Reading 4: Calculus on Manifolds

Due April 9, 2019

REMINDER: The goal of this assignment is to explore how shape analysis can be used in a discipline *you* find interesting. We'll provide a few suggestions for further reading, but do not let these stop you from exploring! The papers we link are not necessarily the best in their respective fields, just places to get you started.

For this reading you can choose any paper on *vector fields*, *exterior calculus*, or the *Laplacian* operator. See the previous reading assignment for pointers for the Laplacian operator, including many in the machine learning literature.

Discrete Exterior Calculus

A lot of the continuous language of exterior calculus translates quite beautifully to the discrete case. Here's a few representative papers:

- **Discrete Exterior Calculus**, Desbrun et al., arXiv 2005.
- **Weighted Triangulations for Geometry Processing**, de Goes et al., TOG 2014.
- **HOT: Hodge-Optimized Triangulations**, Mullen et al., SIGGRAPH 2011.
- **Subdivision Exterior Calculus for Geometry Processing**, de Goes et al., TOG 2016.

An alternative technique applies ideas we've developed in earlier lectures involving the Finite Element Method (FEM). If you're looking for a challenge, this neglected mathematical work likely has applications/interest in computer science that are largely neglected (so far!):

- **Finite Element Exterior Calculus, Homological Techniques, and Applications**, Arnold et al., Acta Numerica 2006.

Vector Fields

Vector fields are fundamental objects in differential geometry. The following papers give an introduction to possible ways of discretizing vector fields on discrete domains, alongside a few applications:

- **Vector Field Processing on Triangle Meshes**, de Goes et al., SIGGRAPH Asia course notes, 2015.
- **Directional Field Synthesis, Design, and Processing**, Vaxman et al., Eurographics 2016.
- **Trivial Connections on Discrete Surfaces**, Crane et al., SGP 2010.
- **Discrete Connection and Covariant Derivative for Vector Field Analysis and Design**, Liu et al., TOG 2016.

- **On Discrete Killing Vector Fields and Patterns on Surfaces**, Ben-Chen et al., SGP 2010. (the topic of your instructor's undergraduate thesis!)
- **An Operator Approach to Tangent Vector Field Processing**, Azencot et al., SGP, 2013.
- **Functional Characterization of Deformation Fields**, Corman & Ovsjanikov, TOG 2019.

The first two papers above have a wealth of references to other work on vector field design/discretization.

Helmholtz-Hodge Decomposition

A few discretizations and applications are available for the Hodge decomposition of fields:

- **The Helmholtz-Hodge Decomposition – A Survey**, Bhatia et al., TVCG 2013.
- **Applications of the Discrete Hodge Helmholtz Decomposition to Image and Video Processing**, Palit, Basu and Mandal, LNCS.
- **Discrete Multiscale Vector Field Decomposition**, Tong et al., TOG 2003.
- **Helmholtz-Hodge Decomposition of Scalar Optical Fields**, Bahl and Senthilkumaran, Opt. Soc. Am. A 2012.

One paper attempts to extend this theory to problems on graphs:

- **Statistical Ranking and Combinatorial Hodge Theory**, Jiang et al., J. Mathematical Programming 127.1, 2011.

Applications

Here is just a sampling of computational applications that use field techniques similar to the ones we've covered:

- **Stable Fluids**, Stam, SIGGRAPH 1999.
- **Realistic Solar Convection Simulations**, Stein and Nordlund, Solar Physics 2000.
- **Learning Divergence-Free and Curl-Free Vector Fields with Matrix-Valued Kernels**, Macedo and Castro, IMPA 2010.
- **Designing N-PolyVector Fields with Complex Polynomials**, Diamanti et al., SGP 2014.
- **Tensor Field Networks: Rotation- and Translation-Equivariant Neural Networks for 3D Point Clouds**, Thomas et al., ArXiv 2018.
- **Mixed-Integer Quadrangulation**, Bommes et al., TOG, 2009.
- **Integer-Grid Maps for Reliable Quad Meshing**, Bommes et al., TOG 2013.
- **From Numerics to Combinatorics: A Survey of Topological Methods for Vector Field Visualization**, Wang et al., Journal of Visualization, 2016.
- **Higher Dimensional Vector Field Visualization: A Survey**, Peng & Laramee, EG Theory and Practice of Computer Graphics, 2009.
- **Efficient Morse Decompositions of Vector Fields**, Chen et al., TVCG, 2008.
- **Statics Aware Grid Shells**, Pietroni et al., CGV, 2015.
- **Bijective Maps from Simplicial Foliations**, Campen et al., TOG, 2016.
- **Vector Field k-Means: Clustering Trajectories by Fitting Multiple Vector Fields**, Ferriera et al., TOG 2013.

- **Bendfields: Regularized Curvature Fields from Rough Concept Sketches**, Iarussi et al., TOG 2015.
- **Instant Field-Aligned Meshes**, Jakob et al., TOG 2015.
- **Singularity-Constrained Octahedral Fields for Hexahedral Meshing**, Liu et al., TOG 2018.
- **An Approach to Quad Meshing Based on Harmonic Cross-Valued Maps and the Ginzburg–Landau Theory**, Viertel & Osting, ArXiv, 2017.
- **Illustrating Smooth Surfaces**, Hertzmann & Zorin, SIGGRAPH 2000.
- **On Learning Vector-Valued Functions**, Micchelli & Pontil, 2003.
- **Geodesic Distance Function Learning via Heat Flow on Vector Fields**, Lin et al., ICML 2014.
- **FPNN: Field Probing Neural Networks for 3D Data**, Li et al., NIPS 2016.
- **Multi-task Vector Field Learning**, Lin et al., NIPS 2012.
- **Rotation Equivariant Vector Field Networks**, Marcos et al., ICCV 2017.

These slides contain many references at the end to interesting papers on using vector field topology to help visualization in scientific computing applications: `https://www3.nd.edu/~cwang11/research/vis13-tutorial-chen.pdf`

Some interesting papers make use of vector field language to analyze recent algorithms in machine learning:

- **The Numerics of GANs**. Mescheder & Nowozin, NIPS 2017.
- **What Regularized Auto-Encoders Learn from the Data Generating Distribution**. Alain & Bengio, ICLR 2013.