Iterated N-Player Games on Small-World Networks

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ABSTRACT

The evolution of strategies in iterated multi-player social dilemma games is studied on small-world networks. Two different games with varying reward values – the N-player Iterated Prisoner's Dilemma (N-IPD) and the N-player Iterated Snowdrift game (N-ISD) – form the basis of this study. Here, the agents playing the game are mapped to the nodes of different network architectures, ranging from regular lattices to small-world networks and random graphs. In a given game instance, the focal agent participates in an iterative game with $N-1$ other agents drawn from its local neighbourhood. We use a genetic algorithm with synchronous updating to evolve agent strategies. Extensive Monte Carlo simulation experiments show that for smaller cost-to-benefit ratios, the extent of cooperation in both games decreases as the probability of re-wiring increases. For higher cost-to-benefit ratios, when the re-wiring probability is small we observe an increase in the level of cooperation in the N-IPD population, but not the N-ISD population. This suggests that the small-world network structure with small re-wiring probabilities can both promote and maintain higher levels of cooperation when the game becomes more challenging.

Categories and Subject Descriptors

I.2 [**Computing Methodologies**]: Artificial Intelligence

General Terms

Theory, Algorithms, Experimentation

Keywords

Iterated N-player games, small-world networks, prisoner's dilemma, snowdrift game

1. INTRODUCTION

Understanding how cooperative behaviour can be promoted and maintained in social communities when the selfish actions available to individuals are clearly favoured is a long-standing scientific endeavour. A great deal of the related research has focused on two-player social dilemma models, such as the Prisoner's Dilemma (PD) and the Snowdrift (SD) games. In these games, the players typically must choose between two actions: to cooperate or to defect. A reward (R) is given if both players cooperate, while punishment (P) is handed out if they both defect. In the situation where one player defects and the other player cooperates, the one who defects is awarded a tempting reward (T) but the one who cooperates will be given the sucker's punishment (S). Accordingly, we have a PD game when T > R > P > S and an SD game when T > R > S > P.

Spatial (or network) reciprocity is now widely regarded as a mechanism that can be used to promote cooperation in social dilemma games [24, 25]. That is, by mapping players onto the nodes of a simple regular lattice, and restricting their interactions to a local neighbourhood, cooperative outcome that might otherwise be impossible in a well-mixed population can emerge. Over the past decade, related studies have suggested that relaxing the rigid local neighbourhood structure imposed by regular lattices could be advantageous. For example, Abramson and Kuperman [1] studied the PD game with players placed on a small-world network, and observed that different topologies ranging from regular lattices to random graphs give rise to a variety of emergent behaviours. Masuda and Aihara [23] carried out a similar study using the PD game, with players mapped to nodes of different networks ranging from regular lattices to random networks. The experiments showed that the small-world topology is the optimal structure as far as speed of convergence is concerned. Hauert and Szabó [16] explored the spatial extension of the PD game on regular lattices, regular small-world networks and random regular graphs, in which all players have the same fixed number of neighbours, and noticed that the parameter range over which cooperators persist on random regular graphs is larger than for regular lattices.

In other work, Tomassini et al. [38] have investigated the Hawk-Dove game – a game with the same payoff ranking but a slightly different matrix structure to the SD game – on different network topologies including regular lattices, small-world networks and random graphs, and showed that cooperation is sometimes inhibited and sometimes enhanced in these network structures depending on the update rules as well as the cost-to-benefit ratio of cooperation and defection. More recently, several authors (e.g., [29, 30, 31]) have found that scale-free networks promote the evolution of cooperation, although this to a certain extent is dependent on

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the types of scale-free networks that are in place (see [22, 34]).

In this paper, we extend the study in this area by investigating population dynamics in *iterated* N-player games on small-world networks. Despite the massive body of literature covering the two-player game, there have only been a limited number of studies that have considered the N-player game using alternative network topologies (e.g., [21, 26, 32]). To the best of our knowledge, the previous work reported in the literature has not examined spatial games that also incorporate the concept of reciprocal altruism via repeated interactions.

In our model, agents are mapped to the nodes of the networks. In a given game instance, the focal agent participates in an iterative game with $N-1$ other agents drawn from its local neighbourhood. That is, the agents play their games over a fixed number of generations, with iterated interactions taking place within each of the generations. An agent's strategy, which is decoded to select an action, is represented by a binary string. We use a genetic algorithm (GA) with synchronous updating to evolve the strategies over time.

Comprehensive simulations based on the N-player Iterated Prisoner's Dilemma (N-IPD) and the N-player Iterated Snowdrift game (N-ISD) show that the agents on nodes with lattice-like neighbourhoods perform significantly better when the cost-to-benefit ratio is small, but agents on nodes modified during the re-wiring process (with a small re-wiring probability) of the construction of small-world networks tend to have surprisingly better performances in the N-IPD population when the cost-to-benefit ratio ranges from medium to high. The results suggest that small-world networks could be a potential cooperation promoter when the game[1] is more challenging.

The remainder of this paper is organised as follows: In the next section, the N-IPD and the N-ISD are briefly introduced. Following which, the fundamentals of small-world networks are described. In Section 3, the details of our model are presented. Sections 4 and 5 discuss the experimental settings, results and findings. Finally, we conclude the paper in Section 6 with a summary of the implications of this work.

2. BACKGROUND

2.1 Iterated N-player Games

Iterated games provide an abstract framework to investigate reciprocal altruism – the situation where players interact on a regular basis, and each player takes into consideration the impact of their current action on the future actions of other players [40]. In the Iterated PD (IPD) game, for example, many rounds of the simple PD are played, thus allowing the players to counteract an opponent's past behaviour. The IPD has been widely studied since the 1980s (see [2]) across various disciplines, including biology, sociology, economics, computer science and artificial intelligence.

A simple extension to the IPD, which we refer to as the N-IPD game (see [4, 7, 8, 9, 11, 43]), is to modify the game so that multiple players play a version of the game at the same time. In the N-IPD, N players repeatedly interact with one another (where $N > 2$), making decisions independently based on two actions – cooperate or defect – without knowing the choices of other players. A rule exists that rewards a social benefit b, which increases when more players are cooperating. There is, however, always a cost c for the cooperators. Here, $b > c$. Boyd and Richerson [4] formally defined the utility function, Π, for this scenario as follows:

$$\Pi = \begin{cases} \frac{b\times i}{N} - c & \text{for cooperators,} \\ \frac{b\times i}{N} & \text{for defectors.} \end{cases} \quad (1)$$

where i is the number of cooperators.

In contrast to the IPD, there has been less research investigating iterated interactions in the SD game. One example can be found in the work of Posch et al. [27] who studied the role of aspiration levels in generalised deterministic strategies in both the SD and PD games. Another example, based on the results of Dubois and Giraldeau [13], suggests that iterations generally promote less aggressive, and hence more cooperative behaviour in the SD game. In a recent study, Kümmerli et al. [20] experimentally examined human cooperation in the Iterated SD (ISD) game and compared it with human cooperation in the IPD. Their results showed that iterations in the ISD game consistently lead to higher levels of cooperation than in the IPD.

Similarly, the N-player versions of the SD game have only been studied in detail very recently (e.g., [5, 6, 10, 17, 21, 35, 45]). Among these studies, only Chiong and Kirley [10] have investigated the N-ISD. In their work, they formulated a generalised utility function for the N-ISD as follows:

$$\Pi = \begin{cases} b \times i - c \times (N-1)/i & \text{for cooperators,} \\ b \times i & \text{for defectors.} \end{cases} \quad (2)$$

As can be seen from Equation 2, the main difference between the N-IPD and the N-ISD is that the cost of cooperation is shared among the cooperators for the latter but this is not the case for the former. In other words, the N-ISD represents an "easier" game than the N-IPD. That is, we would expect higher levels of cooperation to be maintained within the population for a given parameter set as compared to the N-IPD game.

2.2 Small-World Networks

2.2.1 *Definitions*

Formally, a small-world network can be modelled as a graph $G(V, E)$ where V is a finite set of *nodes* (vertices) and E a finite set of *edges* (links), such that each edge is associated with a pair of nodes i and j. G can be represented by simply giving the $V \times V$ adjacency (or connection) matrix whose entry a_{ij} is 1 if there is an edge joining node i to node j and is 0 if otherwise. Typically, two measures are used to characterise the structural properties of the network: a global property–the average path length (L), and a local property–the clustering coefficient (C) [41].

L measures the average separation between two nodes in the graph. The distance d_{ij} between two nodes, labelled i and j respectively, is defined as the number of edges along the shortest path connecting them. Short connecting paths suggest that contagious behaviour can spread more easily across the network.

[1]Throughout this paper, we will be addressing the game as "more challenging", "less challenging", "more difficult" or "easier" in terms of promoting/maximising cooperative behaviour in the population.

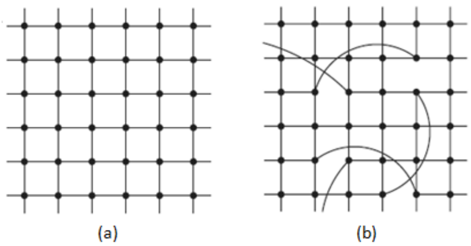

Figure 1: (a) A regular lattice; (b) Some of the links have been re-wired resulting in a small-world network.

C is the probability that two nearest neighbours of a node are also nearest neighbours of each other. C_i of node i is then defined as the ratio between the number E_i of edges that actually exist among these k_i nodes and the total possible number $k_i(k_i - 1)/2$. The clustering coefficient C of the whole network is the average of C_i over all i. Clearly, $C < 1$ and $C = 1$ if and only if the network is globally coupled, which means that every node in the network connects to every other node. Generally speaking, high clustering implies that interaction in the network resembles interaction in a closed group.

Small-world networks are often described as a transition from regular to random networks. In these networks, each link is re-wired with some probability ρ. The effect of re-wiring is the substitution of some short-range connections with long-range connections (see Figure 1). The regular lattice ($\rho = 0$) and the random graph ($\rho = 1$) represent the two extreme cases. Regular networks are highly clustered with relatively high shortest average path lengths. On the other hand, random networks are rather homogeneous, that is, most of the nodes have approximately the same number of links. Random networks have relatively short average path lengths and tend to have low clustering. For intermediate ρ values, small-world networks have a large overlap of neighbourhoods (clustering), and yet only relative short paths connecting any two individuals in the network.

2.2.2 *Evolutionary games on small-world networks*

Since the work of Watts and Stogatz [41], many authors have investigated various aspects/effects of the small-world settings using two-player games with pair-wise interactions (e.g., [1, 14, 15, 16, 18, 23, 33, 37, 38, 39, 42, 44, 46]). Some of the latest studies can be found in [12, 19, 28]. On the contrary, minimal attention has been paid to N-player games on small-world networks. In a recent study, O' Riordan et al. [26] showed that cooperation in N-player games can emerge in a small-world network if the so-called community structure is preserved. Szabó and Fáth [36] provided a comprehensive review of evolutionary games on complex networks for interested readers.

3. THE MODEL

In our model, we consider a population of 30×30 players where interacting individuals (agents) are placed on the nodes (or vertices) of a graph. Every agent is initialised with a random strategy, represented by a binary string (see details below). Each agent – the focal agent – participates in an iterative game with $N - 1$ other agents drawn from its local neighbourhood. For example, in a regular lattice for values of $N < 9$, the non-focal agents are drawn from the Moore neighbourhood. It is important to note that for a given value of N, the agents which form the focal agent's group do not change over the course of the game. The utility (fitness) of each agent is determined by summing its payoffs in the game against the group members. At the end of each generation, all agents are presented with an opportunity to update their strategies according to the payoffs received.

In the following subsections, we describe the individual components of our model and the game in detail.

3.1 Payoffs

The payoff values used in this study are defined below:

$$\Pi_n = \begin{cases} b \times i - c \times \lambda(N-1) & \text{for cooperators,} \\ b \times i & \text{for defectors.} \end{cases} \quad (3)$$

where $n \in [1...N]$.

Based on this utility function, we have, on one hand, the N-IPD game with payoffs equivalent to Equation 1 as defined by Boyd and Richerson [4] when $\lambda = 1$. On the other hand, we have the N-ISD equivalent to Equation 2 when $\lambda = \frac{1}{i}$. By doing so, we are able to study both games as a natural transition from one to another rather than comparing two "disparate" games.

3.2 Strategy Representation

There are various ways to represent the agent's game-playing strategies. The most straightforward method is to use the representation scheme proposed by Axelrod in [3] for the IPD. As pointed out by Yao and Darwen [43], however, Axelrod's representation scheme does not scale well with the increase in the number of players for N-player games. Besides that, it provides redundant information by telling which of the other players cooperated or defected, whereas the only information needed is how many cooperators or defectors there are. As such, we have decided to adopt the representation developed by Yao and Darwen.

Under this representation, a history of l rounds for an agent can be represented as the combination of the following bit strings:

- l bits to represent the agent's l previous actions ('1' = defection and '0' = cooperation)
- $l \times log_2 N$ bits to represent the number of cooperators in the previous l rounds among the agent's social group (N is the group size)

Based on preliminary empirical analysis, we have limited the number of previous actions in memory to 3 (i.e. $l = 3$), as this value can be used to generate a very large set of possible strategies. In the case of $N = 4$, for example, the history for an agent would be $3 + 3 \times log_2 4 = 9$ bits long according to this representation scheme.

Figure 2 illustrates a possible history an agent could have. The initial three bits are the agent's previous three actions. From the figure we can see that the agent defected in the last two rounds and cooperated the round before that. The two-bit sets after the first three bits represent the number of cooperators in the last three rounds from the agent's social group. This agent's history indicates that there were 3,

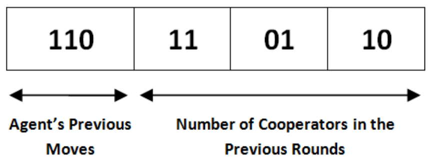

Figure 2: History of an agent.

1 and 2 cooperators in the agent's group in the last three rounds.

An agent's strategy provides a move m, $m \in [1, 0]$, in response to every possible history. Therefore, when $N = 4$ the strategy should be at least $2^9 = 512$ bits in length. Using the example from Figure 2, the history 110 11 01 10 would trigger agent a_n, $n \in [1...N]$, to make a move corresponding to the bit listed in the 438^{th} position of strategy s_n (438 is the decimal number for the binary 110110110).

It should be noted that the larger the group size, the more bits are required for the representation. Due to the fact that there is no memory of previous rounds in the beginning, we have added additional three bits to each strategy to compensate for the lack of complete history in the first three rounds. This means that the actions in the first three rounds of each generation are hard-coded into the strategy. Thereafter, the moves are made based on the history of the agent and its group members.

3.3 Strategy Update

For strategy update, we use a GA to evolve the pool of agents' strategies. Each agent plays the game repeatedly for T iterations at each generation. Every agent a_n uses a unique strategy s_n to decide the action to play at iteration t, where $t \in [1...T]$. At the end of T iterations, they may change their behaviour by comparing their utility to that of neighbouring agents using two GA-based operators crossover and mutation.

For crossover, a random number is generated to determine whether it should take place. Two-point crossover with rank-based selection is used. Note that the crossover operation will happen only when the crossover rate is satisfied and the current strategy is ranked below the elite group (in this study, strategies that rank among the top 50% are considered to be in the elite group). Otherwise, nothing comes about. This elite preserving mechanism ensures that good strategies are being carried forward to the next generation. As with crossover, a random number is generated to determine whether a strategy will be mutated. For mutation, a random position in the strategy's bit representation is selected and the bit at that position is flipped.

3.4 Network Topologies

Three different network architectures provide the game scaffolding to be investigated in this study: regular lattice, small-world network, and random graph. The regular lattice is two-dimensional with periodic boundary conditions. The neighbourhood structure in place is dependent on the value of N. As for the small-world network, we use a version similar to the one introduced by Watts and Strogatz [41]. That is, from our two-dimensional regular lattice substrate we re-wire each link with probability ρ. Here, we allow neither self or repeated links nor disconnected graphs. By doing so, we ensure that the individuals in the population are highly clustered and have relatively short path length. When $\rho = 0$, it is essentially a regular lattice. As $\rho = 1$, we have a random regular graph (note: we have kept the value of N constant each time, hence it is not just a random graph but a random regular graph).

4. EXPERIMENTS AND RESULTS

Extensive computational simulations have been carried out to investigate the population dynamics of the games played. All experiments were performed on a network consisting of $30 \times 30 = 900$ agents, randomly initialised with 50% of cooperators and 50% of defectors at the start of each game. Every agent played iteratively against one another within its social groups for 500 generations, with $T = 25$ rounds of learning process constituting each generation. At the end of each generation, all agents had an opportunity to update their strategies via the crossover and mutation operations of the GA. The rates of crossover and mutation were set to 0.7 and 0.05 respectively. Payoffs of agents were calculated based on Equation 3, using a range of b and c values normalised to a function of cost-to-benefit ratios. All our simulations were repeated for 100 independent runs with appropriate statistical tests. The small-world networks were generated by systematically varying the value of ρ from 0 to 1, starting from a two-dimensional regular lattice base. Synchronous updating of the model was used.

4.1 Levels of Cooperation with Varying ρ

The main goal of this study was to analyse the influence of small-world topologies on the level of cooperation in the population. As such, we first compare equilibrium proportions of cooperators as a function of the cost-to benefit ratio $r \in [0...1]$ in both the N-IPD and N-ISD populations with varying ρ values.

Figures 3 and 4 show the levels of cooperation achieved for the N-IPD and the N-ISD respectively, with $N = 5$, when the values of ρ were varied. In the N-IPD population, we see that the extent of cooperation decreases as the probability of re-wiring increases for small r. When $r > 0.5$, however, we observe an increase in the proportions of cooperators for a wide range of ρ compared to the two extreme cases of $\rho = 0$ and $\rho = 1$. Moving our attention to the N-ISD, we see a much higher level of cooperation throughout. This is largely expected since the N-ISD represents a less challenging game than the N-IPD. As r and ρ increase, we notice a consistent drop in the numbers of cooperators in the N-ISD population.

To verify whether the results obtained were statistically significant, we have performed pair-wise t-tests to compare $\rho = 0$ (i.e., the regular lattice as the baseline) with varying ρ values at specific r intervals based on the 100 individual runs. We report results based on significance levels with α value = 0.05. In the following tables, the scenarios being compared are represented by a symbol in each cell. Three different symbols are used: "=" indicates that there is no statistical significance between the two settings compared, "+" means that the setting in the column has yielded a significantly higher level of cooperation than the baseline case of $\rho = 0$ with confidence, and "−" is used if otherwise.

As can be seen from Table 1, the regular lattice holds a significant advantage over re-wired small-world topologies for promoting cooperative behaviour in the N-IPD popula-

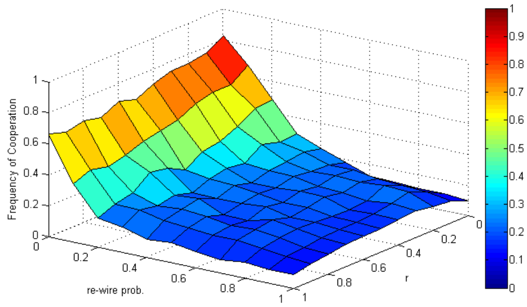

Figure 3: The frequency of cooperation as a function of $r = c/b$ across different values of ρ for the N-IPD population, where $N = 5$. All data points are averages over 100 realisations.

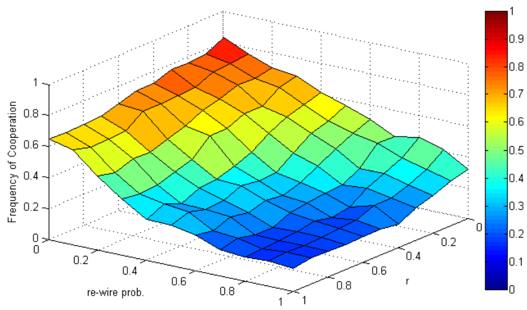

Figure 4: The frequency of cooperation as a function of $r = c/b$ across different values of ρ for the N-ISD population, where $N = 5$. All data points are averages over 100 realisations.

tion when r is small. However, this advantage diminishes as r increases. An interesting observation is that when small re-wiring probabilities are used in the network, the extent of cooperation can actually be enhanced significantly when r is beyond 0.5. In other words, minor re-wiring in the population may be beneficial when the game becomes more challenging.

For the N-ISD, we see from Table 2 that the re-wiring process in general has a detrimental effect on the evolution of cooperation, except for the situation when the re-wiring probability is very small. This indicates that a strong local neighbourhood is favourable for inducing cooperative behaviour.

To probe further on this issue, we have conducted a series of experiments with a specific case where all links of an agent (instead of each link) were re-wired at the same time according to a single ρ value. This means even with a small re-wiring probability, it is possible that an agent could have all its links re-wired resulting in random long-range interactions and thus low clustering in the population. The rationale of this set of experiments is to demonstrate the importance of strong local neighbourhoods in the emergence and maintenance of cooperative behaviour.

Figures 5 and 6 show the levels of cooperation in both the N-IPD and N-ISD populations, with $N = 5$, when all

Table 1: Statistical tests on the N-IPD with $N = 5$

r	$\rho = 0.1$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$
0.0	−	−	−	−	−
0.1	−	−	−	−	−
0.2	=	−	−	−	−
0.3	=	−	−	−	−
0.4	=	=	=	=	−
0.5	=	=	=	=	=
0.6	+	=	=	=	=
0.7	+	=	=	=	=
0.8	+	=	=	=	=
0.9	+	=	=	=	=
1.0	+	=	=	=	=

Table 2: Statistical tests on the N-ISD with $N = 5$

r	$\rho = 0.1$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$
0.0	−	−	−	−	−
0.1	=	−	−	−	−
0.2	=	−	−	−	−
0.3	=	−	−	−	−
0.4	=	−	−	−	−
0.5	=	−	−	−	−
0.6	=	−	−	−	−
0.7	=	−	−	−	−
0.8	=	−	−	−	−
0.9	=	−	−	−	−
1.0	=	−	−	−	−

the links of an agent would be re-wired simultaneously in accordance to the same ρ. As expected, the proportions of cooperators become very low when this is the case. The uncertainties surrounding who to associate with (or play against) are highly destructive to the local dynamics of the population.

4.2 Levels of Cooperation with Varying N

It is well-known in evolutionary game theory that as the size of interacting groups increases, the evolution of cooperation becomes more challenging. To complete the picture, we performed simulation experiments with varying group sizes ranging from $N = 3$ to $N = 9$. The results across the spectrum of ρ values (see Figures 3 and 4) indicate that the most interesting behaviour has been observed with small ρ values. As such, in this series of experiments we have fixed $\rho = 0.1$.

Figures 7 and 8 show the equilibrium proportions of agents playing cooperatively in the small-world N-IPD and N-ISD populations with different group sizes. A close examination of Figure 7 reveals that agents in the N-IPD population can maintain high levels of cooperation only for small r. As r increases, the proportion of cooperators drops regardless of the group sizes. When r is high, the number of cooperators becomes very low. In Figure 8, we again observe the decreasing of cooperators in the N-ISD population as r increases but this time at a much slower rate. Even for high r, around 20% to 30% of cooperators persisted. These results are consistent with our previous findings.

5. DISCUSSION

In the spatial evolutionary games domain, the population structure can, in some instances, help to promote and maintain high levels of cooperation. While the shift from simple

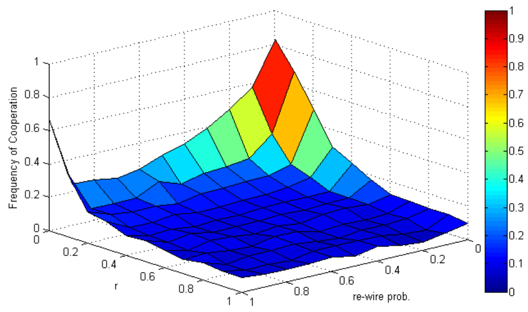

Figure 5: The frequency of cooperation as a function of $r = c/b$ across different values of ρ (where all links of an agent would be re-wired at the same time according to ρ) for the N-IPD population, where $N = 5$. All data points are averages over 100 realisations.

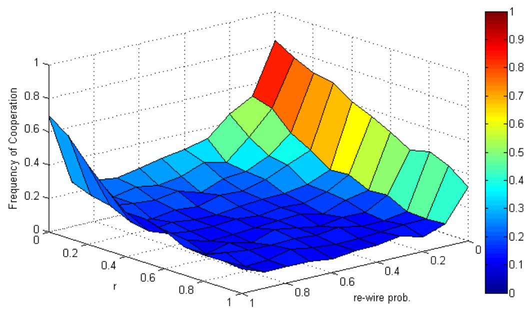

Figure 6: The frequency of cooperation as a function of $r = c/b$ across different values of ρ (where all links of an agent would be re-wired at the same time according to ρ) for the N-ISD population, where $N = 5$. All data points are averages over 100 realisations.

lattice-based models to games on networks can be considered a step towards more realistic conditions, especially in the context of human societies, its real benefits remain unclear (see the discussion in [1]; see also [18] and the lengthy review in [36]). To date, the majority of the studies on evolutionary games and complex networks have been done on two-player pair-wise games. In this paper, our investigation has focused on the impact of alternative network topologies in iterated games with N-player interactions. The specific aim has been to examine the extent of cooperation in the agent population playing the N-IPD and the N-ISD. Here, the role of (a) the cost-to-benefit ratio r, (b) the probability of re-wiring ρ, and (c) the group size N, are of interest.

To meet the objectives of this study, systematic numerical simulations were used to determine how these parameters influenced the population dynamics. In the first set of experiments, we have examined population trajectory for the scenario: r vs. ρ with the group size fixed at 5. If ρ is disregarded, one may expect that when the value of r is sufficiently high defectors would dominate the agent population. On the other hand, when the value of r is small it would be worthwhile to cooperate. For intermediate values of r, the population would settle into a mixed state where we see cooperators striving to form clusters in order to resist the invasion by defectors. Our results in Figures 3 and 4 are consistent with this.

The value of ρ has a direct impact on the level of clustering in the network. The higher its value, there is a corresponding high probability of low clustering among agents. It is reasonable to expect that the level of cooperation should drop when the probability of re-wiring is increased. Our results provide further supporting evidence for this expectation over a wide range of r, except in the N-IPD population when the cost-to-benefit ratio is beyond 0.5. For an N-IPD game with $r > 0.5$, it is extremely challenging to evolve cooperative behaviour. The frequency of cooperators at this level is typically low, as can be seen in Figure 3. In this case, small re-wiring probabilities could have led to isolated clusters of cooperators surrounded by a sea of defectors to rejoin cooperative clusters else where, hence the unexpected increase in their numbers compared to when there is no re-wiring at all.

In the next set of experiments, we have investigated a special case when all links of an agent were re-wired at the same time based on a value of ρ. One may expect this to result in a low-clustering agent population and thus lower levels of cooperation. Our results show exactly this, once again providing supporting evidence for the positive effects that local neighbourhoods have on promoting and supporting cooperation [26].

In the final set of experiments, we have focused on examining the effects of changing the group size for given topologies and payoff values: N vs. r with ρ fixed at 0.1. Naturally, the level of cooperation is expected to drop off with larger group sizes and cost-to-benefit ratios. Our results in Figures 7 and 8 reinforce this well-known fact, where the trend can be observed across different group sizes.

In a nutshell, it is clear that different values of the average path length and clustering coefficient of alternative networks can lead to the establishment and persistence of different types of populations. The simulation results here suggest that a small re-wiring probability could be advantageous when the evolution of cooperation is difficult, while highly modified nodes tend to switch actions too often and thus causing instabilities in the agent population.

6. CONCLUSION

In this paper, we have investigated the evolution of strategies in iterated N-player games on different network architectures. Two games have been considered, namely the N-IPD and the N-ISD. Detailed computational simulations have shown that the regular lattice with strong local neighbourhoods is highly favourable for evolving cooperative behaviour in both games. Within a certain parameter range when the game becomes extremely challenging (especially the N-IPD), however, the small-world network with small re-wiring probabilities could spring a few surprises by providing a way out for the cooperators to persist in the population. The results suggest that the small-world may favour cooperation in more difficult games.

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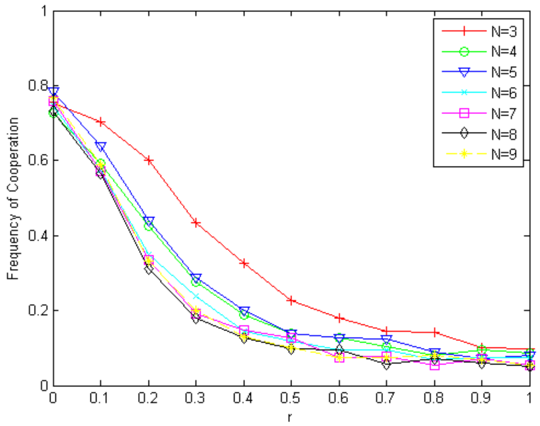

Figure 7: The frequency of cooperation as a function of $r = c/b$ for the N-IPD game with small-world topologies, where N = 3, 4, 5, 6, 7, 8 and 9, and $\rho = 0.1$. Each data point is an average over 100 realisations.

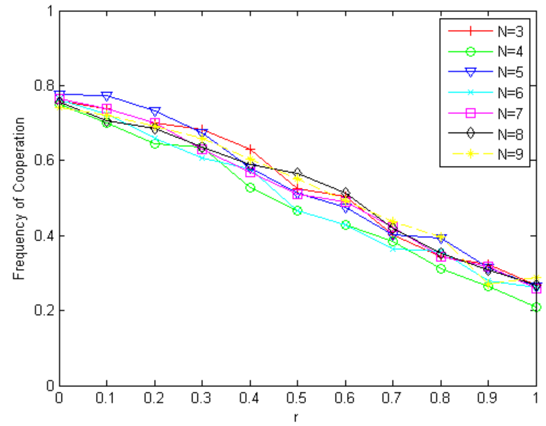

Figure 8: The frequency of cooperation as a function of $r = c/b$ for the N-ISD game with small-world topologies, where N = 3, 4, 5, 6, 7, 8 and 9, and $\rho = 0.1$. Each data point is an average over 100 realisations.

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