Scheduling Synchronous Dataflow Graphs

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Schedule

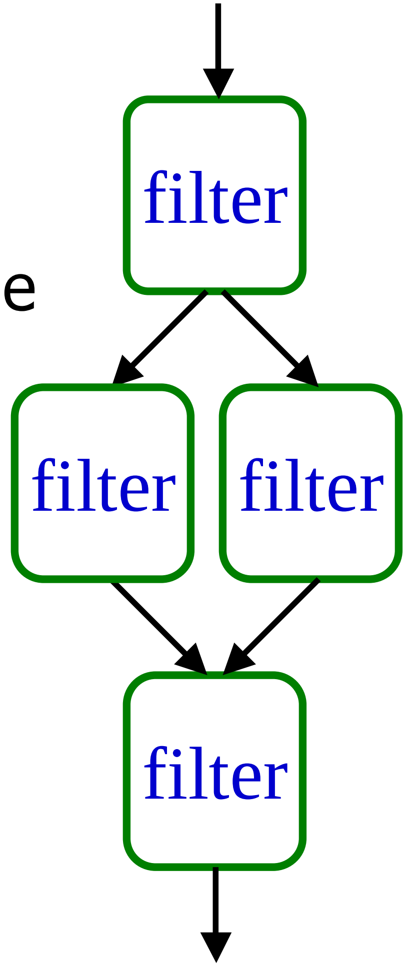
1:30-1:40	Overview (Saman)
1:40-2:20	Stream Architectures (Saman)
2:20-3:00	Stream Languages (Bill)
3:00-3:30	Break
3:30-3:55	Stream Compilers (Saman)
3:55-4:20	Domain-specific Optimizations (Saman)
4:20-5:00	Scheduling Algorithms (Bill)

Outline

- Introduction to Scheduling
  - Finding a Steady State
  - Finding a Schedule
  - Scheduling Tradeoffs
- Phased Scheduling Algorithm
  - Code Size / Buffer Size
  - Hierarchical scheduling
  - Results

Synchronous Dataflow (SDF)

- Consists of Filters and Channels
- Filters perform computation
  - Atomic execution step
  - Number of items produced / consumed on each firing is constant and known at compile time
- Channels act as FIFO queues for data between Filters
- For SDF, can statically determine:
  - Schedule of node firings
  - Buffer sizes
  - Deadlock conditions
- As we saw before, there are many generalizations



The Scheduling Problem

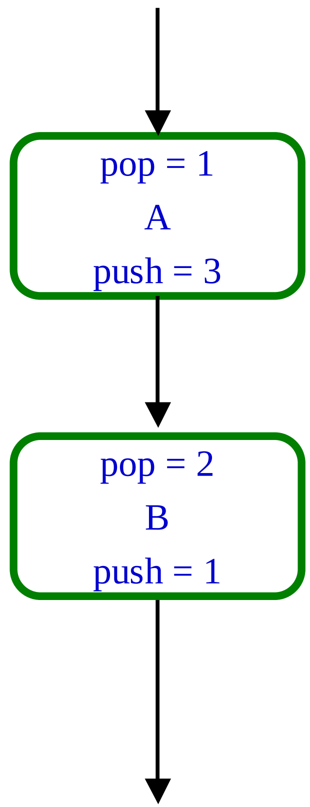
- Find a legal order in which filters can be executed
  - Nodes only fire when their inputs are ready
- Manage mismatched rates between filters
- Minimize data buffered up in channels between filters
- Minimize latency of data processing

Scheduling – Steady State

- Every valid stream graph has a Steady State
- Steady State does not change amount of data buffered between components
- Steady State can be executed repeatedly forever without growing buffers

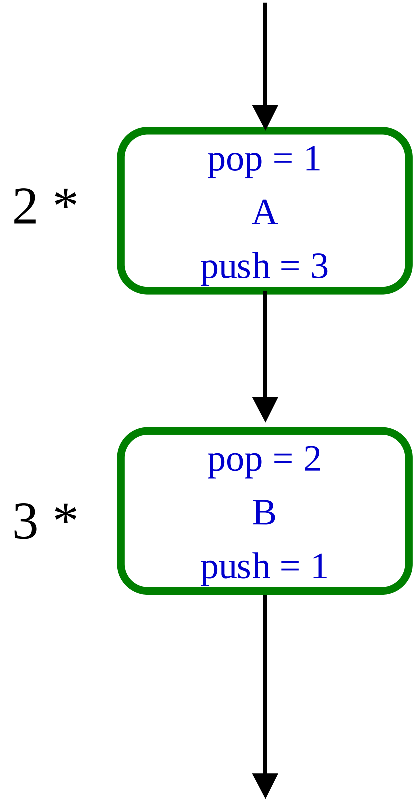
Steady State Example

- 3:2 Rate Converter
- First filter (A) upsamples by factor of 3
- Second filter (B) downsamples by factor of 2



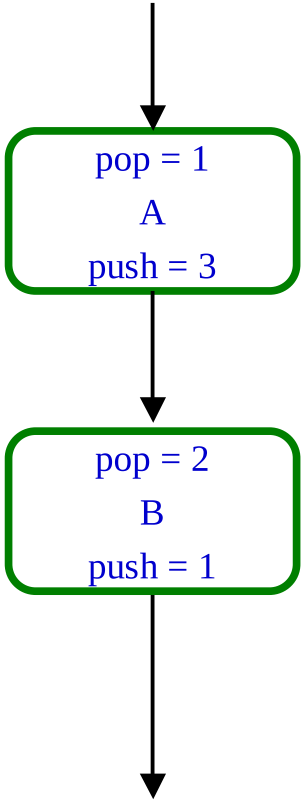
Steady State Example

- A executes 2 times
  - pushes 2 * 3 = 6 items
- B executes 3 times
  - pops 3 * 2 = 6 items
- Number of data items stored between Filters does not change



Computing the Steady State

- Balance equations
  - For each edge (src, dst):
    n(src) * push(src) = n(dst) * pop(dst)
  - Example:



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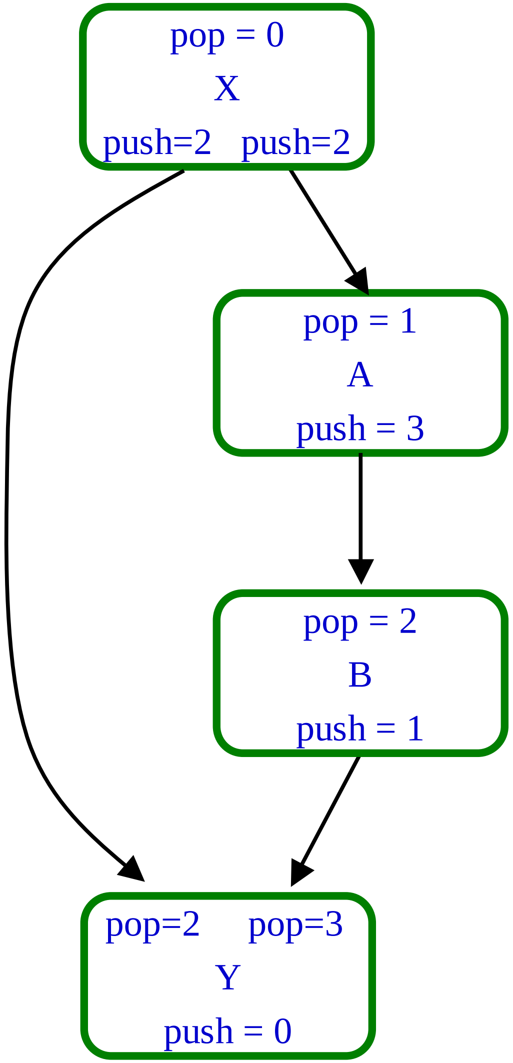

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- Balance equations
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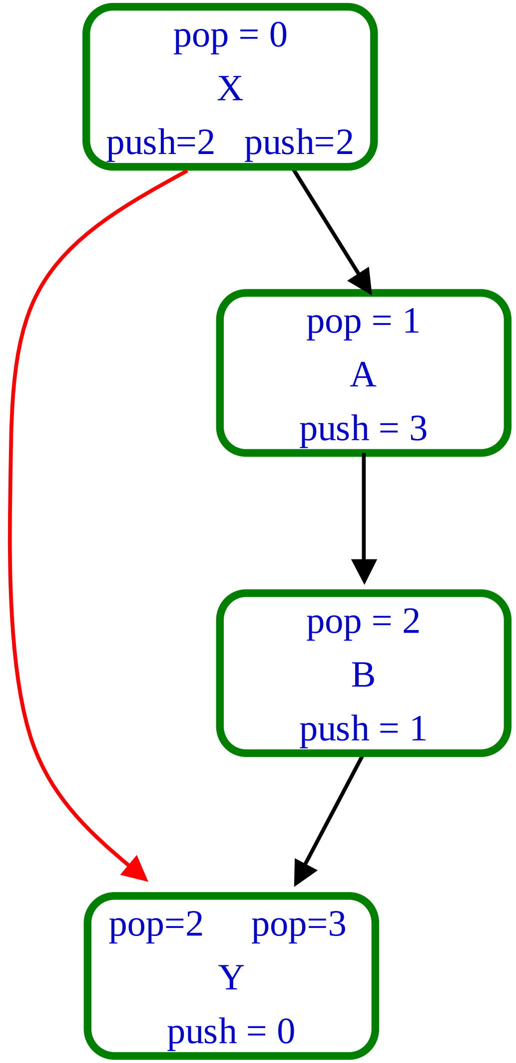
    n(X) * 2 = n(Y) * 2



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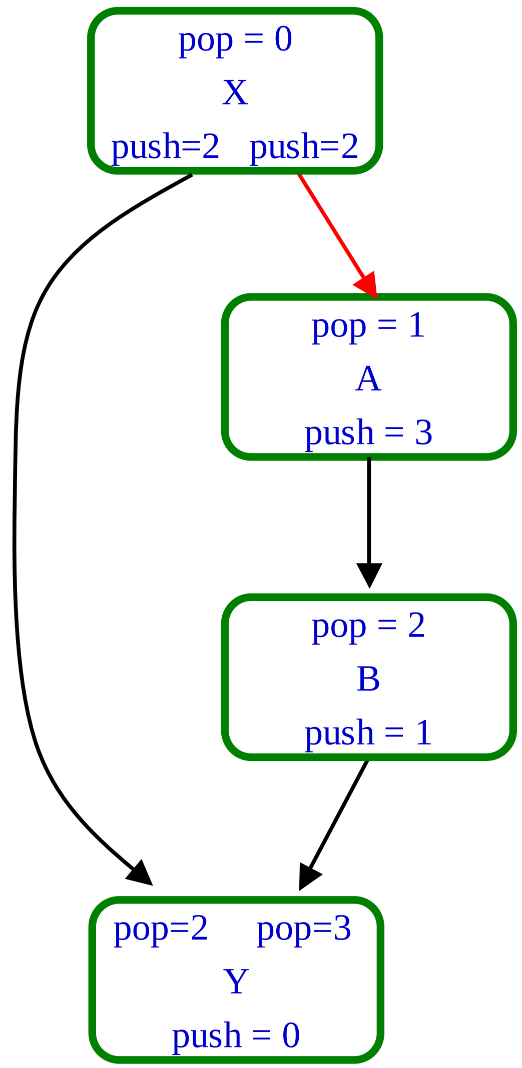
    n(X) * 2 = n(A) * 1



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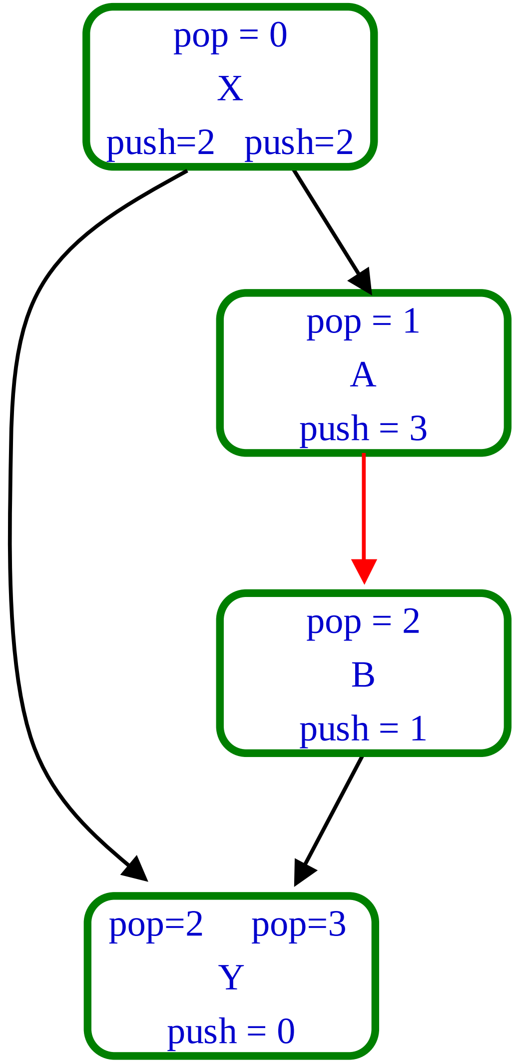
    n(X) * 2 = n(A) * 1

    n(A) * 3 = n(B) * 2



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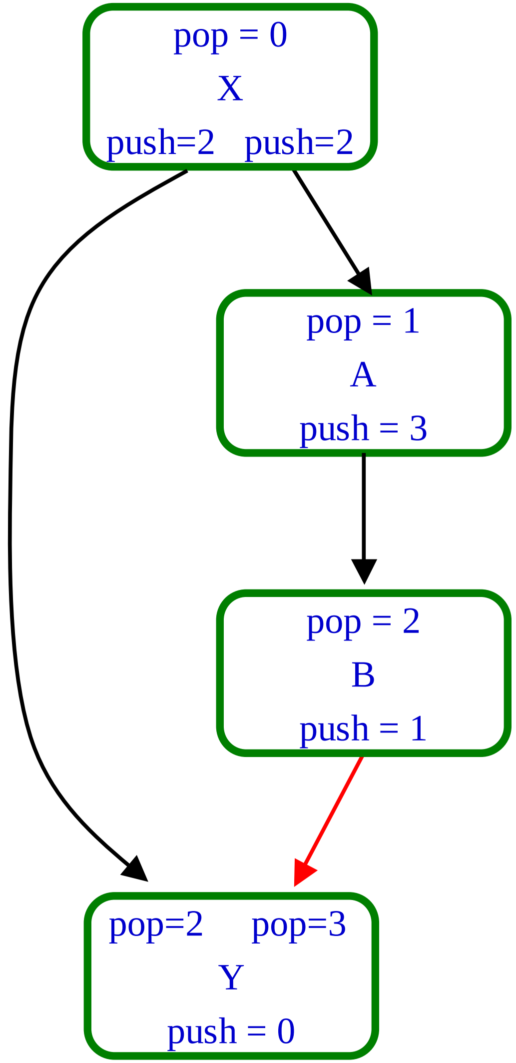
    n(A) * 3 = n(B) * 2

    n(B) * 1 = n(Y) * 3



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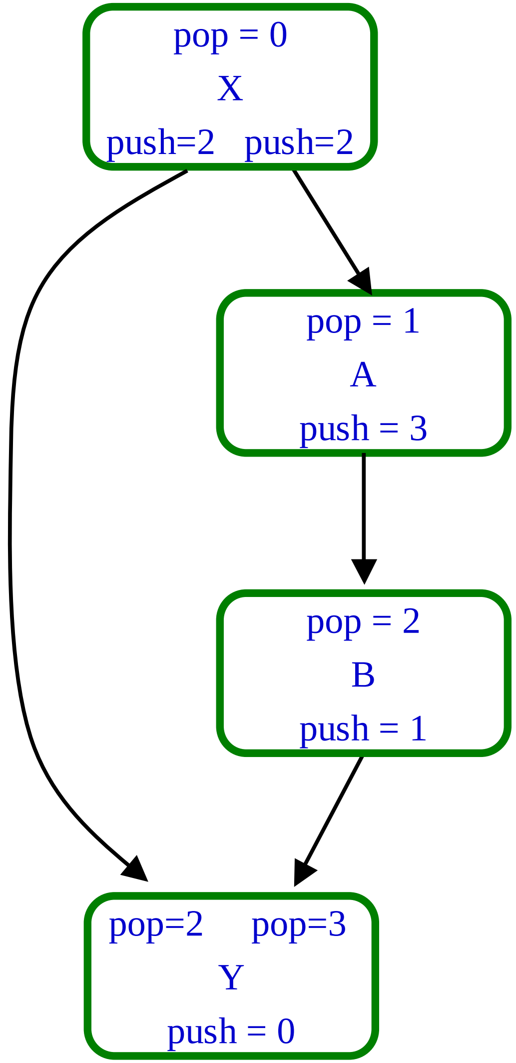
    n(A) * 3 = n(B) * 2

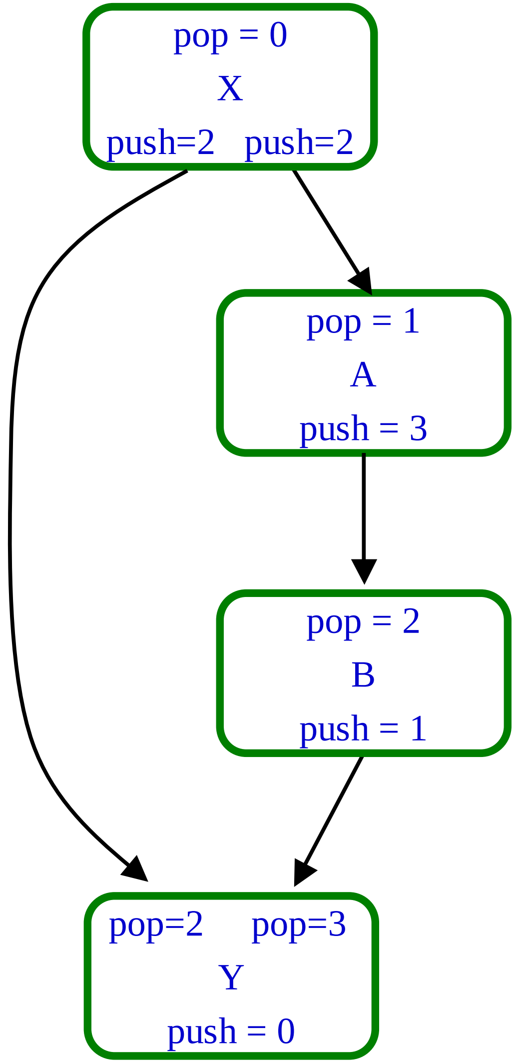
    n(B) * 1 = n(Y) * 3

$$\begin{bmatrix} 2 & 0 & 0 & -2 \\ 2 & 0 & -1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} n(X) \\ n(A) \\ n(B) \\ n(Y) \end{bmatrix} = 0$$



Computing the Steady State

$$\underbrace{\begin{bmatrix} 2 & 0 & 0 & -2 \\ 2 & 0 & -1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix}}_{\text{Topology Matrix, } \Gamma} \begin{bmatrix} n(X) \\ n(A) \\ n(B) \\ n(Y) \end{bmatrix} = \vec{0}$$



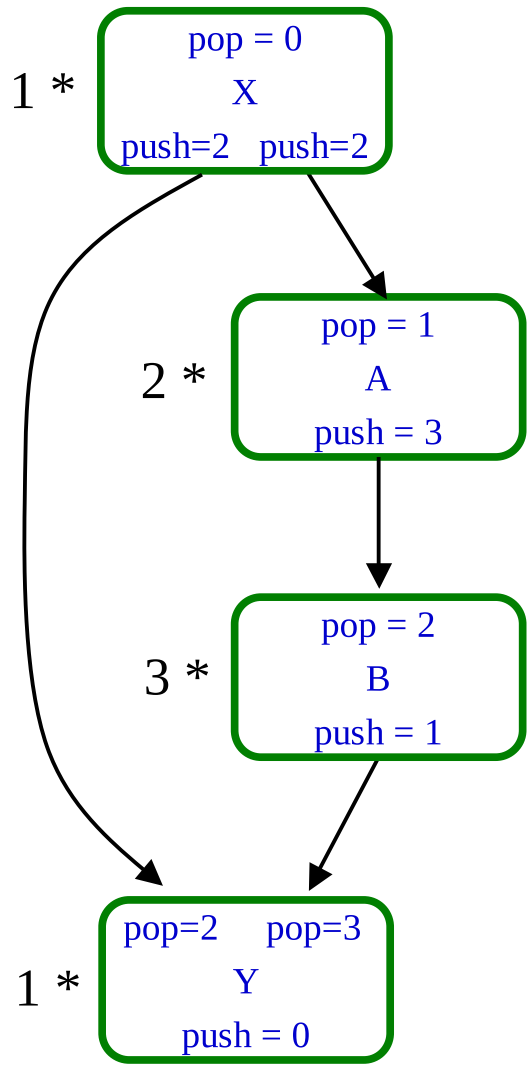
- Theorem (Lee '86):
  - A connected SDF graph with n actors has a periodic schedule iff its topology matrix Γ has rank n-1
    - Rank > n-1 → no periodic schedule
    - Rank < n-1 → graph is not connected
  - If Γ has rank n-1 then there exists a unique smallest integer solution to $\Gamma n = 0$

Computing the Steady State

$$\underbrace{\begin{bmatrix} 2 & 0 & 0 & -2 \\ 2 & 0 & -1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix}}_{\text{Topology Matrix, } \Gamma} \begin{bmatrix} n(X) \\ n(A) \\ n(B) \\ n(Y) \end{bmatrix} = \vec{0}$$

- Minimal solution: $\begin{bmatrix} n(X) \\ n(A) \\ n(B) \\ n(Y) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$

- All multiples are valid steady-states



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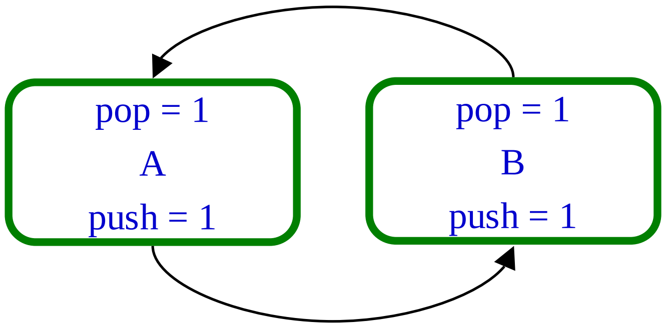
Computing the Schedule

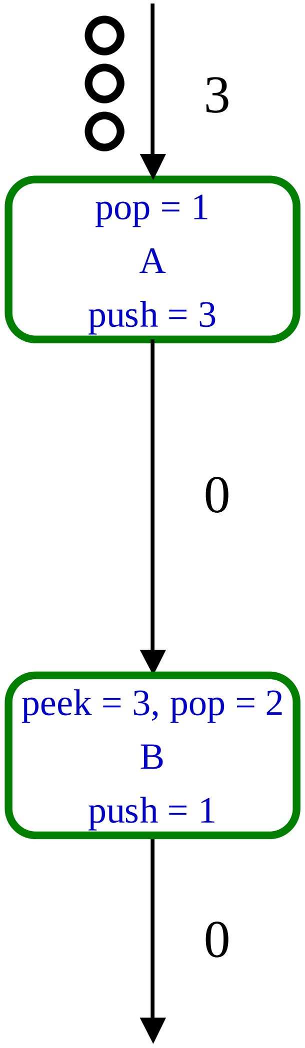
- Schedule indicates exact ordering of nodes
- Steady state indicates only the multiplicity
  - A graph might have a valid steady-state without having any admissable schedule



- To build legal schedule, fire any node that:
  1. Has enough input items to execute
  2. Has not exceeded its multiplicity in the steady state
- If deadlock reached before steady state complete, then no valid schedule exists (Lee '86)

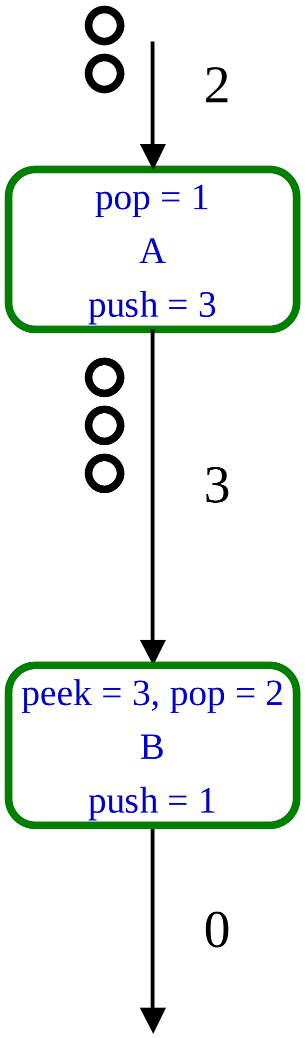
Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  -



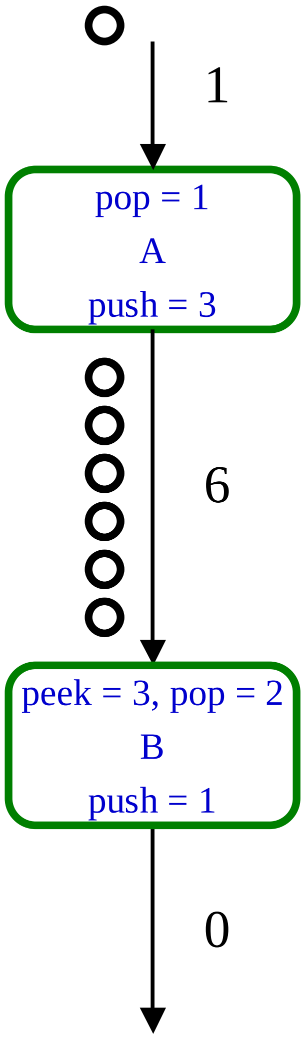
Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  - A



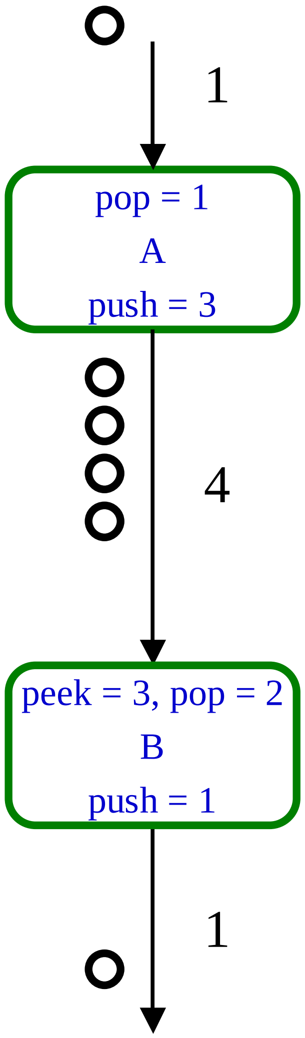
Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  - AA



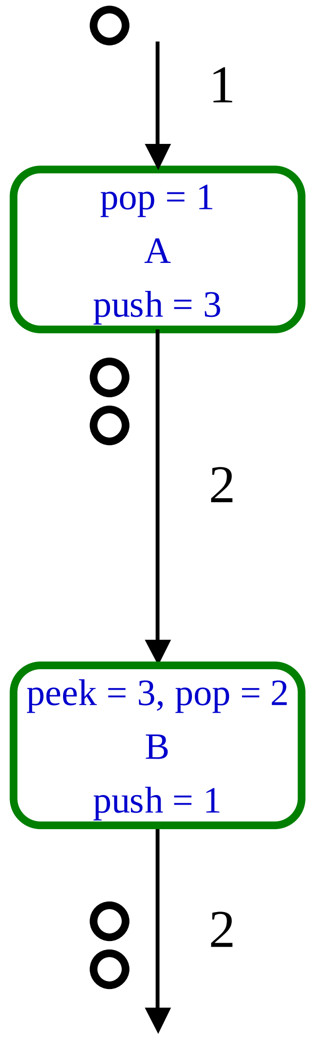
Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  - AAB



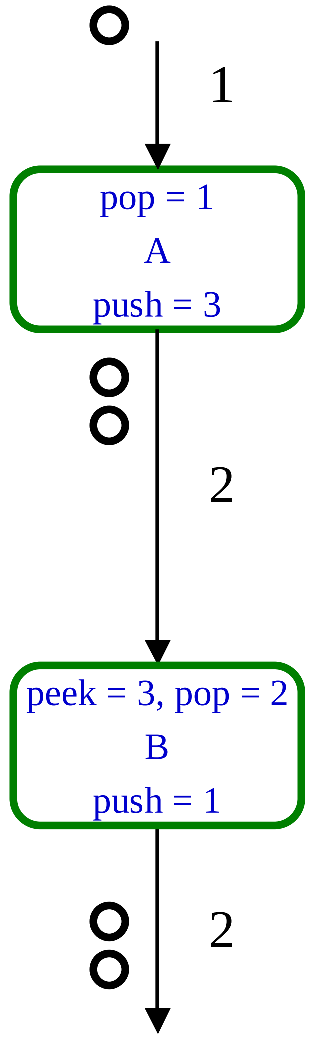
Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  - AABB
  - Can't execute B again!



Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  - AABB
  - Can't execute B again!
- Can't execute A one extra time:
  - AABB



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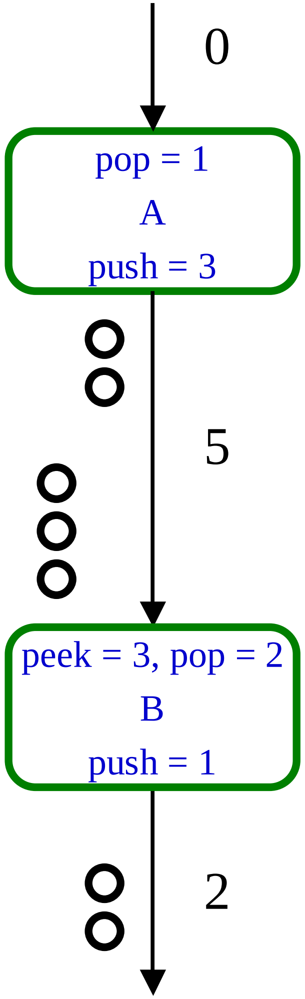
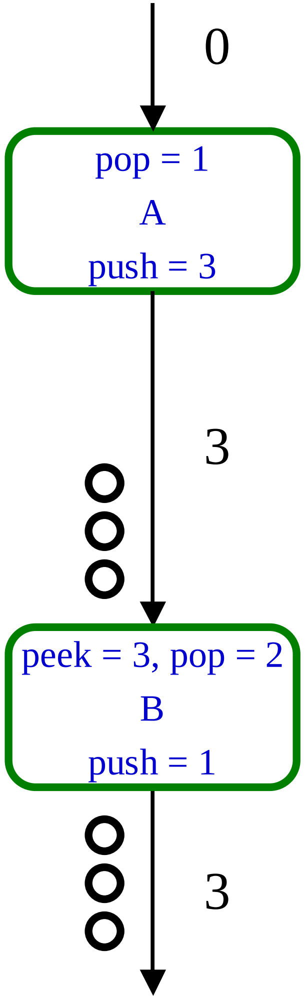


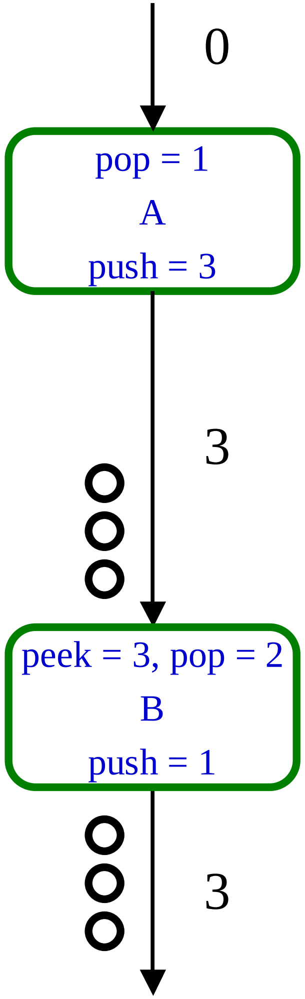
Initialization Schedule

- Filter Peeking provides a new challenge
- Just Steady State doesn't work:
  - AABB
  - Can't execute B again!
- Can't execute A one extra time:
  - AABBAB
  - Left 3 items between A and B!



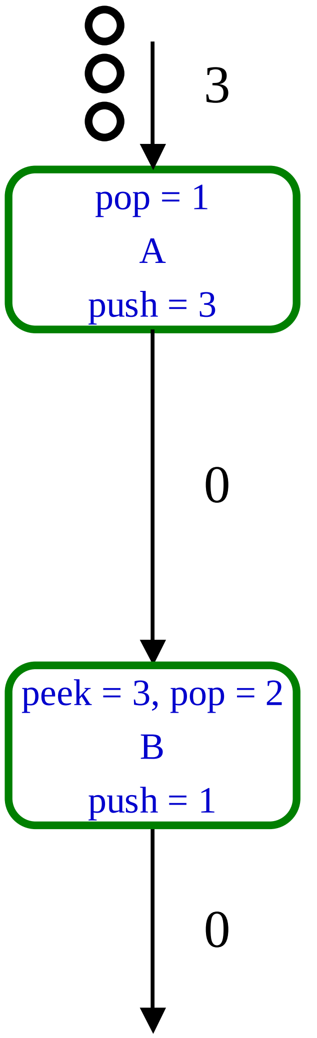
Initialization Schedule

- Must have data between A and B before starting execution of Steady State Schedule
- Construct two schedules:
  - One for Initialization
  - One for Steady State
- Initialization Schedule leaves data in buffers so Steady State can execute



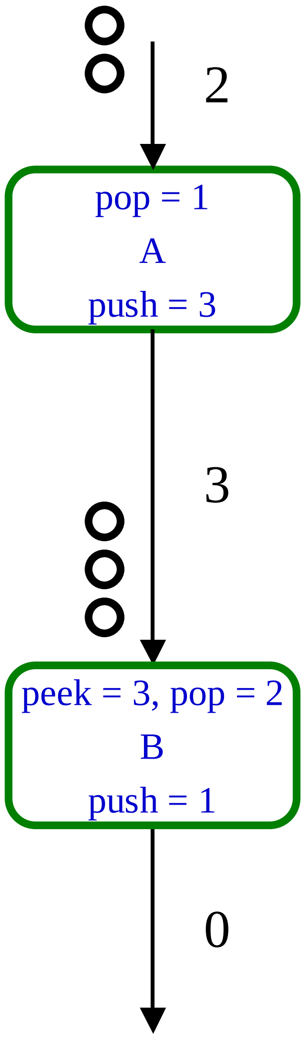
Initialization Schedule

- Initialization Schedule:
  -



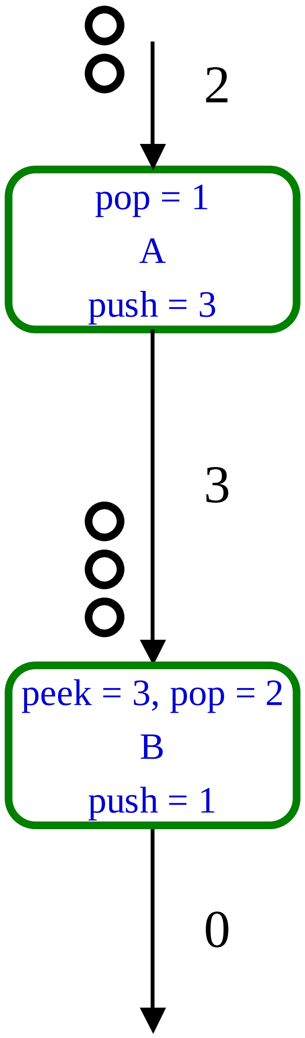
Initialization Schedule

- Initialization Schedule:
  - A



Initialization Schedule

- Initialization Schedule:
  - A
  - Leave 3 items between A and B
- Steady State Schedule:
  -



Initialization Schedule

- Initialization Schedule:
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- Steady State Schedule:
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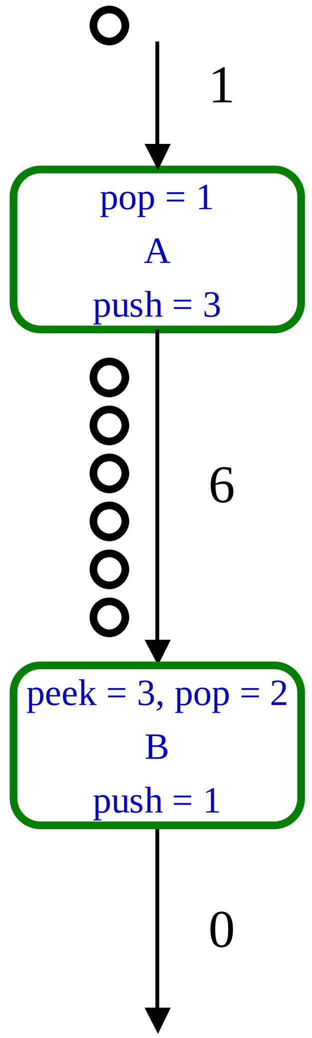
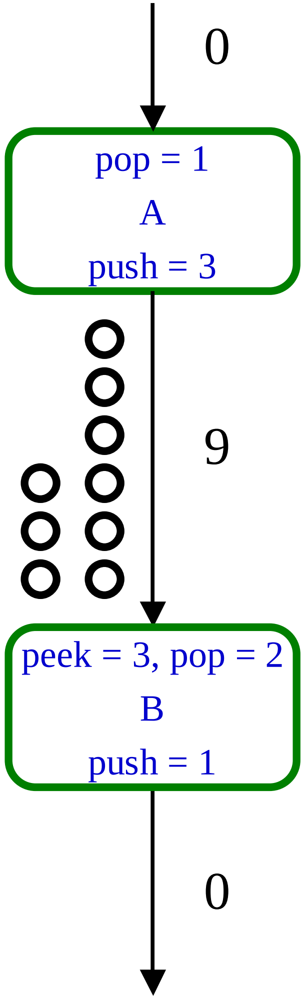


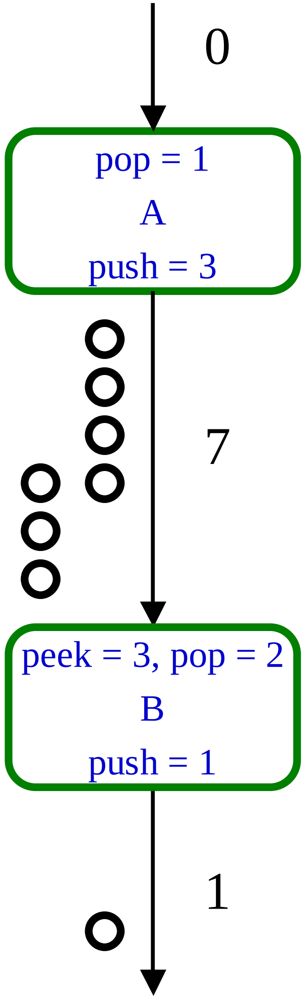
Initialization Schedule

- Initialization Schedule:
  - A
  - Leave 3 items between A and B
- Steady State Schedule:
  - AA



Initialization Schedule

- Initialization Schedule:
  - A
  - Leave 3 items between A and B
- Steady State Schedule:
  - AAB



Initialization Schedule

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  - Leave 3 items between A and B
- Steady State Schedule:
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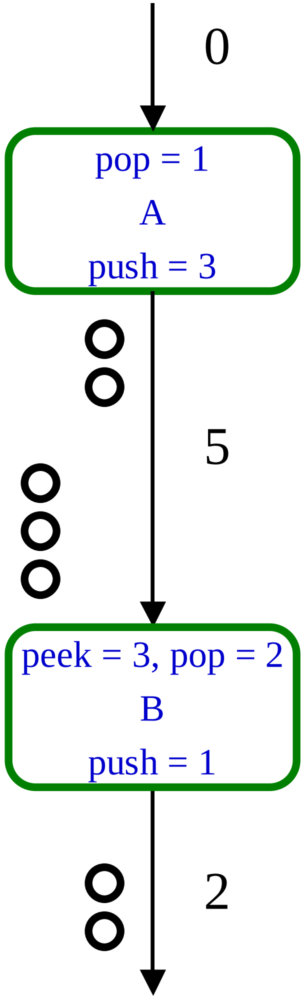

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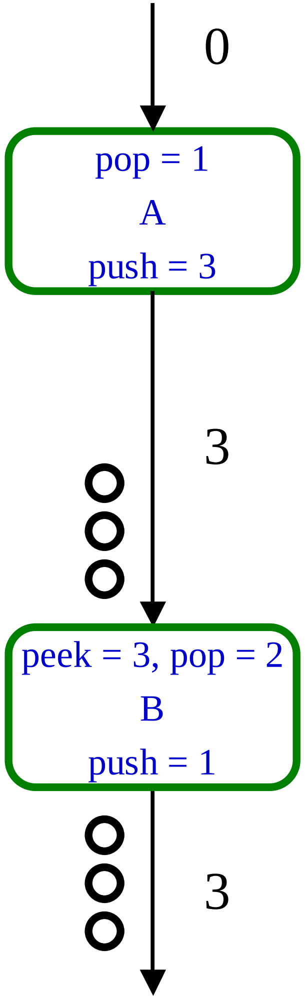

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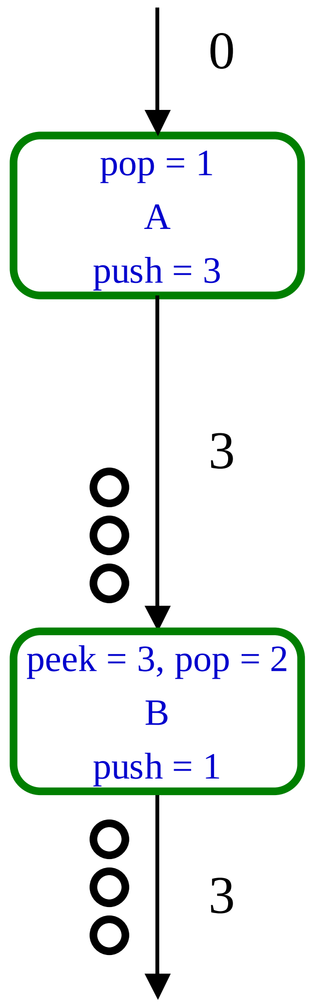
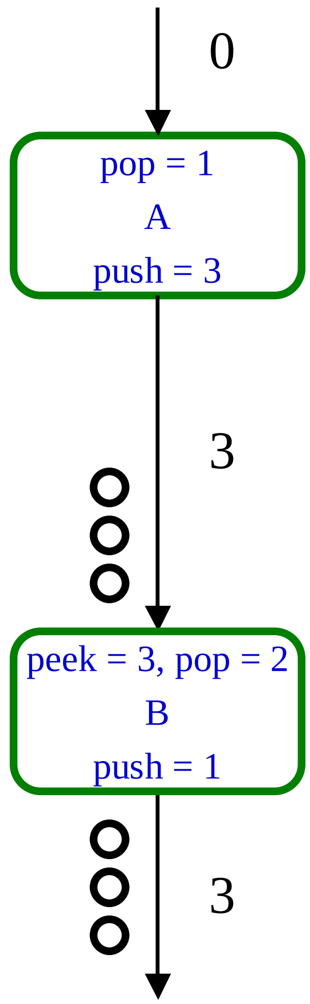
  - Leave 3 items between A and B



Initialization Schedule

- Initialization Schedule:
  - A
  - Leave 3 items between A and B
- Steady State Schedule:
  - AABBB
  - Leave 3 items between A and B
- Number of items preserved



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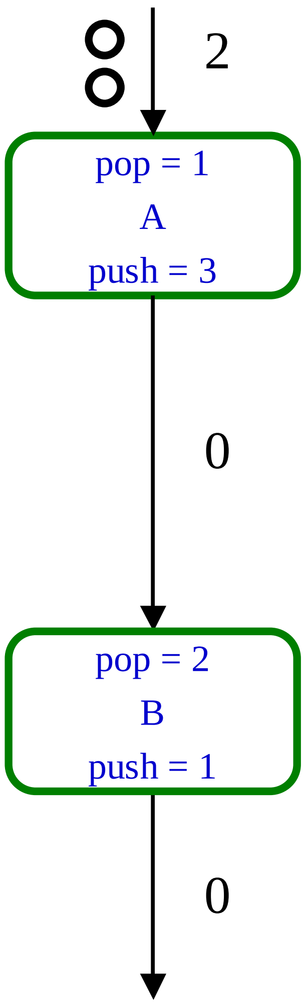
Scheduling Tradeoffs

- There are many possible schedules for a given steady-state
- Order of execution profoundly affects:
  - Latency
  - Buffer size
  - Code Size
- There is a wealth of literature that aims to optimize the schedule by various metrics

Scheduling Tradeoffs Example

- 3:2 Rate Converter
- First filter (A) upsamples by factor of 3
- Second filter (B) downsamples by factor of two
- Schedule:
  -



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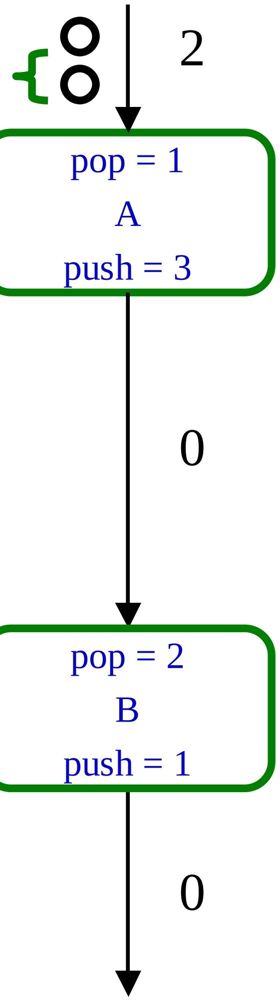

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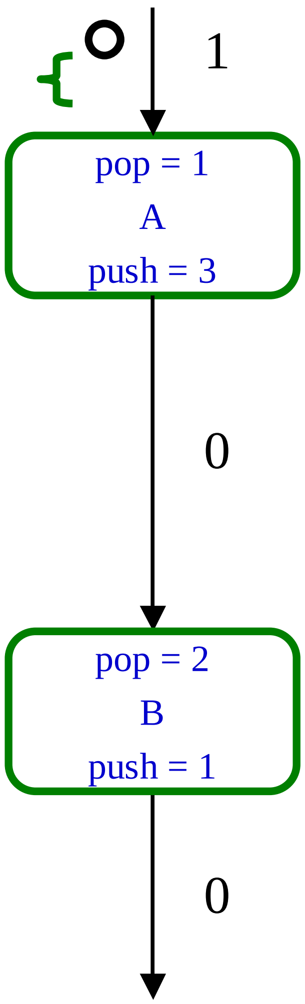

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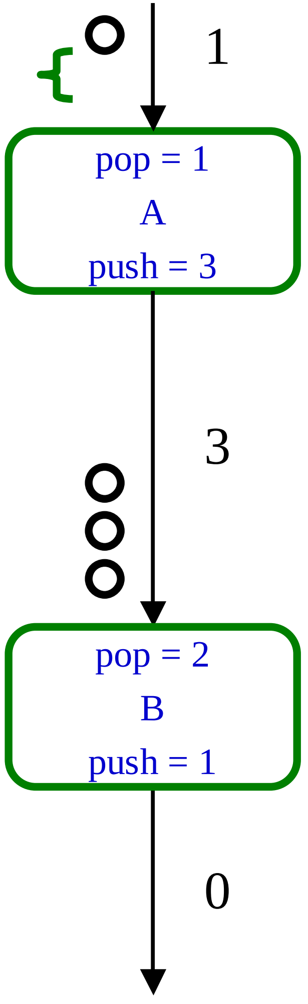

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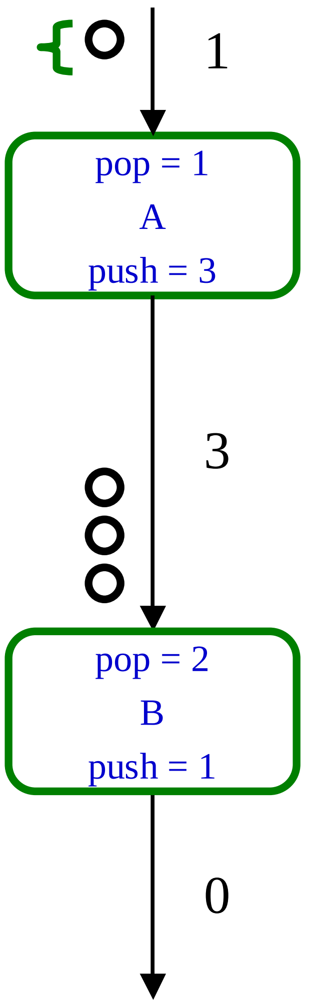

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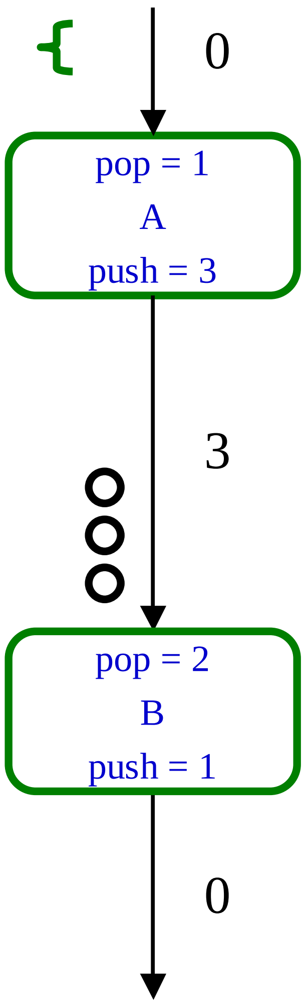

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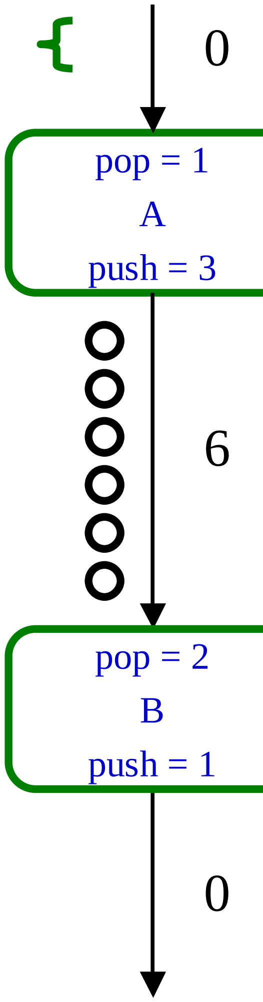

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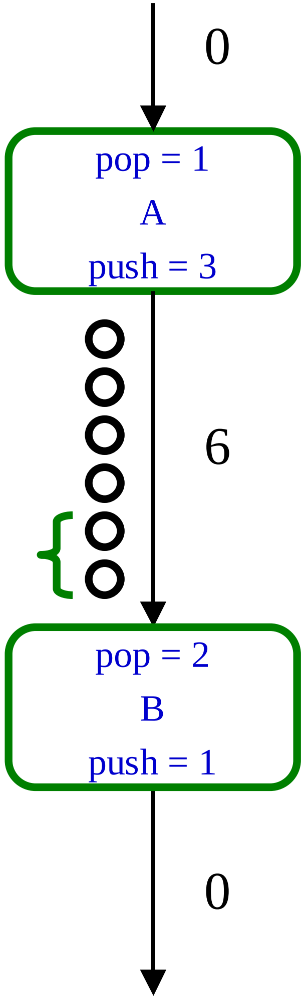

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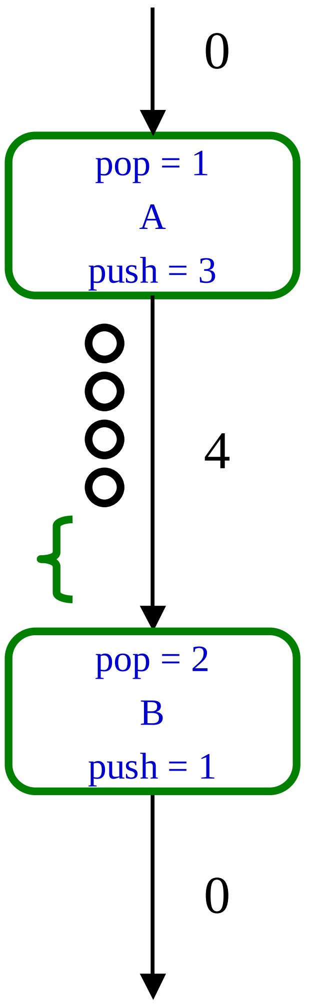

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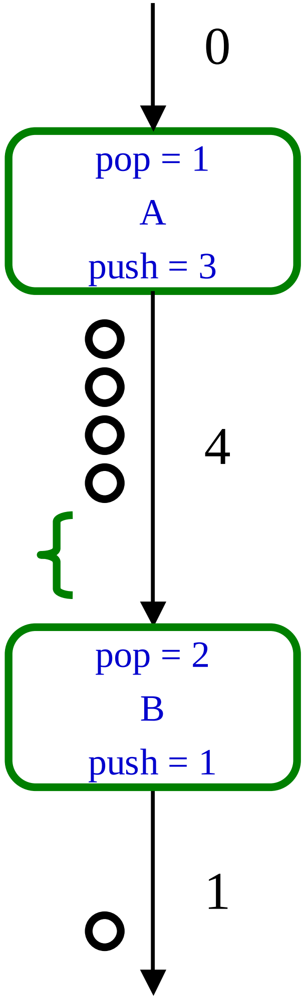

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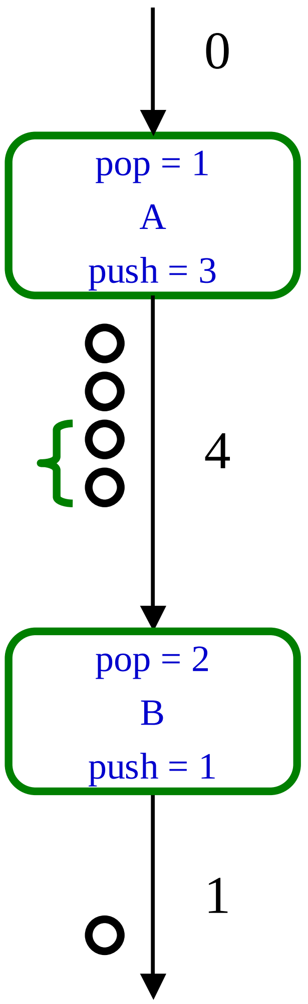

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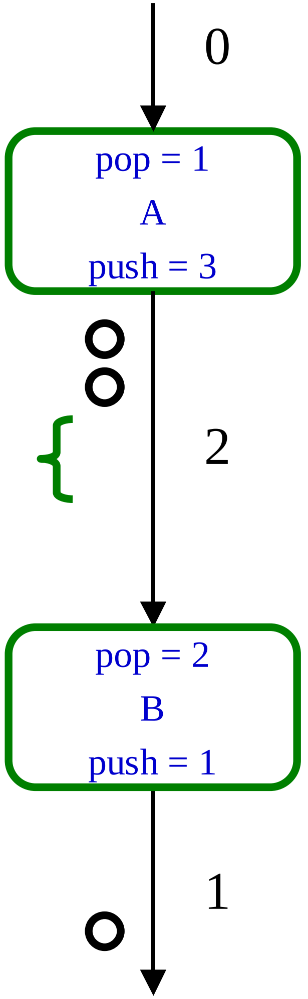

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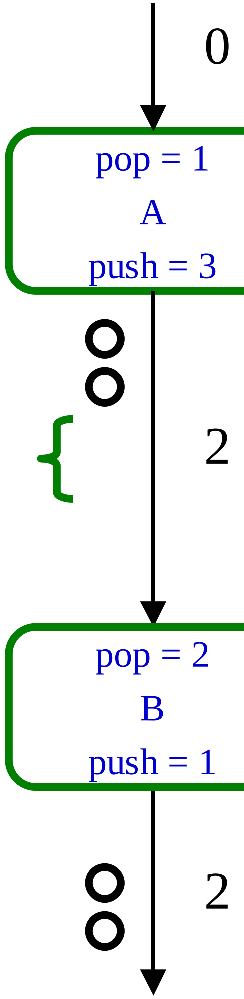

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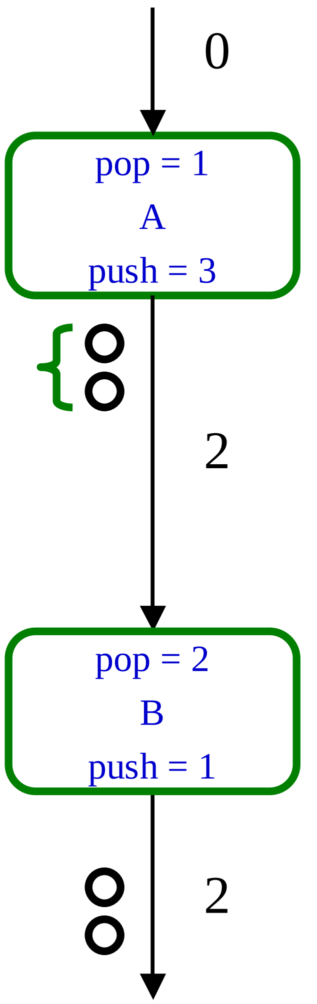

Scheduling Tradeoffs Example

- 3:2 Rate Converter
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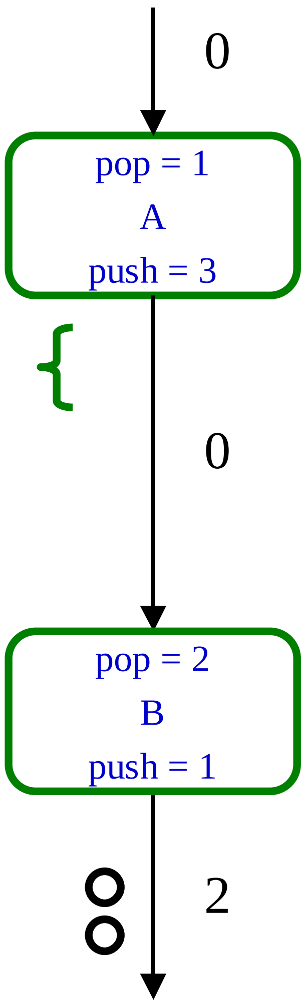

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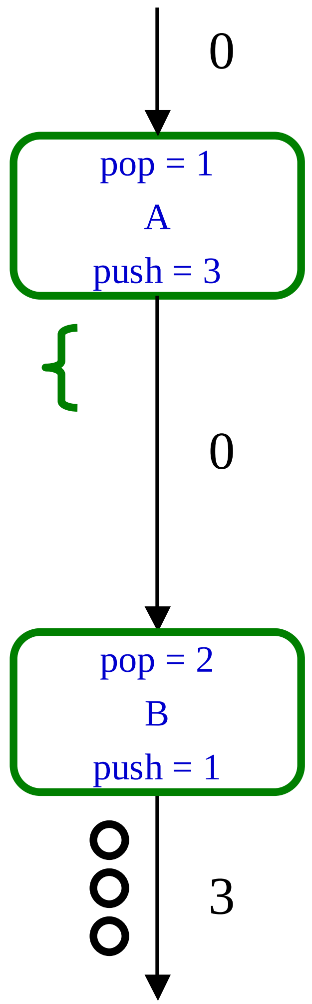

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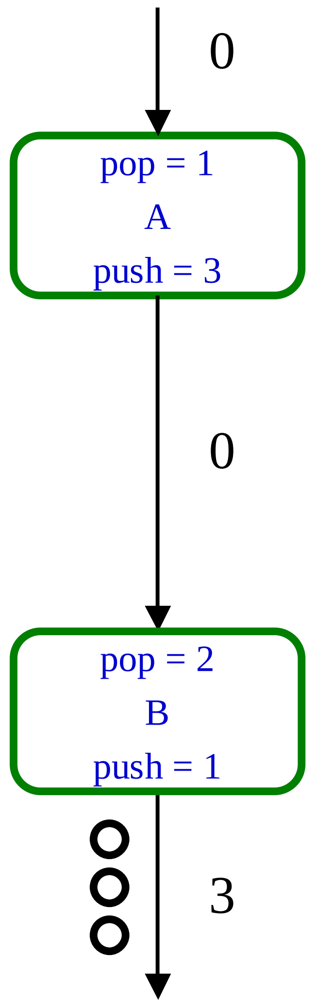

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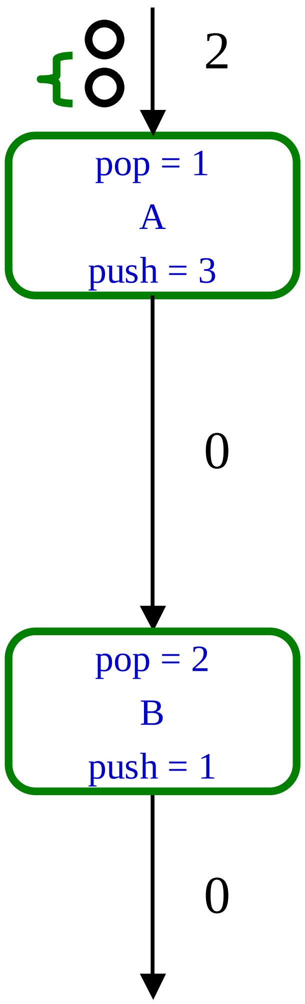

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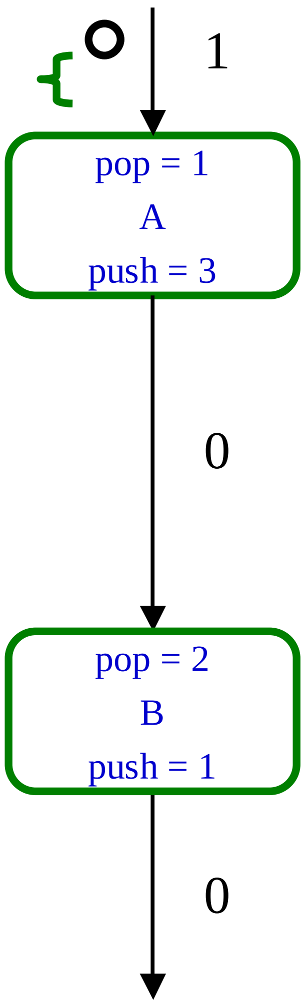

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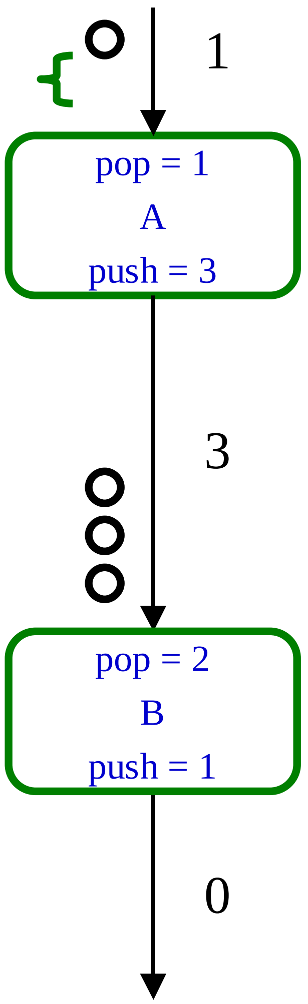

Scheduling Tradeoffs Example

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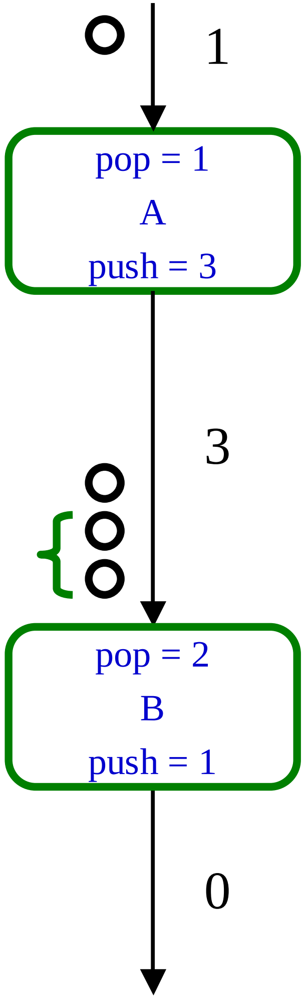

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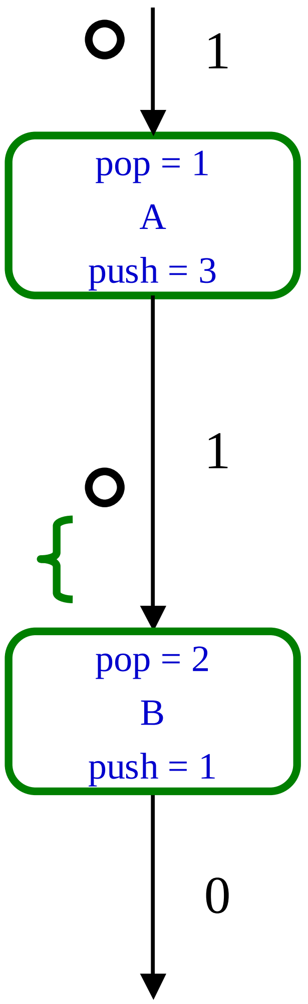

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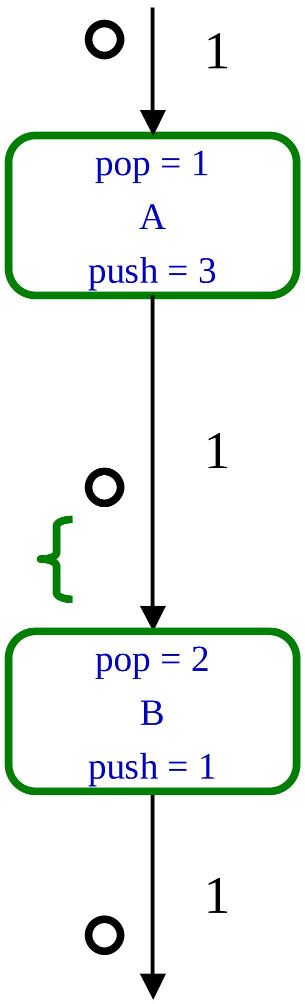

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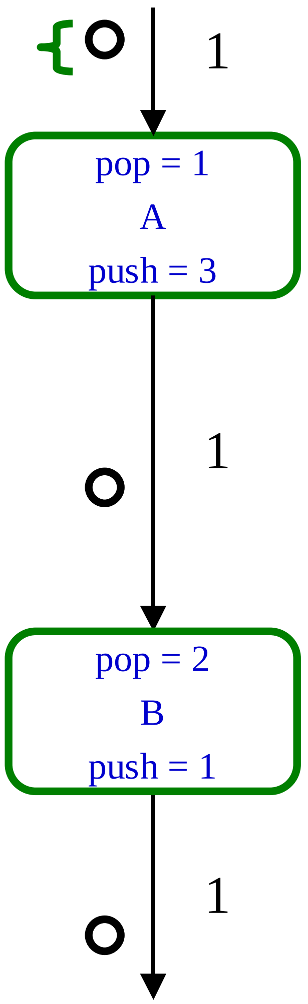

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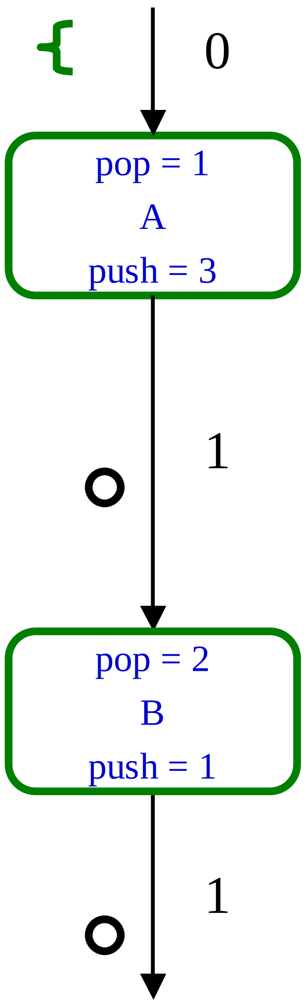

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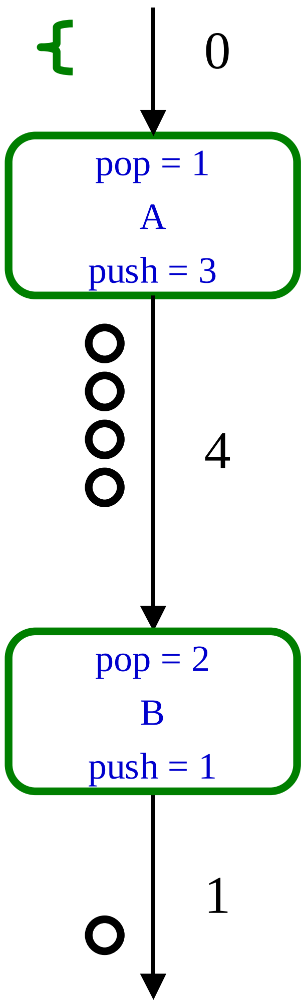

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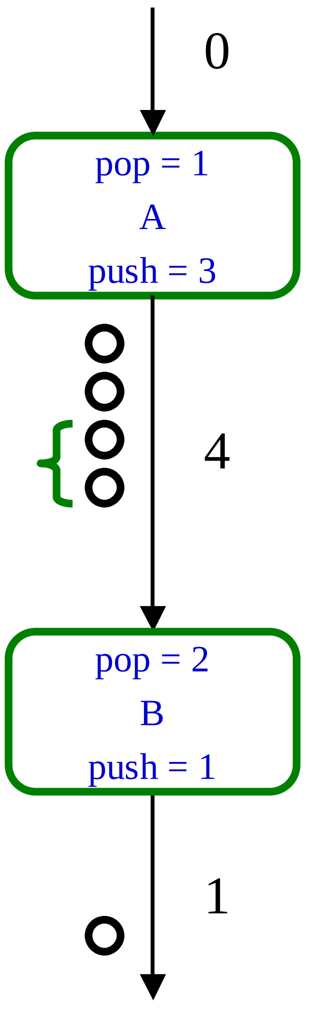

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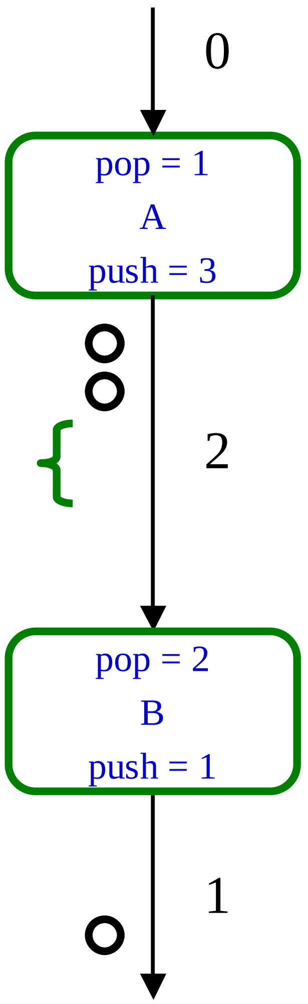

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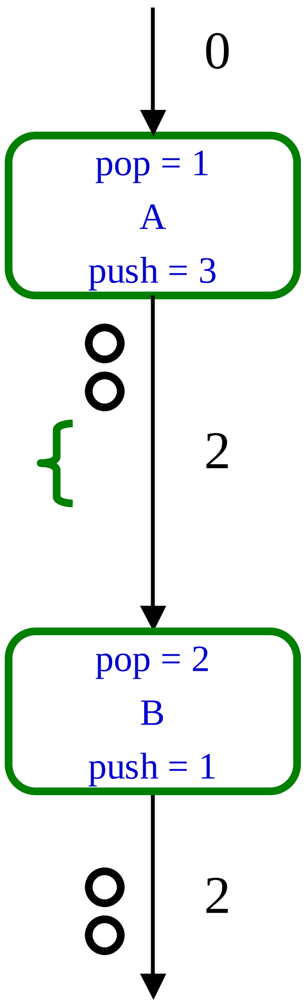

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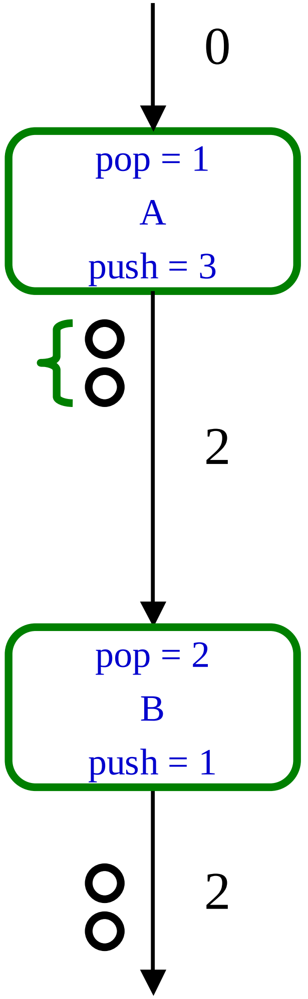

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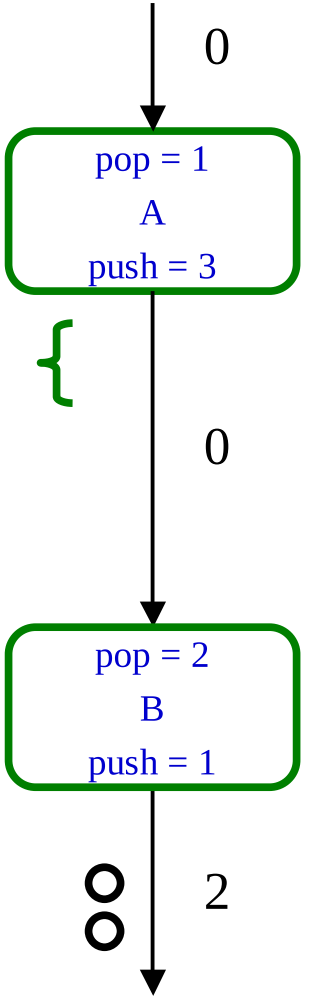

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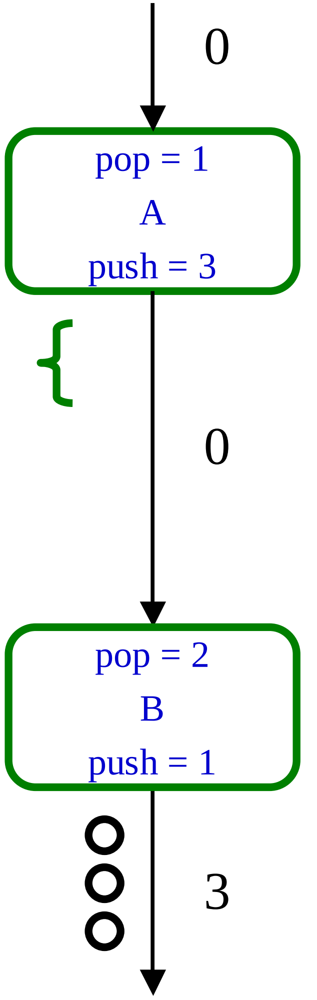

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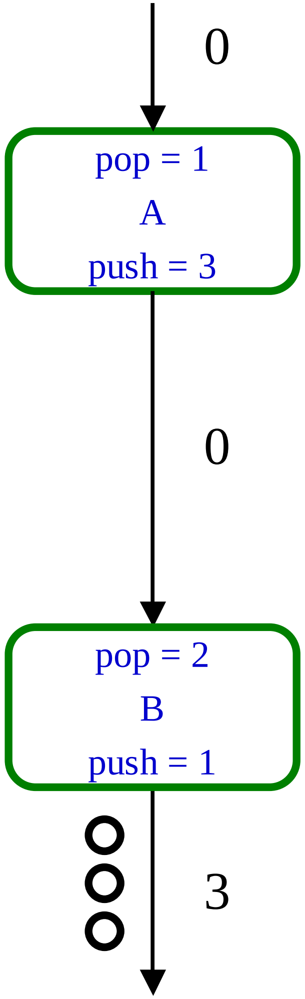
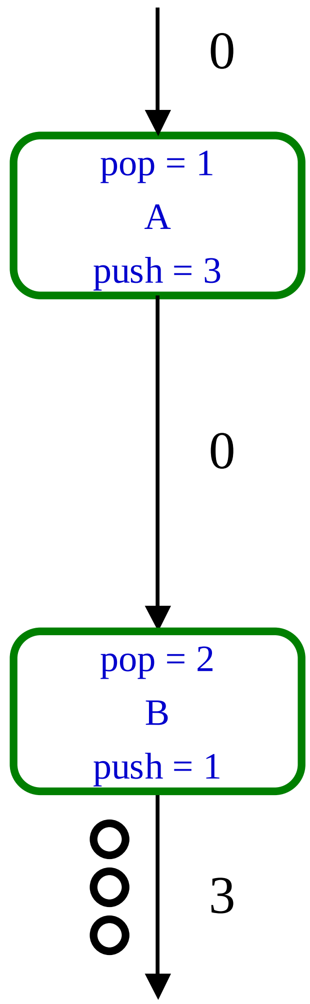


Scheduling Tradeoffs – Latency

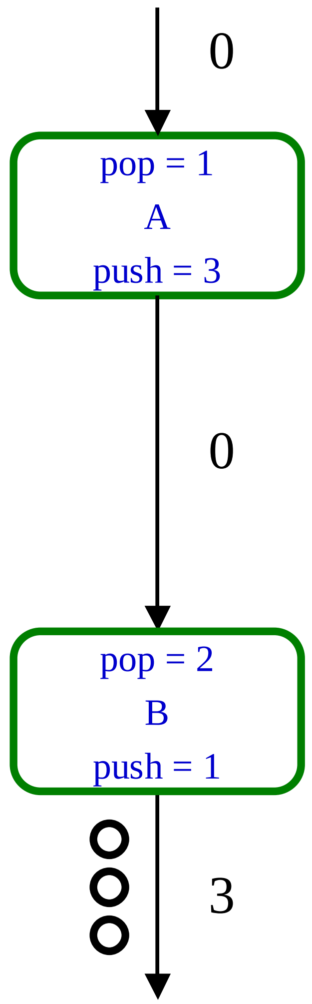
- AABBB – First data item output after third execution of a filter
  - Also A already consumed 2 data items
- ABABB – First data item output after second execution of a filter
  - A consumed only 1 data item



Scheduling Tradeoffs – Buffer Size

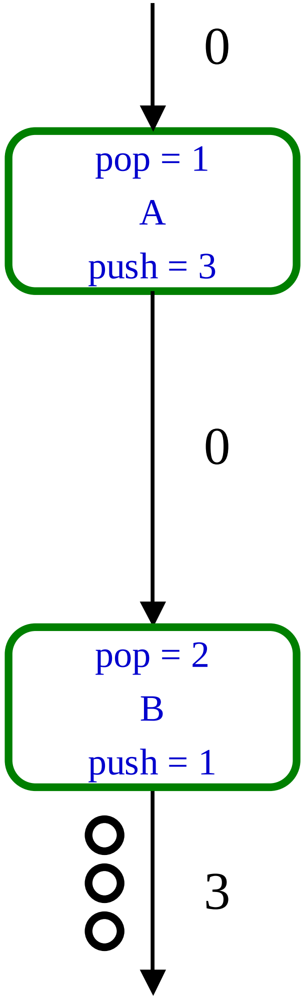
- AABBB requires 6 data items of buffer space between filters A and B

- ABABB requires 4 data items of buffer space between filters A and B



Scheduling Tradeoffs – Code Size

- AABBB – Can be compressed into a loop nest with two appearances of the filters:
  - {5A}{3B}
- ABABB – Requires three appearances of the filters:
  - 2{AB}B

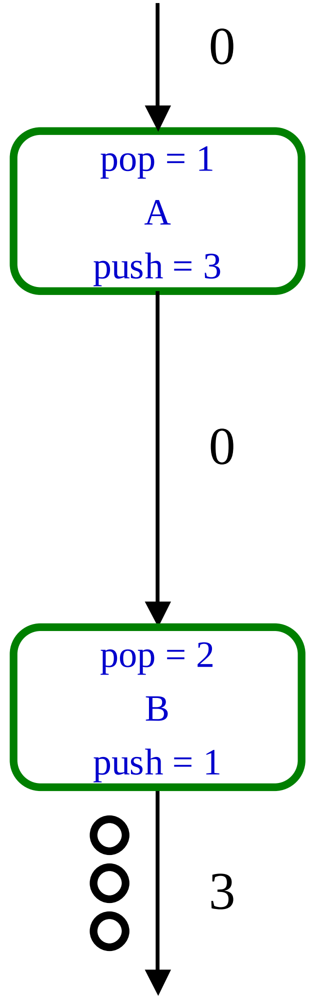


Scheduling Tradeoffs – Code Size

- AABBB – Can be compressed into a loop nest with two appearances of the filters:
  - {5A}{3B}

"Single Appearance Schedule"

- ABABB – Requires three appearances of the filters:
  - 2{AB}B



Single Appearance Scheduling (SAS)

- Every Filter is listed only once in the loop nest denoting the schedule
  - Example: 5{4{AB}} 6{C 3D}
  - There are multiple SAS schedules for a given graph
- By metric of DSP community, SAS schedules guarantee minimal code size
  - Schedule size = # appearances of filters in schedule
  - Filter invocations are often inlined, and consume more space than the loop nests
- Due to their analyzability, SAS schedules have been the target of almost all optimization research
  - Heuristics for finding SAS with minimal buffer size
  - "Buffer merging" for SAS schedules
  - Etc., etc. (see Bhattacharyya '99 for review)

Shortcomings of SAS

- 1. Buffer size explosion for hierarchical components
  - If a large hierarchical component must execute all at once, then its I/O rates are huge
  - Critical consideration for separate compilation

- 2. Restricted space of schedules considered
  - Hampers effectiveness of buffer, latency optimization

➡ Is there a place for multiple-appearance schedules?

Outline

- Introduction to Scheduling
  - Finding a Steady State
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  - Scheduling Tradeoffs
- Phased Scheduling Algorithm
  - Code Size / Buffer Size
  - Hierarchical scheduling
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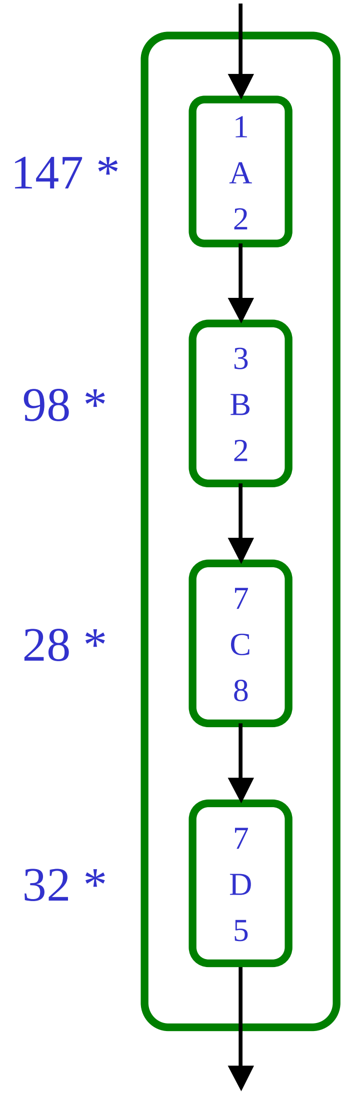
Our recent work: "Phased Scheduling"

- Implements a multiple-appearance schedule
- Approach:
  - Allows code size to grow to a fixed number of SAS "phases"
- Benefits:
  - Small buffer sizes for hierarchical programs
  - Fine grained control over code size vs buffer size
  - Always avoids deadlock in separate compilation

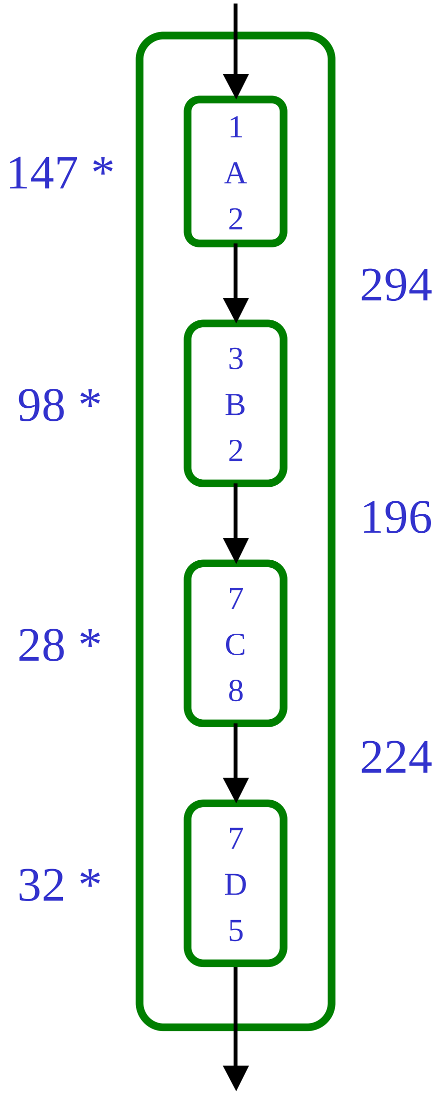
SAS Example – Buffer Size

- Example: CD-DAT
- CD to Digital Audio Tape rate converter
- Mismatched rates cause large number of executions in Steady State



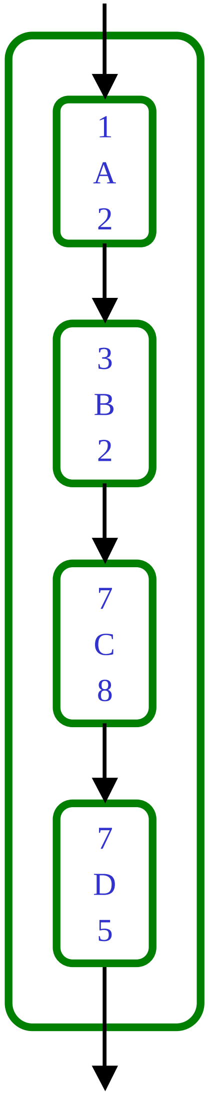
SAS Example – Buffer Size

- Naïve SAS schedule:
  - 147A 98B 28C 32D
  - Required Buffer Size: 714
  - Unnecessarily large buffer requirements!



SAS Example – Buffer Size

- Naïve SAS schedule:
  - 147A 98B 28C 32D
  - Required Buffer Size: 714
  - Unnecessarily large buffer requirements!
- Optimal SAS CD-DAT schedule:
  - 49{3A 2B} 4{7C 8D}
  - Required Buffer size: 258

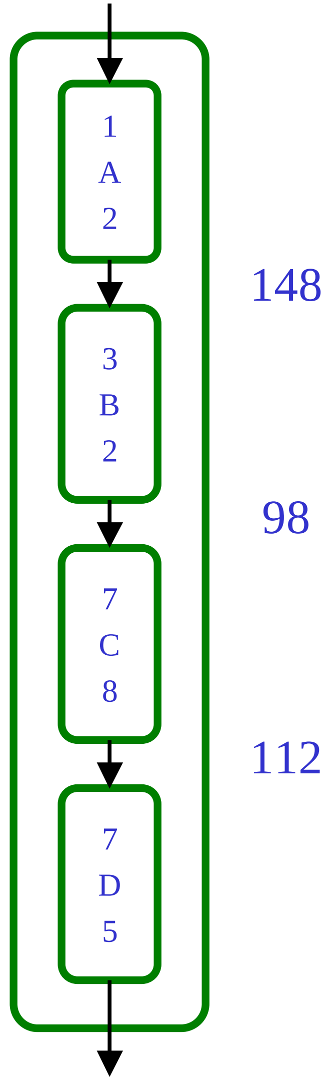


Phased Scheduling

- Idea:
  - What if we take the naïve SAS schedule, and divide it into n roughly equal phases?
- Buffer requirements would reduce roughly by factor of n
- Schedule size would increase by factor of n
- May be OK, because buffer requirements dominate schedule size anyway!

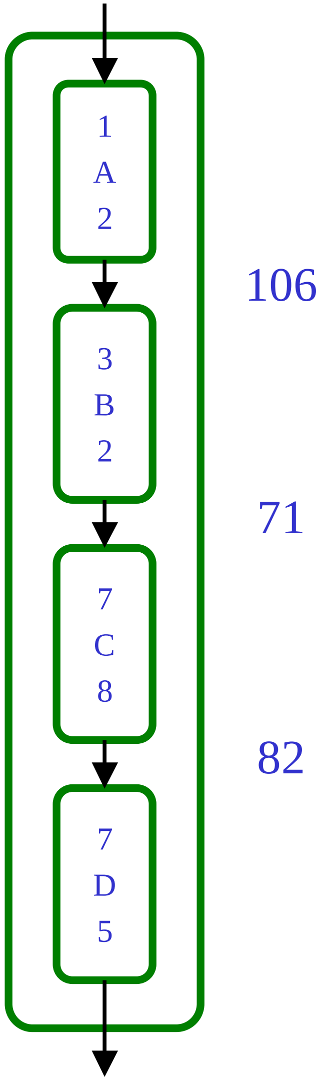
Phased Scheduling

- Try n = 2:
- Two phases are:
  - 74A 49B 14C 16D
  - 73A 49B 14C 16D
- Total Buffer Size: 358
- Small schedule increase
- Greater n for bigger savings



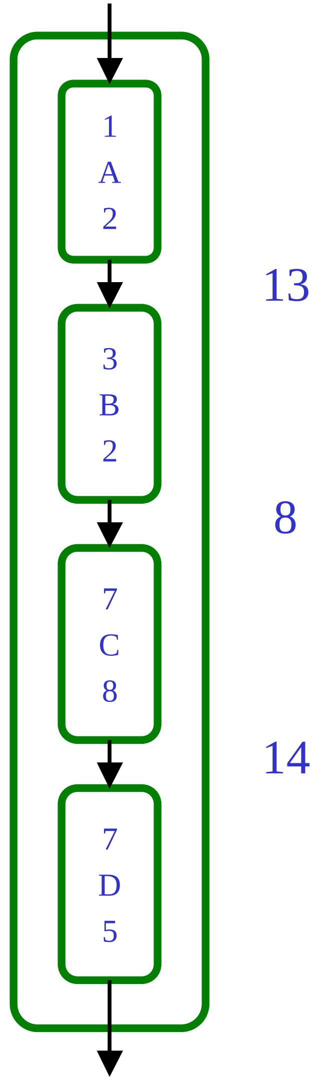
Phased Scheduling

- Try n = 3:
- Three phases are:
  - 48A 32B 9C 10D
  - 53A 35B 10C 11D
  - 46A 31B 9C 11D
- Total Buffer Size: 259
- Basically matched best SAS result
  - Best SAS was 258



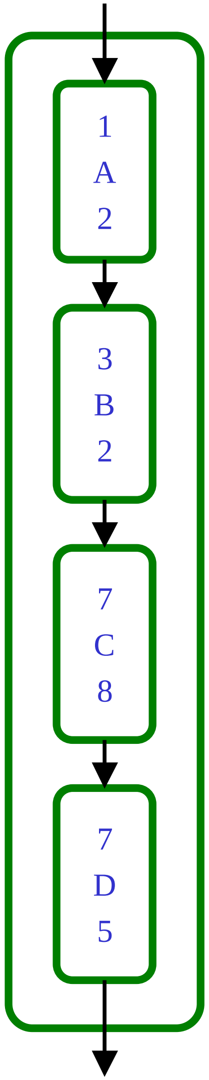
Phased Scheduling

- Try n = 28:
- The phases are:
  - 6A 4B 1C 1D
  - 5A 3B 1C 1D
  - …
  - 4A 3B 1C 2D
- Total Buffer Size: 35
- Drastically beat best SAS result
  - Best SAS was 258



A Lower Bound on Buffer Size: Pull Scheduling

- Pull Scheduling:
  - Always execute the bottom-most element possible
- CD-DAT schedule:
  - 2A B A B 2A B A B C D … A B C 2D
  - Required Buffer Size: 26
  - 251 entries in the schedule
- Hard to implement efficiently, as schedule is VERY large



CD-DAT Comparison: SAS vs Pull vs Phased

	Buffer Size	Schedule Size
SAS	258	4
Pull Schedule	26	251
Phased Schedule	35	52

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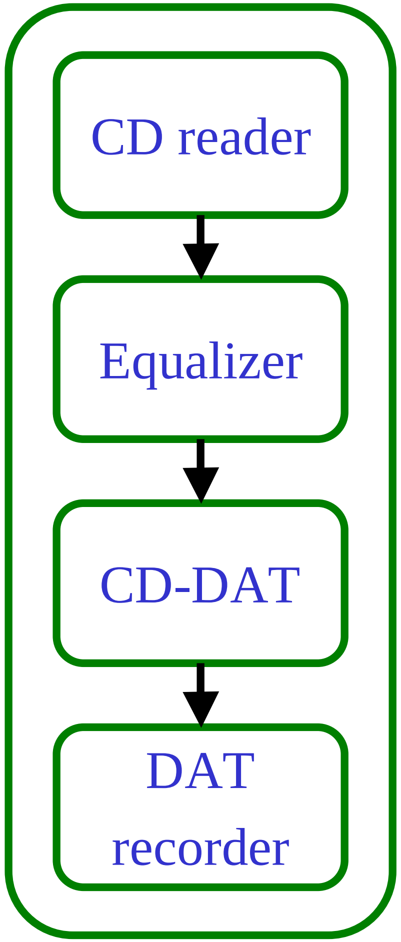
Hierarchical Phased Scheduling

- Apply technique hierarchically
- Children have several phases which all have to be executed
- Automatically supports cyclo-static filters
- Children pop/push less data, so can manage parent's buffer sizes more efficiently



Hierarchical Phased Scheduling

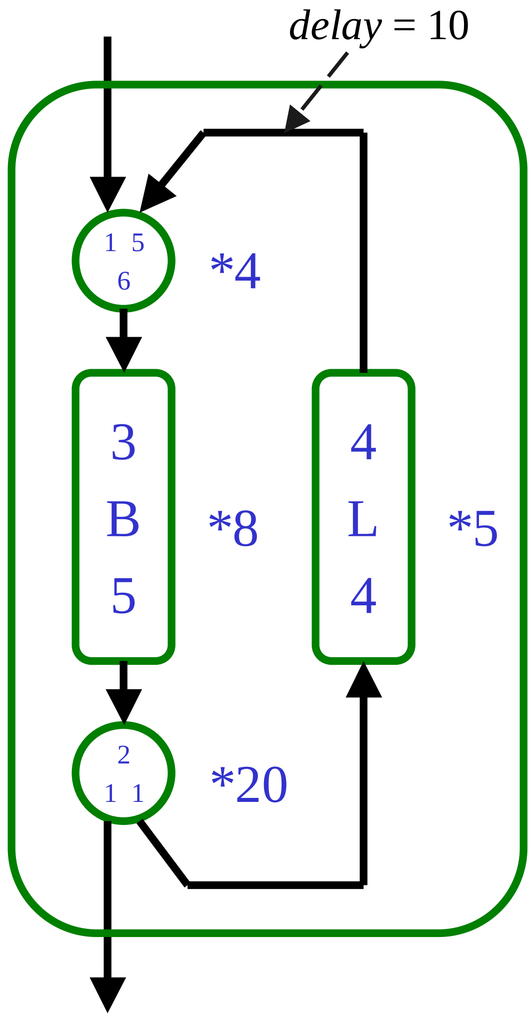
- What if a Steady State of a component of a FeedbackLoop required more data than available?
- Single Appearance couldn't do separate compilation!
- Phased Scheduling can provide a fine-grained schedule, which will always allow separate compilation (if possible at all)

Minimal Latency Schedule

- Every Phase consumes as few items as possible to produce at least one data item
- Every Phase produces as many data items as possible
- Guarantees any schedulable program will be scheduled without deadlock
- Allows for separate compilation
- For details, see LCTES ’03 paper

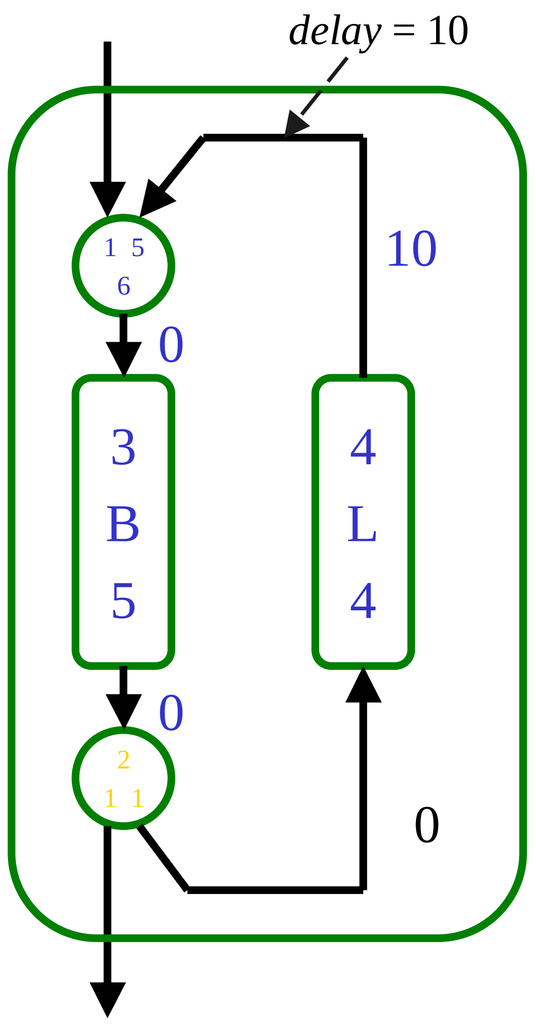
Minimal Latency Scheduling

- Simple FeedbackLoop with a tight *delay* constraint
- Not possible to schedule using SAS
- Can schedule using Phased Scheduling
  - Use Minimal Latency Scheduling



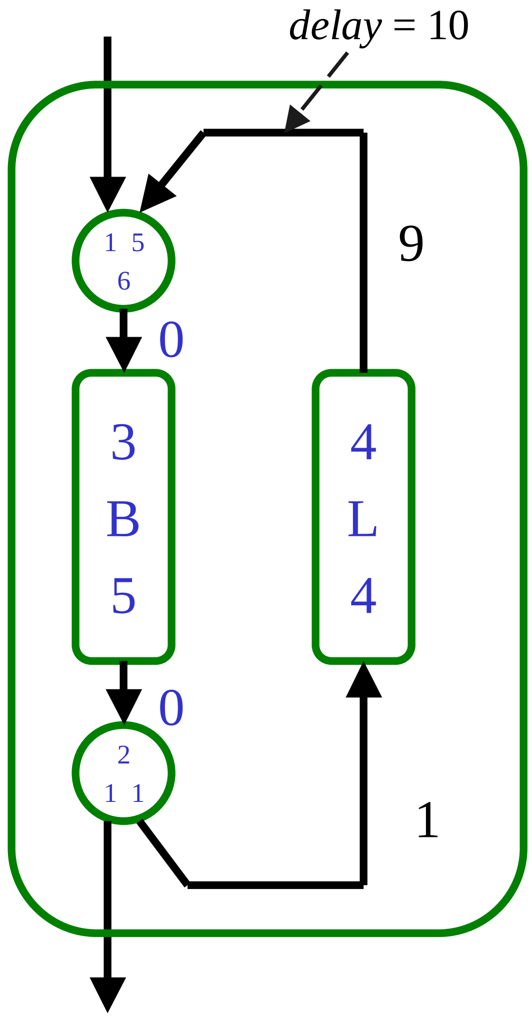
Minimal Latency Scheduling

- Minimal Latency Phased Schedule:



Minimal Latency Scheduling

- Minimal Latency Phased Schedule:
  - join 2B 5split L



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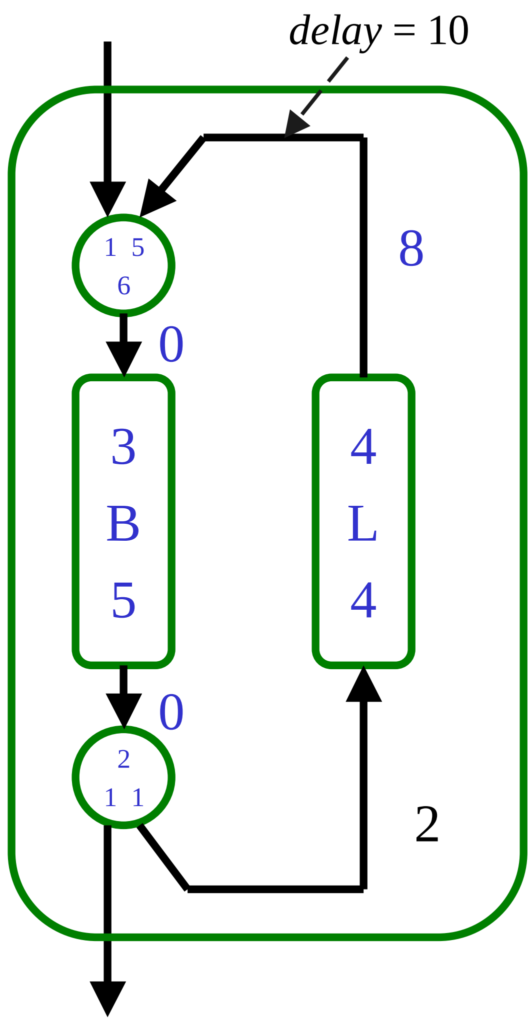

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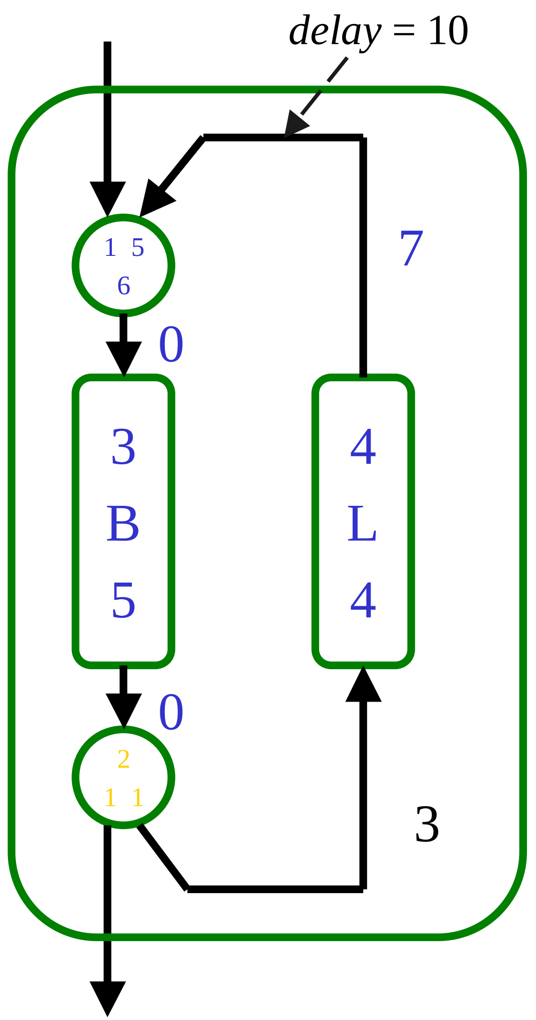

Minimal Latency Scheduling

- Minimal Latency Phased Schedule:
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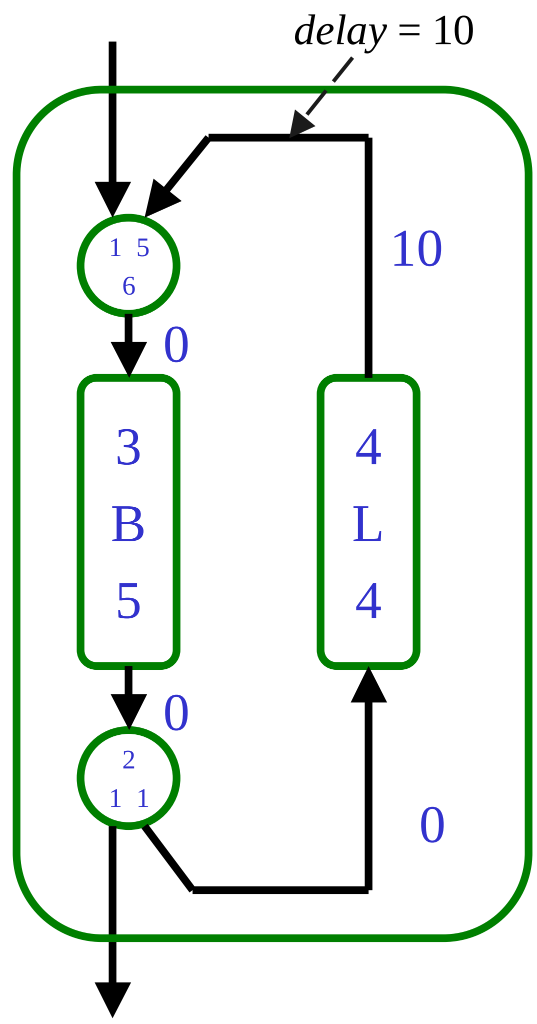
  - join 2B 5split 2L



Minimal Latency Schedule

- Minimal Latency Phased Schedule:
  - join 2B 5split L
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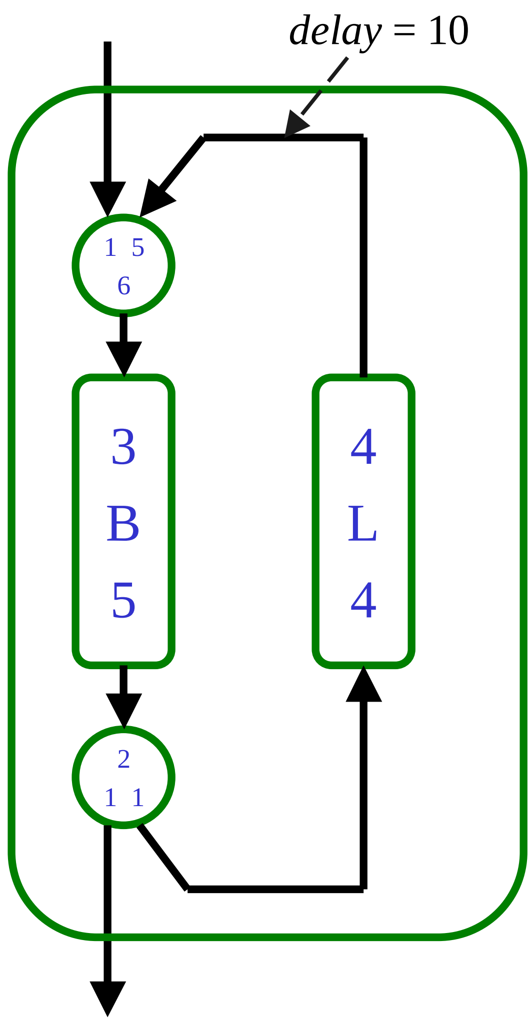
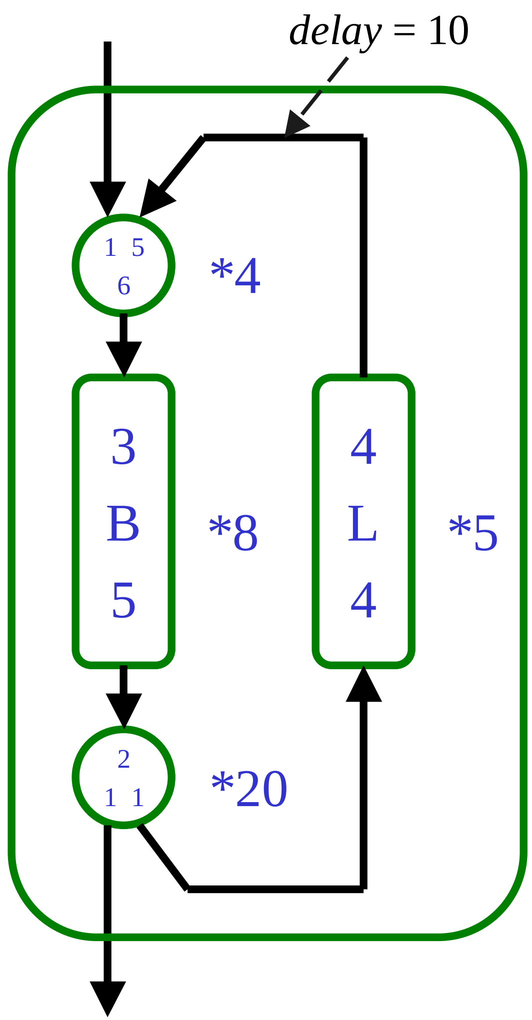
  - join 2B 5split 2L
- Can also be expressed as:
  - 3 {join 2B 5split L}
  - join 2B 5split 2L
- Common to have repeated Phases



Why not SAS?

- Naïve SAS schedule
  - 4join 8B 20split 5L:
  - Not valid because 4join consumes 20 data items
- Would like to form a loop-nest that includes join and L
- But multiplicity of executions of L and join have no common divisors



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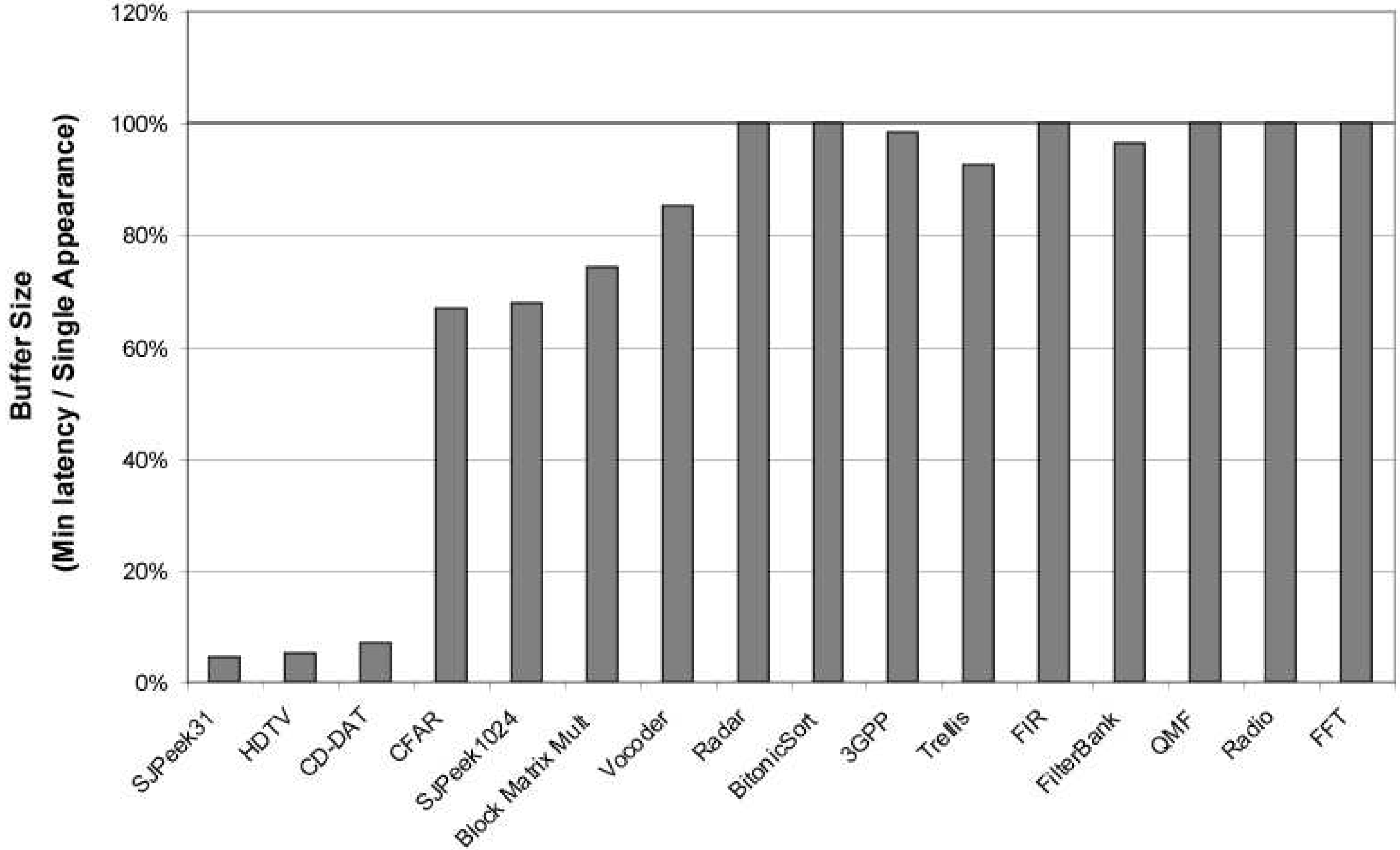
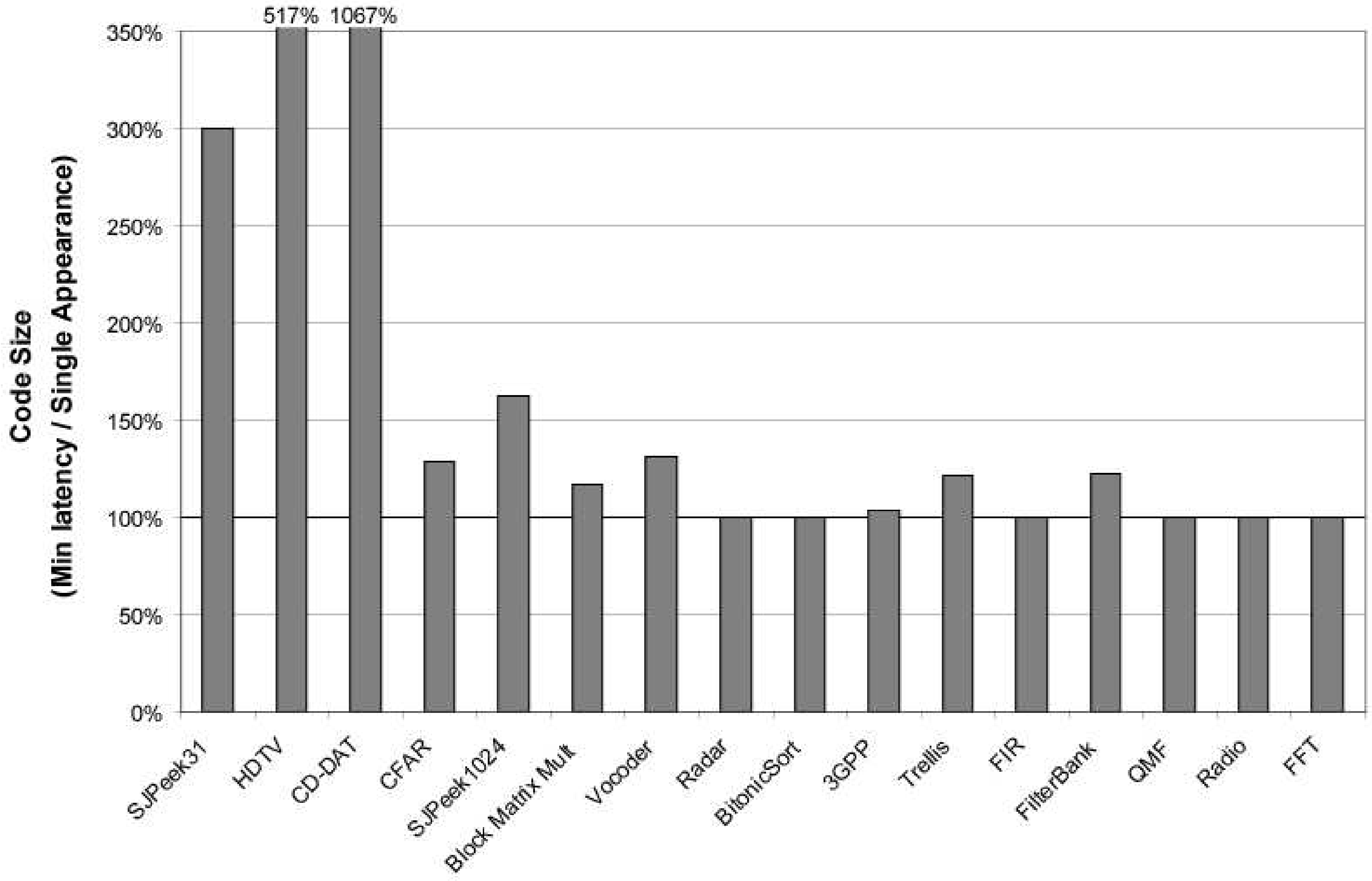
Results

- SAS vs Minimal Latency
- Used 17 applications
  - 9 from our ASPLOS '02 paper
  - 2 artificial benchmarks
  - 2 from Murthy99
  - Remaining 4 from our internal applications

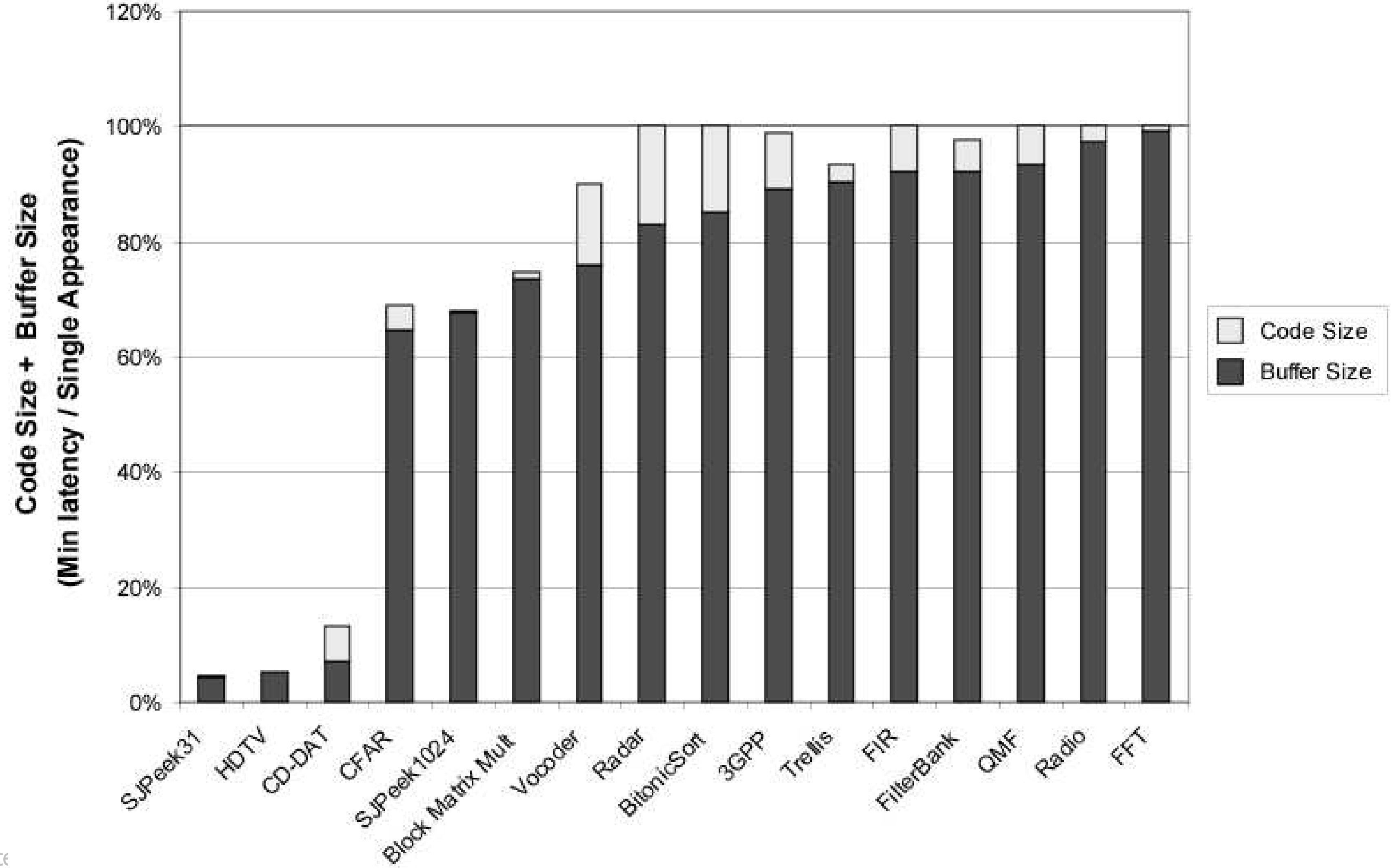
Results - Buffer Size



Results – Schedule Size



Results - Combined



Conclusion

Presented Phased Scheduling Algorithm

- Provides efficient interface for hierarchical scheduling
- Enables separate compilation with safety from deadlock
- Provides flexible buffer / schedule size trade-off
- Reduces latency of data throughput

Step towards a large scale hierarchical stream programming model