Using Pareto-Optimality for Solving Multi-Objective Unequal Area Facility Layout Problem

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ABSTRACT

A lot of optimal and heuristic algorithms for solving facility layout problem (FLP) have been developed in the past few decades. The majority of these approaches adopt a problem formulation known as the quadratic assignment problem (QAP) that is particularly suitable for equal area facilities. Unequal area FLP comprises a class of extremely difficult and widely applicable optimization problems arising in many diverse areas to meet the requirements for real-world applications. Unfortunately, most of these approaches are based on a single objective. While, the real-world FLPs are multi-objective by nature. Only very recently have meta-heuristics been designed and used in multiobjective FLP. They most often use the weighted sum method to combine the different objectives and thus, inherit the well-known problems of this method. As of now, there is no formal approach published for the unequal area multi-objective FLP to consider several objectives simultaneously. This paper presents an evolutionary approach for solving multi-objective unequal area FLP using multi-objective genetic algorithm that presents the layout as a set of Pareto-optimal solutions optimizing multiple objectives simultaneously. The experimental results show that the proposed approach performs well in dealing with multi-objective unequal area FLPs which better reflects the real-world scenario.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search — *Heuristic methods*

General Terms

Experimentation

Keywords

Unequal area FLP, Pareto-optimal solutions, Multi-objective optimization, Quantitative objective, Qualitative objective.

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1. INTRODUCTION

FLP is concerned with the physical arrangement of a number of interacting facilities on the factory floor of a manufacturing system to meet one or more objectives. A facility is an entity that assists to one dedicated tasks. It may be a machine tool, a work centre, a manufacturing cell, a machine shop, a department, a warehouse, etc. [9]. FLP is an emerging problem in the manufacturing industries due to the fact that the computational complexity increases with the number of facilities, which makes it a combinatorial optimization problem. Layout planning is one of the key factors for successful operation in a manufacturing company. A good layout will help any company to improve its business performance and can reduce up to 50% of the total operating expenses [18, 25]. FLPs also arise in other engineering design contexts such as VLSI placement and routing. Unfortunately, all of these problems are known to be NP-hard [7].

Classical approaches to FLP tend to focus on the relative location of equal area facilities on a floor plan. If all facilities are of equal area, or can be physically interchanged without altering the overall adjacency or distance relationship among the remaining facilities, it is easy to specify in advance a finite number of potential sites for these facilities to occupy [24]. FLP can then be modelled as a QAP. However, in most applications and realworld scenarios, equal facility area is a very poor assumption [13]. When layout problems have varying area facilities, it can no longer be treated as the problem of assigning facilities to *n* distinct centroid locations. Instead, the locations of the centroids will depend on the exact configuration selected, making QAP formulations of the unequal area problem less tractable than their equal area counterparts. To handle such unequal area FLPs, early heuristic algorithms are based on discrete models which divide the floor plan into a grid of equal-sized squares. Then, each facility is assigned the number of squares which most closely matches its total area. While conceptually simple, this approach has discouraged users because of the odd-shaped facilities generated [13].

Although the FLP is an inherently multi-objective optimization problem (MOOP), it has traditionally been solved considering only one objective either qualitative or quantitative feature of the layout [18]. Quantitative (distance-based) objective aims at minimizing the total material handling (MH) cost between facilities based on a distance function. Qualitative (adjacencybased) goal looks for maximizing the closeness relationship (CR) scores between facilities based on the placement of facilities that utilize common materials, personnel, or utilities adjacent to one another, while separating facilities for the reasons of safety, noise,

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or cleanliness. Accordingly, practical FLP involves several conflicting objectives, thus requiring a multi-objective formulation. Therefore, like many other real-world optimization problems, it is naturally posed as a MOOP. For that reason, FLP must consider both quantitative and qualitative objectives simultaneously.

Since practical FLPs are multi-objective by nature, they require the decision makers to consider a number of criteria involving both quantitative and qualitative objectives before arriving at any conclusion. A solution that is optimal with respect to a certain given criterion might be a poor candidate for some other criterion. Hence, it is desirable to generate many near-optimal layouts considering multiple objectives according to the requirements of the production order or customer demand. Then, the production manager can selectively choose the most demanding one among all of the generated layouts for specific order or customer demands.

Multi-objective approaches in FLP, which recently have been proposed, in most cases lead to the optimization of a weighted sum of a function. In this method, multiple objectives are added up into a single scalar objective using weighted coefficients. Comprehensive surveys are found in [7, 22]. The poor practicability of weighted sum approach is due to the difficulty of normalizing these functions and of quantifying the weights in advance. As a result, the objective function that has the largest variance value may dominate the multi-objective evaluation. Consequently, inferior non-dominated solutions with poor diversity will be produced. Also, the user always has to specify the weight values for functions and sometimes these will not have any relationship with the importance of the objectives. In addition, a single solution is obtained at a time. If we are interested in obtaining a set of feasible solutions, it has to be run several times. However, there is no warranty that the solutions obtained in different runs are different. Most importantly, the layout designer based on his/her past experience randomly selects the layout having multiple objectives. This restricts the designing process completely designer dependent and thus, the layout varies from designer to designer. Interested readers can find the details of these problems in [19]. To overcome such difficulties, Paretooptimality [5] has become an alternative to the classical weighted sum method.

Problems related to FLP are computationally difficult. In an *n*facility problem, we would have to evaluate *n*! different layouts. Due to the combinatorial nature of the problem, optimal algorithms have been successfully applied only to small problems (>15 facilities), but they require high computational efforts and extensive memory capabilities. As a result, heuristic and metaheuristic algorithms have got the attention in recent years to solve FLPs. This is due to their ability to generate feasible solutions in the least possible computational time. Broad reviews of the different approaches to FLPs are found in [7, 21]. Among these approaches, the genetic algorithm (GA) has found wide application in research intended to solve FLP. Generally speaking, GA outperforms the other heuristic methods due to its capability to generate feasible solutions in a minimum amount of time [10, 18].

To date, there are only a few attempts to tackle the multiobjective FLP using GA. However, they use weighted sum method. Some recent applications of GA using Pareto-optimality can be found in [17, 18, 19]. Unfortunately, in all existing methods, attention has been given to the equal area FLP. Very recently, unequal area FLP has received significant attention by many researchers. Interested readers should consult [12] for a detailed review. The primary difficulties associated with unequal area FLP have to do with the vast number of possible physical layouts, and with the existence of many locally optimal layouts that are poor compared to the global optimum layout. For such a problem, parallel search methods perform better than strictly serial searches, and randomized search methods perform better than greedy or enumerative searches. GA combines both of these attributes in a parallel, stochastic heuristic manner [24]. Similar to the equal area FLPs, various methods for solving unequal area FLPs using GA have been suggested in the literature [3, 24, 26]. However, existing approaches are intended to achieve singleobjective optimization. Thus, these methods essentially ignore the prospects of Pareto-optimal solutions in solving the real-world multi-objective unequal area FLP.

Although the advantages and good performance of multiobjective GA in many combinatorial optimization problems have been demonstrated in the literature, yet there is no formal approach to solve the unequal area FLP considering multiple objectives separately. All these, motivate us to propose a multiobjective GA for solving unequal area FLP that presents the layouts as a set of Pareto-optimal solutions, and to investigate its performance. In an attempt to address multiple objectives simultaneously, we apply material handling (MH) costs and closeness relationship (CR) scores among various facilities as quantitative and qualitative objective, respectively. In this work, we have used the Non-dominated Sorting Genetic Algorithm 2 (NSGA 2) proposed by Deb et. al. [6] as the multi-objective evolutionary algorithm (MOEA).

The remainder of the paper is structured as follows. Section 2 highlights the importance of Pareto-optimality in the FLP. Relevant literatures for the unequal area FLP are reviewed in Section 3. Section 4 presents the mathematical formulation for the multi-objective unequal area FLP. Section 5 contains the proposed approach. To demonstrate the performance of the proposed approach, computational results are presented and analyzed in Section 6. Finally, this paper ends with conclusion in Section 7.

2. IMPORTANCE OF PARETO - OPTIMALITY IN FLP

Real-life scientific and engineering problems typically require the search for satisfactory solution for several objectives simultaneously. It is also common that conflicts exist among the objectives. In the presence of such multiple and conflicting objectives, the resulting optimization problem gives rise to a set of optimal solutions, instead of one absolute optimal solution. Multiple optimal solutions exist because no single solution can be optimal for multiple conflicting objectives. These multiple solutions, namely the Pareto-optimal solutions, are optimal in the wider sense that no other solutions in the search space are superior when all the objectives are considered. Since none of these solutions can be said to be an 'absolute optimum', it is reasonable for the users to find as many different Pareto-optimal solutions as possible. The set of such solutions is called Pareto front. Among these solutions, the designer is free to select any solution that offers the most profitable trade-off among the

objectives. Figure 1 presents a Pareto front for two objectives, which are subject to minimization.

In general, the minimization of the total MH costs is often used as the optimization criterion in FLP. However, the closeness rating, hazardous movement or safety, and the like are also important. Many researchers have questioned the appropriateness of selecting a single-criterion to solve FLP because qualitative and quantitative approaches each have advantages and disadvantages [19]. The major limitations on quantitative approaches are that they consider only relationships that can be quantified and do not consider any qualitative factors. On the other hand, qualitative approaches suffer from the shortcoming of strong assumption made on all qualitative factors that these factors can be aggregated into one criterion. In essence, FLP falls into the category of a MOOP. Thus, instead of offering a single solution, giving options and letting decision makers choose between them based on the current requirement is more realistic and appropriate. Moreover, based on the principle of multi-objective optimization, obtaining an optimal solution that satisfies all of the objectives is almost impossible. It is mainly due to the conflicting nature of objective functions, where improving one objective may only be achieved when worsening another objective. Accordingly, it is desirable to obtain as many different Paretooptimal layout solutions as possible, which should be converged to, and diverse along the Pareto-optimal front with respect to multiple criteria.

3. LITERATURE REVIEW

FLP is one of the truly difficult ill-structured, multi-criteria and combinatorial optimization problems. Since the pioneer work in FLP by Armour and Buffa [2], a number of sub-optimal and intelligent techniques have been proposed to cope with this type of problems. Early heuristic algorithms are mainly based on discrete models. In this model, the total manufacturing floor area is divided into a grid of equal-sized squares, and each facility is assigned the number of squares which most closely matches to its total area. The major drawback of this model is that it generates odd-shaped facility areas. In terms of meta-heuristics, Castillo et al [4] applied a mixed-integer nonlinear programming for solving this problem. In [23], a simulated annealing based approach has been proposed. Hu and Wang [10] applied GA to unequal area FLP for achieving the minimal layout cost. Islier [11] used GA to solve a multi-criteria FLP to name but a few. A genetic search for solving construction site-level unequal area FLP has been proposed in [8]. Tate and Smith [24] presented a GA based model for FLPs with unequal areas and different geometric shape constraints. A solution to the unequal area layout problem by GA has also been given in [26]. A hybridized meta-heuristic for the solution of the unequal area FLP is presented in [15]. Recently, Liu and Meller [14] proposed an approach to solve this problem by using GA and mixed-integer programming (MIP). More recently, a Tabu Search (TS) based approach with slicing tree representation [20] and an Ant System (AS) based method [12] have been proposed to solve unequal area FLPs. A good survey for various approaches to unequal area FLP can be found in [1]. From the existing literature, it can be summarized that unequal area FLP is still an active area. It is also noticeable that all the existing methods use either single objective or weighted sum method to solve the unequal area FLP. As a result, Paretooptimality has not been utilized for solving the unequal area FLP.

4. PROBLEM FORMULATION

In this work, we follow the assumptions described in [12]: facilities must be located within a given area; facilities must not overlap with each other; the layout must fulfill the maximum ratio constraints (or minimum value restrictions) for the dimension of facilities (length and width of each facility). The first fitness function, total material handling (MH) cost, is based on quantitative model. This function is subject to minimization, and measured as

$$
F_1 = \sum_{i=j}^{N} \sum_{j=1}^{N} C_{ij} f_{ij} d_{ij}
$$
 (1)

The second fitness function, the closeness rating (CR) score, is based on qualitative model. This function is subject to maximization, and expressed as

$$
F_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} r_{ij}
$$
 (2)

where

$$
r_{ij} = \begin{cases} CR \text{ value when facility } i \text{ and } j \text{ are neighbors with common boundary} \\ 0, \text{otherwise} \end{cases}
$$

Where, f_{ij} is the material flow between facility *i* and *j*, C_{ij} is the transportation cost between facility *i* and *j*, and d_{ii} is the Euclidean distance between the centres of facility *i* and *j*. Here, we apply the following closeness ranking value: A (absolutely necessary)=6, E (essentially important)=5, I (important)=4, O (ordinary)=3, U (un-important)=2, and X (undesirable)=1.

Figure 2. Chromosome representation for 7- facility problem.

5. PROPOSED APPROACH

5.1 Chromosome Representation

In the proposed approach, a chromosome representation suitable for *slicing tree structure* for unequal area FLP [12] is used. The chromosomes are encoded as $(f_1f_2f_3 \ldots f_N)$ $(s_1s_1s_2 \ldots s_{N-1})$ $(s_0, s_0, s_0, \ldots, s_0, s_1)$, where *N* is the number of facilities; and *f*, *ss*, and *so* represents facility sequence, slicing sequence, and slicing orientation, respectively. In general, the slicing tree representation recursively divides the total floor area (horizontally or vertically), in proportion to the areas of the facilities. The first two parts of the chromosome are represented by integer numbers, whereas the last part is represented by either 1 or 0. The facility sequence will be transformed into a slicing tree form. The slicing sequence is the ordering that slices the facility sequence. The slicing orientation 0 represents a horizontal cut and 1 represents a vertical cut. A chromosome for a 7 facility problem is presented in Figure 2. Figure 3 presents the corresponding solution representation, the slicing tree transformation, and the layout solution for this chromosome.

(c) Solution layout

5.2 Crossover

In this approach, we applied 3-point crossover for performing the crossover operation. For keeping the chromosome valid after the operation, we choose the 3 points separately from each segment of a chromosome. However, for the first two segments (facility sequence and slicing sequence), some repair works are required after the crossover to remove any duplication or absence of facility. In this repair work, first we find and list the duplicate facilities in the first segment according to the occurrence in the chromosome. Then, we check whether any facility is missing in the segment starting from the first to the last facility (from 1 to N). After that we replace the list of the duplicate facilities with missing facilities. The same procedure is repeated for the second segment, except that here the range is from 1 to N-1. Figure 4 depicts the crossover operation.

5.3 Mutation

To apply mutation, we use swap mutation with the restriction that both genes will be chosen from the same segment. As a result, no repair work is necessary for mutation. Unlike the crossover, the genes will be chosen from only one segment of the chromosome and this choice will be random for every chromosome of the population pool. Figure 5 gives an example for the mutation.

Figure 5. Mutation.

The non-dominated sorting strategy, crowding distance mechanism and elitism strategy used in the proposed approach is the same as used in NSGA 2 [6].

6. COMPUTATIONAL RESULT AND ANALYSIS

6.1 Benchmark Problem

To evaluate how the proposed approach performs with respect to solution quality, we run the algorithm on various problem sets taken from the literature. The test problems are composed of 7, 8, 9, 10, 12, 14, 20, 30, 35, and 62 facilities. The details of the problem data can be found in [12]. We used the last digits to indicate the number of facilities in each problem.

Data Sets	AS [12]	GA with flexible bay $[24]$	GA with MIP [14]	TS with slicing tree $[20]$	Best known	Best known reference	Proposed Approach	Dif. $(\%)$
07	131.68	NA	131.63	132.00	131.58	$[4]$	98.6698	25.01
08	243.12	NA	245.41	243.16	242.93	$[4]$	202.7174	16.14
09	236.12	NA	246.26	239.07	236.14	$[12]$	201,7502	14.56
VC10	19967.60	23671.00	19997.00	19994.10	19967.60	$[12]$	19963,7421	0.19
Ba12	8252.67	8768.00	8702.00	8264.00	8180.00	$[4]$	8103.8476	0.93
Ba14	4724.68	5080.00	4852.00	4712.33	4712.33	$[20]$	4790.8354	-1.66
Ab20	4972.56	NA.	5668.00	5225.96	4972.56	$[12]$	4015.2549	19.25
SC30	3868.54	5743.00	3707.00	NA	3707.00	$[14]$	3740,7553	-0.91
SC35	4132.37	NA	3604.00	NA	3604.00	$[14]$	3835.7802	-6.43
Du ₆₂	3720521.13	NA	NA	NA	3720521.13	$[12]$	2977512.9599	19.97

Table 1. Comparison with existing algorithms for MH cost only

For experimental purpose, we set the maximum aspect ratio (height vs. width of a facility) as 4 for *Ba12*, *Ba14*, *SC30*, and *SC35*. Very few benchmark problems are available for unequal area multi-objective FLP, particularly in the case of CR score. As a result, we have created test data sets for CR score for these problems on our own. These problems are chosen because of their variety in size (from small to large), and their wide use in previous studies.

6.2 Experimental Setup

The experiments are conducted using 200 chromosomes and 100 generations for problems with up to 15 facilities; and 1000 chromosomes and 900 generations for problems with more than 15 facilities. The probabilities of crossover and mutation are 0.9 and 0.3, respectively. We use traditional tournament selection with tournament size of 2. Each benchmark problem is tested 30 times with different seeds. Then each of the final generations is combined and non-dominated sorting [6] is performed to constitute the final non-dominated solutions.

6.3 Experimental Analysis

To evaluate our proposed algorithm, first we perform the experiments in a single objective context to justify its capability to optimize MH cost. Then, we show its performance as a multiobjective evolutionary approach by optimizing MH cost and CR score simultaneously. We should note that, for both single and multi-objective comparison, we have used the same results from the same non-dominated solutions obtained by our approach.

In Table 1, the performance of the proposed approach is compared with some existing algorithms for unequal area FLP in term of MH cost. We compare our results with those obtained by Ant System (AS) [12], GA with flexible bay representation [24], GA with mixed integer programming (MIP) [14], Tabu Search with slicing tree representation [20]. This table is partially cited from [12]. In addition, the reference of some best found results so far has been cited from [4]. The best results are bold-faced. As shown in Table 1, the proposed approach outperforms AS and GA with flexible bay for all the test problems. Only 3 out of 10 problems, it performs slightly worse than the best found results so far (2 for GA with MIP and 1 for TS with slicing tree). From the table, it can be found that the proposed approach clearly outperforms the other evolutionary approaches by a significant margin of up to 25.01% better performance. As mentioned earlier, we find only 3 values with negative deviations. It is interesting to observe that all three negative deviations are for the test problems (*Ba14*, *SC30*, and *SC35*) where we set the aspect ratio to 4 in place of the flexibility offered in the original problems. So, it might be the reason for getting negative deviations. However, the margin is relatively insignificant compared to those of the positive deviations (only 1.66%, 0.91%, and 6.43%). Above all, it is worthwhile to mention that our approach performs well in cases of both small and large FLPs.

Data	MH Cost	CR Score			
Sets	Best	Avg	Best	Avg	
07	98.6698	111,009	54	30.825	
08	202.7174	292.602	64	32.267	
09	201.7502	444.672	88	39.897	
VC10	19963.7421	26065.55	120	61.846	
Ba12	8103.8476	8711.847	110	71.25	
Ba14	4790.8354	5500,217	156	98.167	
Ab20	4015.2549	6131.369	201	99.329	
SC30	3740,7553	4444.037	349	241.872	
SC35	3835.7802	4305.899	337	216.043	
Du62	2977512.9599	3220771.01	248	255.25	

Table 2. Results for multi-objective unequal area FLP considering both MH cost and CR score

Multi-objective optimization differs from single objective optimization in many ways. For two or more objectives, each objective corresponds to a different optimal solution, but none of the trade-off solutions is optimal with respect to all objectives. Thus, MOEAs do not try to find one optimal solution but all the trade-off solutions.

Table 2 shows the performance statistics of the evolutionary multi-objective unequal area FLP in the context of MH cost and CR score. Since there are no published papers using the Paretooptimality for unequal area FLP, we could not compare the performance of the Pareto-optimal solutions, particularly, in the case of CR score. On the other hand, we have already shown and discussed the prerformance of the proposed approach in case of MH cost in Table 1. As shown in Table 2, the gaps between the best and average values are a little high in some cases. Despite that it should be mentioned that the main goal of our algorithm is to find the trade-off solutions for unequal area multi-objective FLP, which is very rare in literature. Also, according to the Pareto-optimal theory, the final and average value of one objective may be influenced by the presence of other objective. While considering this, the overall performance of the proposed approach is very promising for all the problems.

Figure 6. Final Pareto-optimal layouts.

To illustrate the convergence and diversity of the solutions, nondominated (Pareto-optimal) solutions of the final generation produced by the proposed algorithm for the test problems *09* and *SC30* are presented in Figure 6. It is worthwhile to mention that in all cases, most of the solutions of the final population are Paretooptimal. In the figures, the occurrences of the same nondominated solutions are plotted only once. From these Figures, it can be observed that the final solutions are well spread and converged. And for this reason, it is capable of finding extreme solutions. As a result, it provides a wide range of alternative layout choices for the designers.

Figure 7. Two objectives over generations of 07 problem.

Figure 8. Two objectives over generations of *Ab20* **problem.**

Figure 7 and Figure 8 demonstrate the optimization behavior of the proposed method over generations for *07* and *Ab20* problem, respectively. These figures also justify that our proposed approach clearly optimizes both of the objectives with generations. From the figures, it can be found that the proposed method is able to optimize both the best and average values for MH cost (minimize) and CR score (maximize) from the first to the last generation very efficiently.

To summarize the results, the proposed approach for solving the unequal area multi-objective FLP is capable of producing nearoptimal and non-dominated layout solutions, which are also the best-known results in many cases. The simulation results clearly show that it is able to find a set of diverse Pareto-optimal solutions, which fulfills the two main goals of the multi-objective FLP algorithm – convergence and diversity. Accordingly, it shows excellent promises as a useful tool in solving unequal area multi-objective FLP.

7. CONCLUSION

Although a considerable amount of work has been done in FLP over the last few decades, almost none of them deal with multiobjective optimization for unequal area FLP. Nevertheless, equal area and single objective FLP is a very poor assumption considering practical situations. This paper presents an evolutionary approach for solving the multi-objective unequal area FLP to find a set of Pareto-optimal layouts, which better reflects the real-world scenarios. A comparative analysis with the previous studies in the literature shows that the proposed method generates better solutions in the context of single-objective optimization (MH Cost). More importantly, in multi-objective cintext, it is capable of finding a set of Pareto-optimal layouts that optimizes both MH cost and CR score simultaneously throughout the entire evolutionary process. Thus, it provides a wide range of alternative layout choices, allowing the decision makers to be more flexible and to make better decisions based on market circumstances. We reckon this method would pioneer in case of multi-objective optimization for solving the unequal area FLP. However, in some cases the gap between the best and average solution may be relatively large. In future, we hope to improve this by applying local search. Also, we would like to apply several MOEAs to test their performance in solving unequal area multi-objective FLP.

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