## Belief Evolution in Heterogeneous Populations

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## The Goal

Modal the evolution of conflict between culturally distinct populations that differ in their conventions (e.g., cultural, legal etc...)
Questions of interest

- How do populations mediate their actions with others whose conventions conflict with their own.
- How do populations of varying size and preferences evolve over time.
- How should an exogenous force intervene to encourage or force desired outcome.


## Running Example

Driving rules for new country.

- Drive on left or right

New immigrants

- prefer to drive on the side that is common to their home country.
- prefer not to crash with other drivers.



## Coordination Game

Groups Gi and G2 have different preferences structures Distinction between

- preference (convention)
- strategy (what to do given agent's preferences and other's actions)
In this model there are four types of populations (e.g., those that prefer to drive on the left, but drive on the right)

Gi (right-side preference)

| aI | o |
| :---: | :---: |
| o | bI |

G2 (left-side preference)

| a 2 | 0 |
| :---: | :---: |
| 0 | b 2 |

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## Evolutionary Paradigm

Consider those who prefer to drive on left. Their benefit from any action is perceived according to their own conventions in game GI.


Replicator dynamics

## RH7

is $\left.P_{A}^{t}\left(g_{1}\right)+P_{A}^{t}\left(g_{2}\right)\right) \cdot a\left(g_{1}\right)+\left(P_{B}^{t}\left(g_{1}\right)+P_{B}^{t}\left(g_{2}\right)\right) \cdot b\left(g_{1}\right)$
At time $t+I$, the proportion of those playing action A who prefer to drive on the left is

$$
P_{A}^{t+1}(g)=\frac{W_{A}^{t}(g) \cdot P_{A}^{t}(g)}{\sum_{g^{\prime} \in G} W_{A}^{t}\left(g^{\prime}\right) \cdot P_{A}^{t}\left(g^{\prime}\right)+W_{B}^{t}\left(g^{\prime}\right) \cdot\left(1-P_{A}^{t}\left(g^{\prime}\right)\right)}
$$

Some Example of Strategic
Evolution









All critical points lie on a plain


$$
G_{1}=\left[\begin{array}{ll}
4 & 0 \\
0 & 7
\end{array}\right]
$$

$$
G 2=\left[\begin{array}{ll}
7 & 0 \\
0
\end{array}\right]
$$

Stable points
all Bis
all $\mathrm{A}_{2}$
Unstable point mixed $A_{1}, B_{2}$ two more saddle points

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## Modeling Intervention

Government can choose to

- encourage immigration from different countries. (preserving conventions)
- intervene and educate citizens to drive on left- or righthand side. (preserving population size)
- Actions may be associated with different costs

A players prefer G2

B players prefer Gi

