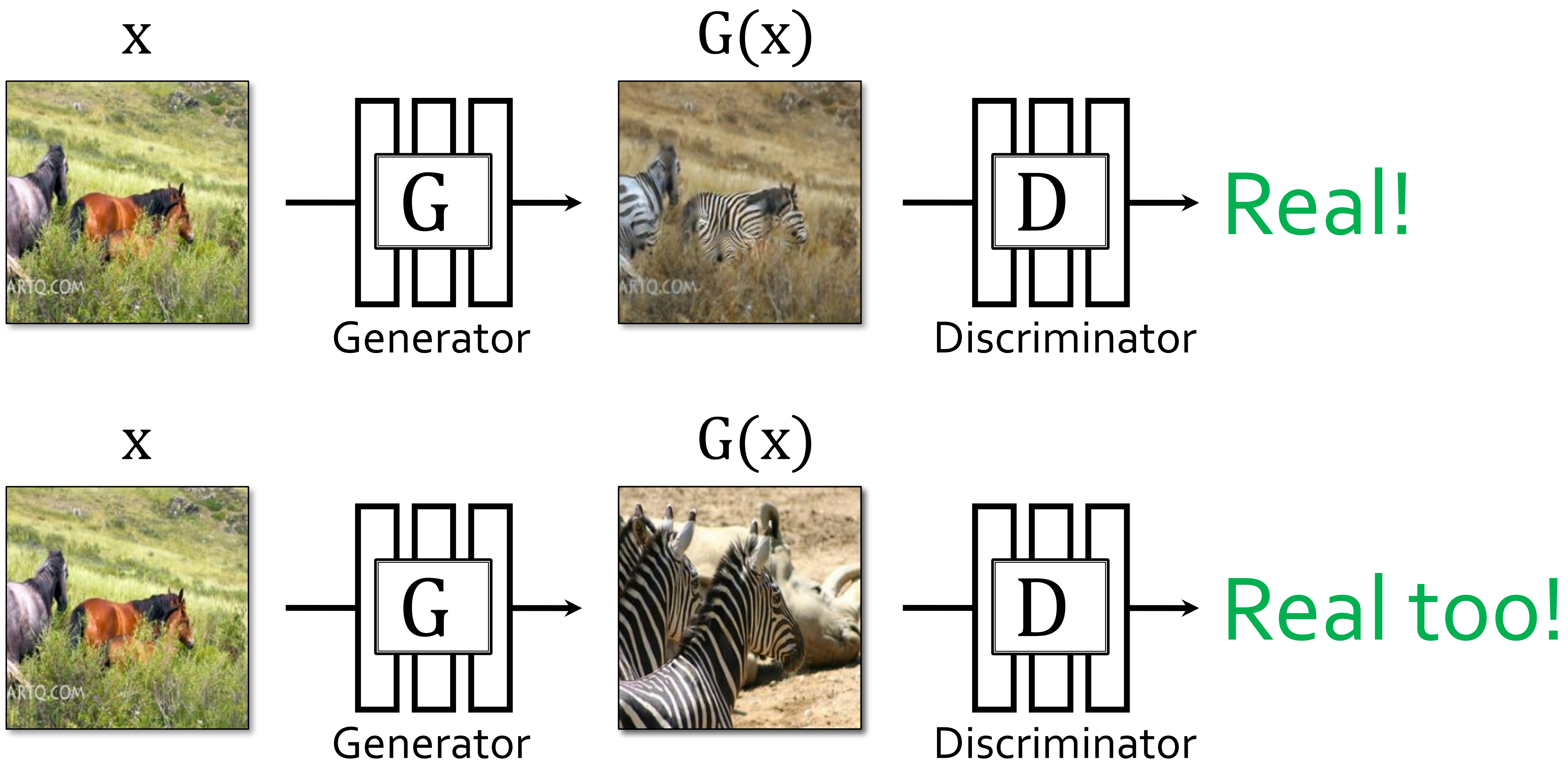
What did Claude Monet see as he place

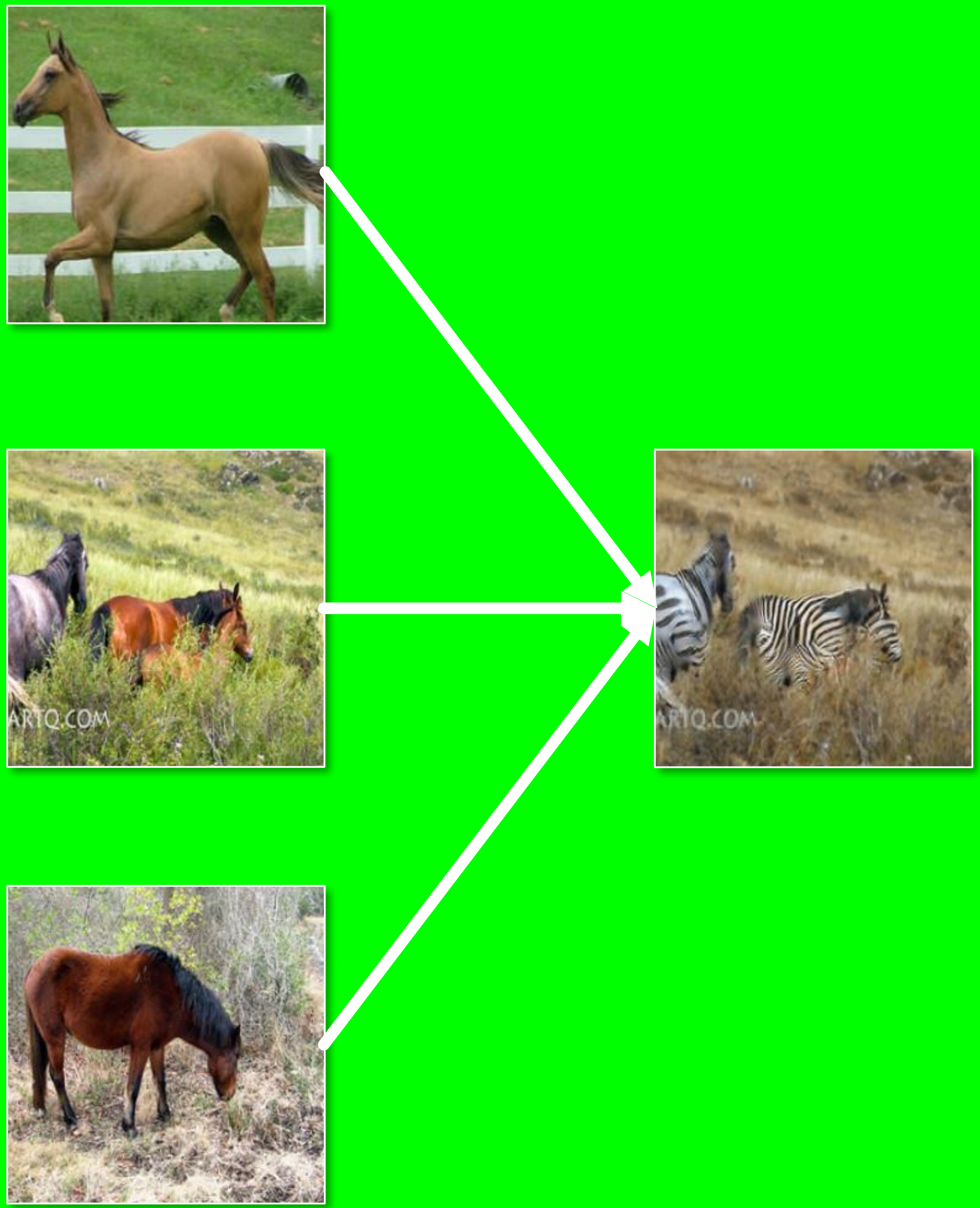
Slides courtesy the authors
https://junyanz.github.io/CycleGAN/

Paired vs. Unpaired Problems



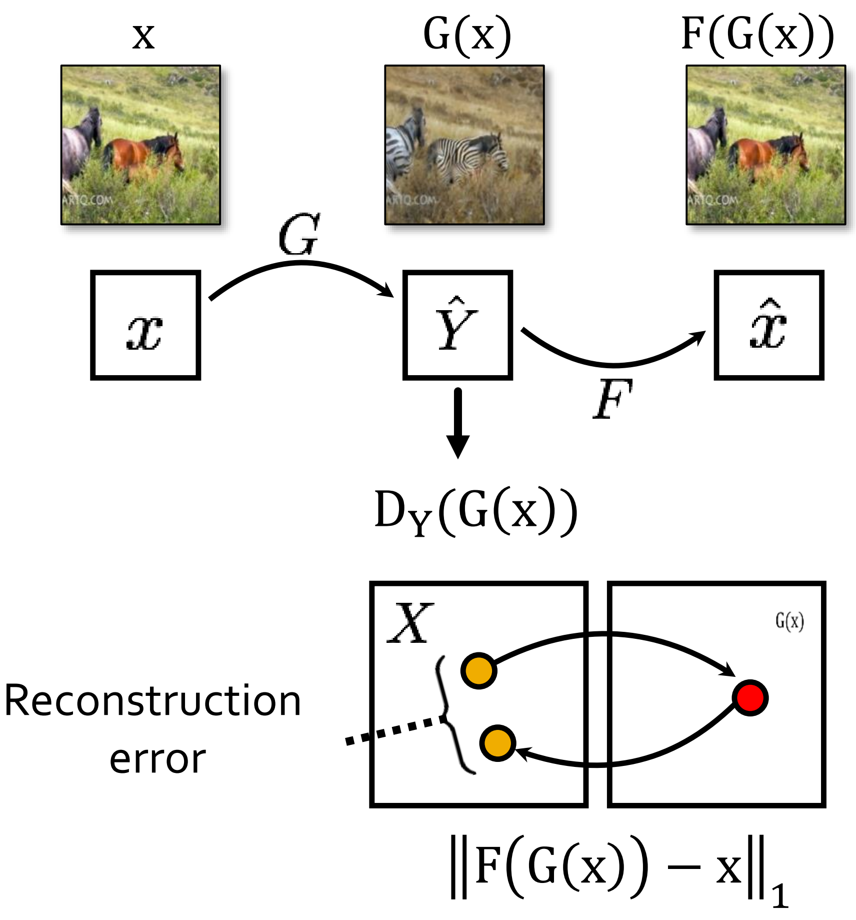
Adversarial Networks: Problem



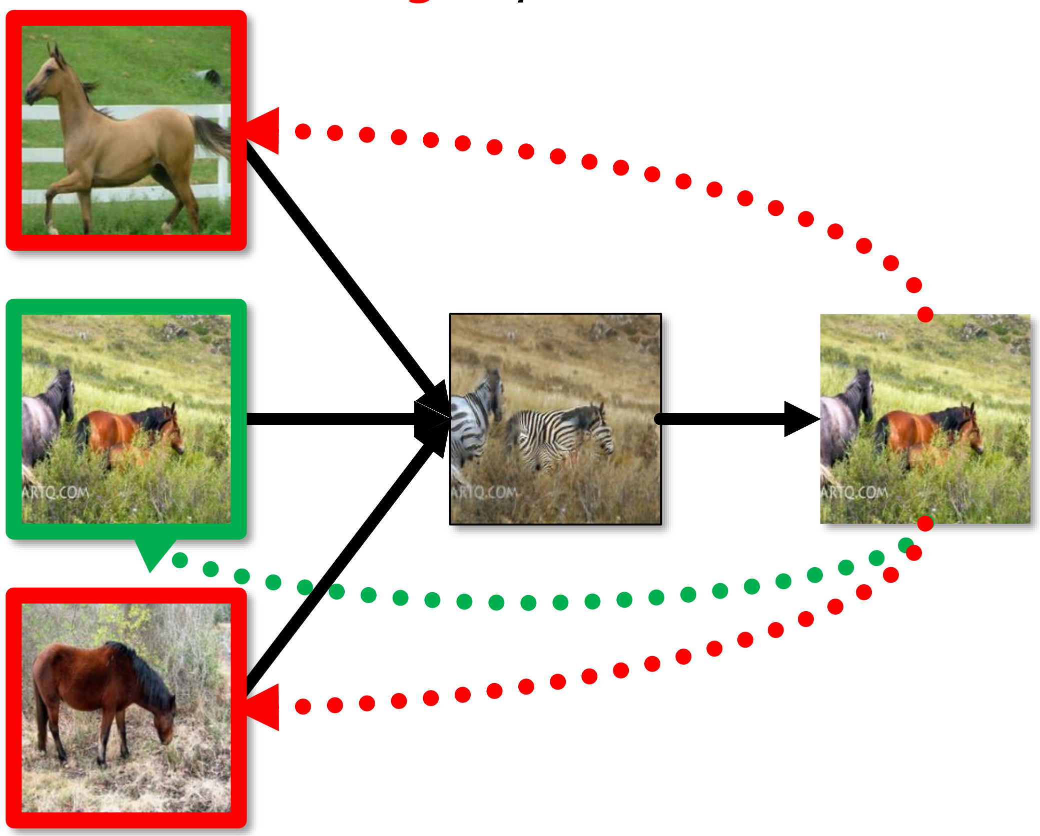


Mode collapse

Cycle-Consistent Adversarial Networks

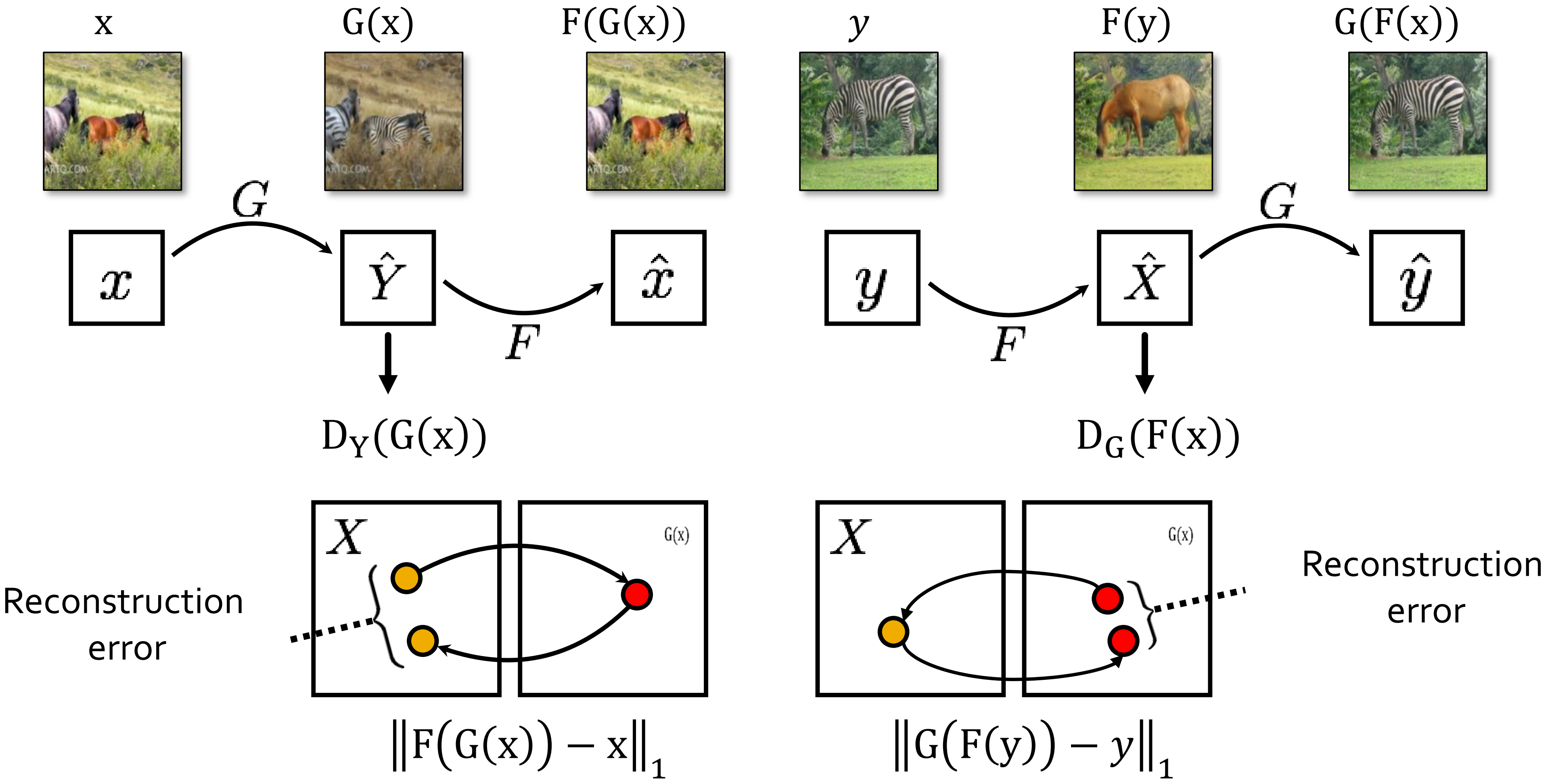


Small cycle loss
Large cycle loss



[Zhu*, Park*, Isola, and Efros, ICCV 2017]

Cycle Consistency Loss



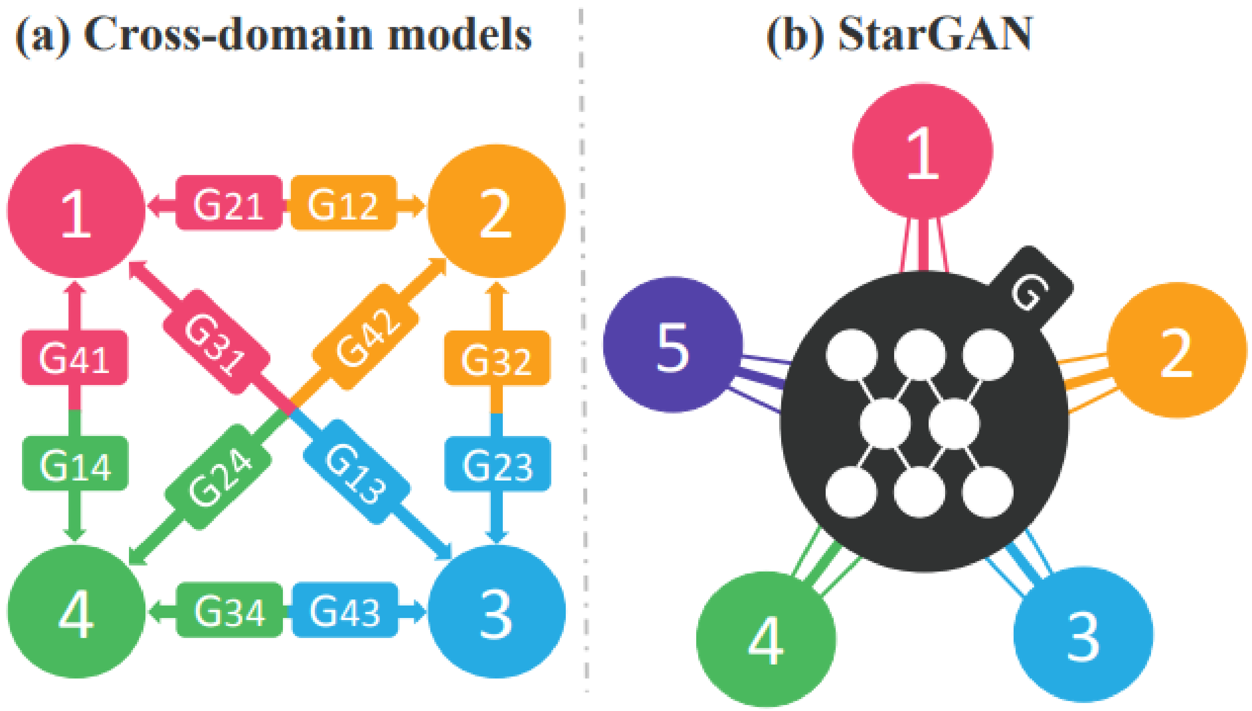
See similar formulations [Yi et al. 2017], [Kim et al. 2017]

[Zhu*, Park*, Isola, and Efros, ICCV 2017]



Input
Monet
Van Gogh
Cezanne
Ukiyo-e

More Than Two Domains?



Choi et al., CVPR 2018

Consistent Correspondence

Justin Solomon

6.838: Shape Analysis

Spring 2021

Extra: Angular Synchronization

Justin Solomon

6.838: Shape Analysis

Spring 2021