

Towards Bilingual Lexicon Discovery From Visually Grounded Speech Audio

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Abstract

In this paper, we present a method for the discovery of word-like units and their approximate translations from visually grounded speech across multiple languages. We first train a neural network model to map images and their spoken audio captions in both English and Hindi to a shared, multimodal embedding space. Next, we use this model to segment and cluster regions of the spoken captions which approximately correspond to words. Finally, we exploit between-cluster similarities in the embedding space to associate English pseudo-word clusters with Hindi pseudo-word clusters, and show that many of these cluster pairings capture semantic translations between English and Hindi words. We present quantitative cross-lingual clustering results, as well as qualitative results in the form of a bilingual picture dictionary.

Index Terms: Low-resource speech processing, multimodal speech processing, cross-lingual speech processing

1. Introduction

With the many languages in the world, people often need to cross language barriers to communicate. Researchers have made huge strides towards making automated machine translation more and more reliable. Current speech-to-speech translation systems rely on a cascade of models that perform automatic speech recognition, machine translation, and text-to-speech synthesis [1]. These models each require large quantities of manually-annotated training data, but transcribing parallel corpora of speech audio in both the source and target languages can be prohibitively costly. The text bottleneck also makes it difficult to automatically translate to and from languages without a written orthography. In this work, we attempt to align semantically equivalent words across languages directly at the speech signal level, without the need for text transcripts. We build on the work presented by [2, 3, 4, 5], which showed that multimodal neural network models could be trained to directly associate speech waveforms with images, resulting in the ability to recognize spoken words in continuous speech signals without the need for conventional ASR. This type of model was generalized to handle speech inputs from two different languages in [5], and was shown to be capable of cross-lingual matching of semantically similar captions. We take the work in [5] a step further by explicitly locating and clustering the words learned by the model in both languages. We then semantically link the discovered English clusters with the Hindi clusters (Figure 1), as well as image regions that they are most similar to. This forms the basis of a picture dictionary, which shows segments of speech from both languages coupled with semantically relevant regions of images.

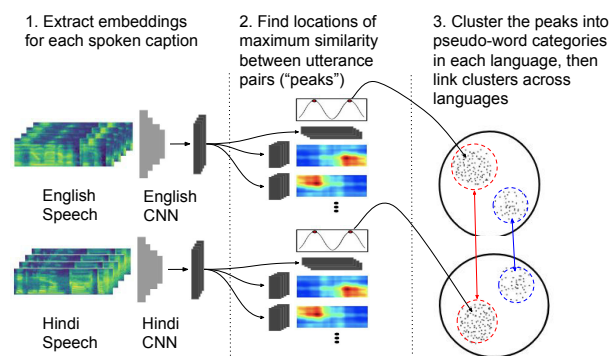


Figure 1: *Speech CNN embeddings (left) are compared to each other using dot product to find and extract regions of high similarity from utterances containing similar concepts (middle). The embeddings at these regions are clustered separately for each language and linked using cross-lingual cluster centroid similarity (right).*

2. Prior work

The high cost of current supervised methods of speech recognition and machine translation has led to several proposed methods for increasingly unsupervised speech recognition. Segmental Dynamic Time Warping proposed by [6] and extended in [7, 8, 9] operates on raw audio to find patterns within speech utterances to automatically discover word categories. Other works such as [10, 11, 12] used Bayesian generative approaches to cluster acoustic segments. Deep learning has been used to learn robust feature representation for speech over varying speaker and background characteristics such as in [13, 14, 15, 16]. [2, 3, 4, 17, 18, 19, 20] explored unsupervised learning of speech features using visual context. Cross-lingual translation research has focused on text-to-text translation [21, 22] as well as speech-to-text from one language to another [23, 24, 25]. [5] recently showed that joint image and speech training performs well on cross lingual caption retrieval using English and Hindi, serving as a basis for speech to speech pseudo translation and [26] confirmed this result using an English-Japanese dataset. A similar line of work was presented in [27], which explored cross-lingual keyword spotting using a visual tagging system.

3. Dataset and visual speech model

3.1. Dataset

We use the same English-Hindi Places Audio dataset in [5]. This dataset is comprised of natural images sampled from the Places 205 image dataset [28], paired with spoken audio captions describing the content of the images. The English speech captions

were collected via Amazon Mechanical Turk, and the collection process is described in [2]. The collection process for the Hindi speech captions, also collected via Mechanical Turk, is described in [5]. The training dataset consists of 84,480 data triplets, and 1000 validation triplets, with each triplet consisting of an image, and English and Hindi spoken captions.

3.2. Model

The model consists of three neural networks - one pretrained VGG16 network [29] for learning image representation and two instances of DAVeNet (audio CNN in [3]) for the audio captions. In this paper, we used an ImageNet-pretrained VGG network finetuned on 400K places images as in [4] and all networks used 1024 final embedding dimension. Training images are augmented using random resized crops to 224x224 and then mean and variance normalized per channel using off-the-shelf Imagenet RGB statistics. The speech waveform is represented by a set of 40 log Mel filterbank energies per 25 ms frame of the speech caption at 10 ms shifts. We train the network for 90 epochs with the 6-way triplet loss $H \leftrightarrow E \leftrightarrow I \leftrightarrow H$ from [5]. We use SGD with batch size 128, and initial learning rate of 0.001 decayed by a factor of 10 every 30 epochs. The model achieved the following recall at 10 scores for image(I), English(E) and Hindi(H) retrieval tasks: $E \rightarrow I$ (0.571), $I \rightarrow E$ (0.545), $H \rightarrow I$ (0.404), $I \rightarrow H$ (0.393), $E \rightarrow H$ (0.192), $H \rightarrow E$ (0.211).

4. Bilingual word discovery

In [5], the authors showed that a visually-grounded model of speech trained to associate both English and Hindi spoken captions with semantically-related images was capable of performing semantic speech retrieval between captions in both languages. Preliminary experiments in that paper suggested that the output feature maps of the English and Hindi speech models could be used to approximately align the segments of both speech signals which referred to the same image region. Our goal in this paper is to automatically extract and cluster these segments into word-like units, and establish pairwise linkages between English and Hindi clusters that capture similar semantics. This section describes the steps we took to discover the word clusters and establish linkages.

4.1. Selecting regions of interest

Given an input log Mel spectrogram spanning 40 filterbanks across T temporal frames, the output of the DAVeNet audio model will be a feature map with d channels spanning $\frac{T}{8}$ frames. During training, mean pooling is used to compress this feature map into a single d dimensional vector, but when using an already-trained model for word discovery we do not apply this pooling so as to preserve temporal information for the purpose of word localization. Although the output of the DAVeNet model captures semantics, it does so by producing a dense embedded representation; it does not explicitly segment or tokenize the speech signal. In order to identify regions of interest with a high likelihood of containing a meaningful word, we use an approach inspired by the ‘‘interval piling’’ step of the Segmental Dynamic Time Warping (S-DTW) pattern discovery algorithm [6], which identifies regions of an utterance which exhibit high similarity with regions in many other utterances. While the S-DTW algorithm computes similarities in the acoustic observation space, our method instead compares pairs of utterances in the d -dimensional multimodal semantic embedding

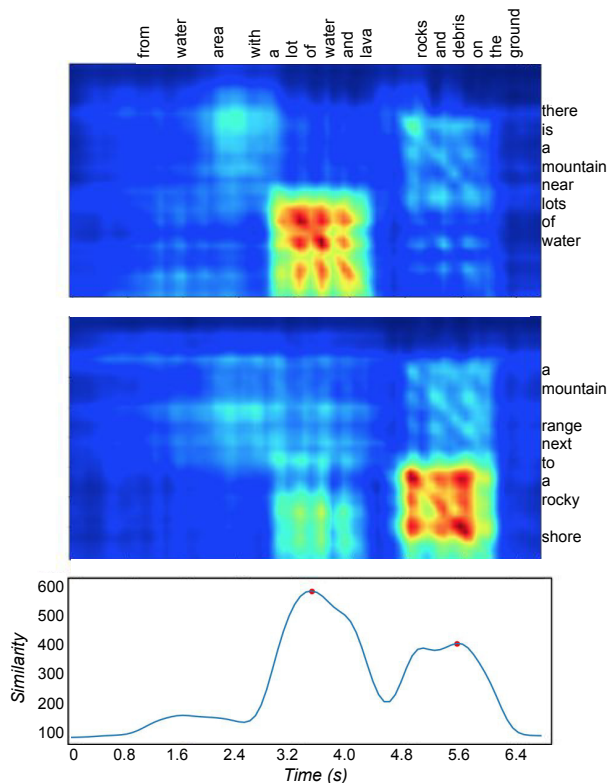


Figure 2: Embeddings at the speech network output for the top utterance are compared with those from its nearest neighbors. Taking the frame-level maximum over all neighbors results in the similarity profile.

space learned by the DAVeNet model. For some reference utterance containing a given word, other utterances containing the same underlying word can help inform the location of the word. We rely on a set of nearest neighbors of the reference utterance in order to perform word localization. This is done by obtaining the 1024-dimensional mean-pooled DAVeNet output for each utterance in the training set, and finding its K nearest neighbors according to the dot product similarity. For our experiments, we used $K = 100$.

After selecting the nearest neighbors for each utterance, we compute a set of similarity maps $\{M^1, \dots, M^K\}$ between the reference and each of its neighbors. To encode temporal localization information in each M^n , we extract the outputs of the last convolutional layer of the DAVeNet model before the global average pooling layer. Then, we compute $M_{i,j}^n = R_i \cdot N_j^n$ where R_i represents the i^{th} frame of the DAVeNet output for the reference utterance, and N_j^n represents the j^{th} frame of the DAVeNet output of its n^{th} nearest neighbor. We then compute a similarity profile p where $p_i = \max_{n,j} M_{i,j}^n$. This gives a profile with peaks at locations of the reference utterance that exhibit high similarity to some region of at least one of its neighbors, indicating the presence of a shared word. This can be seen in Figure 2. To facilitate peak picking, we apply a Gaussian smoothing filter to p with $\sigma = 1$. We use comparison among neighboring values to find local maxima and select peaks with prominence of at least $\max(200, 0.15 \times r)$ where r is the range of values in p .

4.2. Clustering regions of interest

The embedding vectors in the locations of the selected peaks represent regions that may contain words whose semantics have been learned by the cross-modal model. All the utterances in the dataset are processed to get these peak locations and their associated embeddings. Next, we cluster the peaks using a Dirichlet Process Gaussian Mixture Model (DPGMM) [30]. Both English and Hindi peak embeddings were normalized together to have zero mean and unit variance and then projected from \mathbb{R}^{1024} to \mathbb{R}^{300} using PCA. The clustering algorithm used kmeans initialization with 200 components, diagonal covariance matrices for each component, mean precision prior of 30, weight concentration prior (γ) of 1000 and allowed to run for a maximum of 1500 iterations.

4.3. Linking English and Hindi clusters

To control for repeated clusters, we put the cluster centroids into matrices E and H for English and Hindi respectively, each with rows representing all centroids in the language. We constructed an undirected graph whose edges connect rows of E with rows of H and edge weights represented by $E_i H_j^T$. Edges with weight less than a threshold $\tau = 300$ were set to zero. Finally, we ran the Louvain graph community clustering algorithm [31] to group the DPGMM English and Hindi cluster centroids into meta-clusters that capture bilingual concepts.

4.4. Deriving cluster labels for evaluation

Given N peaks in a cluster, each peak has a receptive field of size f . Since the neural network is symmetric for convolution and pooling operations, we simply find the location in the speech caption directly below the peak as $p \times c/n$, where p and n are

the location of the peak and the number of frames respectively in the last convolution layer and c is the caption length in seconds. We then snip $f/2$ seconds on both sides of the the selected peak location within the caption. After performing this operation for each peak in the cluster, we select the ASR text transcripts of these portions to present in this paper. To get a single class label, we calculate the purity of each word selected for the cluster. Purity is the proportion of the selected f -second windows containing a given word. We also compute coverage, the fraction of the total number of instances of a word in the dataset captured by a cluster. To control for stop words that occur next to salient words, we weight the purity scores by the average duration of the word. We then rank the words by the weighted purity and select the top word as the cluster name. In our experiments, f was approximately 2.5 seconds.

5. Experiments

5.1. Bilingual word clustering

Details of the top 35 meta-clusters (rank ordered by similarity) out of 101 discovered meta-clusters in the training dataset are presented in Table 1. Each row of the table shows statistics for English and Hindi clusters grouped together in the same meta-cluster. All texts refer to the ASR transcription of the underlying speech, and Hindi texts are paired with their Google Translate API’s translation to English. The numbers are reported by merging all English clusters whose centroids exist in a meta-cluster and the same is done for Hindi clusters.

5.2. Creating a picture dictionary

To investigate the visual semantics of the bilingual meta-clusters, we find the image regions from the training images



Figure 3: Picture dictionary representing three-way agreement between English speech caption, Hindi speech caption and Image pixels. We present the text transcriptions of the clustered speech segments with their corresponding cluster purities.

Table 1: Bilingual word clusters. E_1 and E_2 correspond to the top two labels for combined English clusters within a meta-cluster and H_1 and H_2 are the Hindi equivalents. P_E and P_H are purity scores while C_E and C_H are coverage fractions using the top 1 label. S represents the similarity score between linked English-Hindi clusters. N_E and N_H are the number of peaks in English and Hindi respectively in the meta-cluster. Average purity and coverage of the top label across all English clusters are 0.53 and 0.45, while for Hindi they are 0.44 and 0.31. The number of clusters with purity greater than 0.5 is 59 for English and 34 for Hindi.

E_1	P_{E1}	E_2	P_{E2}	H_1	P_{H1}	H_2	P_{H2}	S	N_E	N_H	C_E	C_H
lighthouse	0.55	house	0.28	लाइट:light	0.36	हाउस:house	0.30	4898	485	578	0.73	0.30
bed	0.41	bedroom	0.29	बिस्तर:bed	0.37	बेड:bed	0.11	2470	1620	1535	0.85	0.73
guitar	0.79	playing	0.39	गिटार:guitars	0.55	बजाते:playing	0.09	1961	280	411	0.74	0.76
staircase	0.21	stairs	0.23	सीढ़ियाँ:stairs	0.25	चिड़िया:bird	0.14	1925	1115	1305	0.60	0.81
windmill(s)	0.55	turbine(s)	0.14	पवन:air	0.47	चक्की:mill	0.23	1882	569	627	0.69	0.74
kitchen	0.82	cabinets	0.02	रसोईघर:kitchen	0.36	रसोई:kitchen	0.26	1827	680	686	0.57	0.82
bridge	0.77	suspension	0.04	पुल:the bridge	0.27	ब्रिज:the bridge	0.23	1805	1681	895	0.64	0.21
boat(s)	0.64	sailboat	0.04	नाव:the boat	0.24	'जहाज':ship	0.04	1694	1317	1229	0.60	0.76
fish	0.64	aquarium	0.07	मछली:fish	0.49	मछलियाँ:fish	0.22	1617	639	467	0.63	0.57
snow	0.49	snowy	0.13	बर्फ:ice	0.47	बर्फिले:snowy	0.11	1586	3464	3221	0.70	0.55
closet(s)	0.63	cabinet(s)	0.01	अलमारी:cupboard	0.51	अलमारियाँ:shelves	0.11	1537	406	752	0.63	0.49
horse(s)	0.75	riding	0.08	घोड़े:the horse	0.66	सवार:the rider	0.05	1431	674	442	0.62	0.59
track	0.37	running	0.37	दौड़:the race	0.31	प्रतियोगिता:contest	0.13	1422	407	408	0.25	0.43
bus	0.55	inside	0.15	बस:bus	0.40	हवाई:airy	0.24	1319	851	930	0.61	0.31
station	0.52	subway	0.38	स्टेशन:station	0.48	रेलवे:railway	0.45	1296	471	262	0.28	0.28
church	0.64	cathedral	0.12	गिरजाघर:the cathedral	0.27	चर्च:the church	0.20	1213	746	687	0.42	0.74
bird(s)	0.69	flying	0.07	पक्षी:the bird	0.35	चिड़िया:bird	0.27	1171	261	378	0.42	0.54
blue	0.70	sky	0.06	नीले:blue	0.56	रंग:colour	0.65	1167	2242	809	0.29	0.26
shower	0.55	bathroom	0.24	नहाने:bathing	0.30	बाथरूम:bathroom	0.14	1137	583	454	0.55	0.66
man	0.76	standing	0.01	आदमी:man	0.41	व्यक्ति:person	0.11	1068	2803	3435	0.35	0.26
mountain(s)	0.74	range	0.03	पहाड़:the mountain	0.51	पहाड़ियाँ:the hills	0.20	1053	888	1469	0.24	0.21
wooden	0.49	wood	0.17	लकड़ी:the wood	0.86	एक:one	0.24	1017	1858	1007	0.42	0.36
table(s)	0.76	wooden	0.05	टेबल:table	0.24	मेज:the table	0.16	994	2418	2191	0.42	0.22
children	0.57	kids	0.08	बच्चे:children	0.60	बच्चों:the children	0.16	982	723	1406	0.52	0.50
people	0.89	standing	0.09	लोग:the people	0.69	कुछ:some	0.73	943	2683	916	0.31	0.27
forest	0.51	trees	0.10	जंगल:forest	0.48	है:is	0.30	868	821	1198	0.52	0.52
sitting	0.63	people	0.50	बैठे:sitting	0.64	हैं:are there	0.53	862	1509	1183	0.26	0.33
child	0.31	boy	0.27	बच्चा:child	0.34	एक:one	0.52	858	737	611	0.41	0.41
microphone	0.43	stage	0.14	मंच:forum	0.30	माइक:mike	0.27	841	445	327	0.62	0.30
clouds	0.52	sky	0.20	बादल:cloud	0.75	आसमान:sky	0.42	801	1002	928	0.47	0.42
water	0.78	body	0.45	पानी:water	0.51	नदी:river	0.23	779	1838	2828	0.22	0.27
trees	0.83	pine	0.04	पेड़:the trees	0.49	पेड़ों:the trees	0.26	770	1634	1396	0.24	0.13
grass	0.57	green	0.25	घास:grass	0.36	है:is	0.32	728	1273	2306	0.32	0.40
wall(s)	0.73	on	0.40	दीवार:wall	0.64	पर:on	0.47	616	1431	873	0.33	0.23
yellow	0.72	and	0.20	पीले:yellow	0.51	रंग:colour	0.52	586	1220	870	0.36	0.31

most similar to each meta-cluster. We use the mean pooled DAVenet output of the image branch to represent the images and find the top K images that have the highest average (dot product) similarity to all of the DPGMM cluster centroids captured by the meta-cluster. We then compute a binary mask S for each image by taking the inner product of each superpixel region of the DAVenet image model’s output feature map (before global average pooling) with the average of the meta-cluster centroids. We threshold S such that pixels with similarity greater than $.25 * \max(S)$ are set to 1 and 0 otherwise. Finally, we upsample S to the same resolution as the original image using bilinear interpolation, and apply the mask to the image. This results in an audio-visual picture dictionary, from which we show several examples in Figure 3.

6. Conclusion

We presented a method for discovering bilingual word clusters using a visually grounded model of speech audio. We presented numerical clustering results as well as an audio-visual picture dictionary, demonstrating that our method is capable of discovering clusters of word-like units in both English and Hindi that

exhibit a high degree of semantic agreement. In our future work, we plan to develop improved methods for learning a larger number of concepts, especially actions and verbs. Our current approach required us to utilize spoken captions for a common set of images for both languages, but we plan to investigate whether similar results can be achieved when different sets of images are used for each language’s captions. We would also like to extend our method from the bilingual case to the multi-lingual case. Future work should also investigate direct speech-to-speech translation using our discovered meta-clusters. Finally, we believe that the representations learned by our acoustic models could find use in traditional ASR systems, such as in low-resource or code switching scenarios.

7. Acknowledgements

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