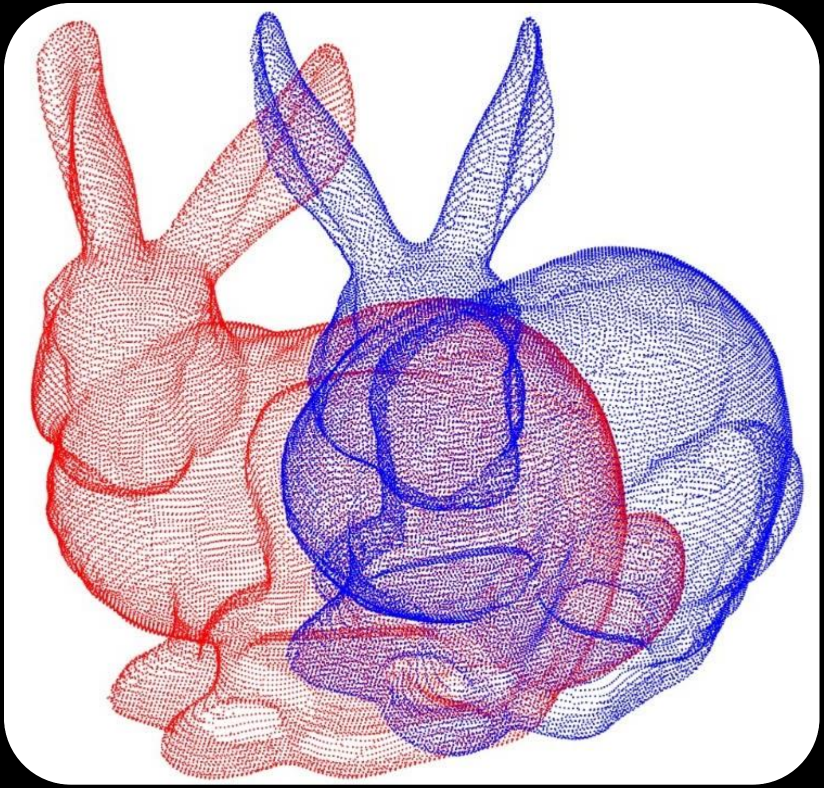
https://i.ytimg.com/vi/uzOCS_gdZuM/maxresdefault.jpg



Registration and Alignment

Justin Solomon

MIT, Spring 2017

MIT EECS

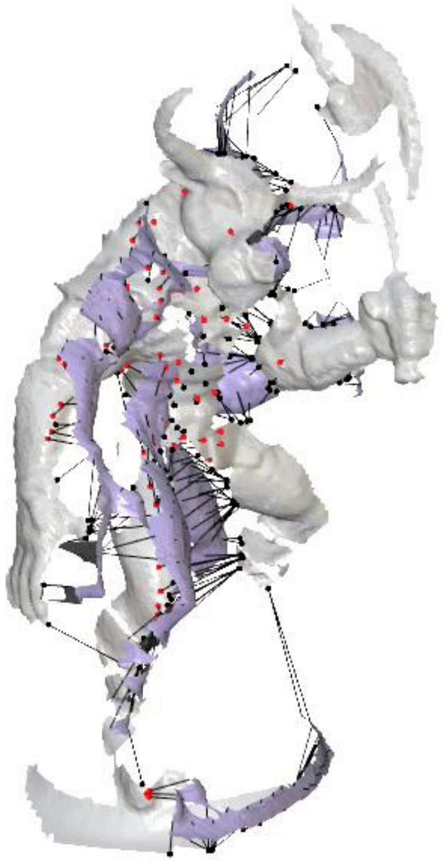
Acknowledgements

Many slides from

- Szymon Rusinkiewicz, Princeton
  ICCV Course, 2005

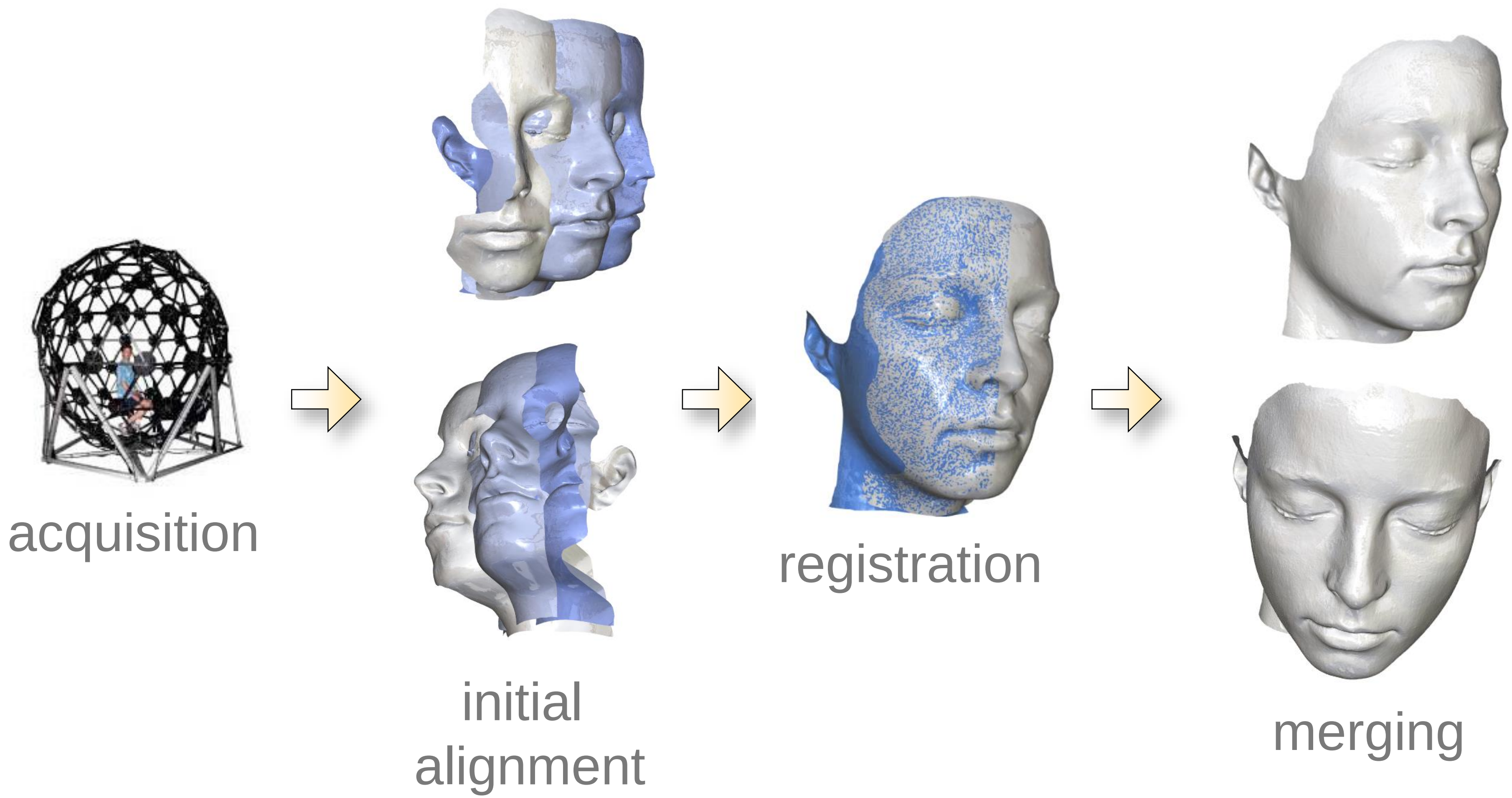
- Hao Li, USC
  CSCI 599, 2015

Registration Problem



Align two overlapping objects

3D Reconstruction Pipeline



Data provided by Paramount Pictures and Aguru Images

Rough Plan

- ICP algorithm

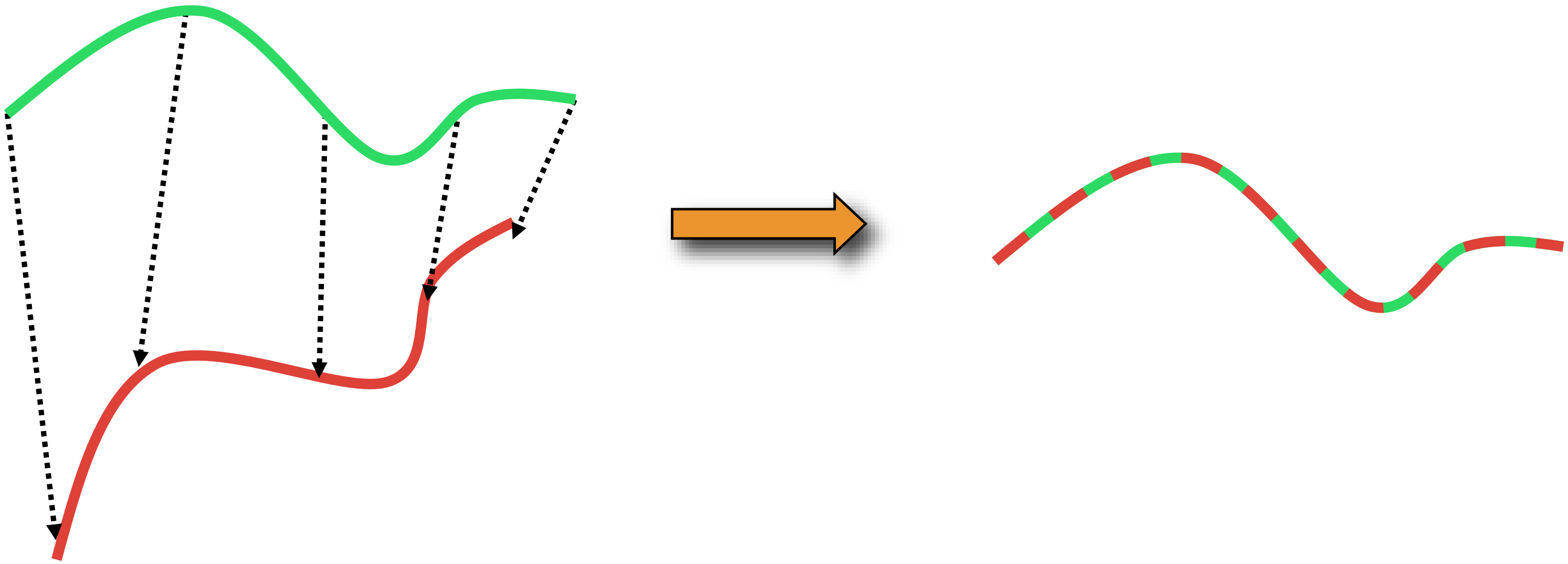
  A classic!

- ICP variants

- Related problems

  Synchronization, non-rigid registration

Starting Point



$$q_i = Rp_i + t$$

Can align given enough matches

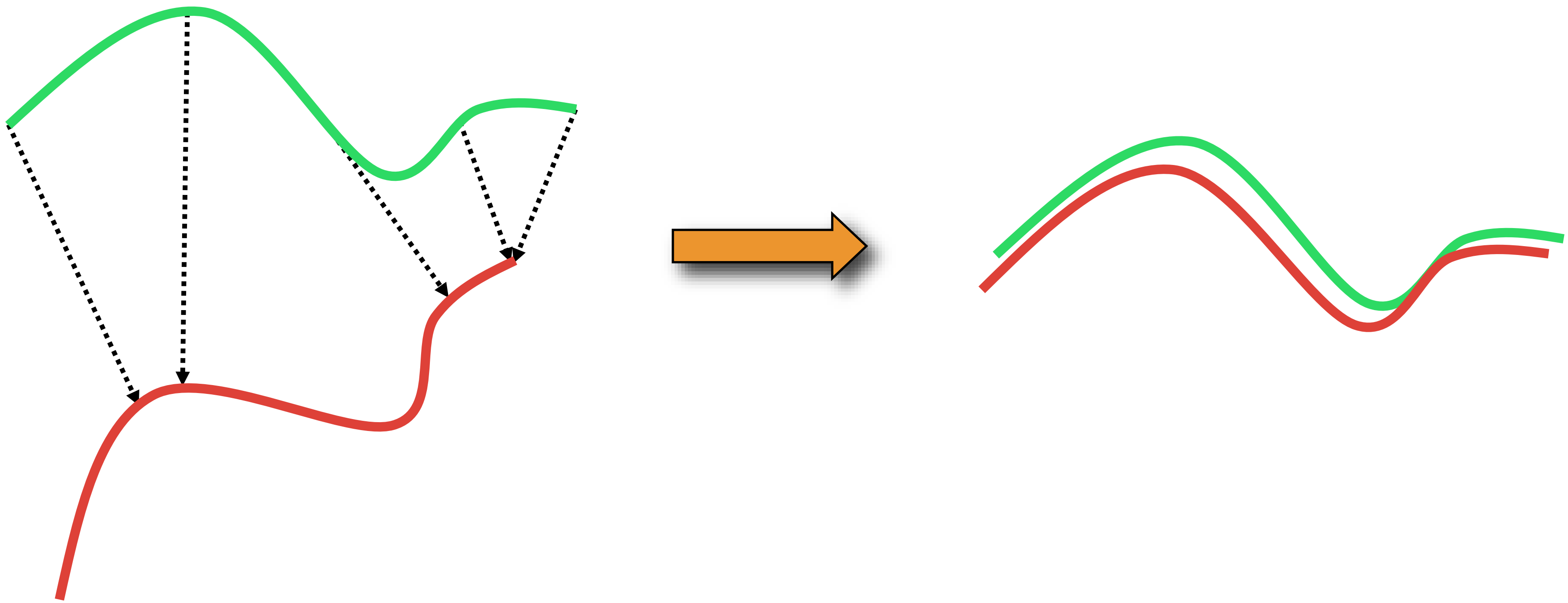
Q:

How many correspondences determine R and t?

Q:

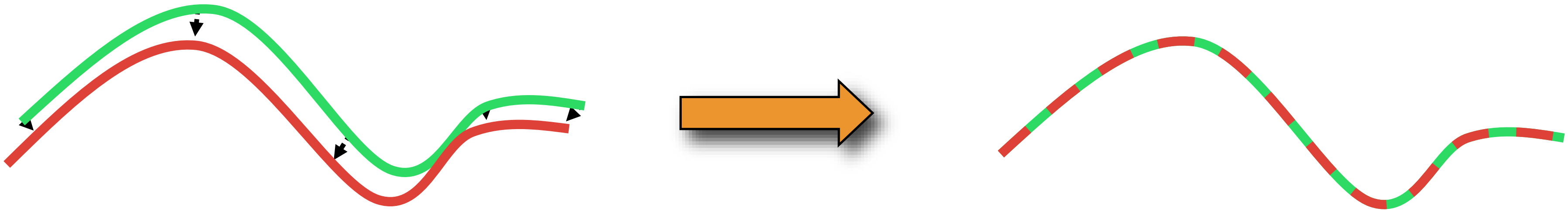
How do you get correspondences?

Rough Approximation



Closest points correspond

Try a Second Time…



Iterative Closest Point (ICP)

- Choose e.g. 1000 random points
- Match each to closest point on other scan
- Reject pairs with distance > *k* times median
- Minimize

$$E[R,t] := \sum_i \|Rp_i + t - q_i\|^2$$

- Iterate

"A method for registration of 3-D shapes."
Besl and McKay, PAMI 1992.

On the Board

$$\min_{t \in \mathbb{R}^3,\ R^\top R = I} \sum_i \|Rp_i + t - q_i\|^2$$

Closed-form formulas!

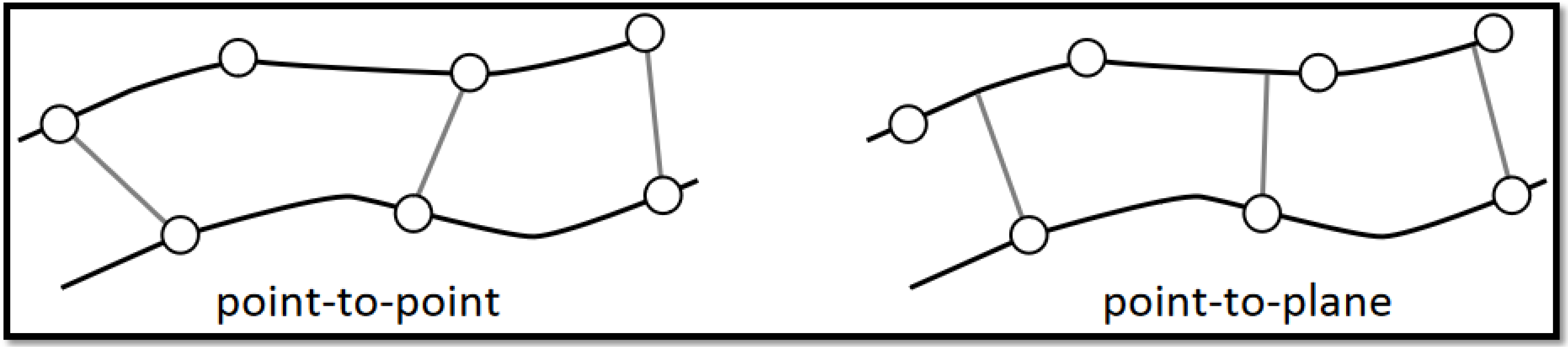
Many (!) Variants of ICP

- Source points from one or both meshes
- Matching to points in the other mesh
- Weighting correspondences
- Rejecting outlier point pairs
- Alternative error metrics

See [Rusinkiewicz & Levoy, 3DIM 2001]

Point-to-Plane Error Metric

Flat parts can slide along each other



$$E[R,t] := \sum_i ((Rp_i + t - q_i)^\top n_i)^2$$

$$\approx \sum_i [(p_i - q_i)^\top n_i + r^\top (p_i \times n_i) + t^\top n_i)^2 \text{ after linearizing}$$

where $r := (r_x, r_y, r_z)$

Least-squares!

"Object modelling by registration of multiple range images"
Chen and Medioni, Image and Vision Computing 10.3 (1992); image courtesy N. Mitra

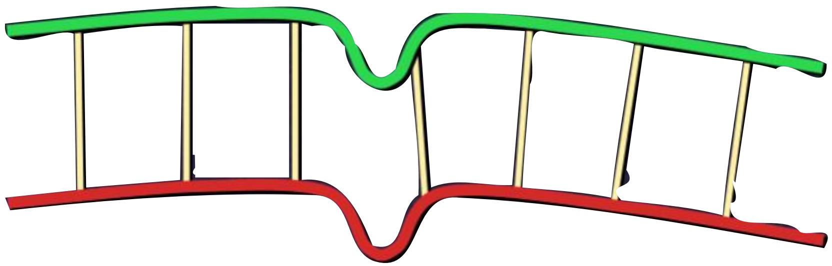
Closest Compatible Point

Can improve matching effectiveness by restricting match to compatible points

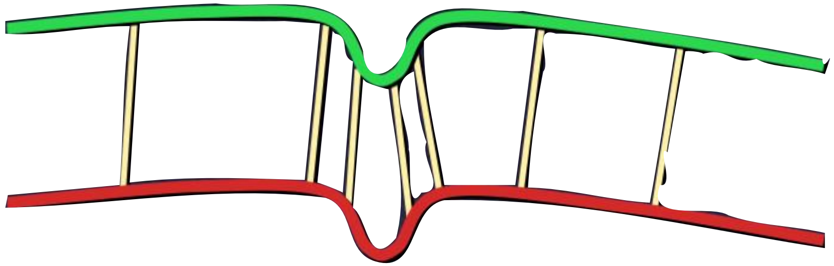
- Compatibility of colors [Godin et al. 94]
- Compatibility of normals [Pulli 99]
- Other possibilities:
  curvatures, higher-order derivatives, and other local features

Slide courtesy N. Mitra

Choose Points to Improve Stability



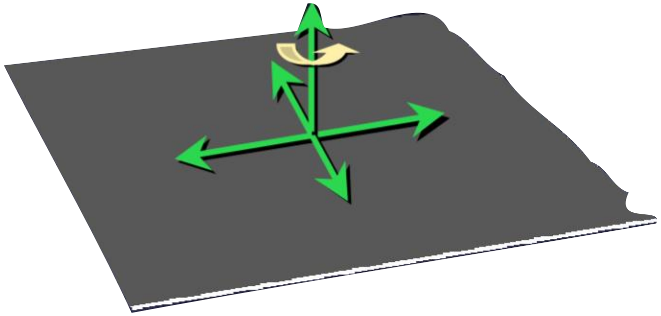
Uniform Sampling



Stable Sampling

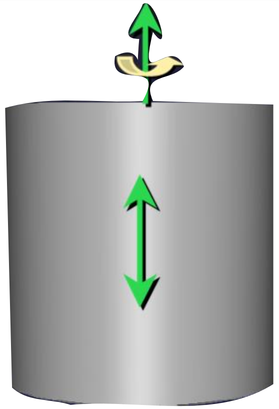
Sample discriminative points

Local Covariance

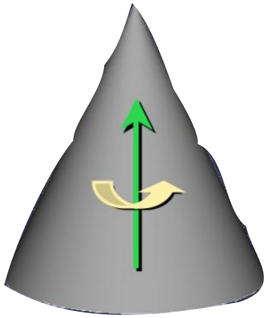


3 small eigenvalues
2 translation
1 rotation

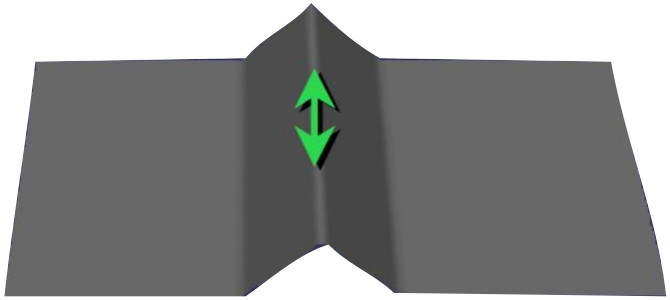
3 small eigenvalues
3 rotation



2 small eigenvalues
1 translation
1 rotation



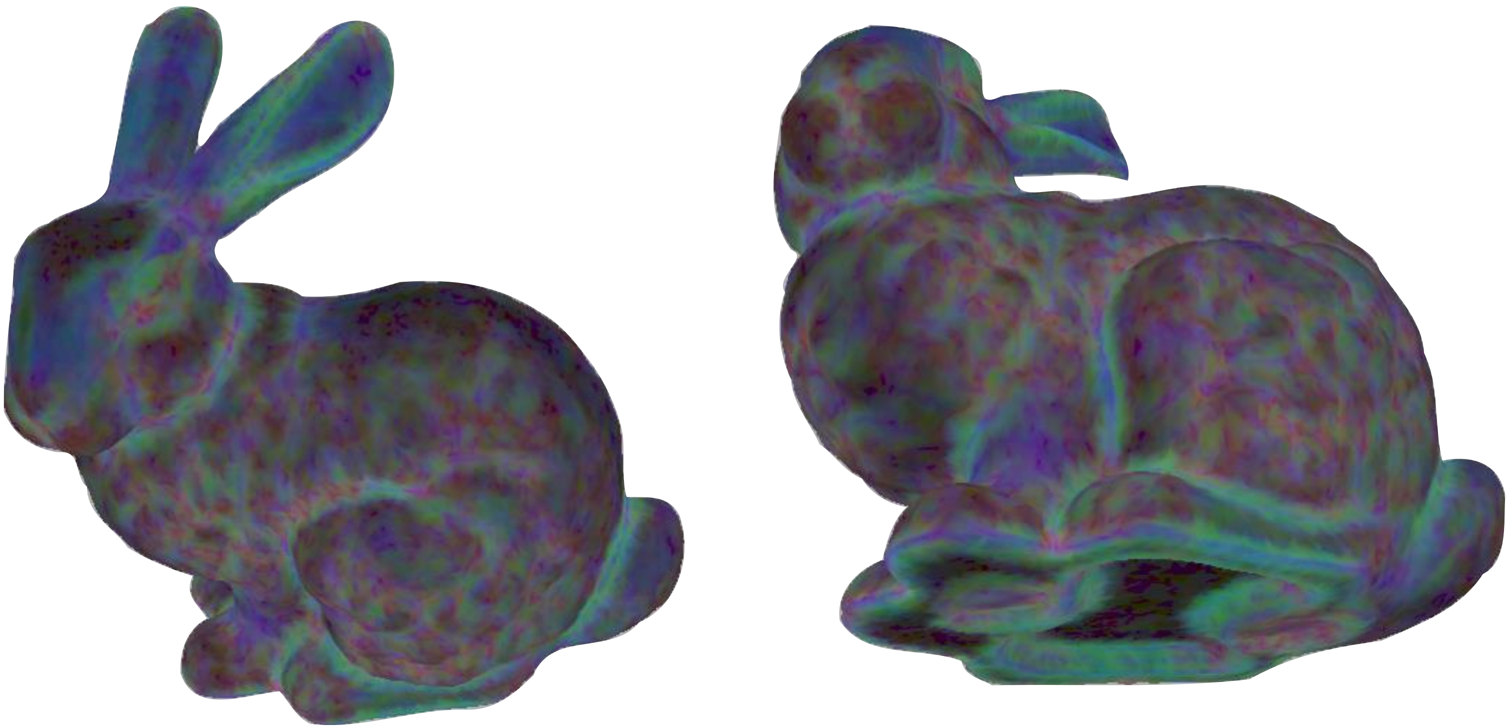
1 small eigenvalue
1 rotation



1 small eigenvalue
1 translation

[Gelfand et al. 2004]

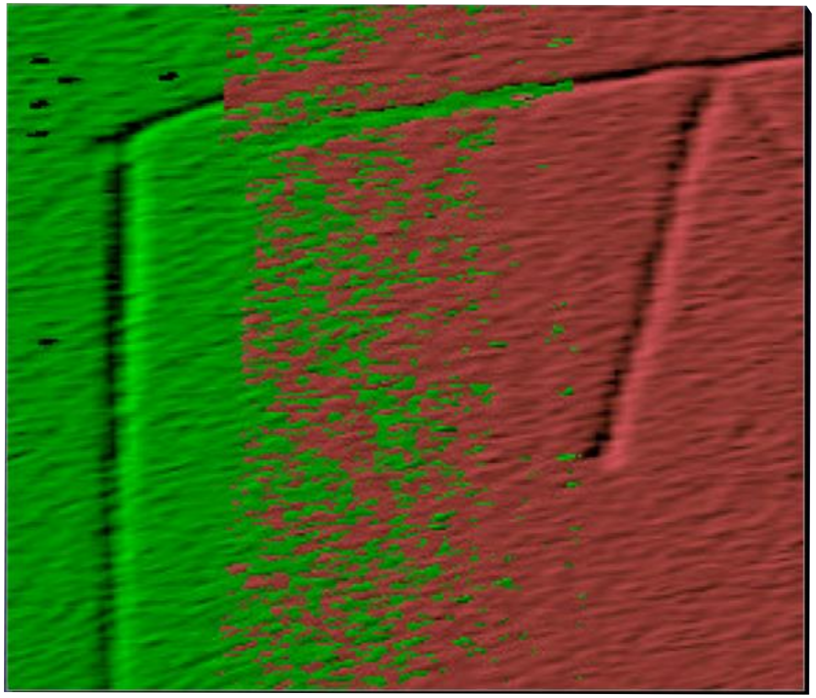
Stability Analysis



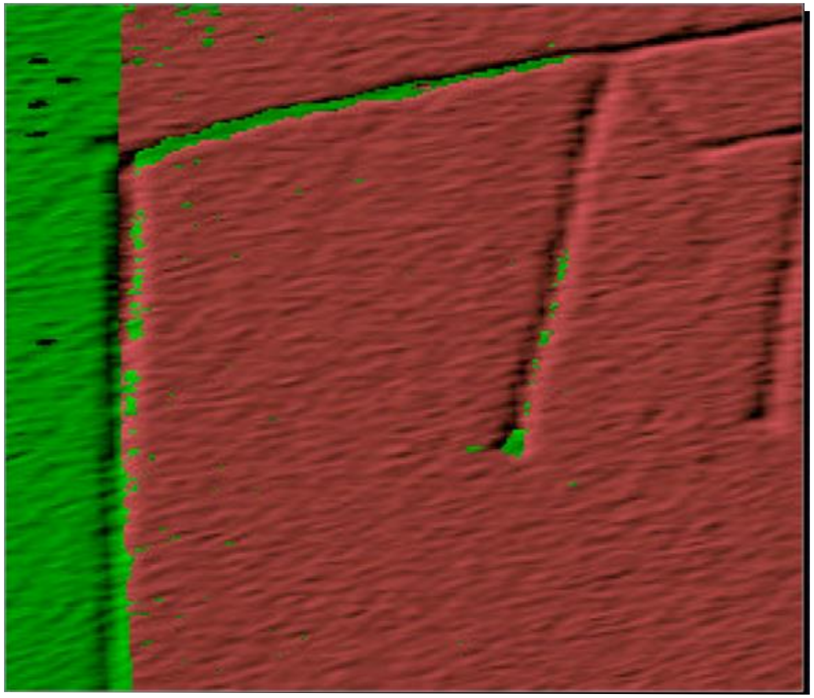
Key:

■ (black)	3 DOFs stable	■ (green)	5 DOFs stable
■ (blue)	4 DOFs stable	■ (red)	6 DOFs stable

Alternative: Uniform Normals



Random Sampling

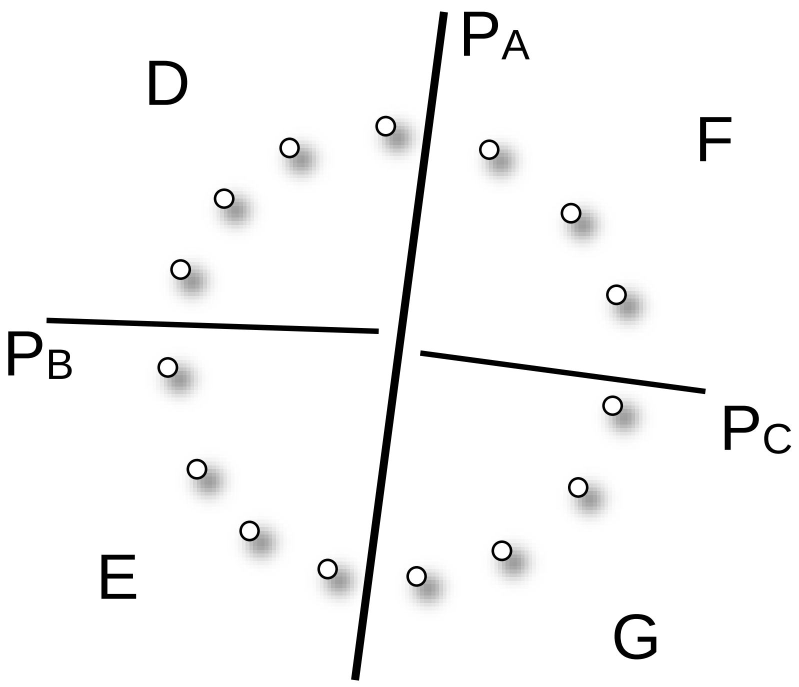


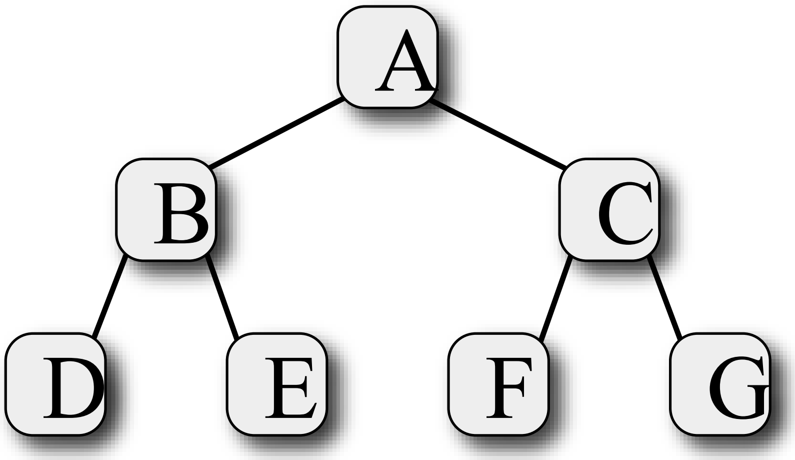
Normal-space Sampling

Q:

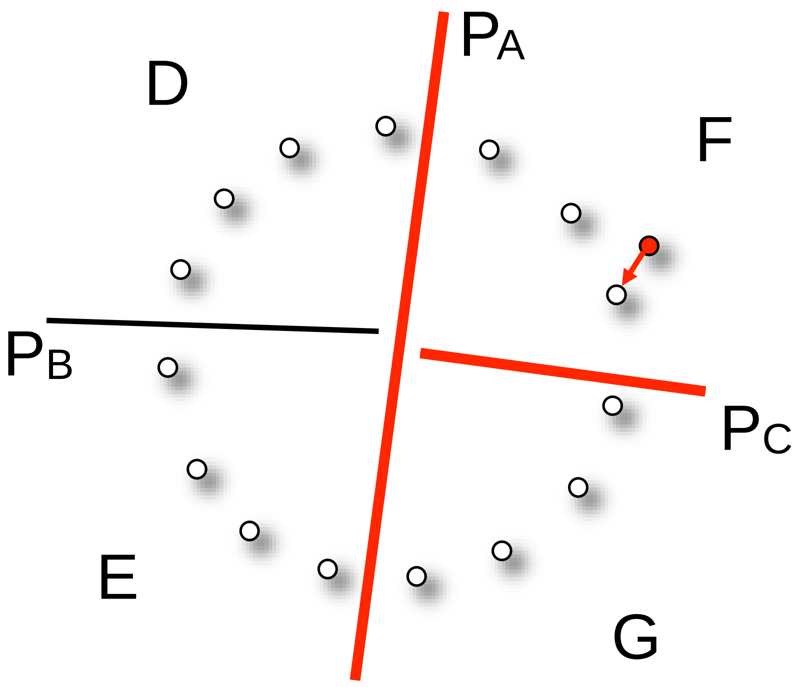
What is the **bottleneck** of ICP iteration?

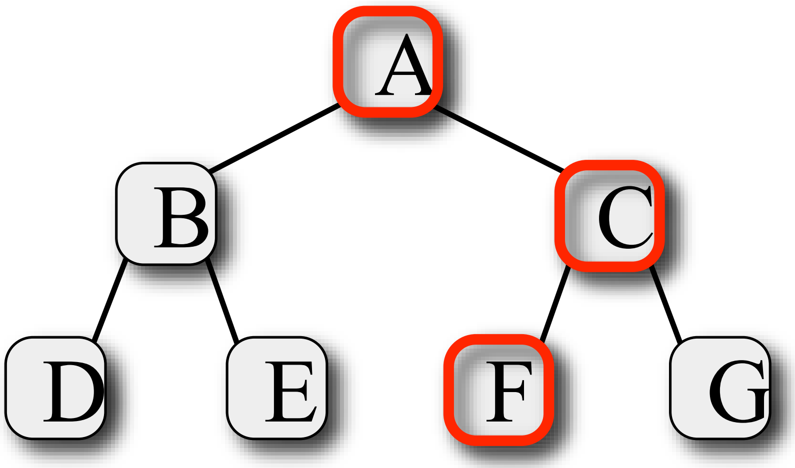
BSP Tree



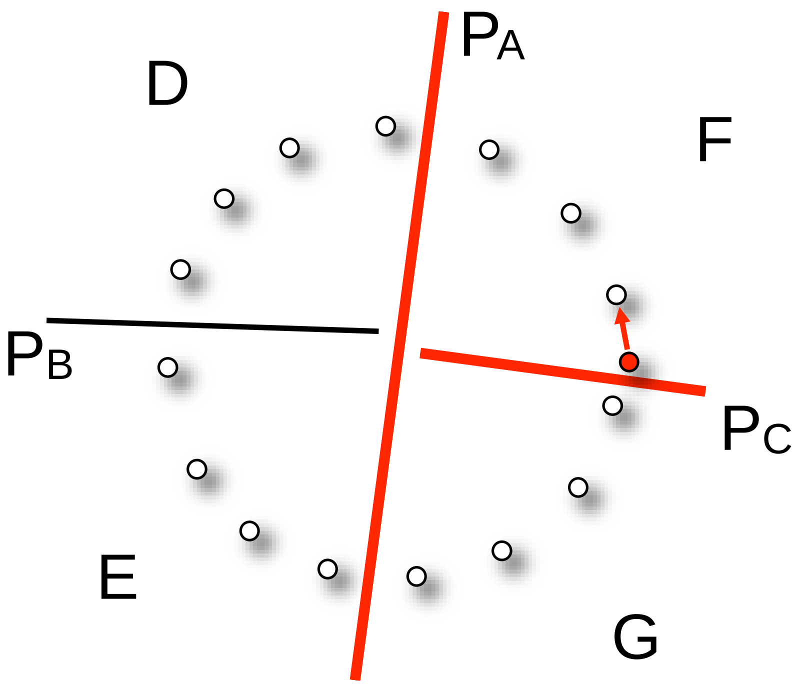


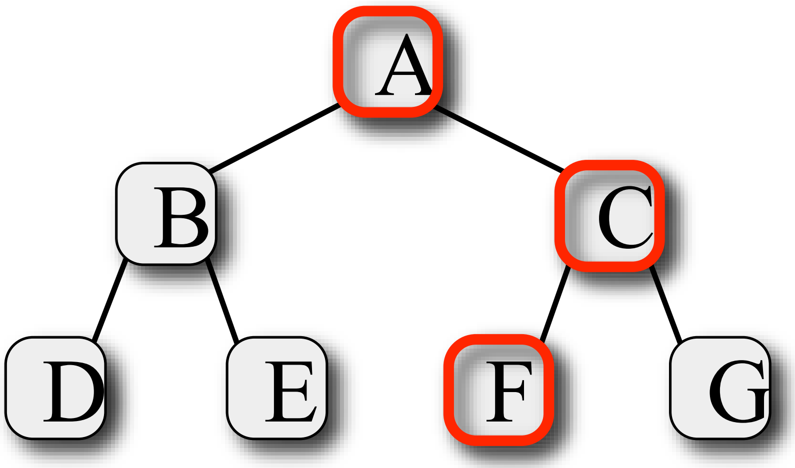
Tree Traversal



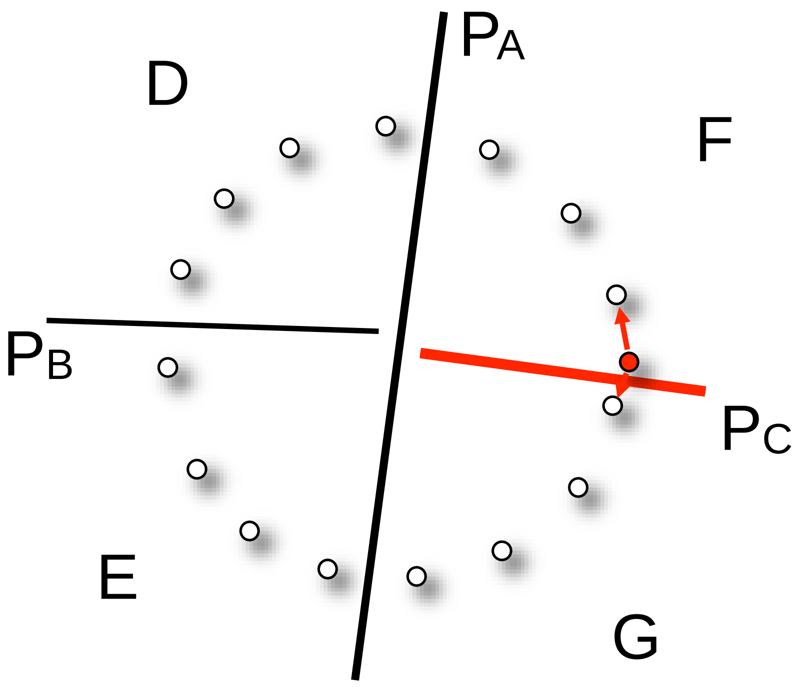


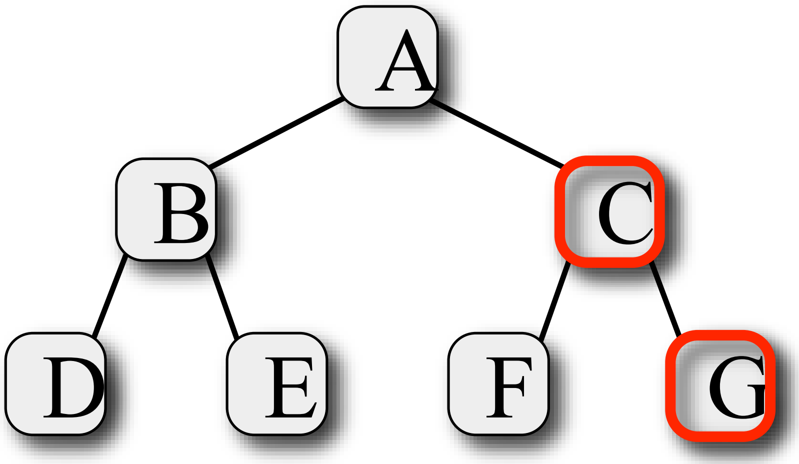
Subtlety: Is this right?





Two Possibilities





Pseudocode: Be Conservative!

```
BSPNode::dist(Point x, Scalar& dmin){
  if (leaf_node())
    for each sample point p[i]
      dmin = min(dmin, dist(x, p[i]));
  else {
    d = dist_to_plane(x);
    if (d < 0) {
      left_child->dist(x, dmin);
      if (|d| < dmin) right_child->dist(x, dmin);
    } else {
      right_child->dist(x, dmin);
      if (|d| < dmin) left_child->dist(x, dmin);
}}}
```

k-d Tree

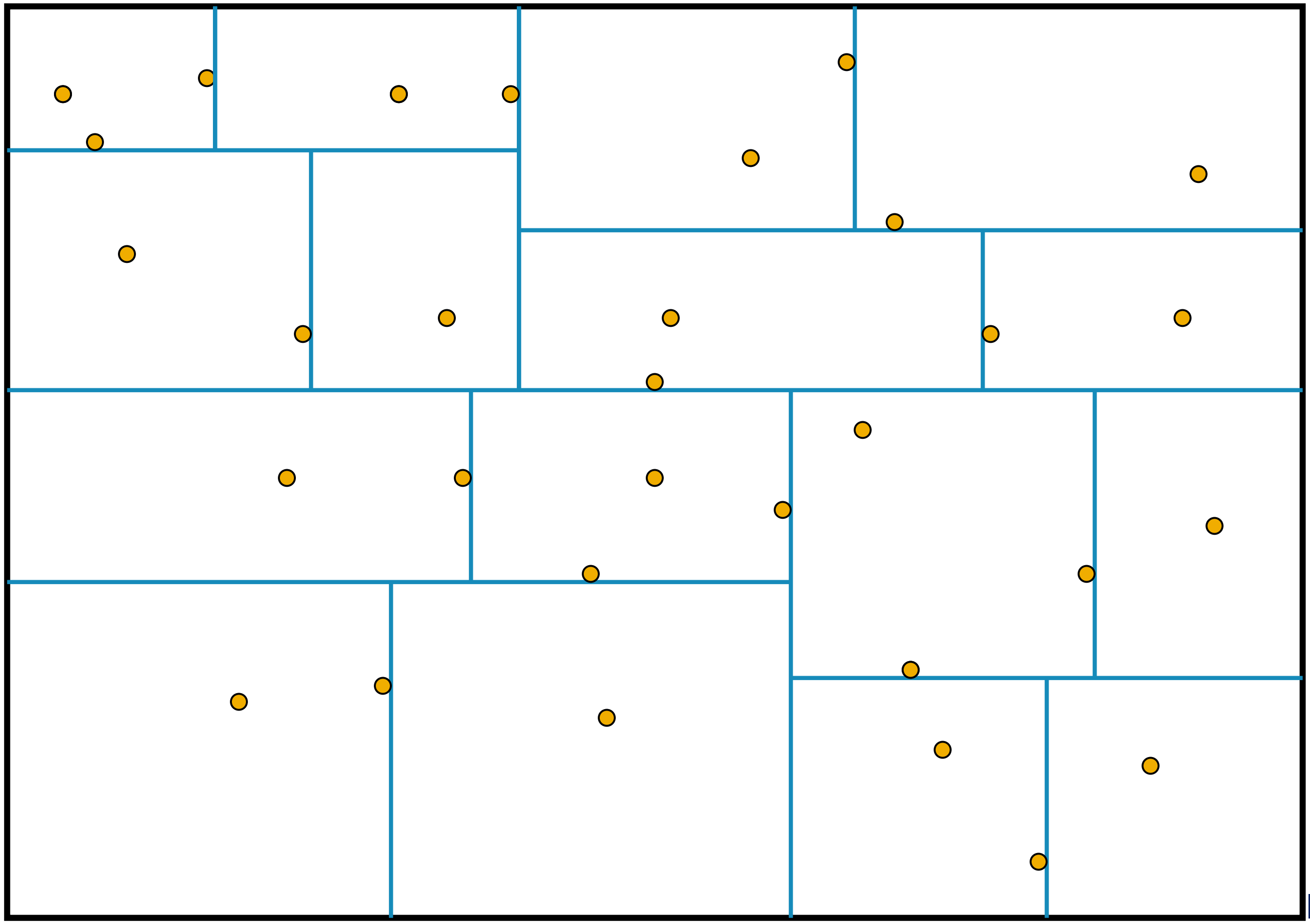


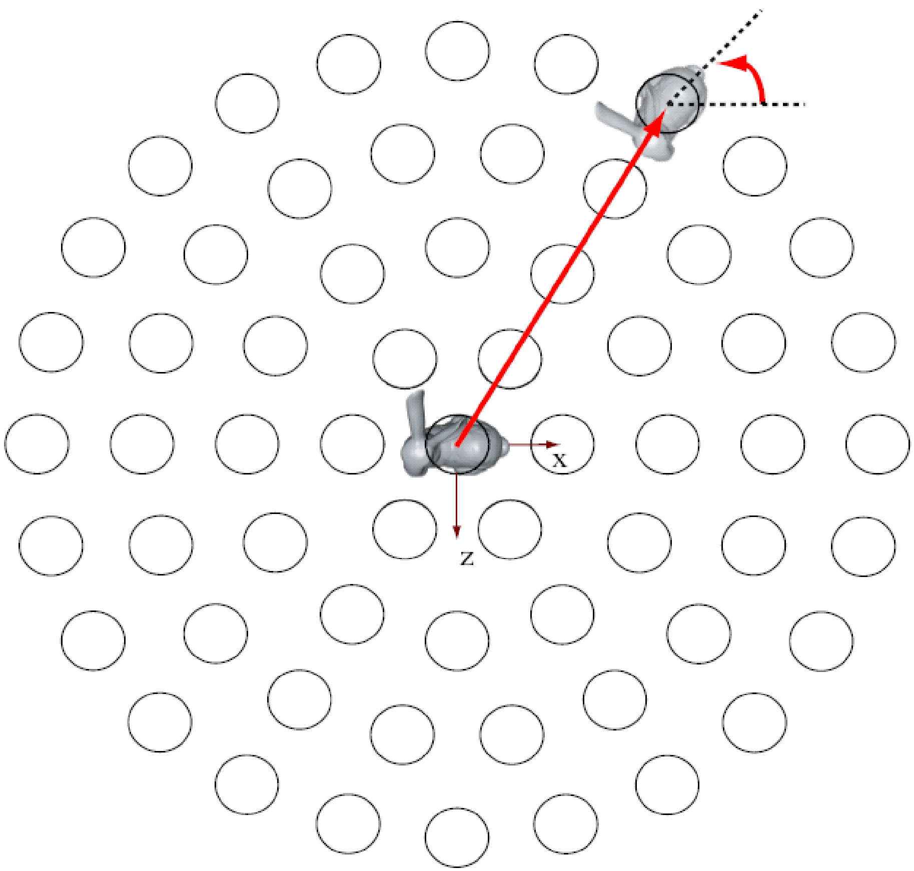
Image courtesy R. Gvili

Axis-aligned tree

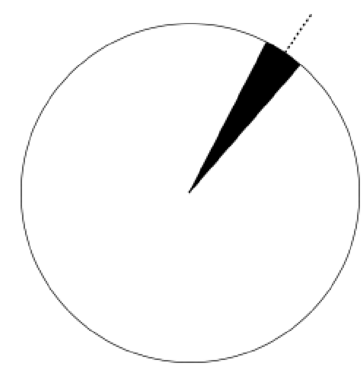
Q:

Speed aside, is ICP **always successful**?

Convergence Funnel Visualization



Translation in xz plane
Rotation about y

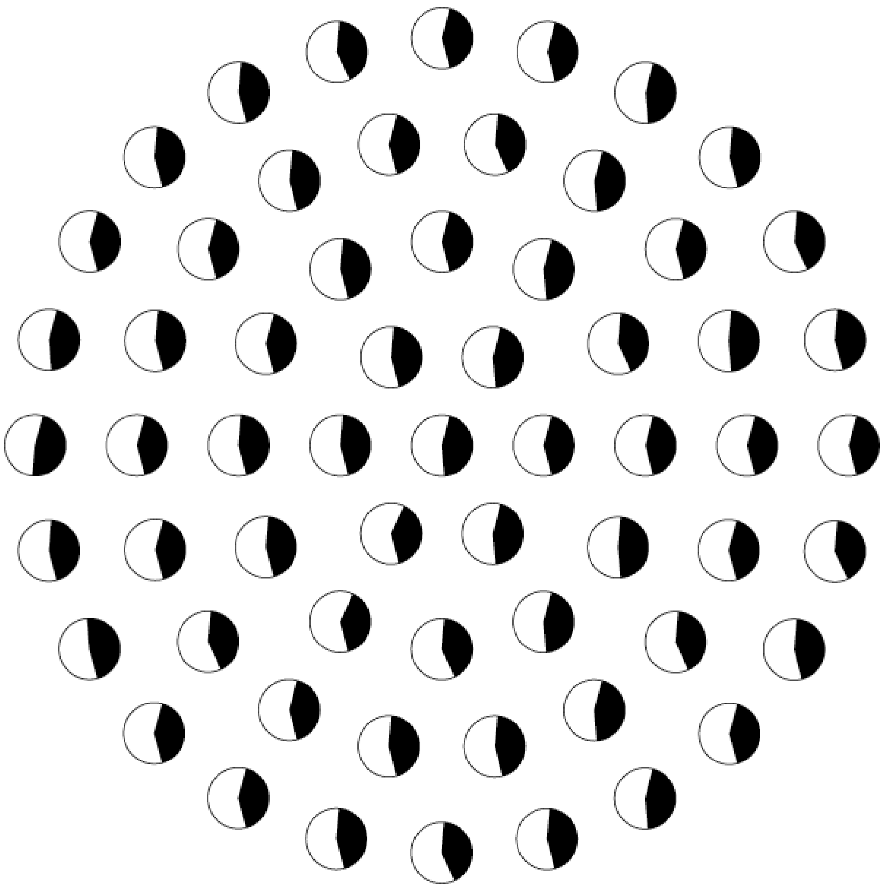


Converges

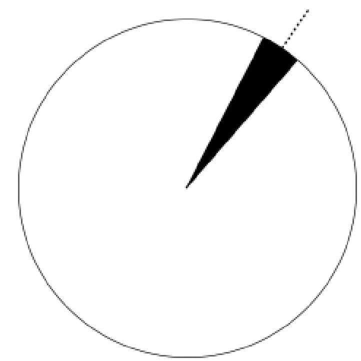
Does not converge

Slide courtesy N. Mitra

Distance Field Method



Translation in xz plane
Rotation about y

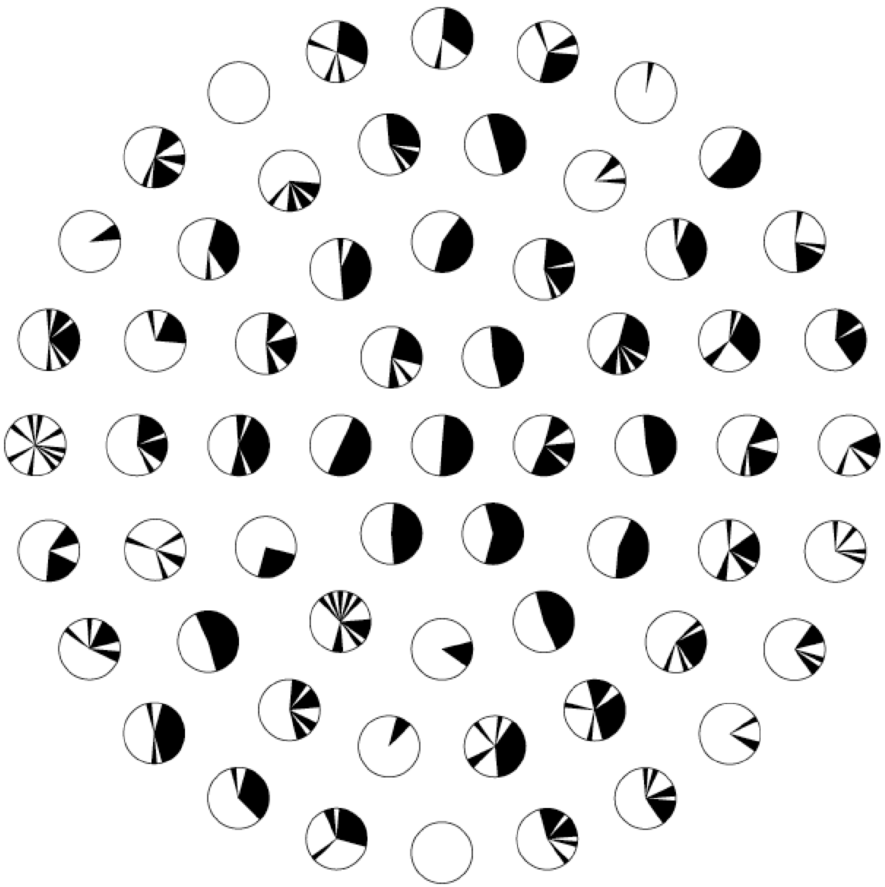


■ Converges

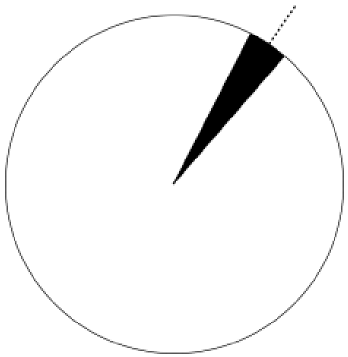
□ Does not converge

Slide courtesy N. Mitra

Point-to-Plane



Translation in xz plane
Rotation about y

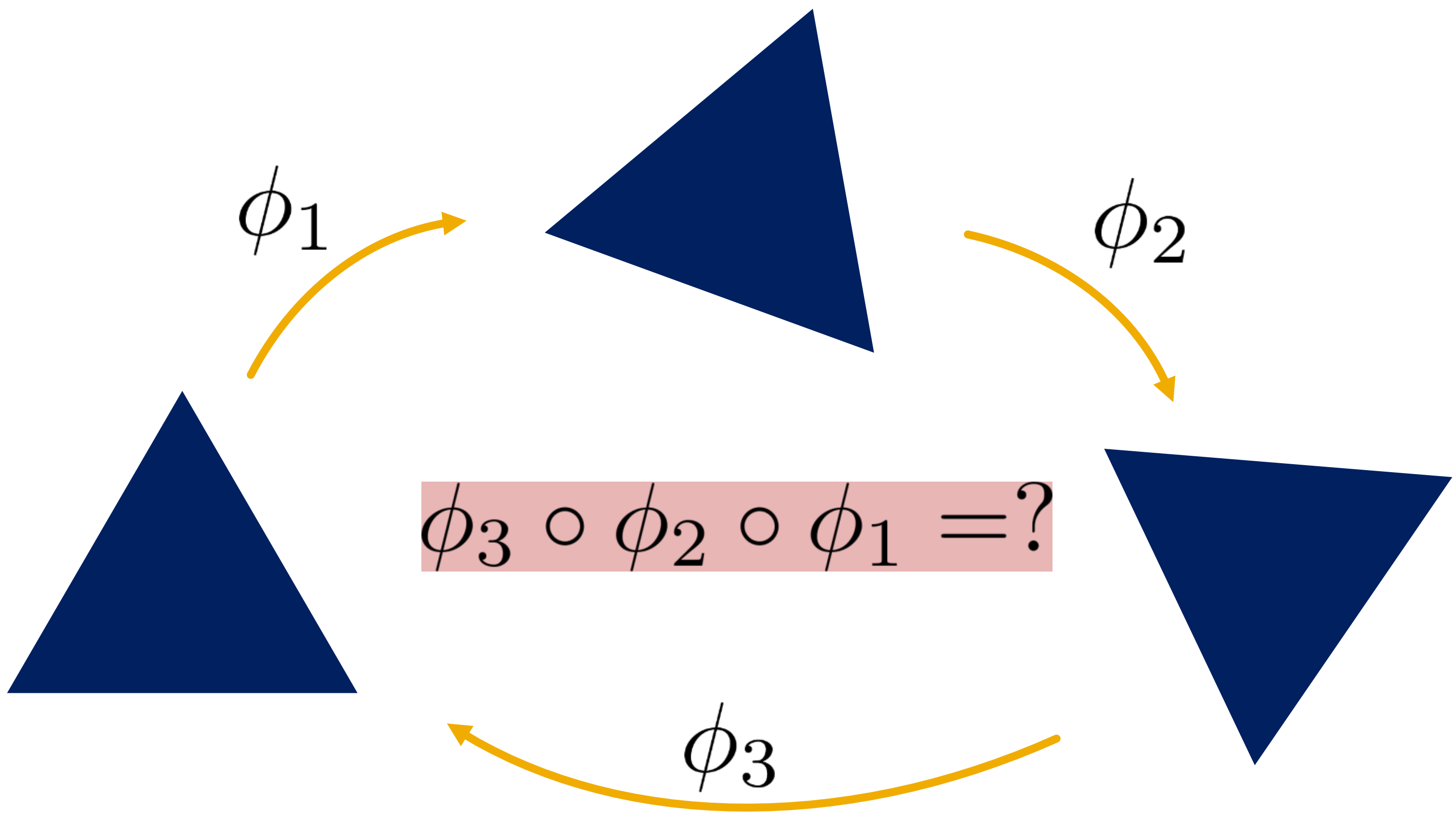


Converges

Does not converge

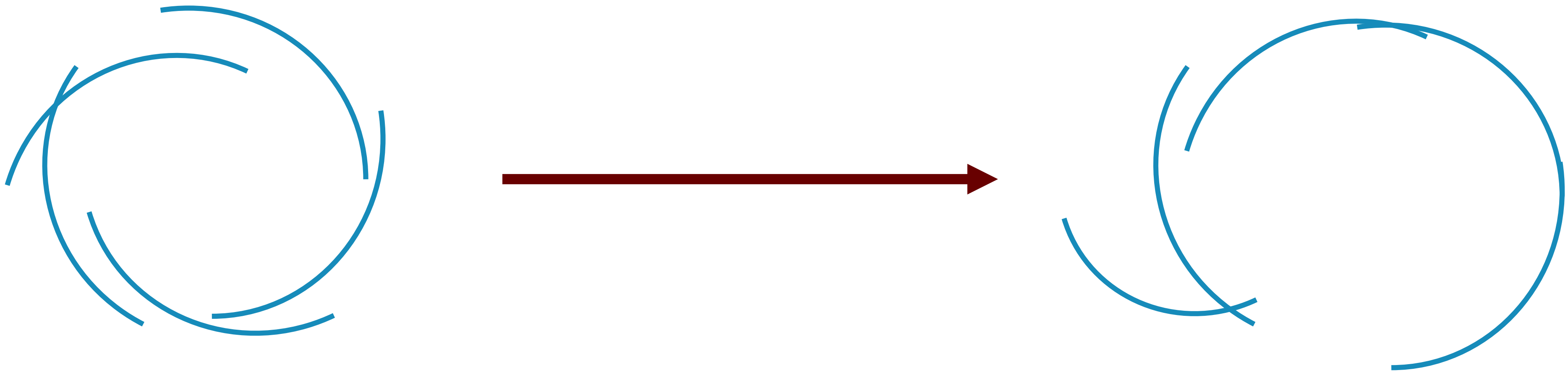
Slide courtesy N. Mitra

Issue: ICP Three Times



Usually have $\geq$ 2 scans

Improve Sequential Alignment?



Prevent "drift"

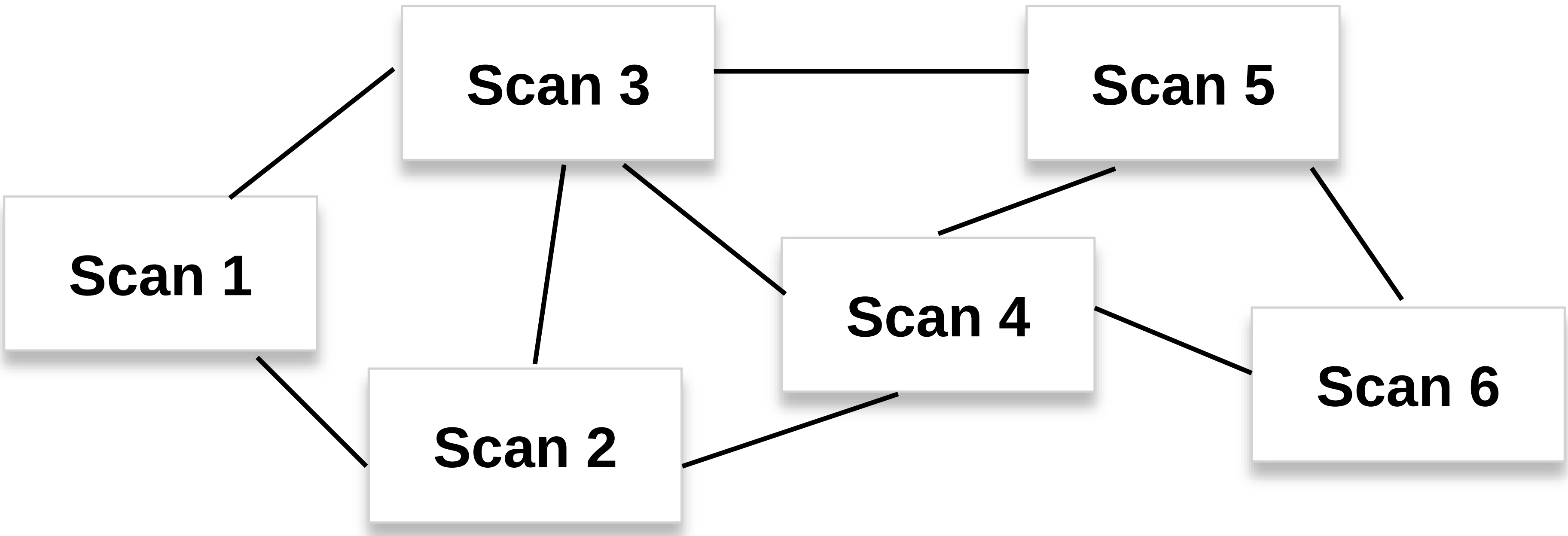
Simple Methods

- Align everything to anchor scan
  Which to choose? Dependence on anchor?

- Align to union of previous scans
  Order dependence? Speed?

- Simultaneously align everything using ICP
  Local optima? Computational expense?

Graph Approach



Align similar scans, then assemble

Lu and Milios

- **Pairwise phase**

  Compute pairwise ICP on graph

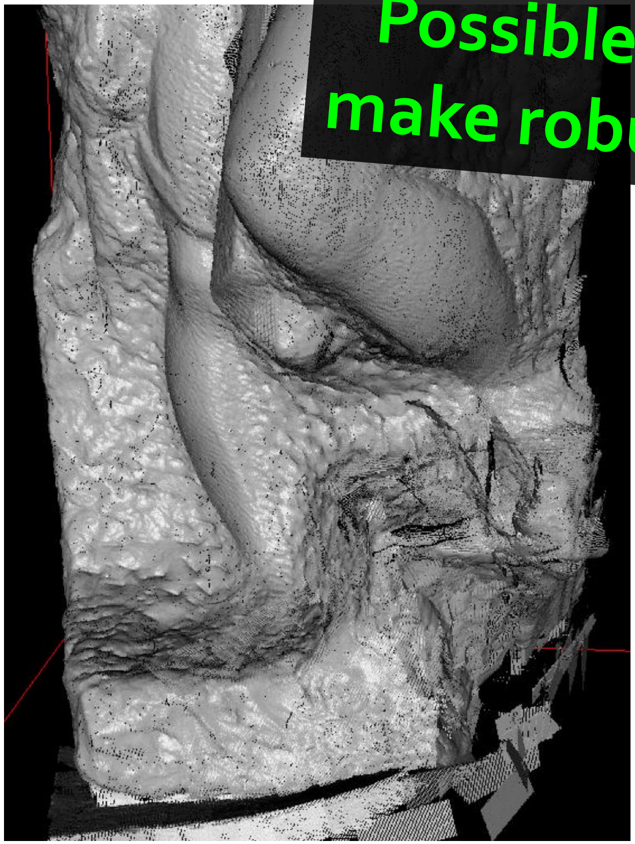
- **Global alignment**

  Least-squares rotation/translation

Linearize for
global alignment

Failed ICP in Global Registration

Correct global registration



Global registration including bad ICP

Possible to make robust?

Digression: Angular Synchronization

$$\text{Given: } \delta_{ij} \approx \theta_i - \theta_j \ (\mathrm{mod}\ 2\pi), (i,j) \in E$$

$$\text{Find: } \{\theta_i\} \text{ up to constant shift}$$

2D version without translation

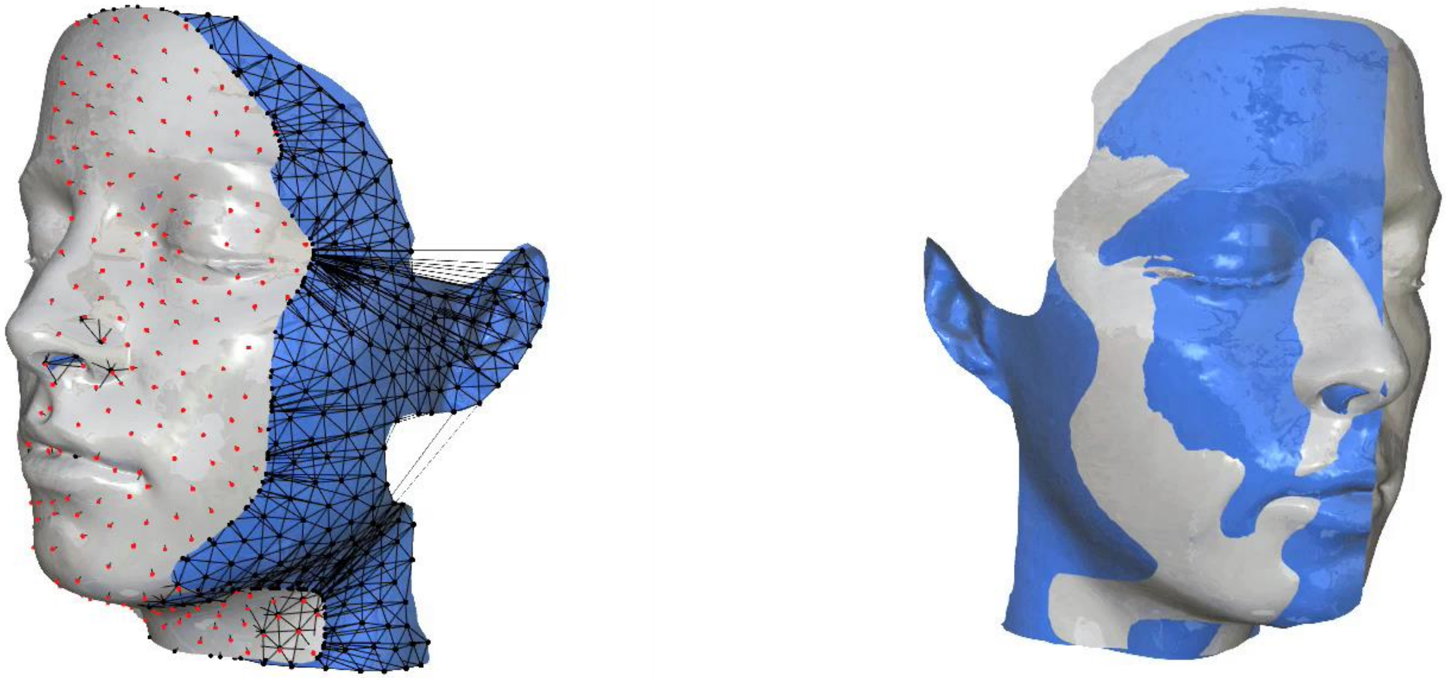
On the board:
Eigenvalue and/or SDP relaxations

Open problem:

Synchronization on non-compact groups (e.g. SE(3)!)

"Angular synchronization by eigenvectors and semidefinite programming."
Singer, ACHA 2010.

Non-Rigid Registration

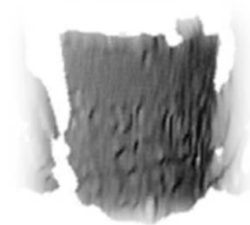


data provided by Paramount Pictures and Aguru Images

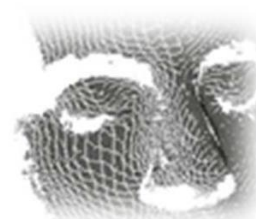
Problems

- Noisy data
- Acquisition holes (incomplete)
- No correspondence
- Deformation

missing correspondences



noise



holes

Example Paper

Eurographics Symposium on Geometry Processing 2008
Pierre Alliez and Szymon Rusinkiewicz
(Guest Editors)

Volume 27 (2008), Number 5

Global Correspondence Optimization for Non-Rigid Registration of Depth Scans

Hao Li Robert W. Sumner Mark Pauly

Applied Geometry Group
ETH Zurich

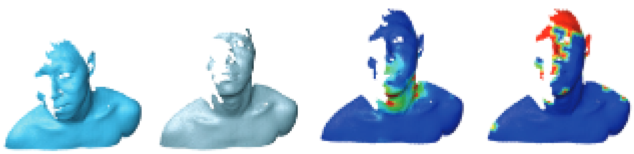
Abstract

We present a registration algorithm for pairs of deforming and partial range scans that addresses the challenges of non-rigid registration within a single non-linear optimization. Our algorithm simultaneously solves for correspondences between points on source and target scans, confidence weights that measure the reliability of each correspondence and identify non-overlapping areas, and a warping field that brings the source scan into alignment with the target geometry. The optimization maximizes the region of overlap and the spatial coherence of the deformation while minimizing registration error. All optimization parameters are chosen automatically; hand-tuning is not necessary. Our method is not restricted to part-in-whole matching, but addresses the general problem of partial matching, and requires no explicit prior correspondences or feature points. We evaluate the performance and robustness of our method using scan data acquired by a structured light scanner and compare our method with existing non-rigid registration algorithms.

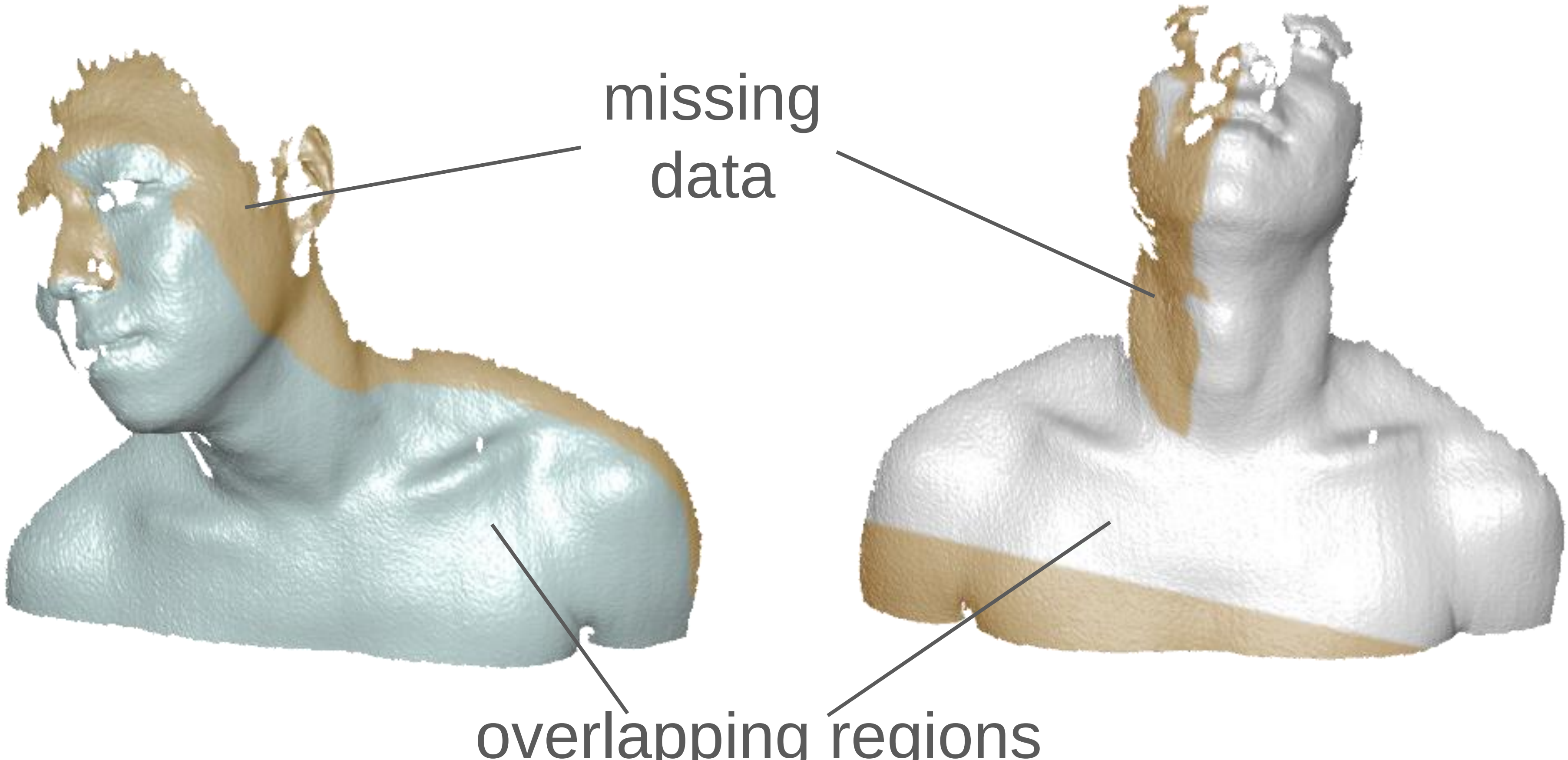
Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling

1. Introduction

Surface registration is a fundamental problem in geometric modeling and 3-D shape acquisition. Most scanning systems provide partial surface data that must be aligned and merged

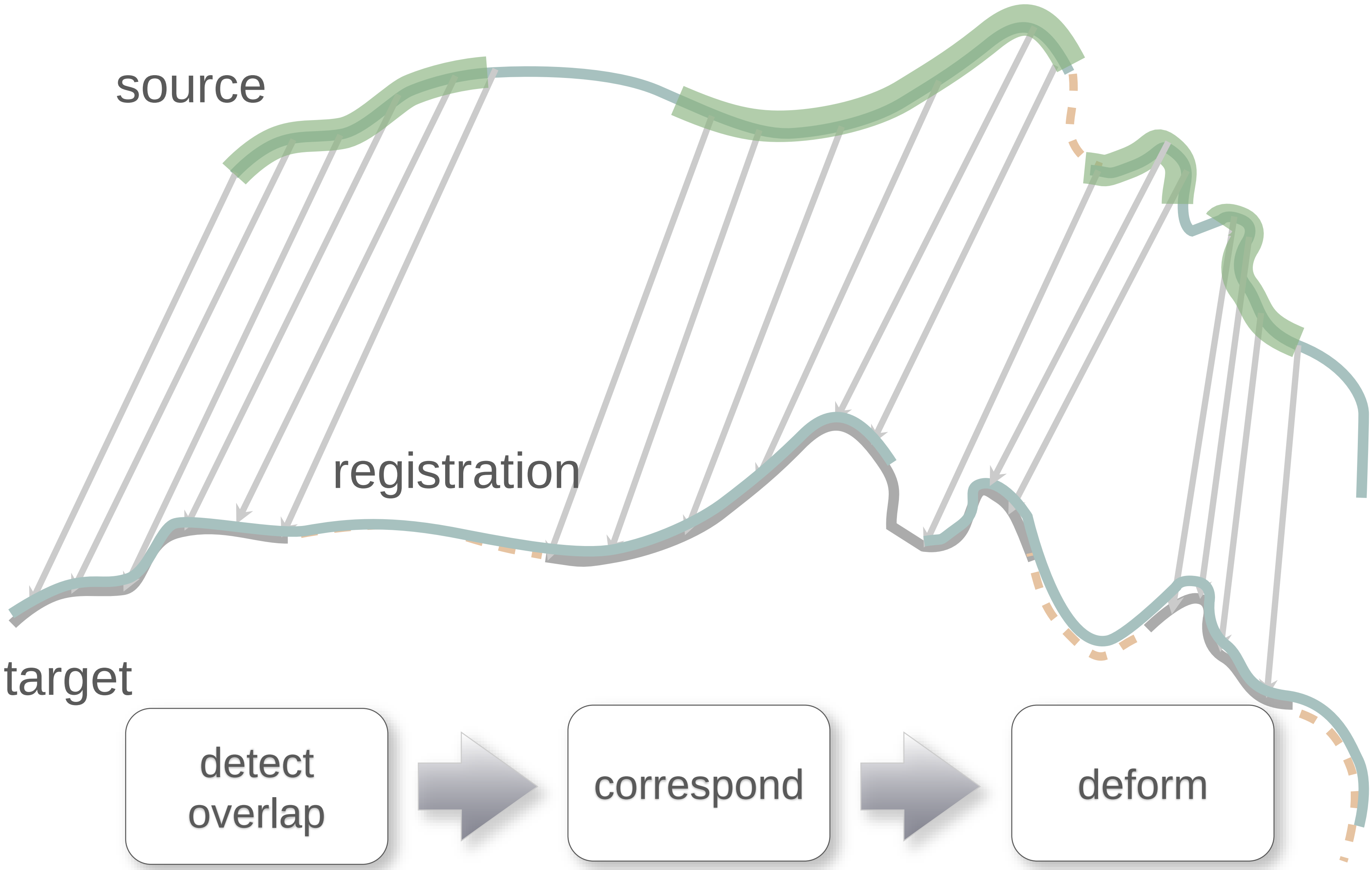


Concrete Example

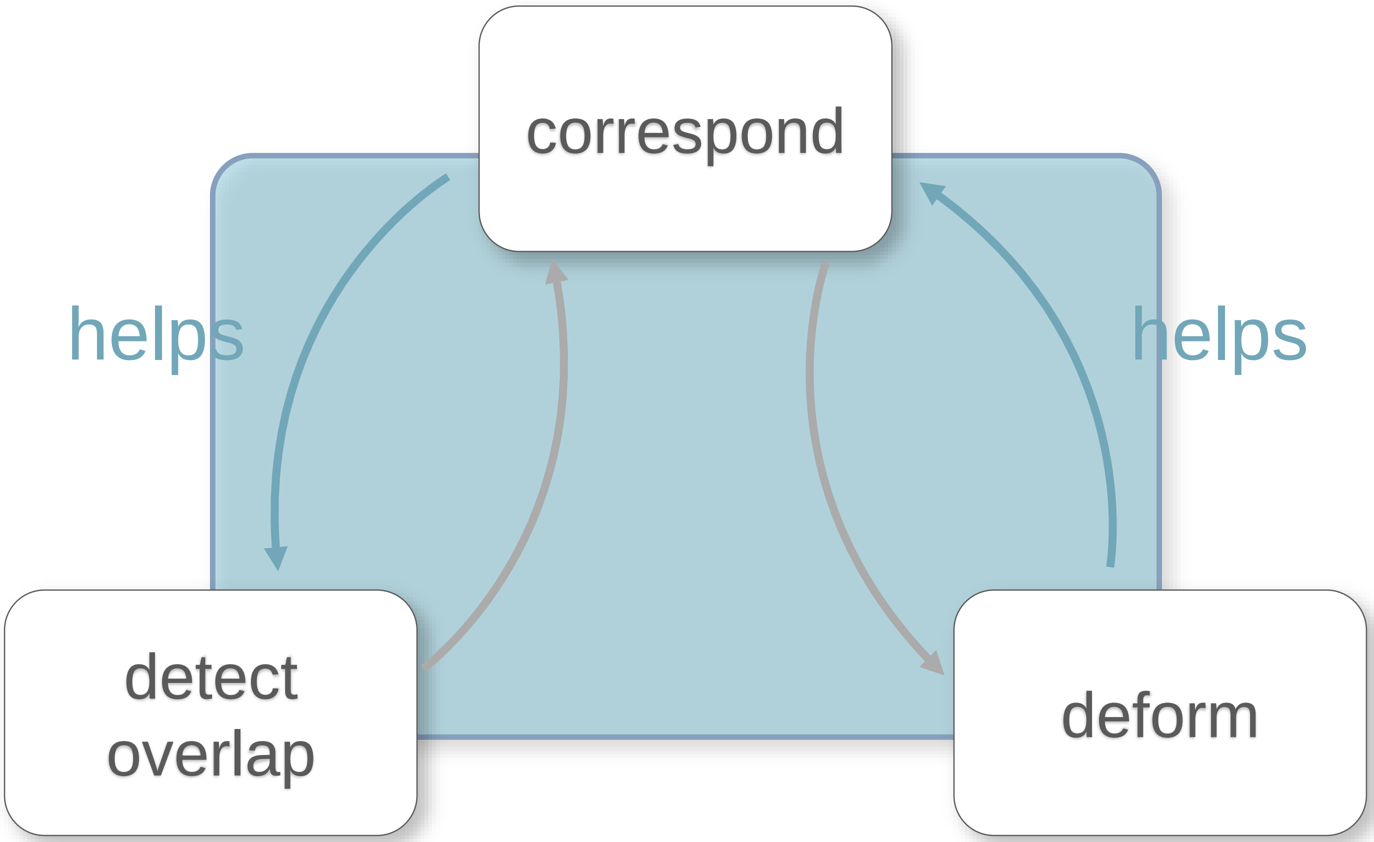


In addition to deformation

Reasonable Approach

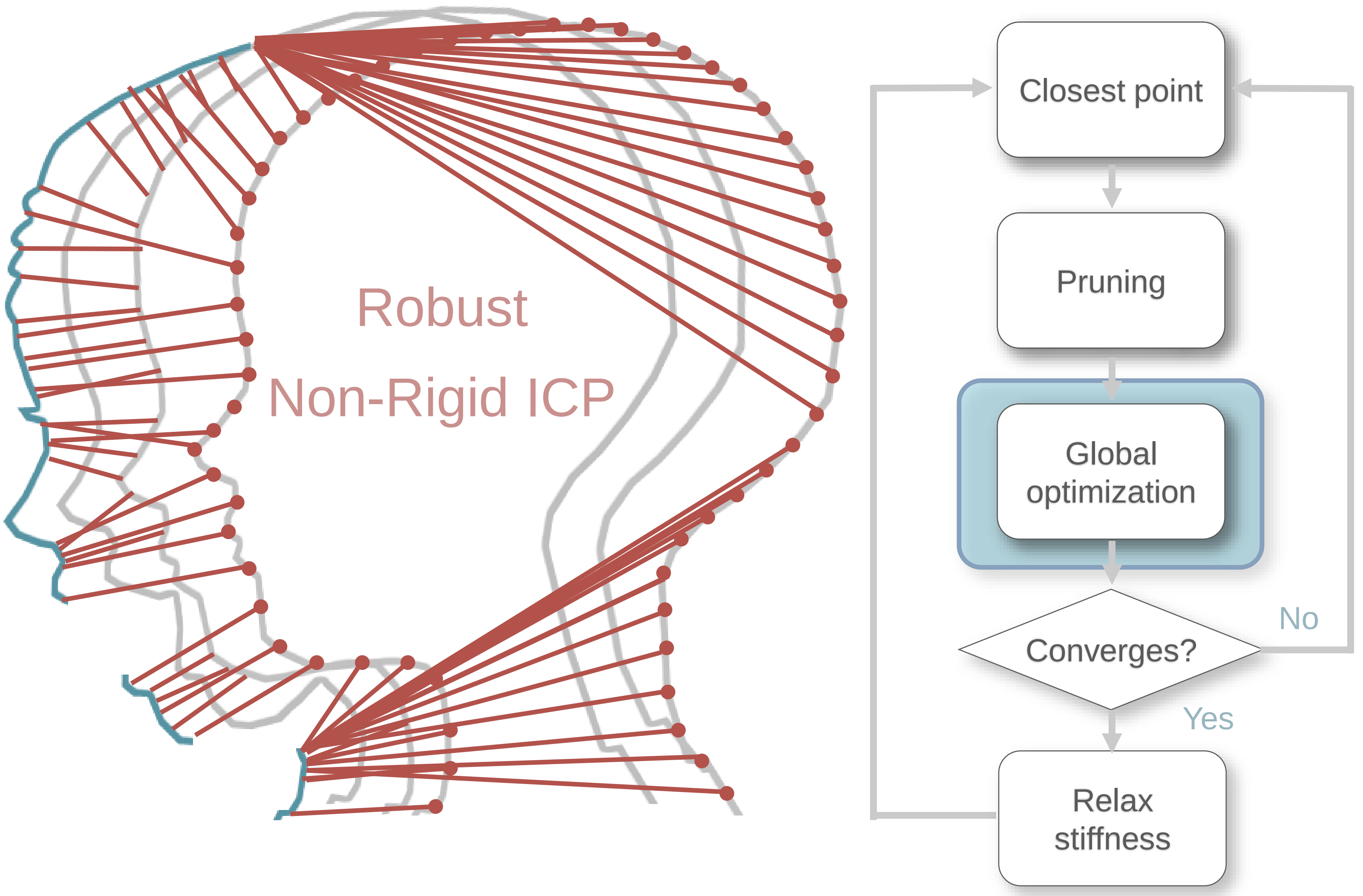


Global Optimization

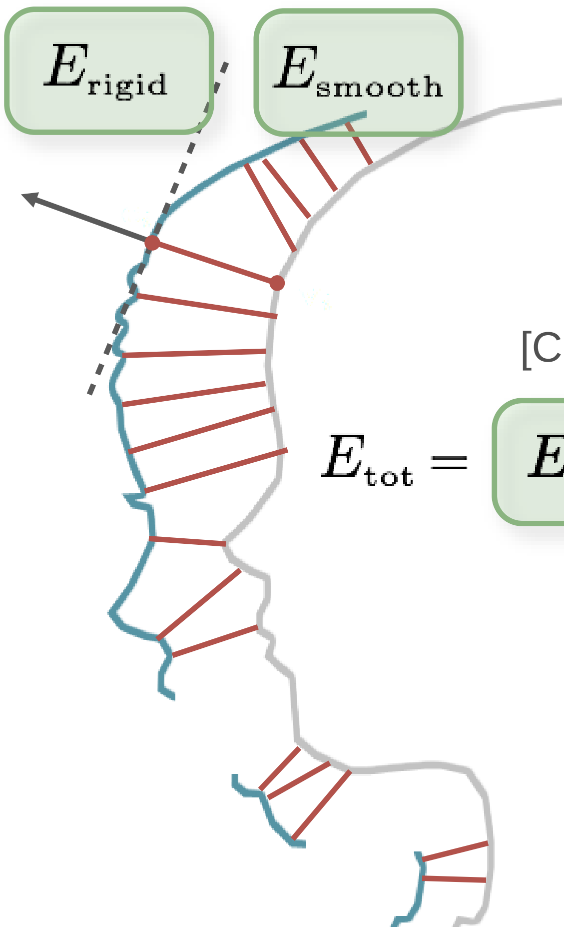


Tasks support each other

Pipeline


Robust
Non-Rigid ICP
Closest point
Pruning
Global optimization
Converges?
No
Yes
Relax stiffness

Rough Summary



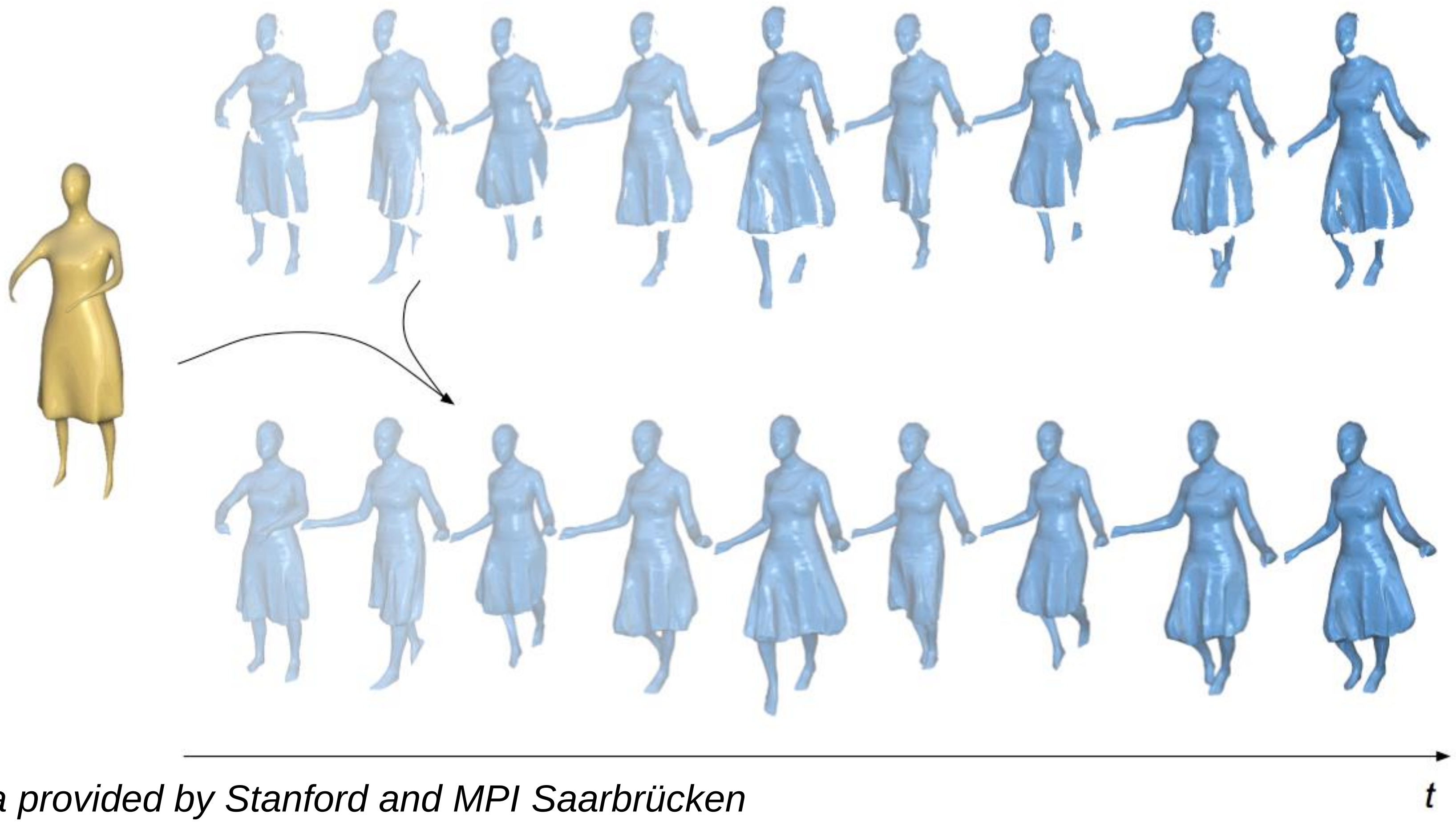
[Chen & Medioni '92]

$$E_{\text{tot}} = \boxed{E_{\text{plane}}} + \alpha_{\text{point}} \boxed{E_{\text{point}}} + \alpha_{\text{rigid}} \boxed{E_{\text{rigid}}} + \alpha_{\text{smooth}}$$

non-linear least squares

Gauss-Newton

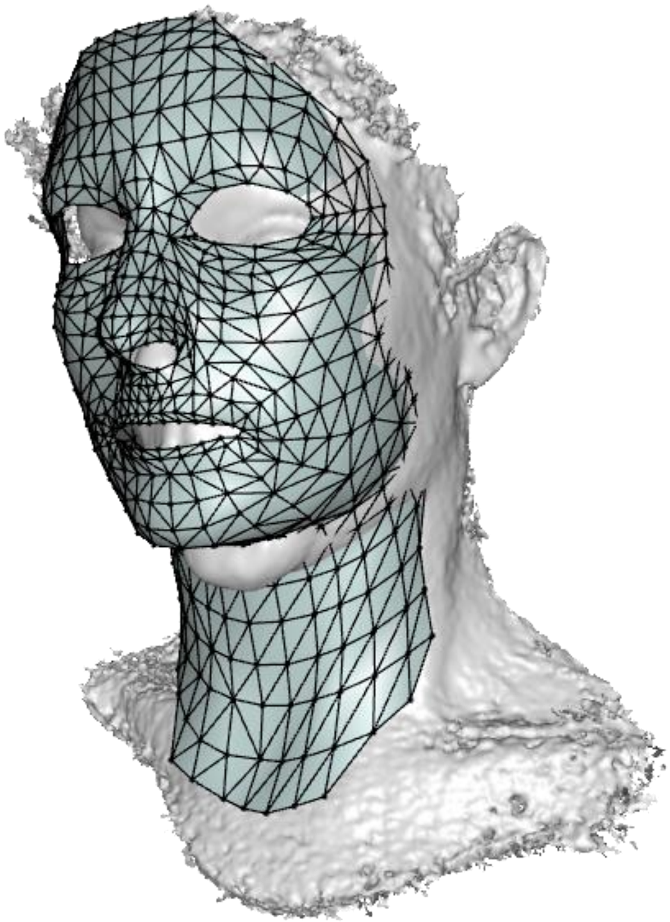
Alternative Approaches



Data provided by Stanford and MPI Saarbrücken

Template-based matching

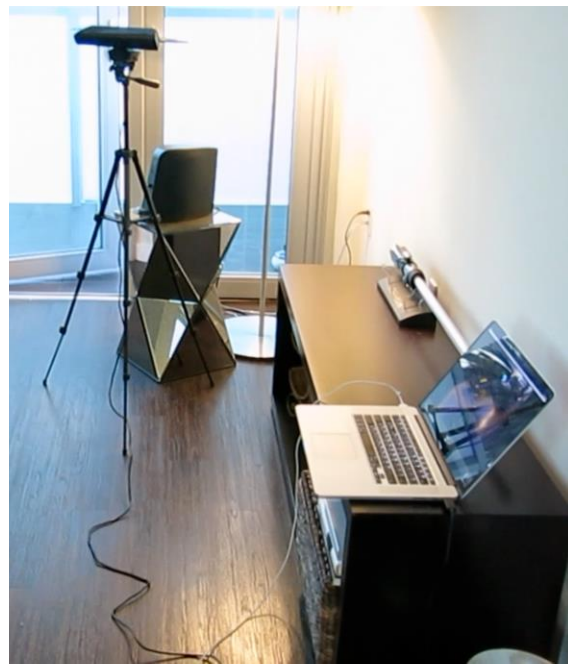
Alternative Approaches



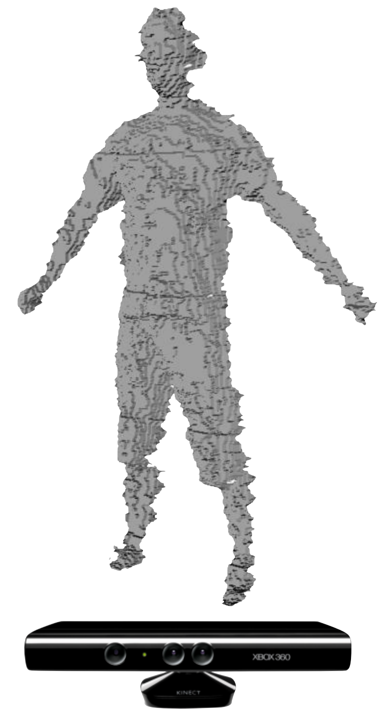
Template alignment, blendshapes

Outstanding Challenges

Deformation, clothing
& props

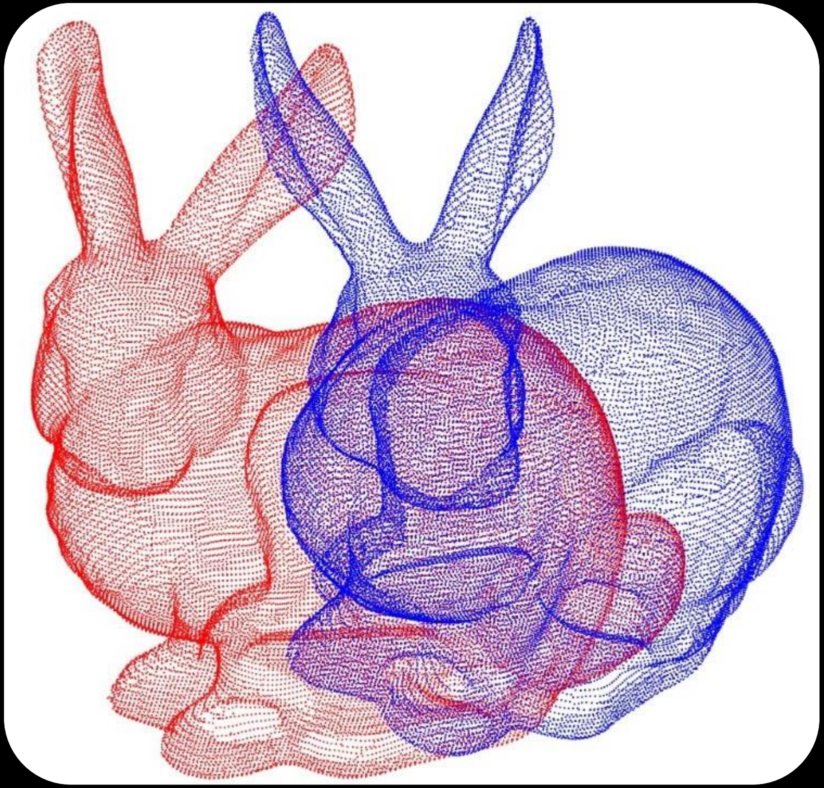


Environment



Low-cost scanners

https://i.ytimg.com/vi/uzOCS_gdZuM/maxresdefault.jpg



Registration and Alignment

Justin Solomon

MIT, Spring 2017