An EA-based Approach to Design Optimization using Evidence Theory

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ABSTRACT

For problems involving uncertainties in design variables and parameters, a bi-objective evolutionary algorithm (EA) based approach to design optimization using evidence theory is proposed and implemented in this paper. In addition to a functional objective, a plausibility measure of failure of constraint satisfaction is minimized. Despite some interests in classical optimization literature, such a consideration in EA is rare. Due to EA's flexibility in its operators, non-requirement of any gradient, its ability to handle multiple conflicting objectives, and ease of parallelization, evidence-based design optimization using an EA is promising. Results on a test problem and a couple of engineering design problems show that the modified evolutionary multi-objective optimization (EMO) algorithm is capable of finding a widely distributed trade-off frontier showing different optimal solutions corresponding to different levels of plausibility failure limits. Furthermore, a single-objective evidence based EA is found to produce better optimal solutions than a previously reported classical optimization procedure. Handling uncertainties of different types are getting increasingly popular in applied optimization studies and more such studies using EAs will make EAs more useful and pragmatic in practical optimization problem-solving tasks.

Categories and Subject Descriptors

J.6 [**Computer Applications**]: Computer-aided engineering—*computer-aided design*

General Terms

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Keywords

Evidence based design, Reliability, NSGA-II

1. INTRODUCTION

Considering the effect of uncertainties is often unavoidable during a design optimization task. This is because the physical realization of the design variables and parameters is generally imprecise and this can lead to an optimal design becoming unusable if any constraints get violated. Reliability Based Design Optimization (RBDO) is the name given to an optimization procedure wherein the reliability of a design is also given due consideration, either as a constraint by limiting it to a minimum value, or as an additional objective.

Uncertainties can be classified as 'aleatory' or 'epistemic' depending on their nature. While an aleatory uncertainty is well defined as a probability distribution, epistemic uncertainty represents our lack of information about the nature of the impreciseness. Thus, a well defined probability distribution may not always be available to handle uncertainties in a design optimization problem. In such a case, the usual RBDO methodologies cannot be used and a new approach which can utilize low amount of information about the uncertainty is required. Few such methods like a Bayesian approach for sample-based information [5] and an evidence theory based approach [7] for interval-based information have been proposed, but only a few studies like [8] have explored them in an EA-context.

In this work, we use the evidence-based approach, assuming that information about the uncertainty is available as evidence of the uncertain variables lying within definite intervals around the nominal value. We propose that instead of constraining the plausibility of failure to a predefined value, a bi-objective approach may be adopted in order to get more useful results, and discuss other benefits of using an evolutionary algorithm for evidence-based design optimization.

2. RBDO PROBLEM FORMULATION

A general formulation of a deterministic optimization problem is:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}, \mathbf{p}) \\ \text{subject to:} \quad & g_j(\mathbf{x}, \mathbf{p}) \geq 0, \; j = 1, \ldots, J \end{aligned} \tag{1}$$

In this formulation, $\mathbf{x}$ are the n design variables that are varied in the optimization while $\mathbf{p}$ are the m design parameters that are kept fixed. Both $\mathbf{x}$ and $\mathbf{p}$ are assumed to be real-valued. The objective is to minimize a function f subject to J inequality constraints. We do not consider equality constraints because reliability is not defined for equality constraints in this context.

In reliability-based optimization, uncertainties in the design are embodied as random design variables $\mathbf{X}$ and random design parameters $\mathbf{P}$, and the problem is formulated as:

$$\begin{array}{ll} \underset{\mu_{\mathbf{X}}}{\text{minimize}} & f\left(\mu_{\mathbf{X}}, \mu_{\mathbf{P}}\right) \\ \text{subject to:} & Pr\left[g_j\left(\mathbf{X}, \mathbf{P}\right) \geq 0\right] \geq R_j,\ j = 1, \ldots, J \end{array} \tag{2}$$

The objective of the problem is to minimize f with respect to the means (μ's) of the random variables given the means of the random parameters. The problem is subject to the constraints that the probability of design feasibility is greater than or equal to R_j, for all $j = 1, \ldots, J$, where R_j is the target reliability for the j^{th} probabilistic constraint. A solution to a reliability-based optimization problem is called an optimal-reliable design.

If the uncertainty in the variables and parameters can be confidently expressed as probability distributions (aleatory uncertainty), the above RBDO formulation is sufficient for reliability analysis. However, it is often found that the uncertainty associated with the variables and parameters of a design optimization problem can not be expressed as a probability distribution. This is because the only information available might be a certain number of physical realizations of the variables, or expert opinions about the uncertainty. The above RBDO formulation can not utilize this type of information and therefore, a different approach to uncertainty analysis is called for.

3. BASICS OF EVIDENCE THEORY

Evidence theory has recently been used for analysis of design reliability when very limited information about the uncertainty is available, usually in the form of expert opinions. Before we proceed to describe an evidence based design optimization (EBDO) procedure using an evolutionary algorithm, we outline the fundamentals of evidence theory as discussed in [7] in this section.

Evidence theory is characterized by two classes of fuzzy measures, called *belief* and *plausibility* measures, respectively, They are mutually dual, and thus each can be uniquely determined from the other. The plausibility and belief measures act as upper and lower bounds of classical probability to measure the likelihood of events without use of explicit probability distributions. When the plausibility and belief measures are equal, the general evidence theory reduces to the classical probability theory. Therefore, the classical probability theory is a special case of evidence theory. A recapitulation of the basics of fuzzy set theory is essential to understand the nature of plausibility and belief measures.

A universe X represents a complete collection of elements having the same characteristics. The individual elements in the universe X, called singletons, are denoted by x. A set A is a collection of some elements of X. All possible subsets of X constitute a special set called the power set $\wp$.

A fuzzy measure is defined by a function $g : \wp(X) \to [0, 1]$. Thus, each subset of X is assigned a number in the unit interval $[0, 1]$. The assigned number for a subset $A \in \wp(X)$, denoted by $g(A)$, represents the degree of available evidence or belief that a given element of X belongs to the subset A.

In order to qualify as a fuzzy measure, the function g must have certain properties. These properties are defined by the following axioms:

- $g(\emptyset) = 0$ and $g(X) = 1$.
- For every $A, B \in \wp(X)$, if $A \subseteq B$, then $g(A) \leq g(B)$.
- For every sequence $(A_i \in \wp(X), i = 1, 2, \ldots)$ of subsets of $\wp(X)$, if either $A_1 \subseteq A_i \subseteq \ldots$ or $A_1 \supseteq A_2 \supseteq \ldots$ (the sequence is monotonic), then $\underset{i\to\infty}{lim}\, g(A_i) = g(\underset{i\to\infty}{lim} A_i)$.

A belief measure is a function $Bel : \wp(X) \to [0, 1]$ which satisfies the three axioms of fuzzy measures and the following additional axiom:

$$Bel(A_1 \cup A_2) \geq Bel(A_1) + Bel(A_2) - Bel(A_1 \cap A_2). \tag{3}$$

Similarly, A plausibility measure is a function $Pl : \wp(X) \to [0, 1]$ which satisfies the three axioms of fuzzy measures and the following additional axiom:

$$Pl(A_1 \cap A_2) \leq Pl(A_1) + Pl(A_2) - Pl(A_1 \cup A_2). \tag{4}$$

Being fuzzy measures, both belief and its dual plausibility measure can be expressed with respect to the non-negative function:

$$m : \wp(X) \to [0, 1]\,, \tag{5}$$

such that $m(\emptyset) = 0$, and

$$\sum_{A \in \wp(X)} m(A) = 1. \tag{6}$$

The function m is referred to as basic probability assignment (BPA). The basic probability assignment $m(A)$ is interpreted either as the degree of evidence supporting the claim that a specific element of X belongs to the set A or as the degree to which we believe that such a claim is warranted. Every set $A \in \wp(X)$ for which $m(A) \geq 0$ (evidence exists) is called a focal element of m. Given a BPA m, a belief measure and a plausibility measure are uniquely determined by

$$Bel(A) = \sum_{B \subseteq A} m(B). \tag{7}$$

$$Pl(A) = \sum_{B \cap A \neq 0} m(B). \tag{8}$$

In Equation (7), $Bel(A)$ represents the total evidence corresponding to all the subsets of A, while the $Pl(A)$ in Equation (8) represents the sum of BPA values corresponding to all the sets B intersecting with A. Therefore,

$$Pl(A) \geq Bel(A). \tag{9}$$

Several theories have been proposed to combine evidence obtained from independent sources or experts. If the BPAs m_1 and m_2 express evidence from two experts, the combined evidence m can be calculated using Dempster's rule of combining:

$$m(A) = \frac{\sum\limits_{B \cap C = A} m_1(B) m_2(C)}{1 - K}, \quad \text{for } A \neq 0, \tag{10}$$

where

$$K = \sum_{B \cap C = 0} m_1(B) m_2(C) \tag{11}$$

represents the conflict between the two independent experts. Other methods of combining evidence may also be used.

4. EBDO PROBLEM FORMULATION AND EA-BASED SOLUTION APPROACH

In the previous section, the fuzzy measure of plausibility was introduced which can be used to represent the degree to which the available evidence supports the belief that an element belongs to a set or overlapping sets. In [7], an evidence theory based approach to handle uncertainties in a design problem is discussed. Each probability constraint of the RBDO problem is replaced with a plausibility constraint, limiting the plausibility of failure for each constraint to a pre-defined value.

In this paper, we propose taking a different approach which treats the problem as a bi-objective problem due to the crucial role of the uncertainty in design. Since the reliability of the design will be an important factor in choosing a design, a trade-off analysis between objective function value and reliability is desirable. Thus, instead of using a plausibility constraint, the plausibility value of any solution $\mathbf{x}$ can be converted to a second objective function. Since we are evaluating the plausibility of failure, the plausibility value should then be minimized. For multiple constraints, the maximum plausibility over all constraints $Pl_{\max}$ is chosen to be the second objective. According to [7], the plausibility measure is preferred, instead of the equivalent belief measure, since at the optimum, the failure domain for each active constraint will usually be much smaller than the safe domain over the frame of discernment. As a result, the computation of the plausibility of failure is much more efficient than the computation of the belief of safe region.

Using an evolutionary algorithm for such a design task has several advantages. Firstly, evidence based design methods have not been investigated adequately due to the high computational complexity of implementation. Since evolutionary algorithms can be efficiently parallelized, a significant amount of computational time and resources can be saved. Also, evolutionary methods are a good choice for EBDO since the optimization task requires an algorithm which is derivative-free and can handle discrete plausibility values. Secondly, as discussed before, using a bi-objective evolutionary approach, a set of trade-off solutions can be obtained which can be very useful to a decision maker. These solutions can in turn be utilized for post optimality analysis and further insights into the nature of the problem can be developed.

Using the bi-objective evolutionary approach discussed above, the RBDO problem stated in Equation (2) can be formulated using an evidence theory based approach as follows:

$$\begin{aligned} &\underset{\mathbf{X},\mathbf{P}}{\text{minimize}} && f(\mathbf{X},\mathbf{P}), \\ &\underset{\mathbf{X},\mathbf{P}}{\text{minimize}} && Pl_{\max}, \\ &\text{subject to:} && 0 \le Pl_{\max} \le 1, \end{aligned} \tag{12}$$

where (for $g \ge 0$ type constraints),

$$Pl_{\max} = \max_{j=1}^{J}(Pl\,[g_j(\mathbf{X},\mathbf{P}) \le 0\,]). \tag{13}$$

The plausibility of failure for each member of the population (each design point) must be evaluated in the bi-objective formulation. A step-by-step method for this evaluation is as follows:

1. The frame of discernment (FD) about the design point is identified first. This is the region in the decision variable space for which evidence is available. In a typical BPA structure, the available evidence is expressed corresponding to various intervals of values near the nominal value (denoted by the subscript N) of each variable and parameter. If a conflict exists, a suitable rule for combining evidence must be used. The FD can be viewed as a Cartesian product of these intervals.

2. Each element of the Cartesian product of intervals is referred to as a focal element. This is the smallest unit in the FD for which evidence can be calculated by combining the BPA structures of the variables and parameters. In Figure 1, the focal elements are the rectangles bounded by dashed lines. When the variables/parameters are independent, the BPA of a focal element is simply the product of BPA values corresponding to the intervals that compose it. The BPA structure for the entire FD is thus obtained.

3. According to Equation 8, the focal elements that completely or partially lie in the failure region contribute to the plausibility calculation. Similarly, the focal elements lying completely in the failure region contribute to the belief calculation (Equation 7). This calculation must be performed for each problem constraint. To check if failure occurs within a focal element, the minimum value of the constraint function in the focal element is searched for. If the minimum is negative (for greater-than-equal-to type constraints), then failure occurs in the focal element. Various issues related to identifying the focal elements for plausibility calculation are discussed in Section 4.1.

4. The combined BPA values for the focal elements identified for plausibility calculation are summed to obtained the plausibility for the failure region according to Equation 8.

5. The maximum plausibility value over all constraints is obtained as the second objective function value to be minimized, according to Equation 13.

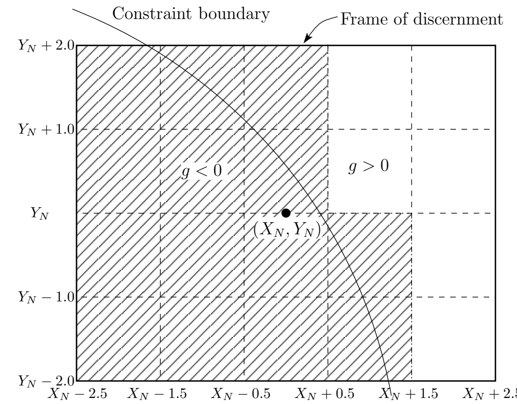

Figure 1: Focal elements contributing to plausibility calculation.

The illustration in Figure 1 shows the FD within which a constraint boundary lies. The dashed lines divide the FD into focal elements according to the intervals of the BPA structure along each axis. In this case, the shaded focal elements contribute to the plausibility calculation since inside these elements, the constraint becomes infeasible.

4.1 Plausibility calculation and related issues

It is evident that the calculation of plausibility is a computation bottle-neck in an evidence-based design optimization task, and this is a major reason that such methods have not been investigated or used as much as others in the past. In this section we discuss some methods to tackle this computational complexity, making the approach less time-consuming and more practical.

An intuitive approach to the problem of reducing computations is to use an algorithm that does not need to search all the individual focal elements for a minimum, so that several focal elements belonging to the feasible or infeasible region can be identified together. Such a method will circumvent the local search effort in each focal element and is thus expected to reduce computation substantially. For this purpose, subdivision techniques such as [3] can be used to eliminate a portion of the search region in each step. A similar algorithm has been demonstrated for evidence-based design optimization in [7]. However, it must be noted that if the computation cost of the identification of focal elements still remains high, then such an algorithm can not be parallelized readily.

Parallelization or distributed computing is certainly a viable option for the search process required for plausibility calculation. Focal elements belonging to a frame of discernment can be searched in parallel threads and all the elements contributing to the plausibility calculation identified simultaneously. Thus, a computational speed-up of the order of the number of focal elements is expected, making the approach most suitable when this number is high, that is, the information available about the uncertainty is more and more crisp. We are currently pursuing such a parallelization and shall communicate the results at a later date.

Whether a subdivision technique is used or not, a subtask of finding the failure plausibility at a design point is to classify a region (containing one or several focal elements) as feasible or infeasible with respect to all the constraints. As previously stated, this can be done by searching for the minimum of the constraint in the region, and checking if it is negative (implying infeasibility for $g > 0$ type constraints). There are various techniques which can be used for this search:

Grid-point evaluation: The constraint function can be evaluated at all points forming a hyper-grid across the dimensions of the search space and then the minimum of the values obtained can be used as the minimum of the constraint function. Although the minimum obtained through this technique will not be accurate, it may be sufficient for checking the negativity condition.

Sampling-based evaluation: Instead of evaluating the constraint at uniform grid-points, techniques like optimum symmetric Latin hypercube sampling [10] may be used to evaluate the constraint at fewer, representative points in the search space. Both the grid-based method using a coarse grid and sampling techniques are suitable when the search regions are not large enough that major variations in the constraint values may be expected.

Local search: A local search for the minimum can be performed using a vertex method, gradient-based method or a solver like KNITRO [1] depending on the complexity of constraints. Some of these methods may significantly raise the computation cost and thus should be carefully chosen. To simplify the evaluation of the constraints, a response surface or kriging models may be generated using sampling techniques as in [11]. For this paper, KNITRO is used to confirm the results obtained from a grid-point evaluation method.

Finally, it is seldom the case that a design optimization task is a black-box operation and no information about the behavior of the constraints is available. Quite often, designers are capable of predicting the behavior of the constraint functions with respect to different variables and parameters, and this information should be utilized effectively to reduce the computation cost for such methods. For example, a deflection constraint might always reach extrema at the variable bounds for the material property, and there is no need to perform a search operation for the region under consideration with respect to this variable. This type of information utilization can significantly reduce the computational effort in practice.

5. NUMERICAL TEST PROBLEM

We first demonstrate the approach on a two-variable test problem described as follows:

$$\begin{aligned}
\underset{\mathbf{X}}{\text{minimize}} \quad & f(\mathbf{X}) = X_1 + X_2, \\
\text{subject to:} \quad & g_1 : 1 - \frac{X_1^2 X_2}{20} \leq 0, \\
& g_2 : 1 - \frac{(X_1 + X_2 - 5)^2}{30} \\
& \qquad - \frac{(X_1 - X_2 - 12)^2}{120} \leq 0, \\
& g_3 : 1 - \frac{80}{X_1^2 + 8X_2 - 5} \leq 0 \\
& 0 \leq X_1 \leq 10, \\
& 0 \leq X_2 \leq 10.
\end{aligned} \tag{14}$$

The deterministic optimum for this problem is at $f^* = 5.179$. To demonstrate evidence based analysis, it is assumed that the variables X_1 and X_2 are epistemic, and the evidence available about the variation of uncertainty corresponding to each can be represented by a BPA structure such as that described in Table 1. Due to the uncertainties, the optimal solution to the evidence based design optimization is likely to be worse than the deterministic optimum. For simulation purposes, the value of BPA assigned to a particular interval may be obtained directly from an assumed probability distribution. The subscript N denotes the nominal value for a decision variable or parameter.

The problem is converted to a bi-objective problem as developed in the previous section, where an additional objective of minimizing the plausibility of failure is now considered. The problem is solved using the well-known NSGA-II [2] procedure. We use a population size of 100 (to obtain as distributed a front as possible) and run it for 100 generations. The resulting trade-off front is shown in Figure 2.

Table 1: BPA Structure for the numerical test problem.

x_1 Interval	BPA
[$x_{1,N}$ - 1.0, $x_{1,N}$ - 0.5]	4.78 %
[$x_{1,N}$ - 0.5, $x_{1,N}$]	45.22 %
[$x_{1,N}$, $x_{1,N}$ + 0.5]	45.22 %
[$x_{1,N}$ + 0.5, $x_{1,N}$ + 1.0]	4.78 %

x_2 Interval	BPA
[$x_{2,N}$ - 1.0, $x_{2,N}$ - 0.5]	4.78 %
[$x_{2,N}$ - 0.5, $x_{2,N}$]	45.22 %
[$x_{2,N}$, $x_{2,N}$ + 0.5]	45.22 %
[$x_{2,N}$ + 0.5, $x_{2,N}$ + 1.0]	4.78 %

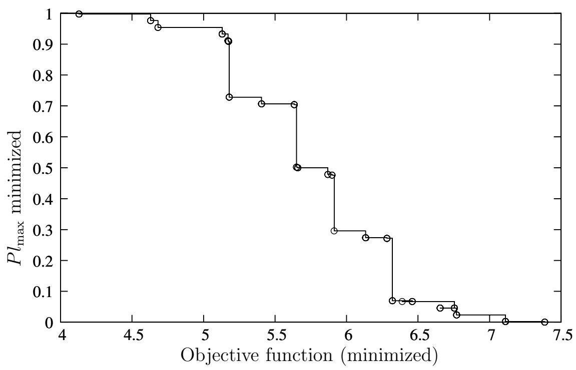

Figure 2: Results obtained for the two variable test problem.

It should be noted that the front is discontinuous due to the discrete nature of the BPA structure; only certain values of plausibility are possible, and the front does not exist for intermediate values. Also, several design points will have the same plausibility of failure, and the algorithm is able to find the points for which the first objective is minimized while the second remains constant. The design points with failure plausibility closer to unity lie in the infeasible region with respect to their nominal values, and it is only due to the uncertainty that a few feasible realizations may be expected for these designs.

The jumps in the plausibility values obtained in the final front are due to the jumps in the BPA structures of the variables. To demonstrate this, we modify the BPA structures to be more *smooth*, distributing the high values of evidence for the central intervals as in Table 2. The final front obtained in Figure 3 is seen to be much smoother and without large jumps in plausibility values. This reflects our intuition that more information about the uncertainty distribution will lead to better decision making.

The mirrored 'S' shaped form of the trade-off frontier is evident from these plots. Solutions near the minimum-f point (deterministic optimum of f) would make large failures due to uncertainties in decision variables and parameters. However, any small improvement in failure probability demands for a large sacrifice of f. Also, when a too tight a failure probability is needed, a very large sacrifice in optimal f has to be accepted. In general, high values of plausibility of failure are undesirable, and should be eliminated using a constraint. Designers can set a limit of failure probability and choose a preferred solution from such a bi-objective trade-off frontier.

Table 2: Modified BPA structure for the numerical test problem.

x_1 Interval	BPA
[$x_{1,N}$ - 1.0, $x_{1,N}$ - 0.5]	4.78 %
[$x_{1,N}$ - 0.5, $x_{1,N}$ - 0.25]	15.22 %
[$x_{1,N}$ - 0.25, $x_{1,N}$]	30 %
[$x_{1,N}$, $x_{1,N}$ + 0.25]	30 %
[$x_{1,N}$ + 0.25, $x_{1,N}$ + 0.5]	15.22 %
[$x_{1,N}$ + 0.5, $x_{1,N}$ + 1.0]	4.78 %

x_2 Interval	BPA
[$x_{2,N}$ - 1.0, $x_{2,N}$ - 0.5]	4.78 %
[$x_{2,N}$ - 0.5, $x_{2,N}$ - 0.25]	15.22 %
[$x_{2,N}$ - 0.25, $x_{2,N}$]	30 %
[$x_{2,N}$, $x_{2,N}$ + 0.25]	30 %
[$x_{2,N}$ + 0.25, $x_{2,N}$ + 0.5]	15.22 %
[$x_{2,N}$ + 0.5, $x_{2,N}$ + 1.0]	4.78 %

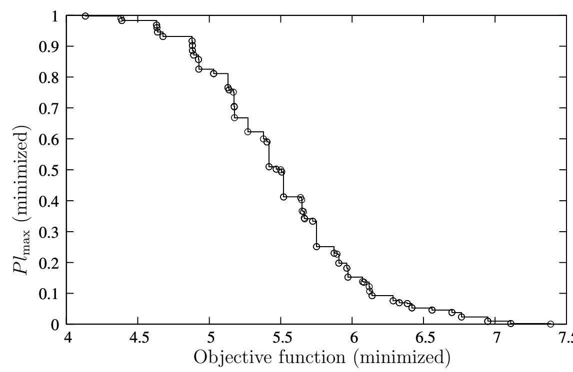

Figure 3: Results obtained for the modified two variable test problem.

6. RESULTS FOR ENGINEERING DESIGN PROBLEMS

6.1 Cantilever beam design

The first engineering design problem we solve is a cantilever beam design for vertical and lateral loading [9]. In this problem, a beam of length 100 in. is subjected to a vertical load Y and lateral load Z at its tip. The objective of the problem is to minimize the weight of the beam which can be represented by $f = w \times t$, where w is the width and t the thickness of the beam.

The problem is modeled with two constraints which represent two non-linear failure modes. The first mode is represented by failure at the fixed end of the beam, while the second mode is the tip displacement exceeding a maximum allowed value of $D_0 = 2.5$ in. The deterministic optimization problem formulation is given as follows:

$$
\begin{aligned}
\underset{w,t}{\text{minimize}} \quad & f = wt, \\
\text{subject to:} \quad & g_1 : y - \left(\frac{600Y}{wt^2} + \frac{600Z}{w^2t}\right) \geq 0, \\
& g_2 : D_0 - \frac{4L^3}{Ewt}\sqrt{\left(\frac{Y}{t^2}\right)^2 + \left(\frac{Z}{w^2}\right)^2} \geq 0, \\
& 0 \leq w \leq 5, \\
& 0 \leq t \leq 5.
\end{aligned} \tag{15}
$$

To demonstrate the principle of the evidence theory based EA approach, the variables w and t are taken to be deterministic, while the parameters Y = vertical load, Z = lateral load, y = yield strength and E = Young's modulus are assumed to be epistemic. In general, expert opinions and prior information about the uncertainty will yield a BPA structure for the epistemic parameters. However, in order to facilitate comparison with an RBDO result, the distributions of the parameters as used in an RBDO study [6] are used to obtain the BPA structure for them. Thus, normal distributions $Y \sim N(1000, 100)$ lb, $Z \sim N(500, 100)$ lb, $y \sim N(40000, 2000)$ psi and $E \sim N(29(10^6), 1.45(10^6))$ psi are assumed and the area under the corresponding PDF for each interval is taken as the BPA for each parameter.

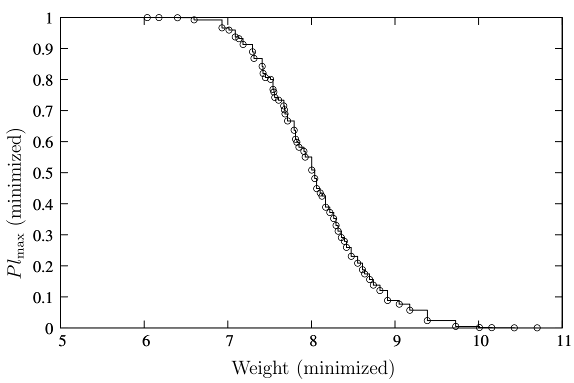

Figure 4: Obtained trade-off front for the Cantilever design problem.

The problem is converted to the bi-objective formulation developed in Section 4, and solved using a population size of 60 for 100 generations of NSGA-II. The resulting trade-off is shown in Figure 4. The mirrored 'S' shaped front is obtained.

6.1.1 Comparison with an earlier study

In order to compare to the results with a previous study [7], which used a single objective of minimizing f alone, NSGA-II is used to solve for the single-objective formulation, setting the limit of failure plausibility for each constraint equal to p_{lim} as follows:

$$
\begin{aligned}
\underset{w,t}{\text{minimize}} \quad & f = wt, \\
\text{subject to:} \quad & Pl_{\max} \leq p_{lim}, \\
& 0 \leq w \leq 5, \\
& 0 \leq t \leq 5.
\end{aligned} \tag{16}
$$

Table 3: BPA Structure for the cantilever design problem

Z Interval	Z BPA	y ($\times 10^3$) Interval	y ($\times 10^3$) BPA
[200, 300]	2.2 %	[35, 37]	6.1 %
[300, 400]	13.6 %	[37, 38]	9.2 %
[400, 450]	15 %	[38, 39]	15 %
[450, 500]	19.2 %	[39, 40]	19.2 %
[500, 550]	19.2 %	[40, 41]	19.2 %
[550, 600]	15 %	[41, 42]	15 %
[600, 700]	13.6 %	[42, 43]	9.2 %
[700, 800]	2.2 %	[43, 45]	7.1 %

Y Interval	Y BPA	E ($\times 10^6$) Interval	E ($\times 10^6$) BPA
[700, 800]	2.2 %	[26.5, 27.5]	10 %
[800, 900]	13.6 %	[27.5, 28.5]	21 %
[900, 1000]	34.1 %	[28.5, 29]	13.5 %
[1000, 1100]	34.1 %	[29, 29.5]	13.5 %
[1100, 1200]	13.6 %	[29.5, 30.5]	21 %
[1200, 1300]	2.4 %	[30.5, 31.3]	21 %

Table 5: Cantilever design: Comparison of results for $\mathbf{p_{lim}}$ = 0.0013. NSGA-II solution makes $Pl(g_1)$ constraint have a value close to this limit.

Variable	DIRECT [7]	NSGA-II
w	2.5298	2.414244
t	4.1726	4.124685
	Values obtained	
$Pl(g_1)$	0.000032	**0.001274**
$Pl(g_2)$	0.000000	0.000000
f	10.556	**9.957998**

The results, comparing with the previous results obtained using the DIRECT optimizer, are shown in Table 4. It is notable that NSGA-II finds a better design point for all three values of p_{lim}, demonstrating the better performance of an evolutionary algorithm for this complex optimization task. The plausibility and objective function values obtained for $p_{lim} = 0.0013$ corresponding to the design point reported in [7] and the point obtained by NSGA-II are compared in Table 5. The superiority of the NSGA-II solution comes from the fact that NSGA-II solution is able to find a solution for which the plausibility failure of first constraint is almost same as the desired p_{lim}, whereas the solution reported in the existing study is far from this limit.

As expected the design points obtained under uncertainty are worse than the deterministic optimum, since objective function value must be sacrificed in order to gain in the reliability of the design. Also, the value of weight obtained for $p_{lim} = 0.0013$ (corresponding to $R = 0.9987$) for the RBDO study is worse than the reliable optimum. This is because evidence theory based optimization is a case of lower information than RBDO, and this loss of information about the uncertainty affects the optimum weight value that can be obtained.

Table 4: Comparison of results for the Cantilever design problem.

	Evidence theory based results				Reliable Optimum	Deterministic optimum
	Algorithm	$p_{lim} = 0.2$	$p_{lim} = 0.1$	$p_{lim} = 0.0013$	$R = 0.9987$	
Weight	DIRECT [7]	8.6448	10.217	10.556	9.5212	7.6679
	NSGA-II	8.5751	8.8832	9.9580		

6.2 Pressure vessel design

The second engineering design problem is design of a pressure vessel [4]. The objective of the problem is to maximize the volume of the pressure vessel which is a function of design variables R, the radius and L, the mid-section length. A third design variable is the thickness t of the pressure vessel wall. The problem is modeled with five constraints which represent failure in the form of yielding of material in circumferential and radial directions, and violation of geometric constraints. The deterministic problem formulation is as follows:

$$
\begin{aligned}
\underset{R_N, L_N, t_N}{\text{maximize}} \quad & f = \frac{4}{3}\pi {R_N}^3 + \pi {R_N}^2 L_N, \\
\text{subject to:} \quad & g_1 : 1.0 - \frac{P(R + 0.5t)SF}{2tY} \geq 0, \\
& g_2 : 1.0 - \frac{P(2R^2 + 2Rt + t^2)SF}{(2Rt + t^2)Y} \geq 0, \\
& g_3 : 1.0 - \frac{L + 2R + 2t}{60} \geq 0, \\
& g_4 : 1.0 - \frac{R + t}{12} \geq 0, \\
& g_5 : 1.0 - \frac{5t}{R} \geq 0, \\
& 0.25 \leq t_N \leq 2.0, \\
& 6.0 \leq R_N \leq 24, \\
& 10 \leq L_N \leq 48.
\end{aligned} \tag{17}
$$

Besides the design variables, the parameters internal pressure P, and yield strength Y are also assumed to be epistemic for this problem (making a total of five epistemic variables). The assumed normal distribution for P and Y are $P \sim N(1000, 50)$ and $Y \sim N(260000, 13000)$, respectively, while the normal distributions for R, L and t are assumed to have standard deviations equal to 1.5, 3.0 and 0.1 respectively. Table 6 shows the BPA structure used.

The bi-objective formulation of the problem is solved using a population size of 60 for 100 generations of NSGA-II and the resulting trade-off is shown in Figure 5. The observed jumps in the front are expected due to the jumps in the BPA structure of the problem variables, as explained earlier in Section 5.

A similar comparison as in the previous example shows that NSGA-II is able to obtain better solution in terms of objective function value for the same value of p_{lim} than DIRECT. Table 7 shows that the results obtained are consistent with expected values, being worse the deterministic optimum as well as the corresponding reliability based optimum ($p_{lim} = 0.015$ corresponding to $R = 0.985$), reflecting the lack of information about the uncertainty. Table 8 compares the design point reported previously [7] and the design point obtained by NSGA-II in terms of plausibility and objective function values.

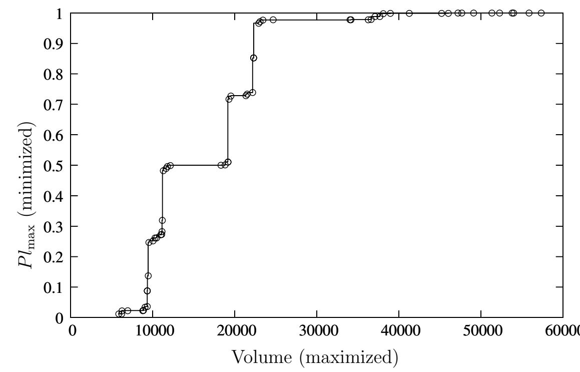

Figure 5: Obtained trade-off front for the Pressure Vessel design problem.

Table 8: Pressure vessel design: Comparison of results for $\mathbf{p_{lim} = 0.015}$. NSGA-II solution makes $Pl(g_2)$ constraint have a value close to this limit.

Variable	DIRECT [7]	NSGA-II
R	7.074	7.127
L	29.626	30.019
t	0.413	0.364
	Values obtained	
$Pl(g_1)$	0.001298	0.001301
$Pl(g_2)$	0.001318	**0.014732**
$Pl(g_3)$	0.012020	0.012020
$Pl(g_4)$	0.012050	0.012050
$Pl(g_5)$	0.012020	0.012020
f	6.137e3	**6.307e3**

7. CONCLUSIONS

In this paper, we have proposed a new evolutionary approach to design optimization using evidence theory. The use of evidence theory for uncertainty analysis in design optimization has been limited in the past primarily due to the high computation cost involved. However, due to availability of better optimization algorithms and parallel computing platforms, they are getting more and more tractable and such methods are bound to become more popular. An evolutionary approach also offers the possibility of parallelization and an algorithmic flexibility which can drastically reduce the computation time required for analysis.

Our proposed approach is based on a bi-objective formulation and the solution methodology is based on the NSGA-II approach. Since a number of trade-off solutions can be found by such an approach, designers will have a plethora of information relating to designs having different plausible failure

Table 6: BPA Structure for the pressure vessel design problem.

R	L	t	BPA	P	Y	BPA
[R_N - 6.0, R_N - 4.5]	[L_N - 12, L_N - 9]	[t_N - 0.4, t_N - 0.3]	0.13%	[800, 850]	[208000, 221000]	0.13%
[R_N - 4.5, R_N - 3.0]	[L_N - 9, L_N - 6]	[t_N - 0.3, t_N - 0.2]	2.15%	[850, 900]	[221000, 234000]	2.15%
[R_N - 3.0, R_N]	[L_N - 6, L_N]	[t_N - 0.2, t_N]	47.72%	[900, 1000]	[234000, 260000]	47.72%
[R_N, R_N + 3.0]	[L_N, L_N + 6]	[t_N, t_N + 0.2]	47.72%	[1000, 1100]	[260000, 286000]	47.72%
[R_N + 3.0, R_N + 4.5]	[L_N + 6, L_N + 9]	[t_N + 0.2, t_N + 0.3]	2.15%	[1100, 1150]	[286000, 299000]	2.15%
[R_N + 4.5, R_N + 6.0]	[L_N + 9, L_N + 12]	[t_N + 0.3, t_N + 0.4]	0.13%	[1150, 1200]	[299000, 312000]	0.13%

Table 7: Comparison of results for the Pressure Vessel design problem.

	Evidence theory based results				Reliable Optimum	Deterministic optimum
	Algorithm	$p_{lim} = 0.3$	$p_{lim} = 0.2$	$p_{lim} = 0.015$	$R = 0.985$	
Volume	DIRECT [7]	1.101e4	0.9053e4	0.6137e4	1.605e4	2.240e4
	NSGA-II	1.126e4	0.9495e4	0.6307e4		

limits. Such information is not only important to choose a single design using a post-optimality analysis, the knowledge of variation of solutions for different limiting failure levels would be most valuable to the designers. Results for numerical and engineering design problems show the effectiveness of the proposed approach, and the capability of an EA to find better solutions in the discrete objective space. Moreover, due to better handling of constraints within NSGA-II, the proposed approach has been able to find a better design compared to an existing classical optimization algorithm.

Handling uncertainties in design variables and parameters is a must if optimization methods have to be used routinely in engineering design activities. The uncertainties in some parameters may be known by means of a probability distribution and uncertainties in some other parameters may not not be known with any particular distribution. A methodology for handling combined such cases (combining reliability-based and evidence-based design) would be the next step. Computation of plausibility values are computationally demanding and the use of parallel computing is important for such problem-solving tasks. The use of optimization algorithms that allow such parallel computations, such as EAs, will also remain as key methodologies for solving such problems.

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