Interactive MOEA/D for Multi-objective Decision Making

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ABSTRACT

In this paper, an interactive version of the decomposition based multiobjective evolutionary algorithm (iMOEA/D) is proposed for interaction between the decision maker (DM) and the algorithm. In MOEA/D, a multi-objective problem (MOP) can be decomposed into several single-objective sub-problems. Thus, the preference incorporation mechanism in our algorithm is implemented by selecting the preferred sub-problems rather than the preferred region in the objective space. At each interaction, iMOEA/D offers a set of current solutions and asks the DM to choose the most preferred one. Then, the search will be guided to the neighborhood of the selected. iMOEA/D is tested on some benchmark problems, and various utility functions are used to simulate the DM's responses. The experimental studies show that iMOEA/D can handle the preference information very well and successfully converge to the expected preferred regions.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search-*Heuristic Methods*

General Terms

Algorithms, Experimentation

Keywords

Multi-objective optimization, evolutionary algorithm, decision making, preference incorporation, decision maker, interaction

1. INTRODUCTION

Multi-objective optimization problems (MOPs) arise in many engineering areas. There is no single optimal solution which can optimize all the objectives at the same time. Since Schaffer's seminal work published in 1985 [1], numerous multi-objective evolutionary algorithms (MOEAs) have been proposed [2]-[6].

Most MOEAs approximate the entire Pareto-optimal frontier (PF). However, multiobjective optimization, by nature, is to help a human decision maker (DM) to find her preferred solutions [7]. Approximation of the whole PF is not very computationally economic in some application. Preference information from the DM can be used for guiding the search to a PF part of interest.

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Several preference-based algorithms are proposed in many literatures [8]-[13]. The DM's preference can be obtained and refined via interaction between the DM and algorithms. Some effort has been made to design interactive MOEAs [14]-[18].

MOEA/D (multiobjective evolutionary algorithm based on decomposition) [5] is a recent MOEA framework with many successful applications. In this paper, we propose an interactive version of MOEA/D, called iMOEA/D. In iMOEA/D, the MOP in question is converted into a number of scalar optimization problems by the Tchebycheff approach [19] with even spread weight vectors. During optimization process, the DMs are asked to compare some current solutions and select ones which please them most at each interactive stage. The weights of selected solutions will be used to guide the following optimization for finding the finial preferred region.

The remainder of this paper is organized as follows: Section 2 presents the related background. In section 3, the structure of iMOEA/D is given. The experimental studies are presented in section 4. Section 5 concludes the paper.

2. BACKGROUND

2.1 Multi-objective Optimization

The minimized multi-objective is broadly involved in our study, and can be stated as follows:

$$
\begin{cases} \min F(x) = (f_1(x), f_2(x), \cdots, f_m(x))^T \\ \text{subject to} \ \ g_i(x) \le 0 \ i = 1, 2, \cdots, p \end{cases} \tag{1}
$$

where $x=(x_1,x_2,\dots,x_k)\in\Omega$ is decision vector and continuous in search space. $m \geq 2$ is the number of objective functions and *p* is the number of constraints.

Very often, the objective functions in (1) contradict one another. It is impossible for one single solution in Ω to minimize all these objectives simultaneously. One has to make a trade-off among them.

The best trade-off candidate solutions can be defined based on dominance. $x_{\text{A}} \in \Omega$ is said to dominance $x_{\text{B}} \in \Omega$ iff:

$$
\forall i \in \{1, \cdots, k\}, \quad f_i(x_A) \le f_i(x_B)
$$

$$
\land \exists j \in \{1, \cdots, k\}, \quad f_j(x_A) < f_j(x_B) \tag{2}
$$

If there is no vector $x \in \Omega$ such that $F(x) > F(x^*)$, x^* is called a Pareto optimal solution. The Pareto set (PS), is the set of all the Pareto solutions, which is denoted as *P** in this paper, its image in the objective function space is called the Pareto front (PF).

2.2 Tchebycheff Approach

MOEA/D requires a decomposition (aggregation) approach to transform the MOP into *N* sub-problems. We use the Tchebycheff approach in this paper. Each sub-problem is as follows [5][19]:

$$
\min g^{te}(x | \lambda, z^*) = \max_{1 \le i \le m} \{ \lambda_i | f_i(x) - z_i^*| \}
$$

subject to $x \in \Omega$ (3)

where z^* is the reference point and $z_i^* = \min\{f_i(x)|x \in \Omega\}, i=1,\dots,m$. $\lambda = (\lambda_1, \dots, \lambda_m)$ is the weight vector. Different sub-problems in MOEA/D have different weight vectors. Their optimal solutions can collectively approximate the PF.

2.3 Utility Function

Modeling the DM's preferences is a challenging task in interactive algorithms. Several mathematical models have been proposed [20][21]. The generic polynomial utility function [21] can be described as:

$$
U(f) = \sum_{i=1}^{m} w_i f_i + \sum_{i=1}^{m} \sum_{j \ge i} w_{ij} f_i f_j
$$

+
$$
\sum_{i=1}^{m} \sum_{j \ge i} \sum_{h \ge j} w_{ij} f_i f_j f_h + \cdots
$$

+
$$
\sum_{i=1}^{m} \cdots \sum_{k \ge m-1} w_{i \cdots km} f_i \cdots f_k f_m
$$
 (4)

where f_i is the *i*-th objective value; and *w* is the parameter to control the preference weights of DM on different combination of objectives. This paper uses (4) as utility functions.

3. PROPOSED ALGORITHM

3.1 Interactive Model

iMOEA/D involves the DM in the process of optimization for investigating the problem and offering feedbacks to the search.

Figure 1. Interactive model in this study.

Figure 1 shows the structure of interactive model used in our approach, and a similar structure has been proposed in [22]. In this model, the MOP is the input to iMOEA/D, which exchanges information with DMs in each interactive stage. During the optimization process, iMOEA/D provides the current best optimal solutions to the DM, which contains information about the MOP. The DM plays a role like a brain in this model. Analyzing the solutions, the DM (brain) can provide some feedbacks to iMOEA/D for guiding its search.

This basic idea of interaction can accommodate many multiobjective optimization algorithms.

3.2 Interactive MOEA/D

The details of iMOEA/D are given as follows:

In algorithm 1, the uniformly spread weight vectors are obtained for decomposing the MOP. *t* is the generation counter.

The process of interacting with DMs will happen periodically. In our implementation, it happens once every *H* generations. At each interaction, *P* individuals are presented to DMs. After estimating the utility function values of these individual solutions, the best solution *y** will be selected as the center of the new preferred region.

3.3 Dynamic Preference Renewing

The information feedback from the DM is used in the process of renewing the preferred weight region in every interactive stage. In iMOEA/D, the idea of favorable weights will be employed to identify the location of solutions.

Assuming the weight vector of the favorite solution y^* is $w^*=(w_1^*, w_2^*, \ldots, w_m^*)$, and indicate the preferred weight region S^h as a sphere (or a hyper-sphere with more than 3 objectives) in this paper, denoted as (w^c, r^h) at the interaction stage *h*. According to this, the center of the weight region is just the *w**, and the radius r^h can be obtained by calculating $R^h/2$. *R* is a reduction factor which can control the shrinking rate of the boundary of the preferred weight region. The higher the *R* is, the faster the boundary shrinks. Based on [23], *R* should meet:

$$
\sqrt[m]{\frac{1}{P}} \le R < 1\tag{5}
$$

where *m* is the number of objectives; *P* is the number of the individuals presented to the DM, and *P>m*.

3.4 Preference Incorporation Mechanism

In MOEA/D, the solutions in each generation are the current best one of corresponding sub-problems. Therefore, guiding solutions converging to DM's preference can be realized by changing the density of the spread weight vectors. Figure 2 illustrates how the weight vectors are renewed based on preferred weight region $S^h = (w^*, r^h)$ in the biobjective problem .

Figure 2. Weight vectors are renewed in the case of two objectives.

The detailed steps of this process are as follows:

Return: *N* new weight vectors

4. EXPERIMENTAL STUDY

In this section, the simulation runs are conducted to demonstrate the performance of the new algorithm. Two 2-objecive tests of ZDT problem and two 3-objecive tests of DTLZ problem will be discussed respectively. And all the simulations were run on a personal computer with P-IV 2.33G CPU and 2G RAM.

4.1 Parameter Settings

In all these experimental studies, the parameter settings are as follows: the total number of interaction stage *H*=4; the reduction factor *R*=0.6; the number of solutions to be presented to the DM in each interaction stage *P*=10. In addition, the population size *N* is set to be 100 for 2-objective test instances and 300 for 3-objecive test instances. The algorithm will stop after 500 generations and the results are based on 30 independent runs.

4.2 Results on 2-objective MOPs

First, the 2-objective test problems ZDT1 and ZDT2 with 30 variables developed by Zitzler et al. in 2000 [24] will be used to test this interactive algorithm. Four different utility functions are used in each problem for demonstrate the performance.

4.2.1 Results on ZDT1

This problem has a concave Pareto-optimal frontier spanning continuously in $f_1 \in [0,1]$ and follows a function relationship: $f_2 = 1$ - $(f_1)^{0.5}$. To sufficiently verify the ability of converging to the PF guided by simple or complicated utility functions, we establish four cases with different combination of relative weight of objectives. However, all these function are coming from (4).

Case 1: In the first case, the first objective holds a higher relative weight; and a simple utility function is adopted described as:

$$
U = \min f_1 \tag{6}
$$

Case 2: On the contrary of the first test, the supposed DM favors the second objective more than the first one; so this simple utility function can be described as:

$$
U = \min f_2 \tag{7}
$$

Case 3: In this case, the supposed DM prefers almost the center of the PF; and a more complicated utility function is designed as:

$$
U = \min(f_1^2 + f_2^2 - 0.8f_1 - 0.8f_2 + 0.32)
$$
 (8)

Restricted by the real PF of ZDT1, the theoretical minimum value $(f_1=0.4, f_2=0.4)$ may not be reached. The final obtained solutions are always inside a region approximating to PF and possess smaller value of (8) .

Case 4: In case four, a more complicated utility function is used to guide the optimization process with the following formula:

$$
U = \min(0.28f_1^2 + 0.29f_1f_2 + 0.38f_2^2 + 0.05f_1) \tag{9}
$$

This utility function is proposed in [21] for dealing with DTLZ1 problem with only two objectives and the exact minimum value of (9) is $(f_1 = -0.111, f_2 = 0.042)$, which will be used to compare the result in the following experiment.

Take case 1 as an example; we display the gradually changed process of interactive in Figure 3. The first interaction occurs after 100 generations when *H=*4 and the maximal generation equal to 500. Sub figure (a) is just the result after the first interaction and the current generation is 200. It clearly shows that more solutions have already begun to be partial to the first objective, but it still has a gap with expectation. Sub figure (b) shows the result after the second interaction and the current generation is 300. In this stage, more and more solutions have moved near the top of the PF; and a part of these solutions almost arrive the expectation. After the third interaction, current solutions in generation 400 can almost satisfy the utility function shown in sub figure (c). And sub figure (d) depicts the finial solutions with preference of the first objective. Shown by Figure 3, we can prove the feasibility of this new approach initially.

Figure 3. Obtained solutions in different interactive stages of ZDT1 about case 1: (a) 1st interaction; (b) 2nd interaction; (c) 3rd interaction; (d) 4th interaction.

Figure 4 displays the plots of all these four kinds of cases separately in sub figure (a) (b) (c) and (d). These plots are shown the final preferred solutions guided by their own utility function. All of these results show that the algorithm not only can converge to the Pareto-optimal frontier, but also can find the preferred region or preferred objective decided by DMs.

Figure 4. Front obtained against theoretical PF of ZDT1 problem: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

4.2.2 Results on ZDT2

ZDT2 has a convex PF spanning continuously in $f_1 \in [0,1]$ and follows a function relationship: $f_2=1-(f_1^2)^{0.5}$. Similar with tests of ZDT1 problem, we adopt four different kinds of utility functions to guide the optimization. Owning to the convex characteristic of ZDT2 problem, the utility function of case 4 will be changed a little.

Case 1: Supposed DMs are preferred the first objective more than the second one; and the utility function are also described as (6).

Case 2: The second objective gets more preferences. So (7) will be used as the utility function.

Case 3: The center of the PF is preferred by supposed DMs; and the complicated utility function similar as (8) can be described as:

$$
U = \min(f_1^2 + f_2^2 - f_1 - 1.1f_2 + 1.3025) \tag{10}
$$

In order to compare the result in the following experiment, the theoretical minimum value in each objective has been calculated as $(f_1=0.5, f_2=0.55)$.

Case 4: In this case, we continue to use the utility function proposed in [21]. However, due to the convex characteristic, small changes are need.

$$
U = \max(0.28f_1^2 + 0.29f_1f_2 + 0.38f_2^2 + 0.05f_1)
$$
 (11)

As we know, the theoretical minimum value of (9) is $(f_1 = -0.111)$, f_2 =0.042), so the maximizing the function can provide another perspective to help DMs finding the region in the middle part of the whole PF when encounter a convex problem as ZDT2.

Sub figures (a) (b) (c) (d) of Figure 5 display the final obtained solutions of all these cases. We can see clearly that the interactive method can well dispose the convex problems with well converged preferred solutions, no matter guided by simple or complicated utility functions.

Figure 5. Front obtained against theoretical PF of ZDT2 problem: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

4.3 Results on 3-objective MOPs

In this part, some 3-objective test problems will be used to test the new approach. The widely used benchmark problems DTLZ1 and DTLZ2 denoted as 3-DTLZ1 and 3- DTLZ2 are proposed by Deb in 1999 [25]. Similar to the tests of 2-objective problems, four utility functions with different complexity will be used for guiding the evolving process.

4.3.1 Results on DTLZ1

The PF of DTLZ1 is $\Sigma_{i \in [1, m]} f_i = 0.5$, which just is a geometric plane with 0.5 as the maximum value in each objective; and DTLZ1 discussed here contains 7 variables. As for 3-objective problem, we make a small change of simple utility functions, which will be discussed in the following case1 and case 2.

Case 1: Supposed DMs always hold a high probability for favoring more than one objective in 3-objective problem. So, in the first case, both first and second objectives are set an equal higher relative weight. And the utility function can be easily described as:

$$
U = \min(f_1 + f_2) \tag{12}
$$

Case 2: Similar with the first case, another two objectives are preferred in this case: the first and the third one.

$$
U = \min(f_1 + f_3) \tag{13}
$$

Case 3: The center of the PF is preferred by supposed DM. Therefore, a more complicated utility function can be described as follows:

$$
U = \min(f_1^2 + f_2^2 + f_3^2 - 0.4f_1 - 0.4f_2 - 0.4f_3 + 0.12)
$$
 (14)

In this case, solutions with smaller value of (14) are preferred.

 However, the real PF may not reach the theoretical minimum, so the exact minimum value $(f_1=0.2, f_2=0.2, f_3=0.2)$ is only used to estimate the finial preferred region.

$$
U = \min(f_1^2 + f_2^2 + f_3^2 - 0.2f_1 - 0.2f_2 - 0.6f_3 + 0.11) \tag{15}
$$

Case 4: In the last case, another region is preferred by DM, which is also in the middle part of the PF with a complicated utility function:

According to (15), the first and the second objective are preferred morn than the third one. And the finial preferred region is partial to the top of the PF. The theoretical minimum value $(f_1=0.1,$ $f_2=0.1$, $f_3=0.3$) is also pointed out for estimate the result obtained by iMOEA/D.

Take the complicated case 4 as an example, the gradually changed solutions at all four interactive stages are shown in Figure 6. It can be seen clearly that more and more solutions are focusing on the very preferred region with the time of interaction increased. In addition, before the finial interaction, solutions always maintain certain diversity for offering the possibility to correct errors.

Figure 6. Obtained solutions in different interactive stages of DTLZ1 about case 1: (a) 1st interaction; (b) 2nd interacttion;(c) 3rd interaction; (d) 4th interaction.

The final results of these four cases are shown in Figure 7. In 3 objective problems, our algorithm also can well converge to the PF within the preferred region. Sub figure (a) and (b) demonstrate the situation in which two objectives are equally preferred. The obtained solutions are nearly the bisector of these two objectives and maintain a good diversity. The shape of obtained solutions shown by sub figure (c) is almost a circle in the center of PF, which just display the mechanism of setting the boundary of preferred weights. However, the shape displayed in sub figure (d) is almost an ellipse. That is because the preferred region is linked to the boundary of weights which not directing to the center of the PF.

4.3.2 Results on DTLZ2

The PF of DTLZ2 with 12 variables satisfy $\sum_{i \in [1, m]} f_i^2 = 1$. And two simple and two complicated utility functions can be set as follows.

Figure 7. Front obtained against theoretical PF of DTLZ1 problem: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

Figure 8. Front obtained against theoretical PF of DTLZ2 problem: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

Case 1: Similar with DTLZ1, in this case, both first and second objective are preferred more than the third one. So the utility function is equal to (12) .

Case 2: The utility function is as same as (13), when the first and the third objective are preferred.

Case 3: The center of PF is preferred in this case, and the complicated utility function is similar as (14).

$$
U = \min(f_1^2 + f_2^2 + f_3^2 - 1.2f_1 - 1.2f_2 - 1.2f_3 + 1.08) \tag{16}
$$

The center of the real PF are preferred in this case, all three objectives hold an equal preferred weight; and the exact theoretical minimum value is $(f_1=0.6, f_2=0.6, f_3=0.6)$.

Case 4: In case four, the third objective is preferred more, but the other two objectives still maintain their own relative weights.

$$
U = \min(f_1^2 + f_2^2 + f_3^2 - 0.4f_1 - 0.4f_2 - 1.6f_3 + 0.72)
$$
 (17)

Utility function (17) slightly leads the preferred region close to the top of the PF. The theoretical minimum value $(f_1=0.2, f_2=0.2,$ $f_3=0.8$) will be used to compare the finial preferred region of iMOEA/D.

Figure 8 has displayed the front obtained against theoretical PF within the preferred region. Sub figure (a) (b) (c) and (d) correspond to different four utility functions, which can well demonstrate the well performance of our interactive mechanism. As for the specific of the final solutions are as same as the results of DTLZ1.

4.4 Results on Welded Beam Design Problem

The welded beam design problem (WBDP) is a typical multiobjective problem in the real world [26]. It has four real parameter variables denoted as $x=(h, l, t, b)$, four non-linear constraints and two objective functions. One objective is about minimizing the cost of fabrication, while the other one is about minimizing the end deflection of the welded beam. The specific expression is shown as follows:

$$
\min F(x) = (f_1(x), f_2(x))
$$

\n
$$
f_1(x) = 1.10471h^2l + 0.04811tb(14.0 + l)
$$

\n
$$
f_2(x) = 2.1952/l^3b
$$

\nsubject to g₁(x) = 13,600 - τ (x) \ge 0
\ng₂(x) = 30,000 - σ (x) \ge 0
\ng₃(x) = b - h \ge 0
\ng₄(x) = P_c(x) - 6,000 \ge 0
\n0.125 \le h, b \le 5.0, 0.1 \le l, t \le 10.0

The specific illustration of the constraint can refer to [26]. One thing we should know is that a violation of any four constraints will make the design unacceptable. Furthermore, the related stress and buckling term also illustrated in [26].

These objectives are confliction in nature, and iMOEA/D is applied to find the trade-off solutions within the preferred region decided by DM. Here, DM supposed prefers both objectives in an almost equivalent rate. So the utility function is designed as follows:

$$
U = \min(f_1^2 + f_2^2 - 10f_1 - 0.006f_2 + 25.000009) \tag{19}
$$

Figure 9 shows the obtained solutions after each interaction. And the total interactive time is four and the maximal generation equal to 500. So sub figure (a) shows the result after the first interaction and the current generation is 200. By analogy, sub figure (b) (c) and (d) just depict the obtained solutions in the 300-th, 400-th and 500-th generation respectively. Seeing from sub figure (d), the finial preferred solution successfully converge to the region which can well balance two objectives.

Figure 9. Obtained solutions in different stages of interaction of WBDP: (a) 1st interaction; (b) 2nd interaction; (c) 3rd interaction; (d) 4th interaction.

For better demonstrating the performance, four different utility functions are used in this part, and the finial results are shown in Figure 10 respectively.

Case 1: Let the first objective hold higher priority. The simple utility function can be described as (6).

Case 2: Similar as (7), DM prefers the second objective more than the first one in this case.

Case 3: In the third case, a more complicated utility function is designed, which has already shown in (19).

Case 4: In the last case, another complicated utility function is used to guide the optimization process, described as:

$$
U = \min(f_1^2 + f_2^2 - 24f_1 - 0.004f_2 + 144.000004) \tag{20}
$$

Sub figure (a) (b) (c) and (d) of Figure 10 correspond to different cases; and we can see clearly that, iMOEA/D holds a good performance for all these four cases. For both simple and complicated utility functions, the algorithm can obtain the most reasonable results which exactly near the preferred region made by DM. Furthermore, another important phenomenon should be paid more attention. When only one objective is preferred, such as case1 and case 2, the obtained solutions may hold a larger boundary. That means the solutions can reach a smaller value for the preferred objective. In such cases, the search intensity is enhanced to some extent; and the search purpose may not be satisfied until meet a more rigorous evaluation criteria.

5. SUMMARY AND CONCLUSIONS

In this paper, decomposition based interactive evolutionary algorithm named iMOEA/D for interacting with DMs has been proposed. Inspired by MOEA/D, the decomposition method is used in the beginning of proposed algorithm for decomposing the MOP into several sub-problems with uniform weight vectors. In addition, each individual solution belongs to different corresponding sub-problem.

Figure 10. Front obtained of WBDP: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

By dealing with the sub-problem, the computational complexity has been saved in a great extent.

During the stage of interactive, *P* current solutions are randomly presented to DM for choosing their favorite one; and the selected solution becomes the center of preferred weight region in the following optimization process. One thing should be emphasized which is all the operators facing to renew preference region are based on weight vectors. With this kind of information exchanging process, DMs will learn more direct and precise knowledge about the possible PF of MOP, and better determine the optimizing direction for converging.

With the interactive process, the algorithm can better imitate the pattern of decision-making process of human brain. The proposed algorithm holds the ability of providing feedbacks and correcting error decisions in some extent. The utility function is used for representing the human DM in this paper. And both two and three objectives MOPs are adopted to test the performance of iMOEA/D with different utility functions. According to the result of experimental study, iMOEA/D can successfully converge to different regions of all the problems guided by utility function with varying difficulties.

As we known, the research on imitating the human brain is still in its infancy. And iMOEA/D just provides a novel angle of solving preference based MOPs by exchange information during interactive stages. There are still many aspects worth further exploration. Such as a comprehensive way for estimating the performance, operable utility function for imitating the human DM, and so on.

6. ACKNOWLEDGMENTS

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