Brief Announcement:
On Simple Back-Off in Unreliable Radio Networks

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Abstract

In this paper, we study local broadcast in the dual graph model, which describes communication in a radio network with both reliable and unreliable links. Existing work proved that efficient solutions to these problems are impossible in the dual graph model under standard assumptions. In real networks, however, simple back-off strategies tend to perform well for solving these basic communication tasks. We address this apparent paradox by introducing a new set of constraints to the dual graph model that better generalize the slow/fast fading behavior common in real networks. We prove that in the context of these new constraints, simple back-off strategies now provide efficient solutions to local broadcast in the dual graph model. These results provide theoretical foundations for the practical observation that simple back-off algorithms tend to work well even amid the complicated link dynamics of real radio networks.

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1 Introduction

Existing papers proved that it is impossible to solve standard broadcast problems efficiently in the dual graph model without the addition of strong extra assumptions [3]. In real radio networks, however, which suffer from the type of link dynamics abstracted by the dual graph model, simple back-off strategies tend to perform quite well.

These dueling realities seem to imply a dispiriting gap between theory and practice: basic communication tasks that are easily solved in real networks are impossible when studied in abstract models of these networks.

What explains this paradox? This paper tackles this fundamental question.

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As detailed below, we focus our attention on the *adversary* entity that decides which unreliable links to include in the network topology in each round of an execution in the dual graph model. We introduce a new type of adversary with constraints that better generalize the dynamic behavior of real radio links. We then reexamine simple back-off strategies originally introduced in the standard radio network model (which has only reliable links) [1], and prove that for reasonable parameters, these simple strategies *now do* guarantee efficient communication in the dual graph model combined with our new, more realistic adversary.

Dual Graph Model. This model describes the network topology with two graphs $G = (V, E)$ and $G' = (V, E')$, where $E \subseteq E'$. The $n = |V|$ vertices in V correspond to the wireless devices in the network, which we call *nodes* in the following. The edge in E describe reliable links (which maintain a consistently high quality), while the edges in $E' \setminus E$ describe unreliable links (which have quality that can vary over time). For a given dual graph, we use Δ to describe the maximum degree in G', and D to describe the diameter of G.

Time proceeds in synchronous rounds that we label $1, 2, 3...$ For each round $r \geq 1$, the network topology is described by $G_r = (V, E_r)$, where E_r contains all edges in E plus a subset of the edges in $E' \setminus E$. The subset of edges from $E' \setminus E$ are selected by an *adversary*. The graph G_r can be interpreted as describing the high quality links during round r. That is, if $\{u, v\} \in E_r$, this mean the link between u and v is strong enough that u could deliver a message to v, or garble another message being sent to v at the same time.

With the topology G_r established for the round, behavior proceeds as in the standard radio network model. That is, each node $u \in V$ can decide to transmit or receive. If u receives and exactly one neighbor v of u in E_r transmits, then u receives v's message. If u receives and two or more neighbors in E_r transmit, u receives nothing as the messages are lost due to collision. If u receives and no neighbor transmits, u also receives nothing. We assume u does not have collision detection, meaning it cannot distinguish between these last two cases.

The Fading Adversary. We parameterize the adversary with a *stability factor* that we represent with an integer $\tau \geq 1$. In each round, the adversary must draw the subset of edges (if any) from $E' \setminus E$ to include in the topology from a distribution defined over these edges. The adversary selects which distributions it uses and it can change this distribution at most once every τ rounds.

Problem. In this paper, we study the *local* broadcast problem. The problem assumes a set $B \subseteq V$ of nodes are provided with a message. Let $R \subseteq V$ be the set of nodes in V that neighbor at least one node in B in E. The problem is solved once every node in R has received at least one message from a node in B.

Uniform Algorithms. In this paper focus on *uniform algorithms*, which require nodes to make their probabilistic transmission decisions according to a predetermined sequence of broadcast probabilities that we express as a repeating cycle, $(p_1, p_2, ..., p_k)$ of k probabilities in synchrony.

Our results In standard Dual Graph Model, where the adversary can arbitrarily change the state of all the unreliable edges in every step, the time of local broadcast can be lower bounded by $\Omega(n/\log n)$ [3]. On the other hand, in reliable networks, *decay* algorithm solves local broadcast in time $O(\log \Delta \log(n/\varepsilon))$ [1] with probability at least $1 - \varepsilon$ and this time is optimal [2]. Thus there is an exponential gap between the reliable model and worst-case unreliable model. Our fading adversary can be (for large τ) seen as an average-case unreliable model. For smaller τ the model becomes similar to the standard dual graph model (in particular, for $\tau = 1$ model with fading adversary is stronger than the dual graph model).

We show that for $\tau \geq \log \Delta$, the optimal time of local broadcast for reliable networks can be achieved in the model with fading adversary. Secondly we prove a tradeoff between the optimal time of local broadcast in the model with fading adversary and the value of τ. We show that factor $\Delta^{1/\tau}$ is necessary in the time complexity of any uniform local broadcast algorithm. This shows how quickly the optimal time increases between both extremes depending on τ.

2 Results

Our algorithm is a simple back-off style strategy inspired by the *decay* routine from [1]. We use notation $\bar{\tau} = \min\{\lceil \log_{2e} \Delta/2 \rceil, \tau\}$.

```
Procedure: Uniform($k, p_1, p_2, \ldots, p_k$)
for $i = 1, 2, \ldots, k$ do
    if has message then
        with prob. $p_i$ Transmit else Listen
    else
     Listen // without a message listen
```

```
Algorithm: FRLB($r$)
for $i \leftarrow 1$ to $\bar{\tau}$ do $p_i \leftarrow \frac{\log_{2e} \Delta}{\Delta^{i/\bar{\tau}} \bar{\tau}}$
repeat $r$ times
    Uniform ($\bar{\tau}, p_1, p_2, \ldots, p_{\bar{\tau}}$)
```

▶ **Theorem 1.** *For any error bound* $\epsilon > 0$, *algorithm* **FRLB**$(2\lceil \ln(n/\epsilon) \rceil \cdot \lceil 4\Delta^{1/\bar{\tau}}\bar{\tau}/\log \Delta \rceil)$ *solves local broadcast in* $O\left(\frac{\Delta^{1/\bar{\tau}} . \bar{\tau}^2}{\log_{2e} \Delta} \cdot \log(n/\epsilon)\right)$ *rounds, with probability at least* $1 - \epsilon$.

Notice, for $\tau \geq \log \Delta$ this bound simplifies to $O(\log \Delta \log(n/\epsilon))$, matching the performance of *decay* algorithm [1] and the lower bound in the standard reliable radio network model [2]. This performance, however, degrades toward the polynomial lower bounds from the existing dual graph literature [3] as τ reduces from $\log \Delta$ toward a minimum value of 1. We show this degradation to be near optimal by proving that *any* local broadcast algorithm that uses a fixed sequence of broadcast probabilities requires $\Omega(\Delta^{1/\tau}\tau/\log \Delta)$ rounds to solve the problem with probability 1/2 for a given τ. For $\tau \in O(\log \Delta/\log\log \Delta)$, we refine this bound further to $\Omega(\Delta^{1/\tau}\tau^2/\log \Delta)$, matching our upper bound within constant factors.

▶ **Theorem 2.** *Fix a maximum degree* $\Delta \geq 10$, *stability factor* τ *and uniform local broadcast algorithm* $\mathcal{A}$. *Assume that* $\mathcal{A}$ *solves local broadcast in expected time* $f(\Delta, \tau)$ *in all graphs with maximum degree* Δ *and fading adversary with stability* τ. *It follows that:*

1. *if* $\tau < \ln(\Delta - 1)/(12 \log\log(\Delta - 1))$ *then* $f(\Delta, \tau) \in \Omega(\Delta^{1/\tau}\tau^2/\log \Delta)$,
2. *if* $\tau < \ln(\Delta - 1)/16$ *then* $f(\Delta, \tau) \in \Omega(\Delta^{1/\tau}\tau/\log \Delta)$.

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