Proceedings of the American Control Conference
Albuquerque, New Mexico June 1997
0-7803-3832-4/97/$10.00 © 1997 AACC

Tools for Safety-Throughput Analysis of Automated Highway Systems[1]

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Abstract

The development of two tools to study the performance of automated highways is presented. The first is a spacing tool that produces the minimum spacing necessary for two vehicles not to collide, as a function of their initial velocities and their deceleration capabilities. The second tool investigates the multiple collisions that may occur in a string of vehicles if the requirements made by the first tool are violated. We show how the tools can be used to obtain estimates of the safety and throughput that can be expected if various automated highway concepts are implemented.

1. Introduction

Highway congestion is an ever increasing problem on most of the highways in and around urban areas. One of the most promising solutions suggested so far for this problem is to automate the traffic on highways so as to increase highway safety and capacity. Various alternatives have been suggested for organization of automated traffic. The effectiveness of an Automated Highway System (AHS) design should be judged by performance metrics in the areas of safety, throughput, fuel economy, environmental impact, vehicle and infrastructure cost, social fairness, etc. Design and analysis tools are needed to evaluate candidate AHS designs and to synthesize an optimum AHS. In this paper, we describe two such tools for safety and throughput evaluation of an AHS design. The tools can be used to perform trade off studies between safety and throughput and to compare different design alternatives. The design attributes that can be analyzed using the tools include:

1) Inter-vehicle coordination and communication architecture: the tools can model autonomous vehicle operation as well as inter-vehicle cooperation at different levels.

2) Vehicle separation policies: the tools consider individual vehicle operation as well as platooning. In a platooning architecture, vehicles travel in closely spaced platoons (intra-platoon separation of the order of 1 m) of up to 20 vehicles. Different platoons are isolated from each other by a larger distance of the order of 60m. Such close packing of vehicles can achieve dramatic increase in highway capacity. It is conjectured that even in case of a failure in a platoon collisions will happen at low relative velocity (because of the tight spacing) resulting in minor damage.

Here we concentrate on the safety and capacity aspects of the design. We describe the theory behind the tools and the tool development process. We also present results of how these tools can be used to investigate trade-offs between throughput and safety and compare between design alternatives.

2. Tool Development

2.1. Vehicle Model

Consider three vehicles (labeled A, B and C) moving along a single lane highway (Figure 1). Assume that vehicles A and B have lengths L_A and L_B and let x_A and x_B denote their positions with respect to a fixed reference on the road. Assume that vehicle B is leading while vehicle C comes last, i.e. $x_B > x_A > x_C > 0$. Following [1], assume that the longitudinal dynamics of vehicle A can be modeled by a third order system and that the acceleration of vehicle B can not be measured by vehicle A. If we let $D = x_B - x_A - L_B$ the system A-B can be described by the state vector $x = [\dot{x}_A \;\; \ddot{x}_A \;\; D \;\; \dot{D}]$. After feedback linearization the evolution of the state is described by:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \ddot{x}_B \\ x(0) &= x^0 \end{aligned}$$

where u is the jerk applied by the controller of vehicle A. The dynamics are constrained by the engine, tire and road conditions. More specifically it is required that vehicles do not go backwards and their accelerations and jerks are

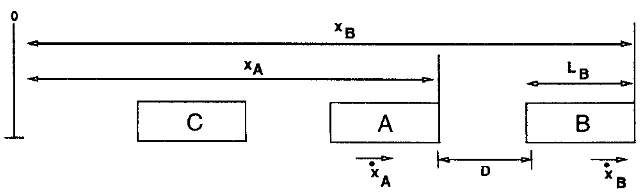

Figure 1: Vehicle Following

[1]Research supported by PATH program, Institute of Transportation Studies, University of California, Berkeley, under MOU-238.

bounded. The values of these bounds will be parameters for the safety tools.

If vehicle B happens to collide with the vehicle in front of it, or vehicle C happens to hit vehicle A from behind the state of the system will undergo an almost instantaneous jump. If the change in velocities because of the impact are δv_B and δv_C and the collision times are T_B and T_C respectively, then:

$$\begin{aligned} x_4(T_B^+) &= x_4(T_B^-) + \delta v_B \\ x_1(T_C^+) &= x_1(T_C^-) - \delta v_C \\ x_4(T_C^+) &= x_4(T_C^-) + \delta v_C \end{aligned}$$

T_B^- and T_B^+ denote the time right before and right after the collision of vehicle B (similarly for C). In the coordinate system considered here, $\delta v_B \leq 0$ and $\delta v_C \leq 0$. Assume that vehicle B can hit the vehicle ahead of it with relative velocity at most v_B and vehicle C can hit vehicle A from behind with relative velocity at most v_C. If one collision of each kind takes place in the time interval of interest then the effect of vehicles B and C on vehicle A can be summarized as a disturbance:

$$\begin{aligned} d \in \{d \quad | \quad & \ddot{x}_B(t) \in [a_{min}^B, a_{max}^B], \\ & 0 \leq T_B, \delta v_B \in [\max\{v_B, x_4(T_B) + x_1(T_B)\}, 0] \\ & 0 \leq T_C, \delta v_C \in [v_C, 0]\} \end{aligned} \tag{1}$$

The complicated bound on δv_B is dictated by the fact that $x_4(T_B^+) + x_1(T_B^+) \geq v_{min}^B = 0$. This formalism can also be used to model the situation where no collisions take place by setting $v_B = v_C = 0$.

2.2. Spacing Tool

For the purposes of safety we would like vehicles to avoid collisions whenever possible. For vehicle A this requirement can be encoded by a cost function:

$$J(x^0, u, d) = -\inf_{t \geq 0} x_3(t) \tag{2}$$

If for a given initial condition x^0 and a given choice of u and d, $J(x^0, u, d) \leq 0$ m, vehicle A will never collide with vehicle B (it may still be hit from behind by vehicle C though). We would like vehicle A to remain safe in this sense whatever vehicles C and B decide to do. We therefore seek the worst possible action of vehicles B and C and the best possible action of vehicle A. In other words we are seeking a *saddle solution* (u^*, d^*) to the two player, zero sum game between u and d with cost function J. The saddle solution satisfies:

$$J(x^0, u^*, d) \leq J(x^0, u^*, d^*) \leq J(x^0, u, d^*)$$

For our example consider the candidate saddle strategy:

$$u^*(t) = \begin{cases} j_{min} & \text{if } t \leq T_1 \\ 0 & \text{if } t > T_1 \end{cases} \tag{3}$$

$$d^*(t) = \{\ddot{x}_B^*, (T_B^*, \delta v_B^*), (T_C^*, \delta v_C^*)\} \tag{4}$$

where:

$$\begin{aligned} \ddot{x}_B^*(t) &= \begin{cases} a_{min}^B & \text{if } t \leq T_2 \\ 0 & \text{if } t > T_2 \end{cases} \\ T_B^* &= 0 \\ \delta v_B^* &= \max\{v_B, x_4^0 + x_1^0\} \\ T_C^* &= 0 \\ \delta v_C^* &= v_C \end{aligned} \tag{5}$$

T_1 is the time when the acceleration of vehicle A reaches a_{min}^A under j_{min} and T_2 the time when vehicle B stops under a_{min}^B. The candidate saddle solution simply dictates that both vehicles decelerate as hard as possible and both collisions take place at time $t = 0$ with the maximum allowable change in velocity. In [1] it was shown that:

Lemma 1 *(u^*, d^*) is globally a saddle solution for cost $J(x^0, u, d)$.*

Using the saddle solution, we calculate optimal cost $J^*(x^0)$ for a given initial condition x^0. In particular we can distinguish safe situations ($J^*(x^0) < 0$) from unsafe ones ($J^*(x^0) > 0$) and determine the boundary between them ($J^*(x^0) = 0$). Note that for all safe initial conditions vehicle A is guaranteed not to collide with vehicle B as long as it starts decelerating if the state reaches the boundary (i.e. whenever $J^*(x(t)) = 0$). For unsafe initial conditions, on the other hand, there exist actions of vehicles B and C where a collision between vehicles A and B is unavoidable, whatever vehicle A does.

The above principle was used in the development of a computational spacing tool. The user of the tool is asked to provide the minimum deceleration rates, a_{min}^A and a_{min}^B, the minimum jerk of vehicle A, j_{min}, and the maximum allowable relative velocities at collisions of vehicle B with the vehicle ahead of it (v_B) and of vehicle C with vehicle A (v_C). The tool then calculates the minimum spacing, x_3^0, required to guarantee no collisions between vehicles A and B, for a given initial condition x_1^0, x_2^0 and x_4^0.

In addition to the third order vehicle model described above, the calculation was also generalized to a second order integrator plus first order lag model. The lag represents lumped sensing and actuation delay. For both these models, the calculations and tool code were also extended to address the case where the acceleration information of vehicle B is communicated to vehicle A.

2.3. Collision Tool

The spacing tool allows us to obtain spacing requirements to guarantee that two vehicles will not collide. To analyze all possible situations we would also like to know what happens if these requirements are violated[1]. Consider two vehicles A and B of Figure 1. To keep the calculations tractable we simplify the model for vehicle A, by assuming that its acceleration can be directly controlled.

Assume that the acceleration of A and B follow the trajectories of Figure 2 until the vehicles stop or collide[2].

[1]This may be the case in a platoon of vehicles undergoing emergency deceleration due to a fault for example.

[2]For more elaborate deceleration profiles refer to [2].

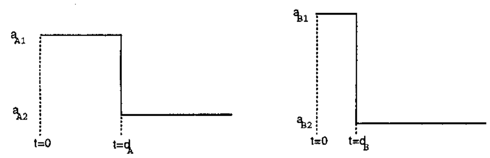

Figure 2: Assumed acceleration trajectories

As $\dot{x}_B \geq 0$ and we are interested in investigating the cases where the vehicles collide, we restrict our attention to the interval of time when $\dot{x}_A > 0$. It is easy to show [3] that under these conditions the spacing and relative velocity between vehicles A and B is given by:

$$x_3(t) = \frac{a}{2}t^2 + bt + c + x_3^0 \tag{6}$$

$$x_4(t) = at + b \tag{7}$$

The values of a, b, and c depend on the parameters of the problem (a_{ij} and d_i). If a collision takes place, equation (6) allows us to determine the time T at which it happens while equation (7) gives us the relative velocity at impact.

To analyze cases where multiple collisions occur we also want to determine the vehicle velocities after the collision. We model collision elasticity by a coefficient of restitution γ. If the collisions are centered, γ relates the longitudinal velocities before and after collision as follows:

$$\gamma = \frac{\dot{x}_B(T^+) - \dot{x}_A(T^+)}{\dot{x}_A(T^-) - \dot{x}_B(T^-)} \tag{8}$$

$\gamma = 1$ models perfectly elastic collisions whereas $\gamma = 0$ models plastic collisions. Conservation of linear momentum provides another relationship between collision speeds. Let m_A and m_B be the masses of the two vehicles and define $M = m_B/m_A$. Then:

$$\dot{x}_A(T^-) + M\dot{x}_B(T^-) = \dot{x}_A(T^+) + M\dot{x}_B(T^+) \tag{9}$$

The coefficient of restitution depends on the design of the vehicle body and bumpers. Given a particular value of γ, the above set of equations can be solved for $\dot{x}_A(T^+)$ and $\dot{x}_B(T^+)$ (which in turn gives $x_2(T^+)$ and $x_4(T^+)$) and the process can be repeated.

The choice of trajectories for the accelerations of the two vehicles is motivated by physical considerations (such as actuator and communication delays) relating to the operation of the platoons. In addition the class of trajectories characterized in this way can be shown to contain trajectories which are in some sense optimal (see [4]).

Based on these calculations a computational collision tool was developed. The tool accepts as input the acceleration levels a_{ij}, $j = 1, 2$, the delays d_i, the masses m_i and the coefficients of restitution γ_i for each vehicle in the platoon (i denotes the i^{th} follower and $i = 0$ denotes the leader). Then, for a given set of initial velocities $x_{i_1}^0$ and initial spacings $x_{i_3}^0$ the tool calculates all collisions that will occur and the corresponding relative velocities. To accomplish this the tool solves equation (6) for all vehicles, determines the smallest collision time, T, and the vehicles involved, j and $j-1$, calculates the state of all vehicles right before the collision, $x_i(T^-)$ for all i, solves equations (9) and (8) to obtain $x_j(T^+)$ and $x_{j-1}(T^+)$, and repeats the process. The iteration terminates when no more collisions are possible.

3. Throughput/Safety Tradeoff Analysis

We use a probability distribution on the minimum acceleration of vehicles, constructed in [5] by averaging over manufacturers specifications for a number of current production models (Figure 3). The values of throughput and

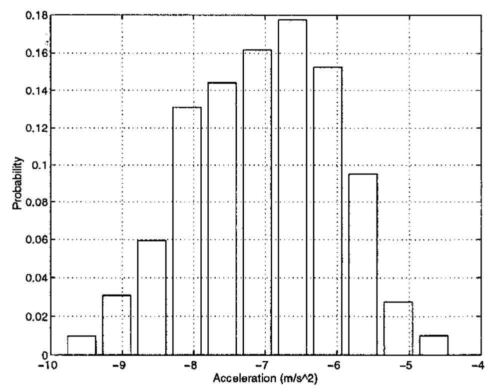

Figure 3: Minimum acceleration probability distribution for passenger vehicles

safety obtained here are essentially the expected values over this probability distribution.

3.1. Normal Operation

The throughput estimates we produce here are the so called "pipeline" estimates. Assume that the traffic on the highway is arranged in platoons, all of them consisting of N vehicles and traveling at a velocity x_1^0. If the average length of a vehicle is L the average intra-platoon spacing is F and the average inter-platoon spacing is x_3^0, then the throughput, Q, of the highway in vehicles per lane per unit time is given by:

$$Q = \frac{N x_1^0}{x_3^0 + NL + (N-1)F} \tag{10}$$

We fix $L = 5m$ and $F = 1m$ and use the spacing tool to obtain inter-platoon spacings so as to satisfy *no collisions in the absence of faults*. For platoons of length $N > 1$ the deceleration exerted by the leader should be limited by the deceleration capabilities of the followers, to guarantee the *string stability* of the platoon. Motivated by the work of [6] we use the following formula to calculate the minimum allowable deceleration, a_{allow}, for a leader:

$$a_{allow} = \max\left\{a_{min}^0, \frac{a_{min}^1}{1.05}, \frac{a_{min}^2}{1.1}, \frac{a_{min}^3}{1.15}, \frac{a_{min}^i}{1.2} \text{ for } i \geq 4\right\} \tag{11}$$

where a_{min}^i is the deceleration capability of the i^{th} follower.

A C-program was written to obtain the expected value of the throughput for a range of velocities x_1^0. The throughput depends on the information structure assumed by the AHS. Each platoon may or may not have access to its own deceleration capability (a_{min}^A) or the deceleration capability of the platoon ahead (a_{min}^B). We consider three different cases. Every platoon may:
1) have access to both pieces of information,
2) have access to neither,
3) have access to its own deceleration capability but not that of the platoon ahead
In order to guarantee safety for alternatives 2 and 3 a platoon has to assume the worst case for the missing pieces of information. Figures 4, 5 represent results for information structures 2 and 3. From the figures it seems that if

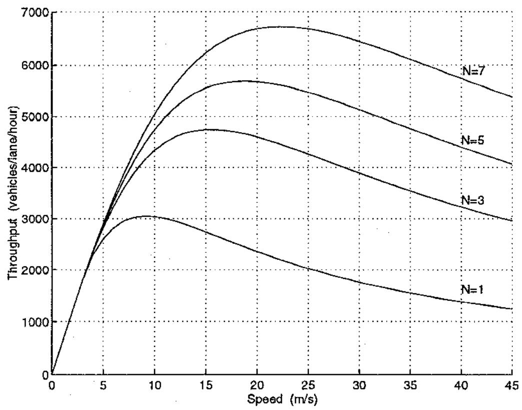

Figure 4: Throughput when no deceleration capability is available, platoon size N

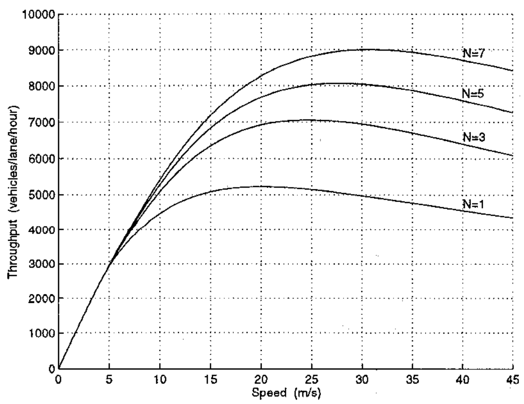

Figure 5: Throughput when own deceleration capability is available, platoon size N

partial information is available much higher throughputs are possible using large platoons. Results for information structure 1 are similar (refer to [3]).

It should be noted that this pipeline calculation will produce the maximum throughput that can be obtained under the assumed conditions. In practice lane changes, formation and dissipation of platoons and entry and exit from the AHS will introduce disturbances that may produce significantly lower throughputs than the ones predicted here. This problem is going to be more pronounced for larger platoon sizes[3]. Moreover, extra safety margins may need to be added to the spacings produced by the spacing tool to account for sensing, processing and actuation delays[4].

3.2. Degraded Operation

For the normal mode analysis it was required that no collisions occur in the absence of faults. There are still, however, situations where collisions can arise. One such situation is emergency braking by the leader of a platoon. This may occur because of a "brakes on" failure of the leader or because of emergency maneuvers initiated in response to other failures or obstacles. For single vehicle platoons such braking will not result in any collisions under the spacing rules of Section 3.1. Multiple collisions may occur for larger platoons, however, because of the possible mismatch in deceleration capabilities between the followers.

To investigate this effect the collision tool is used. As an emergency braking system for platoons has not been yet designed, a simple control scheme is considered: follower i ($i = 0$ for the leader) keeps a constant acceleration $x_{i_2}^0$ until a time d_i when it switches to its minimum deceleration a_{min}^i. The time d_i may depend on the processing and actuation delays as well as the communication architecture within a platoon. For hop-by-hop communication a delay of d is added for each follower (i.e. $d_i = i \cdot d$). For broadcast communication, on the other hand, the delay is d for all the followers. Here we restrict our attention to hop-by-hop communication with $d = 0.05s$.

Assume that all the followers in a platoon are initially at steady state, have equal mass and that all collisions are elastic ($\gamma = 1$). The collisions can be classified according to their relative velocity at impact, which is a measure of their severity. Figure 6 shows collision statistics collected using the collision tool. The results indicate that even though most of the collisions occur at small relative velocities, there is a significant probability of high relative velocity collisions. This probability increases with the size of the platoon. Other statistics [3] reveal that the average number of collisions per vehicle due to hard braking by the platoon leader increases roughly linearly with the platoon size.

The collision calculation imposes a restriction on the throughput that can be achieved safely. To ensure that the collisions that occur because of emergency braking affect only the platoon that executes the maneuver, additional inter-platoon spacing is needed. The amount of extra spacing can be calculated by obtaining the relative velocities of the most severe collision experienced by the leader and the last vehicle in the platoon (using the collision tool) and using them as v_C and v_B respectively in the

[3]For the impact of such "transient" effects on throughput the reader is referred to [7, 8].

[4]A feature already exists in the current version of the tool that allows for the addition of such safety margins.

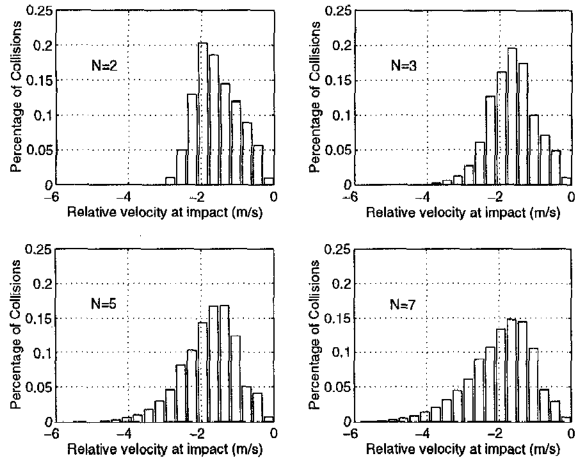

Figure 6: Classification of collisions by relative velocity

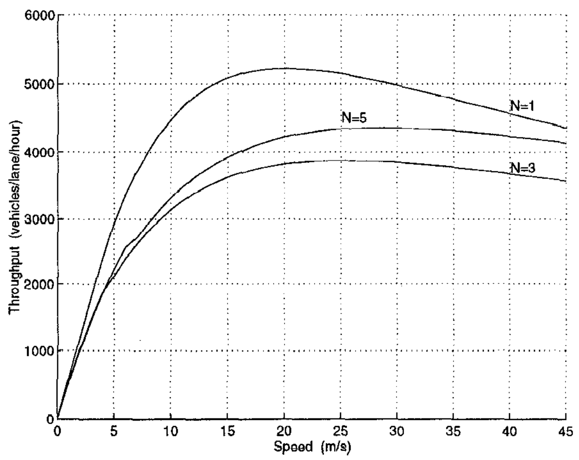

Figure 7: Reduction in throughput because of collisions

spacing tool. The throughput values obtained assuming that a platoon leader has information only about its own platoons deceleration capabilities are summarized in Figure 7. As expected the throughput is severely reduced in all cases except single vehicles; in fact, because the severity of collision increases with the platoon size, relatively small throughputs are obtained with platoon sizes up to $N = 5$.

It should be noted that the throughput calculated in this way will probably be unnecessarily restrictive. The spacings produced by the above calculation will guarantee that collisions do not propagate from one platoon to the next in any situation (even if both platoons have to undergo emergency braking and the distribution of deceleration capabilities is the worst possible). Moreover, the collision statistics were collected for a rather ad-hoc emergency braking scheme. Significant improvement, both in the number and the severity of collisions, may be possible with a better design. On the other hand, the throughput calculations assume that the first and last vehicles experience exactly one collision per incident when in fact they may experience multiple collisions.

4. Concluding Remarks

We presented the design of a spacing and a collision tool that can be used to perform safety and throughput analysis for automated highway systems. We showed how these tools can be used to compare the performance of various proposed AHS architectures. We envision these tools as a first step towards abstracting the macroscopic, emergent behavior of the automated highway system as a whole from the the microscopic, local interactions between the vehicles. The tools are also a first step towards the probabilistic verification of the hybrid control architectures for automated highways, both under normal conditions and in the presence of faults.

We are currently working on understanding the sensitivity of the above results to changes in the system parameters. For this purpose the tools were extended to allow for the introduction of delays, safety margins, measurement uncertainties, different deceleration capability distributions, different coefficients of restitution, etc. Preliminary results indicate that the normal mode throughput is relatively insensitive to most of these parameters. However, the collision statistics (and consequently the throughput in the presence of collisions) seem to be very sensitive to changes in the intra-platoon spacing, deceleration distribution and coefficient of restitution. We hope that these results will be useful as guidelines in determining design and policy specifications for the AHS.

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