Spanning the Pareto Front of a Counter Radar Detection Problem

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ABSTRACT
Radar system design and optimization are complex problems recently cast in the framework of multi-objective evolutionary algorithms. However, in the problem of counter radar detection and tracking, the state-of-the-art multi-objective optimization algorithm NSGA-II is unable to span the complete 2D Pareto front of the asymptotic and convex problem domain, leaving out vital information on the radar-jammer system dynamics. Common modifications to the domination principle employed will to some degree increase the span of the Pareto front, at the expense of slower convergence and a less dense front. In this paper, the new Surface Evolutionary Algorithm (SEA) is introduced to overcome these problems. The SEA characterizes all solutions by one single metric and uses interpolated attraction points along the boundary of the solution set as basis for selecting and evolving solutions in the optimizer. The SEA is proposed and analyzed in the context of the conflicting multi-objective optimization criteria of search efficiency, density distribution and span of the complete Pareto front of the counter radar detection problem. The SEA is shown to produce high performance solutions not easily obtained using the well-established optimization methods of NSGA-II and ε-MOEA.

Categories and Subject Descriptors
G.1.6 [**Numerical Analysis**]: Optimization; I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search; J.7 [**Computer Applications**]: Military

General Terms: Design, Performance

Keywords: Jamming technique design, Digital RF Memory, pulse-Doppler radar, constant false alarm rate, genetic algorithm, multi-objective optimization

1. INTRODUCTION
Evolutionary Algorithms (EAs) like Genetic Algorithms (GAs) [9], [10], Genetic Programming (GP) [15] and Particle Swarm Optimization (PSO) [14] are powerful methods for efficient and robust global optimization. Numerous complex radar applications

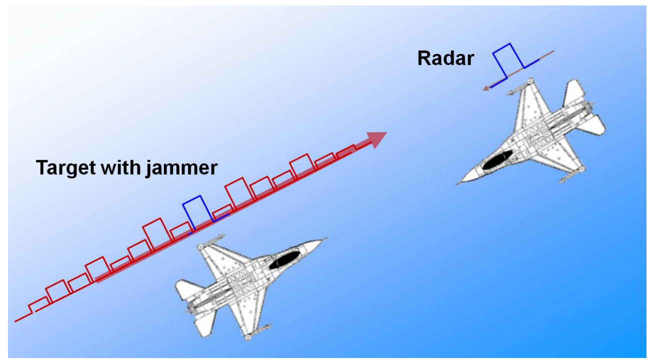

Figure 1. Radar-jammer system dynamics of an air-to-air radar scenario.

have benefited from such evolutionary optimization ranging from radar antenna design [21] to radar waveform development [12] and now also radar electronic warfare applications [11], [16], [18], [19], [20]. Furthermore, radar system design and analysis are naturally framed as multi-objective optimization problems where optimal trade-off solutions between the different optimization objectives are sought. This is known as the search for the optimal Pareto front [4]. EAs are especially suited for multi-objective optimization problems due to their population based approach [4] seeking to spread their solutions uniformly onto the entire optimal Pareto front.

In the optimization problem of counter radar detection and tracking against systems using the Constant False Alarm Rate (CFAR) detection algorithm [16], [19], the goal is to find the optimal trade-off between the radar detection probability and the transmitted energy of the radar jammer. In Fig. 1 the radar-jammer system dynamics is illustrated for an air-to-air radar scenario. CFAR is also used in land and sea applications of moving or static radar-jammer situations. The radar transmits a waveform and receives echoes from the target and a self-protection jammer on-board the target. Furthermore, the clutter reflections and radar system thermal noise enforces some kind of signal filtering for target detection and tracking, for example the CFAR detection algorithm. The on-board jammer is used for manipulating the received radar signal in order to deny target detection and tracking. Recently, single-objective and multi-objective evolutionary algorithms have proven to be highly efficient in solving the complex and challenging Counter-CFAR (C-CFAR) optimization problem [16], [19]. However, the state-of-the-art Multi-objective Evolutionary Algorithm (MOEA) NSGA-II [5] is reported [19] to fail in spanning the complete Pareto front of the 2D problem domain of radar detection probability versus jammer transmitting power. The ability to find a complete Pareto front is important since choice of implemented solution is dependent on scenario and user preferences. The NSGA-II will converge onto parts of the optimal Pareto front, but have difficulties spanning the entire asymptotic and convex front, leaving out all the important solutions at peripheral parts of the Pareto front. Asymptotic and convex problem domains are found in many low dimensional design and engineering problems, as exemplified by [1], [7], [21], [23]. Modifying the principle of Pareto Domination (PD) in a MOEA by using Guided Domination (GD) [3] or ε-Domination (εD) [17] is expected to increase the span of the Pareto front [8] though at the cost of lower search efficiency and a less dense front.

In this paper, a proposed multi-objective optimization method called the Surface Evolutionary Algorithm (SEA) is used to directly span the bi-objective Pareto front of the C-CFAR problem without the cost of lower search efficiency and a less dense Pareto front. The main idea behind SEA is that each point in objective space is characterized according to some kind of single value metric and together with a set of evenly spaced attraction points on the boundary of the solution set forms the basis for selecting and reproducing the next generation. Three different types of single value metrics are proposed and analyzed in the context of the multi-objective optimization goals of the C-CFAR problem. These are domination-based metrics, distance-based metrics and angular-based metrics. The different metrics are tested on the boundary surface of the solution set interpolating attraction points on the convex hull.

The SEA is compared to the NSGA-II and the ε-MOEA [6] high-performance multi-objective optimizers using the concept of guided domination and ε-domination on the C-CFAR problem. The algorithms are tested on their ability to simultaneously converge onto the optimal Pareto front, uniformly distribute solutions along the optimal Pareto front and at the same time span the extremes of the optimal Pareto front.

In section 2, an introduction to multi-objective optimization is given, reviewing the NSGA-II and the ε-MOEA in terms of Pareto domination, guided domination and ε-domination. In section 3, the SEA procedure is introduced along with the different single value metrics employed. In section 4, the SEA is used to solve the complete 2D counter radar detection problem. In the end, conclusions are drawn and further work is proposed.

2. MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization is the analysis of optimization problems consisting of two or more often conflicting objectives [4]. Whereas single-objective optimization seeks only one globally optimal solution, multi-objective optimization seeks the optimal Pareto front [4]. The Pareto front is a mathematical description of the trade-offs between the different objectives in the multi-objective optimization, derived from the principle of domination. Formally, in a minimization problem, solution $\bar{x}_i$ is said to dominate solution $\bar{x}_j$ if solution $\bar{x}_i$ is no worse than solution $\bar{x}_j$ for all k=1,2,…,M objective values $f_k(\bar{x}_i) \leq f_k(\bar{x}_j)$ and at the same time solution $\bar{x}_i$ is strictly better than solution $\bar{x}_j$ in at least one objective value $f_k(\bar{x}_i) < f_k(\bar{x}_j)$ where $k \in \{1,2,\ldots,M\}$. All solutions that are not dominated by any other solutions in the problem domain constitute the optimal Pareto front. In this regard all possible trade-off solutions on the optimal Pareto front are considered to be of equal importance and each of them constitutes a globally optimal solution. Ideally, the optimal Pareto front found by a multi-objective optimization can be characterized by its ability to converge on the true optimal Pareto front and its ability to span the entire true optimal Pareto front.

2.1 Multi-objective Evolutionary Algorithms

An analytical expression for the true optimal Pareto front is often difficult to obtain in a multi-objective optimization problem. Population based multi-objective evolutionary algorithms are powerful methods for finding approximations to the true optimal Pareto front [4]. In a MOEA one seeks to disperse the population of solutions in the objective space as close as possible to the true optimal Pareto front. The Pareto Optimal (PO) set of solutions is an approximation of the optimal Pareto front. Formally a MOEA can be stated as follows [4]. Find the N population vectors $\bar{x}^* = [x_1, x_2, \ldots, x_n]^T$ of n decision variables that simultaneously optimize the M objective values $\{f_1(\bar{x}^*), f_2(\bar{x}^*), \ldots, f_M(\bar{x}^*)\}$, while at the same time satisfying any constraints. The non-dominated set of the entire search space is the globally Pareto optimal set. Ideally, the globally Pareto optimal set approximating the true optimal Pareto front would:

1. be on the true optimal Pareto front.
2. have evenly distributed samples on the true optimal Pareto front.
3. span the entire true optimal Pareto front, including the extremes.

Additionally, the globally Pareto optimal set should be found in the shortest time possible.

Moreover, these search goals are conflicting objectives in a multi-objective evolutionary algorithm. For instance, spanning the entire Pareto front as opposed to parts of the front would result in a less dense Pareto optimal set possibly decreasing local search efficiency in certain regions. This makes it harder to reach the true optimal Pareto front in reasonable time. Also, an unevenly distributed Pareto optimal set would correspondingly increase local search efficiency at densely populated parts of the Pareto front and lower local search efficiency at sparse parts of the front, making it hard to find the whole true optimal Pareto front in reasonable time as well.

Maximizing these search goals simultaneously, in a multi-objective fashion, is therefore the main concern when using a multi-objective evolutionary algorithm on a multi-objective optimization problem.

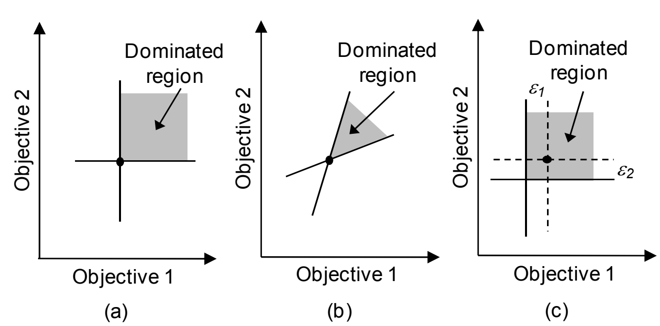

Figure 2. Different domination principles. (a) Pareto dominance, (b) guided domination and (c) ε-domination

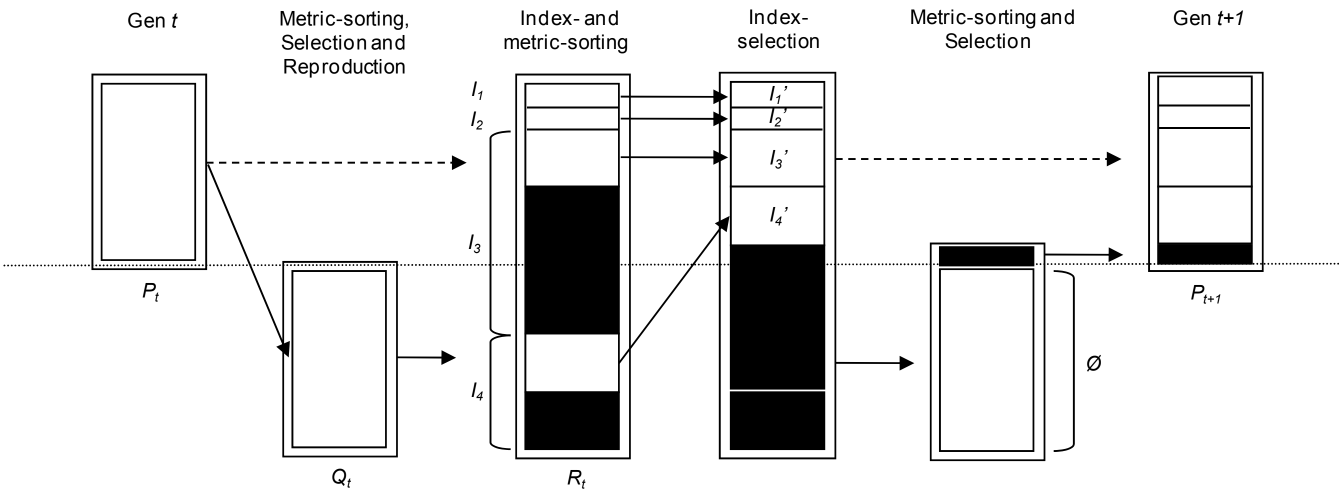

Figure 3. The SEA generation-to-generation procedure.

2.2 Dominance-based MOEAs

Multi-objective evolutionary algorithms that on a generation-to-generation basis admit non-dominated solution sets can be classified as dominance-based MOEAs. Starting from the pioneering work of Goldberg in 1989 [9] and up to today a number of dominance-based MOEAs have been proposed and studied [4]. Among these, the most well-known and perhaps most popular multi-objective evolutionary algorithm, is the NSGA-II proposed by Deb et al. [5] in 2002. The ε-MOEA [6] is a high-performance multi-objective optimizer which employs the principle of epsilon domination to obtain a set of well distributed optimal solutions. In our paper these established optimization methods are used for the purpose of comparing the Pareto front span of the counter radar detection problem.

2.2.1 Pareto domination

Pareto domination is the scheme employed in the original NSGA-II procedure. Using the concept of Pareto domination on a set of solution points is equivalent to finding the Pareto front or Pareto optimal set of a population. In Fig. 2a the orthogonal linear contour lines defining the regions of domination and non-domination using Pareto domination is illustrated for a bi-objective minimization problem.

2.2.2 Guided domination

Guided domination [3] modifies the regions of domination and non-domination by altering the orientation of the linear contour lines of the Pareto domination. Fig. 2b illustrates how a rotation of the Pareto domination contour lines alters the region of domination. A narrower region of domination is expected to increase the span of the Pareto front, whereas a wider region of domination is expected to focus the span of the Pareto front.

2.2.3 Epsilon domination

In ε-domination [17] the principle of Pareto dominance is changed to make sure that a particular solution dominates another solution with at least ε_k in the k-th objective. A solution x_i said to dominate solution x_j if $f_k(x_i) \le f_k(x_j) + \varepsilon_k$ for all objectives. Also, the

strict inequality must be true for at least one objective. In Fig. 2c the altered region of ε-domination is shown. Appropriate values of the parameter ε_k must be set by the user.

3. SURFACE EVOLUTIONARY ALGORITHM

The SEA procedure presented here for solving the C-CFAR problem is mainly based on three important concepts: a suitable single value metric, explicit control of Pareto front span and density and an elite preserving mechanism.

3.1 The SEA procedure

In Fig. 3 the SEA generation-to-generation procedure is illustrated.

Initially SEA creates a random parent population P_t of size N and the starting point for evolving a new generation is the sorting of this parent population according to its single value metric. A stochastic uniform rank fitness selection operator is used to select a children population Q_t, also of size N. Q_t is then crossed and mutated upon.

The recombination of parent and children population R_t=P_t+Q_t ensures elite preservation in the algorithm, like the fast and efficient recombination operator found in NSGA-II [5]. The population R_t of size $2N$ is formed as a selection pool for the next generation P_{t+1} of N individuals again. The selection procedure is as follows. The single value metric of all individuals in R_t is calculated. Then each solution is indexed according to its nearest index point on the interpolated hypersurface I^* constructed using a stepwise linear interpolation of the convex hull solution points in the objective scene. Alternatively the Pareto optimal solution set could be used instead of the convex hull solution set. A number of evenly spaced interpolation points I^* are given an index number and each solution point in R_t is assigned the index number of the closest point in I^*. In Fig. 4 an illustration of the interpolated hypersurface in a 2D minimization problem is given. In Fig. 4 the line segments *1* and *2* connected by solutions *A*, *B* and *D* are on the convex hull. It is also possible to extend this interpolated hypersurface to include the global extremes of the population. This extended convex hull hypersurface can be constructed by including line segment 3. In Fig. 4 the interpolated

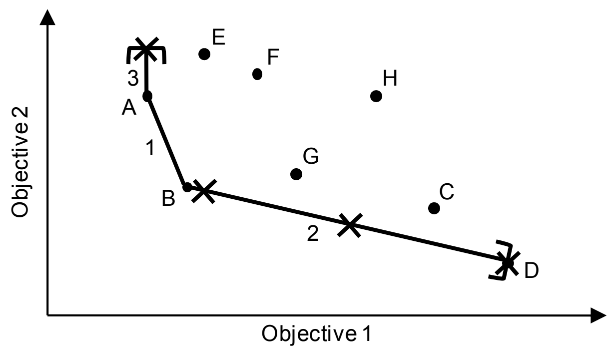

Figure 4. The depiction of I^*=4 interpolation points on the extended convex hull of the solution set used for explicit control of Pareto front density and span in the SEA procedure.

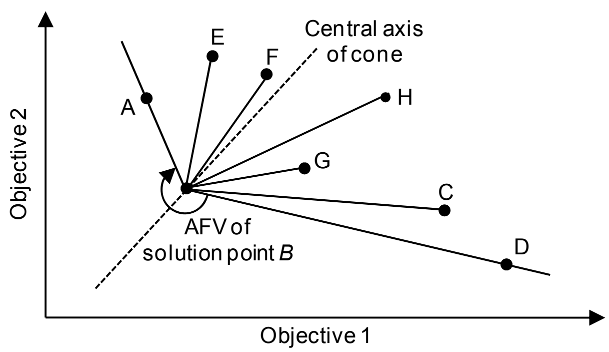

Figure 5. A 2D illustration of the angular-free-view (AFV) single value metric for solution B relative to 7 other random solution points. The AFV is defined as the largest hypercone apex angle, in this case spanned by point B as the fixed point on the central axis of the cone and point A and D lying on the surface of the cone.

and indexed points of the extended convex hull is shown for 4 evenly distributed interpolation points. Now, by selecting $n_c=N/I^*$ best solution points in terms of the single value metric for each index cohort, a piecewise balanced population is selected upon. If there are still unoccupied slots in P_{t+1}, i.e. cohorts with less than n_c indexed solution points, they are filled up using global competition among the remaining solution points, finalizing the process of producing a new generation.

In a bi-objective problem and leaving out the single value metric calculations, the SEA has a complexity of $O(N\log(N))$, where N is population size.

3.1.1 Dominance-based metrics

The domination-based single value metric used in this paper is the domination count (DC) [4]. The domination count of a particular solution is simply the number of solutions in the population that it dominates. Alternatively, one could use the non-domination level of the consecutive Pareto fronts of a non-dominated sorted population.

3.1.2 Distance-based metrics

The distance to the nearest interpolation point in I^* (DI) is used as the distance-based metric in the SEA. As an alternative one could also use the distance to the nearest line segment spanned by neighboring points in I^*. Also worth mentioning is that distance-based metrics will degenerate to the usual fitness evaluation of a single objective evolutionary algorithm in the 1D case.

3.1.3 Angular-based metrics

A 2D illustration of an angular-based metric, here called the Angular-Free-View (AFV), is depicted in Fig. 5. The AFV of a solution point B is found by finding the largest cone angle spanned by the solution point under investigation and any two other solution points lying on the surface of the cone, without any points in the interior. In Fig. 5 the AFV of point *B* relative to 7 other random solution points is spanned by points *A* and *D* on the cone surface. This angular metric was originally conceived and introduced by Hughes [13] in 2008 for boundary surface identification of irregularly shaped regions of multi-objective problems.

The AFV has a general complexity of $O(MN^M)$ where N is population size and M is the number of objectives, impeding optimization of high dimensional problems. Nevertheless, the AFV complexity is comparable to the complexity of the NSGA-II when optimizing 2D problems and the convex hull solutions of the solution set is computed directly using AFV-values$\geq 180°$.

In order for AFV to be an effective search and optimization property, one has to exclude the free-view as seen from the solution under test in the direction of the dominated hypervolume defined in the objective search space. For instance, in an overall minimization problem, all view direction vectors relative to each solution point under test having strictly positive objective values would be excluded from the AFV-calculation.

4. The Radar Problem

The complex radar problem of counter detection and tracking is described and analyzed using the NSGA-II evolutionary algorithms in [16] and [19]. The goal of this 2D C-CFAR problem is to find the *complete* optimal set of signal matrices that minimizes the probability of CFAR-detection as a function of transmitted signal energy of the counter radar system employed. In the following, the radar CFAR-filtering for detection is described and the corresponding optimization objectives of the C-CFAR-signals are stated mathematically.

4.1 Radar Problem Formulation

Radar CFAR-filtering [22] is the transformation of a radar Range-Doppler amplitude matrix into a radar Range-Doppler CFAR-ratio matrix given formally by the 2D convolution

$$RD_{CFAR-ratio} = C_{CFAR-filter} \otimes RD_{amplitude} \qquad (1)$$

where $\otimes$ is the convolution symbol, the transformation $C_{CFAR\text{-}filter}$ contains the morphology of the CFAR-filter, $RD_{amplitude}$ is the radar amplitude matrix of size $m\times n$ and $RD_{CFAR\text{-}ratio}$ is the transformed radar ratio matrix also of size $m\times n$. In this problem the generalized cell averaging 2D $C_{CFAR\text{-}filter}$ of a 5×5 CFAR-frame is used. This filter is depicted in Fig. 6 showing an empty guard gate around the cell-under-test. The CFAR-filter is operated on the $RD_{amplitude}$ matrix on a pixel-by-pixel basis and the amplitudes are positive real values. In effect, this filtering transforms the radar RD matrix from the raw input energy domain into the decision domain, detecting and tracking a fixed number of the highest CFAR-ratio pixels in the $RD_{CFAR\text{-}ratio}$ matrix. The CFAR-ratios are calculated as follows

$$r_{ij} = \frac{a_{ij}}{\bar{a}_{ij,CFAR-frame}} \quad (2)$$

where r_{ij} is the CFAR-ratio of pixel *i,j* in the $RD_{CFAR\text{-}ratio}$ matrix, a_{ij} is the amplitude of the cell-under-test pixel *i,j* of the $RD_{amplitude}$ matrix and $\bar{a}_{ij,CFAR\text{-}frame}$ is the mean average of the pixels found on the CFAR-frame relative to the cell-under-test.

The C-CFAR-signal matrix $RD_{C\text{-}CFAR}$ is an amplitude matrix of size $p \times q$, typically centered on the radar $RD_{amplitude}$ matrix adding signal to the radar $RD_{amplitude}$ matrix on a pixel-by-pixel basis. The amplitudes of the $RD_{C\text{-}CFAR}$ matrix are positive real values mapped from the GA chromosome using linear indexing. The optimal C-CFAR-signal matrix is found by minimizing the highest ratio-value found in the $RD_{CFAR\text{-}ratio}$ matrix and at the same time minimizing the energy expenditure J given by the sum of all squared RD cells in the $RD_{C\text{-}CFAR}$ matrix. This 2D multi-objective minimization problem can formally be stated as

$$C-CFAR: \begin{cases} Minimize \quad f_1 = r_{\max} \\ Minimize \quad f_2 = J \\ 1 \le f_1 \le 50 \wedge 0 \le f_2 \le 50 \end{cases} \quad (3)$$

where r_{max}=max($RD_{CFAR\text{-}ratio}$) and J=sum($RD_{C\text{-}CFAR}^2$) is the summed squared pixels assuming centre pixel of $RD_{amplitude}$ is assigned a constant target value of 1 restricting the energy output of the $RD_{C\text{-}CFAR}$ matrix.

In correspondence with [19] the $RD_{C\text{-}CFAR}$ matrix is set to size 11×11 using 16 bit binary chromosome coding. The radar $RD_{amplitude}$ matrix is set to size 21×21 formatted with realmin values leaving the outermost pixels unaffected by the $RD_{C\text{-}CFAR}$ matrix. The objective search space is bounded by the intervals $r_{max} \in [1, 50]$ and $J \in [0, 50]$ restricting the optimization into the relevant and interesting regions of the problem domain [19]. Any solution having values out of bound will be set to nearest bounding value. Mutation rate is set to the baseline inverse chromosome length and crossover fraction is set to 0.9 throughout all simulation runs. The C-CFAR radar problem is formulated using a population size of N=121 and each simulation is run for 500 generations or equivalent in the case of the ε-MOEA. All test configurations have been run 100 times, reporting the mean average and standard deviation for all relevant performance measures. Due to time constraints the comparison simulations are restricted in contrast to the original study [19] having population

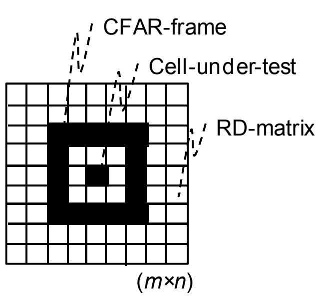

Figure 6. The generalized cell averaging 2D CFAR-frame of 5×5 operating on the central pixel of an *m*×*n* radar RD-matrix as the cell-under-test.

size of N=1815 and approximately $3 \cdot 10^4$ generations, producing several weeks of computing time. All simulations have been run in the Matlab R2009a programming environment using a HP xw8600 Workstation of 3 GHz Xeon 5450 CPU single core operation, 4 Gb memory and Windows XP 32 bit.

The C-CFAR comparison results are reported using the *Hypervolume* (*HV*) [24], [26] performance metric as the search efficiency metric. The *HV* is used for characterizing the search efficiency towards the optimal Pareto front when the true optimal Pareto front *is not known* in advance, as is the case with the C-CFAR-problem. The *HV* metric calculates the hypervolume in the objective space spanned by the Pareto optimal set *PO* relative to some reference point *W* in the objective space. The reference point *W* is set to the bounding values of *W*(50,50) making the radar problem almost nominally quadratic in objective space. A larger value of *HV* is desired.

Minimal Spacing (*MS*) [2] is used for evaluating the uniformity of consecutive solutions found on the Pareto optimal set. Smaller value of *MS* is better.

Span (*D*) [26] of the Pareto optimal set is defined as the length of the diagonal of the hyperbox formed by the extreme values of the Pareto optimal set *PO*. Large *D* values are desired.

4.2 Results

In Fig. 7 the final populations of the C-CFAR problem are plotted for the different optimization methods employed. In Fig 7a the flawed span of the obtained Pareto front using NSGA-II is clearly demonstrated having a span of D=12.3, as shown in Table I. The revised NSGA-II using guided domination and ε-MOEA, in Fig. 7b and 7c respectively, have an increased span of D=38.8 and D=19.3 but still do not sufficiently span the Pareto front of the C-CFAR problem. The guided domination is run rotating the linear Pareto domination contour lines 22.5° inwards on both sides, narrowing the region of domination into 45° of the upper left quadrant of the objective space. The guided domination has been run using 11.25° and 5.625° rotation but results are not reported here due to lower performance. The ε-MOEA is run using ε_{Rmax}=1 and ε_J=3 and strong ε-domination for extending the span of the method. Extensive parameter variation studies have been conducted, but they are too lengthy to be included here.

The SEA methods, on the other hand, widely spans the Pareto front, ranging from span D=43.2 to D=46.3, as seen in Table I. The SEA is run using $I^*=N$ interpolation points on the extended convex hull and a rank fitness scaling of $1/\text{rank}^{0.1}$. Also here, extensive parameter variation studies have been conducted too lengthy to be included. The good spanning properties of the SEA are confirmed by Fig. 7d-e where the final populations of the C-CFAR problem almost completely span the objective space. All the SEA algorithms tested have significantly better span than the NSGA-II and the ε-MOEA when testing on a 99.9 percent confidence interval using Welch's corrected *t*-test [25]. In between the SEA using distance to nearest interpolation point and SEA using angular-free-view there are found no significant differences in terms of span. The SEA using domination count is found to perform significantly poorer than the other SEA methods tested when testing on a 99 percent confidence interval. This might indicate that the good spanning properties of the SEA is related to the concept of boundary surface attraction points and not solely dependent on choice of single value metric employed.

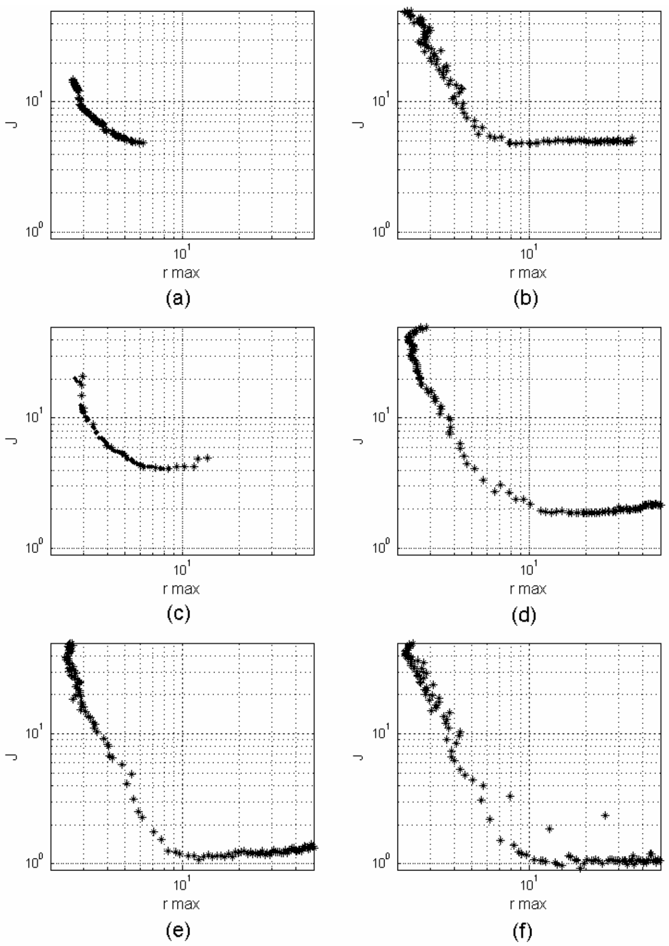

Figure 7. Examples of the resulting C-CFAR populations after 500 generations for (a) NSGA-II using Pareto domination, (b) NSGA-II using guided domination, (c) ε-MOEA, (d) SEA using domination count, (e) SEA using distance-to-interpolation point and (f) SEA using angular-free-view.

Moreover, whereas the MOEAs using revised domination principles suffer from decreased search efficiency, the SEA methods show a significant improvement in convergence when testing on a 99.9 percent confidence interval. Also, the SEA using distance to nearest interpolation point is significantly better than the SEA using angular-free-view and the SEA using domination count when testing on a 99.9 percent confidence interval. This indicates that SEA convergence property is dependent on single value metric selected.

Table 1. C-CFAR radar problem performance measures averaging 100 simulations of 500 generations per run.

Method	*Search eff.* *HV*	*Density* *MS*	*Span* *D*	*Time* [s]
NSGA-II PD	2141.5±50.2	0.5 ±0.4	12.3±6.1	42.2±0.1
NSGA-II GD	1996.3±99.6	1.8±0.6	38.8±4.5	41.97±0.09
ε-MOEA	2092.2±58.8	0.8±0.4	19.3±6.3	37.7±0.9
SEA DC	2275.0±21.7	1.6±0.9	43.2±7.4	41.9±0.1
SEA DI	2295.3±9.0	1.8±0.9	46.3±6.9	43.31±0.05
SEA AFV	2271.5±19.6	1.5±0.7	46.2±6.0	43.11±0.04

The small span of the NSGA-II using Pareto domination and ε-MOEA compared to the SEA makes a direct evaluation of the *MS* distribution performances difficult. However, the NSGA-II using guided domination is found to be outperformed by the SEA using angular-free-view and domination count when testing on a 95 percent confidence interval. No significant differences in *MS* were found between SEA using distance to nearest interpolation point and NSGA-II using guided domination. Furthermore, when comparing among SEA methods none of them have been found to excel in terms of the Pareto front density distribution when testing on a 90 percent confidence interval. This indicates that the distribution prosperity of the SEA is independent of single value metric employed.

These results, in terms of convergence and span of the Pareto front, show that the SEA is *Pareto preferred* to the NSGA-II and the ε-MOEA on this 2D asymptotic and convex radar problem. Furthermore, the simulation time results in Table I indicate that the algorithm efficiency of the different methods are comparable when optimizing this 2D problem.

5. CONCLUSION

In this paper, the 2D counter radar detection and tracking problem has been successfully solved using the novel SEA multi-objective optimizer. The Pareto preferred SEA, when compared to the NSGA-II and ε-MOEA, is shown to span all of the relevant and interesting parts of the optimal Pareto front while maintaining efficient search and density distribution characteristics. The performance of the SEA reveals the important high performance C-CFAR solutions not easily obtained using the NSGA-II or the ε-MOEA. These high energy solutions along with the complete Pareto front are vital when trading-off objectives according to decision maker's preferences or changing scenarios.

The good spanning and density properties of the SEA are attributed to the concept of interpolating attraction points on the boundary surface of the solution set. However, the good convergence properties of the SEA are not solely explained by the concept of boundary surface attraction points but are significantly affected by the single value metric employed in the SEA method.

Extending the work presented here, the high-dimensional properties of the SEA could be worth studying in the future, exploring different strategies for interpolating attraction points on the boundary surface as well as experimenting with different single value metrics. By extending the SEA into optimization of higher dimensional problems, other radar-jammer relevant dimensions could be included in the analysis, like time variations and the noisiness of the solutions in the radar image.

6. ACKNOWLEDGMENTS

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7. REFERENCES

[1] Agrawal, S., Panigrahi, B. K. and Tiwari, M. K., "Multiobjective Particle Swarm Algorithm With Fuzzy Clustering for Electrical Power Dispatch", *IEEE Trans. Evolutionary Computation*, vol. 12, no. 5, pp. 529–541, Oct. 2008.

[2] Bandyopadhyay, S., Pal, S. K. and Aruna, B., "Multi-objective GAs, Quantitative Indices and Pattern Classification", *IEEE Transactions on Systems, Man and Cybernetics - B*, vol. 34, no. 5, pp. 2088-2099, 2004.

[3] Branke, J., Kaußler, T. and Schmeck, H., "Guidance in evolutionary multi-objective optimization", *Advances in Engineering Software*, vol.32, iss. 6, pp. 499-507, June 2001.

[4] Deb, K., *Multi-Objective Optimization using Evolutionary Algorithms*. West Sussex, England: John Wiley & Sons Ltd, 2001.

[5] Deb, K., Pratap, A., Agarwal, S. and Meyarivan, T., "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II," *IEEE Trans. on Evolutionary Computation*, vol. 6, no. 2, pp. 182-197, Apr. 2002.

[6] Deb, K., Mohan, M. and Mishra, S., "Evaluating the ε-Domination Based Multi-Objective Evolutionary Algorithm for a Quick Computation of Pareto-Optimal Solutions", *Evolutionary Computation Journal*, vol. 13, iss. 4, pp. 501-525, 2005.

[7] Deb, K. and Srinivasan, A., "Innovization: Innovating Design Principles Through Optimization", *Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation, ACM GECCO '06*, pp. 1629-1636, New York, NY, 2006.

[8] Deb, K., "A Robust Evolutionary Framework for Multi-Objective Optimization", *Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation, ACM GECCO '08*, pp. 633-640, New York, NY, 2008.

[9] Goldberg, D. E., *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley Longman Inc., 1989.

[10] Holland, J. H., *Adaption in Natural and Artificial Systems*, Ann Arbor, The University of Michigan Press, 1975.

[11] Hong, S. et *al*., "Investigation on Genetic Algorithm for Countermeasures Technique Generator", *Proceedings on ISSSE'07*, pp. 351-354, 2007.

[12] Hughes, E. J., "Radar Waveform Optimisation as a Many-Objective Application Benchmark", S. Obayashi et *al*. (Eds.): *EMO 2007*, LNCS 4403, pp. 700-714, 2007.

[13] Hughes, E. J., "Many Objective Optimization: Direct Objective Boundary Identification", *Lecture Notes in Computer Science* 5199, pp. 733-742, 2008.

[14] Kennedy, J. and Eberhart, R., "Particle swarm optimization", *Proceedings of the IEEE International Conference on Neural Networks*, Piscataway, NJ, pp. 1942–1948, 1995.

[15] Koza, J. R., *Genetic Programming: On the Programming of Computers by Means of Natural Selection*, The MIT Press, 1992.

[16] Kristoffersen, S. and Moen, H. J. F., "Denial Jamming Technique Development Against Pulse-Doppler Radars Using Genetic Algorithms", *Proceedings of the 2008 International Conference on Radar, IEEE RADAR '08*, pp. 253-258, 2008.

[17] Laumanns, M., Thiele, L., Deb, K. and Zitzler, E., "Combining convergence and diversity in evolutionary multi-objective optimization", *Evolutionary Computation*, vol. 10, iss. 3, pp. 263-282, 2002.

[18] McGrath, M., "ECM Techniques Generator", *Proceedings of 48th Midwest Symposium on Circuits and Systems*, vol. 2, pp. 1749-1752, 2005.

[19] Moen, H. J. F. and Kristoffersen, S., "Multi-Resistant Radar Jamming Using Genetic Algorithms", *Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation*, *ACM GECCO '08*, pp. 1595-1602, New York, NY, 2008.

[20] Nunez, A. S. et *al*., "ECM Techniques Generator", *Proceedings of the SPIE Modeling and Simulation for Military Applications*, vol. 6228, pp. 62280Z, 2006.

[21] Rahmat-Samii, Y. and Christodoulou, C. (Eds.): *IEEE Transactions on Antennas and Propagation: Special Issue on Synthesis and Optimization Techniques in Electromagnetics and Antenna System Design*, vol. 55, no. 3, 2007.

[22] Skolnik, M., Radar Handbook. 3rd edition, McGraw-Hill, Inc., 1999.

[23] Smith, K. I., Everson, R. M., Fieldsend, J. E., Murphy, C. and Misra, R., "Dominance-Based Multiobjective Simulated Annealing", *IEEE Trans. Evolutionary Computation*, vol. 12, no. 3, pp. 323–342, Jun. 2008.

[24] Van Veldhuizen, D. A., *Multiobjective Evolutionary Algorithms: Classifications, Analyses, and New Innovations*, PhD Thesis, UMI Order Number: AAI9928483, Air Force Institute of Technology, 1999.

[25] Welch, B. L., "The Generalization of "Student's" Problem When Several Different Population Variances are Involved", *Biometrika*, vol. 34, no. 1–2, pp. 28–35, 1947.

[26] Zitzler, E. and Thiele, L., "Multiobjective Optimization Using Evolutionary Algorithms - A Comparative Case Study", *Proceedings of the 5th international Conference on Parallel Problem Solving From, Lecture Notes In Computer Science*, vol. 1498, pp. 292-304, Springer-Verlag, London, 1998.