Fundamental Frequency Modeling for Speech Synthesis
Based on a Statistical Learning Technique

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SUMMARY

This paper proposes a novel multi-layer approach to fundamental frequency modeling for concatenative speech synthesis based on a statistical learning technique called additive models. We define an additive $F_0$ contour model consisting of long-term, intonational phrase-level, component and short-term, accentual phrase-level, component, along with a least-squares error criterion that includes a regularization term. A backfitting algorithm, that is derived from this error criterion, estimates both components simultaneously by iteratively applying cubic spline smoothers. When this method is applied to a 7,000 utterance Japanese speech corpus, it achieves $F_0$ RMS errors of 28.9 and 29.8 Hz on the training and test data, respectively, with corresponding correlation coefficients of 0.806 and 0.777. The automatically determined intonational and accentual phrase components turn out to behave smoothly, systematically, and intuitively under a variety of prosodic conditions.

key words: speech synthesis, fundamental frequency, additive models, statistical learning

1. Introduction

In recent years, corpus-based concatenative methods for speech synthesis have received increasing attention within the research community as well as the speech technology industry, because of their ability to generate natural sounding speech output\cite{1}–\cite{3}. In general, for synthesized speech to be natural and intelligible, it is crucial to have a proper $F_0$ contour that is compatible with linguistic information such as lexical accent (or stress) and phrasing in the input text. In the corpus-based concatenative speech synthesis setting, target $F_0$ features (e.g., mean frequency, ending frequency, amount of movement) are generated for each synthesis unit. Distance metrics can then be used to compute a cost between the unit target values, and those available in a speech corpus. Overall cost is minimized during search to find the best matching sequence of synthesis units from the corpus.

In some systems, $F_0$ target is predicted by an independent rule-based front-end\cite{4}, while regression tree-based approaches are popularly used to predict $F_0$-related measures from a set of linguistic features\cite{3},\cite{5},\cite{6}. A regression tree approach is advantageous in that it is simple to implement yet very powerful. It has a few drawbacks, however. For example, the predicted values do not have a smooth contour, since it essentially represents a piecewise constant function of the input features. It also has a drawback that it cannot capture additive structure when the data does have such a structure.

In this work, we propose a simple yet novel multi-layer additive model approach to $F_0$ contour prediction, and a method to estimate the component functions through the minimization of a residual sum-of-squares error criterion that includes a regularization (or penalty) term. Additive Models\cite{7},\cite{8} are a class of nonlinear regression models, which can be regarded as a generalization of linear models (or multiple linear regression). It and its extension by link function, called Generalized Additive Models, are described in detail in the monograph\cite{8}, and have been applied to various statistical modeling practices such as weather forecast\cite{9} and public health research\cite{10}, among others.

In the next section we define a two-layer additive $F_0$ model, along with a penalized least-squares criterion from which we derive a backfitting algorithm as the minimizer of the criterion. We then describe experimental results applying the proposed method to a large corpus of Japanese speech in the following section.

2. Additive Model Approach

The basic formulation for the $F_0$ contour is similar to previous work that models $F_0$ in a superpositional way, e.g.,\cite{11} that models $F_0$ generation mechanism with second-order linear systems, and\cite{12} that uses multiple linear regression with indicator variables. In our two-layer additive modeling approach, the $F_0$ contour, $Y$, is regarded as the output of a statistical model that combines a long-range intonational phrase level component, $g$, and a shorter accentual-phrase level component, $h$:

\[
Y = \alpha + g(I, U) + h(A, V) + \epsilon
\]

\[
= \alpha + g_I(U) + h_A(V) + \epsilon,
\]

where $\alpha$ is a constant, $I$ is a discrete input variable that represents a type of intonational phrase, and indexes the relevant function $g_I$. $U$ is a continuous variable representing a time point relative to the starting point of the phrase of type $I$. Similarly, discrete variable $A$ designates a type of accentual phrase, and $V$ represents a time point relative to the starting point of the accentual phrase of type $A$. The random error term, $\epsilon$, is zero mean. Figure 1 shows how the three terms form the entire $F_0$ contour function.

2.1 Penalized Least Square and Backfitting algorithm

A unique characteristics of the additive model approach, as
(a) alpha (constant)

\[
\begin{align*}
\text{(I) } \alpha & = \frac{1}{n} \sum_{i=1}^{N} y_n, \\
\hat{\alpha} & = 0, \\
\hat{h}_\alpha & = 0, \quad \text{for all } i \in r(I), \quad a \in r(A).
\end{align*}
\]

(b) \( g_I(U) \) (I='10m'

\[
\begin{align*}
\text{(2g) Partition the set of training data } [(u_n, a_n, v_n, y_n)] & \quad n = 1, \ldots, N, \quad \text{into } |r(I)| \text{ subsets } [(u_l, a_l, v_l, y_l)] \quad l = 1, \ldots, N_l \quad (i \in r(l)), \\
\text{so that each training point has the same value of } & \quad i \text{ if in the same subset. Note that } \sum_{i \in r(I)} N_l = N.
\end{align*}
\]

\[
\begin{align*}
\hat{g}_l & \leftarrow S_a \left[ y_v - \hat{\alpha} - \hat{h}_{\lambda_A}(v_l) \right]_{l=0}^{N_l}.
\end{align*}
\]

(2h) Repartition the training data \([(u_l, a_l, v_l, y_l)] \quad n = 1, \ldots, N, \quad \text{into } |r(A)| \text{ subsets } [(u_{a_l}, a_{a_l}, v_{a_l}, y_{a_l})] \quad l = 1, \ldots, N_{a_l} \quad (a \in r(A)), \quad \text{so that each training point has the same value of } a \text{ if in the same subset. As before, } \sum_{a \in r(A)} N_{a_l} = N.
\]

\[
\begin{align*}
\hat{h}_a & \leftarrow S_a \left[ y_{a_l} - \hat{\alpha} - \hat{h}_{\lambda_A}(v_{a_l}) \right]_{l=0}^{N_{a_l}}.
\end{align*}
\]

(c) \( h_A(V) \) (A='4m4n'

\[
\begin{align*}
\end{align*}
\]

\[
\begin{align*}
\text{Fig. 1} \quad \text{A schematic diagram of the additive } F_0 \text{ model } f(I, U, A, V) = \alpha + g_I(U) + h_A(V). \text{ A constant } \\
\text{\( \alpha \) and component functions } g \text{ and } h \text{ are summed up to form the } F_0 \text{ contour } f. \text{ In this example, a 6-mora } \\
\text{accentual phrase with the accent nucleus at the third mora ("6m3n", for short), e.g. "monowa'kareni" is followed by an } \text{4m4n" } \text{accentural phrase, e.g. "owatta" to form a 10-mora intonational phrase, e.g.} \\
\text{"monowa'kareni owatta".}
\end{align*}
\]

\[
\begin{align*}
\text{Fig. 2} \quad \text{A backfitting algorithm for the additive } F_0 \text{ model.}
\end{align*}
\]

compared to previous work, is that we do not have to assume any parameterized functional form. Instead, we assume a smoothness defined in terms of curvature, and use an estimation scheme derived from a least-squares error criterion with a regularization term, or roughness penalty [7], [8]. We define the penalized residual sum-of-squares (PRSS) error of the model with regard to the overall training data in the following form:

\[
\begin{align*}
\text{PRSS} & = \text{RSS} + \lambda_g J(g) + \lambda_h J(h) \\
& = \sum_{n=1}^{N} \left[ y_n - \alpha - g_a(a_n) - h_{\lambda}(v_n) \right]^{2} \\
& + \lambda_g \sum_{x \in r(I)} \int g''_a(w)^2 dw + \lambda_h \sum_{a \in r(A)} \int h''_{\lambda}(x)^2 dx, \quad (2)
\end{align*}
\]

where \((i_n, u_n, a_n, v_n, y_n) \quad (n = 1, \ldots, N)\) are a set of training data corresponding to the variables \((I, U, A, V, Y)\), and \(\lambda_g, \lambda_h\) are fixed smoothing parameters. The number \(N\) represents the total number of the available training data points. \(r(I)\) and \(r(A)\) represents the set of possible values (or range) for \(I\) and \(A\), respectively. The number of elements in a set will be denoted by vertical bars, e.g., \(|r(I)| \) meaning the number of different values for \(I\). The first term in (2) measures the closeness to the data, while the second and third terms penalize the curvatures in the functions, and smoothing parameters \(\lambda_g\) and \(\lambda_h\) establish a tradeoff between them. Large values of \(\lambda\)'s yield smoother curves, while smaller values result in more fluctuation.

It can be shown that the minimizer of (2) is an additive cubic spline model, where \(g_a\)'s and \(h_{\lambda}\)'s are natural cubic splines in the predictor variables \(U\) and \(V\), with knots, or break points, at each of the unique values of \((i_n, u_n)\) and \((a_n, v_n)\). To make the solution unique, we assume that \(\sum_{n=1}^{N} g_i(a_n, v_n) = \sum_{N} h_i(a_n, v_n) = 0\), therefore \(\alpha\) will be the overall mean of \(y_n\) \((n = 1, \ldots, N)\). We can find the solution for (2) with a backfitting algorithm [7], a simple iterative procedure depicted in Fig. 2.

In the algorithm, we apply a natural cubic-spline...
smoother matrix, e.g., $S_i$, to the vector of partial residual, 
$(y_{i,l} - \hat{a} - \hat{h}_a(v_{i,l}))_{l=1}^{N_i}$, which is regarded as a function of $u_{i,l}$, to obtain a new estimate \( \hat{g}_l \). Smoothing of partial residual is done for $g_i$’s and $h_a$’s in turn, using the current estimate of the other component function. The iteration is continued until the estimates $\hat{g}_l$’s and $\hat{h}_a$’s stabilize. In the rest of the section, we briefly describe how this backfitting algorithm, with natural cubic spline smoothers, is derived as a iterative procedure equivalent to a blockwise Gauss-Seidel algorithm for solving a system of linear equations emerging from the minimization of the penalized least-square criterion (2).

### 2.2 Natural Cubic Spline and Its Property

In solving the penalized least square problem, we use a unique property of a family of functions called natural cubic splines. A natural cubic spline is a piece-wise cubic function defined in terms of the points called knots $x_1, x_2, \ldots, x_K$ on some interval $[a, b]$. It is known that among all twice differentiable functions $g(x)$ defined on $[a, b]$ that passes through the points $(x_1, z_1), (x_2, z_2), \ldots, (x_K, z_K)$ where $a < x_1 < \ldots < x_K < b$, the one minimizing a “roughness” measure defined as an integrated squared second derivative,

$$
\int_a^b g''(x)^2 \, dx,
$$

is a natural cubic spline with knots at $x_1, x_2, \ldots, x_K$. We take advantage of this property in solving the penalized least square problem. For convenience of the reader, we review the definition of natural cubic spline and the proof of this property in Appendix A and Appendix B, respectively.

### 2.3 Derivation of the Backfitting Algorithm

Now, by paying attention to different intonational phrase types, we can partition the entire set of training data into $|r(I)|$ subsets in such a way that the points in a subset have the same value of $i_n$, i.e., they belong to the same type of intonational phrase. We can then express the entire training data, $D$, as a union of $|r(I)|$ nonoverlapping subsets:

$$
D = \{ (i_n, u_{i_n}, a_{i_n}, v_{i_n}, y_{i_n}) \mid n = 1, \ldots, N \}
= \bigcup_{i \in r(I)} \{ (i, u_{i,l}, a_{i,l}, v_{i,l}, y_{i,l}) \mid l = 1, \ldots, N_i \},
$$

(3)

where $\sum_{i \in r(I)} N_i = N$. Similarly, we can partition the training data based on the identity of the value of $a_n$:

$$
D = \{ (i_n, u_{i_n}, a_{i_n}, v_{i_n}, y_{i_n}) \mid n = 1, \ldots, N \}
= \bigcup_{a \in r(A)} \{ (i_{a,l}, u_{a,l}, a, v_{a,l}, y_{a,l}) \mid l = 1, \ldots, N_a \},
$$

(4)

where $\sum_{a \in r(A)} N_a = N$. By using partitionings in (3) and (4), the expression for the penalized residual sum of squares (2) can now be rewritten in two ways:

$$
PRSS = RSS + \lambda_g J(g) + \lambda_h J(h)

= \sum_{i \in r(I)} \sum_{l=1}^{N_i} \left[ y_{i,l} - \alpha - g(u_{i,l}) - h_a(v_{i,l}) \right]^2
+ \lambda_g \sum_{i \in r(I)} \int g''(w)^2 dw + \lambda_h \sum_{a \in r(A)} \int h''_a(x)^2 dx
$$

(5)

$$
= \sum_{a \in r(A)} \sum_{l=1}^{N_a} \left[ y_{a,l} - \alpha - g(u_{a,l}) - h_a(v_{a,l}) \right]^2
+ \lambda_g \sum_{a \in r(A)} \int g''(w)^2 dw + \lambda_h \sum_{a \in r(A)} \int h''_a(x)^2 dx.
$$

(6)

Now, let us consider searching for the optimal function $\hat{g}_l$ that minimizes the penalized least square criterion (5) for a certain value $i_0$ of $i$, when other $g_i$’s ($i \neq i_0$) and $h_a$’s are fixed to certain functions.

Assume we are given any twice continuously differentiable function $g$ that is not a natural cubic spline which passes through the points $(u_{i_0,l}, g(u_{i_0,l}))$ ($l = 1, \ldots, N_{i_0}$). Let $\bar{g}$ be the natural cubic spline that interpolates the same points $(u_{i_0,l}, g(u_{i_0,l}))$ ($l = 1, \ldots, N_{i_0}$). Since $g(u_{i_0,l}) = g(\bar{u}_{i_0,l})$ by definition, it immediately follows that the residual sum of squares error term is exactly the same for these two functions, i.e.,

$$
\sum_{l=1}^{N_{i_0}} [y_{i_0,l} - \bar{g}(u_{i_0,l}) - h_a(v_{i_0,l})]^2
= \sum_{l=1}^{N_{i_0}} [y_{i_0,l} - g(u_{i_0,l}) - h_a(v_{i_0,l})]^2.
$$

On the other hand, due to the property of the natural cubic spline interpolant that we saw in 2.2, it holds that $\int g''(t)^2 dt < \int \bar{g}''(t)^2 dt$. We can therefore conclude that $PRSS(\bar{g}_l = \bar{g}) < PRSS(g_l = g)$. This means that, unless $g$ itself is a natural cubic spline, we can find a natural cubic spline which yields a smaller value of PRSS in (5). It immediately follows that the minimizer $\hat{g}_l$ of (5) must be a natural cubic spline with knots at each of the unique values of $u_{i_0,l}$ ($l = 1, \ldots, N_{i_0}$). Extending the discussion above to all the instances of $g_i$ in (5) and all instances of $h_a$ in (6), we see that each of $g_i$’s and $h_a$’s has to be a natural cubic spline.

We can now write each of $g_i$ as the linear combination of $K_i$ natural cubic spline basis functions $N_{i,j}^0$ (cf. Appendix A):

$$
g_i(u) = \sum_{j=1}^{K_i} N_{i,j}^0(u) \theta_j^i, \quad i \in r(I),
$$

(7)

where $K_i$ is the number of distinct values for $u_{i,l}$, the time points within the intonational phrase function $g_i$. Then the vector of the values of $g_i$ at the training data points $u_{i,l}$ ($l = 1, \ldots, N_i$) can be written as

$$
g_i = N_i \theta_i
$$

(8)
An accentual phrase function \( h_a \) can also be written as a linear combination of natural cubic spline basis functions \( N_j^{(a)} \):

\[
\begin{align*}
    h_a(y) &= \sum_{j=1}^{K_a} N_j^{(a)}(y) \theta_j^{(a)}, \quad a \in r(A),
\end{align*}
\]

and we can derive the component roughness penalty for \( h_a \) in the same way:

\[
\begin{align*}
    \int h_a''(x)^2 dx &= \theta_a^T \Omega_{N_a} \theta_a,
\end{align*}
\]

where \( \theta_a \) is a cubic spline coefficient vector \( (\theta_1^{(a)}, \ldots, \theta_{K_a}^{(a)})^T \), and \( (\Omega_{N_a})_{jk} = \int N_j^{(a)''}(x) N_k^{(a)''}(x) dx \).

PRSS in (5) can now be written in a matrix form:

\[
\begin{align*}
    \text{PRSS} &= \sum_{i \in r(I)} (y_i - \alpha - g_i - h_i)^T (y_i - \alpha - g_i - h_i) \\
    &+ \lambda_g \sum_{i \in r(I)} \theta_i^T \Omega_{N_i} \theta_i + \lambda_h \sum_{a \in r(A)} \theta_a^T \Omega_{N_a} \theta_a \\
    &= \sum_{i \in r(I)} (y_i - \alpha - N_i \theta_i - h_i)^T (y_i - \alpha - N_i \theta_i - h_i) \\
    &+ \lambda_g \sum_{i \in r(I)} \theta_i^T \Omega_{N_i} \theta_i + \lambda_h \sum_{a \in r(A)} \theta_a^T \Omega_{N_a} \theta_a,
\end{align*}
\]

where \( y_i = (y_{i1}, \ldots, y_{iN_a})^T, \alpha = (\alpha, \ldots, \alpha)^T, h_i = (h_{i0}, (v_{i1}), \ldots, h_{i0}, (v_{iN_a}))^T \). By differentiating (12) with respect to the coefficient vector \( \theta_{a0} \) of one component function \( g_a(u) \) \((a_0 \in r(A))\), and setting the partial derivative to zero, we obtain:

\[
\begin{align*}
    \hat{\theta}_{a0} &= (N_i^T N_{a0} + \lambda_g \Omega_{N_{a0}})^{-1} N_i^T (y_{i0} - \alpha - \hat{h}_{i0}).
\end{align*}
\]

Similarly, we can derive another matrix form of the penalized least square criterion from (6):

\[
\begin{align*}
    \text{PRSS} &= \sum_{a \in r(A)} (y_a - \alpha - g_a - N_a \theta_a)^T (y_a - \alpha - g_a - N_a \theta_a) \\
    &+ \lambda_g \sum_{i \in r(I)} \theta_i^T \Omega_{N_i} \theta_i + \lambda_h \sum_{a \in r(A)} \theta_a^T \Omega_{N_a} \theta_a,
\end{align*}
\]

where \( y_a = (y_{a1}, \ldots, y_{aN_a})^T, g_a = (g_{a0}(u_{a1}), \ldots, g_{a0}(u_{aN_a}))^T, \) and \( (N_a)_{ij} = N_j^{(a)}(v_{ai}) \). As before, differentiating with respect to the coefficient vector \( \theta_{a0} \) of one component \( h_{a0} \) \((a_0 \in r(A))\), and setting the partial derivative to zero, we obtain:

\[
\begin{align*}
    \hat{\theta}_{a0} &= (N_i^T N_{a0} + \lambda_g \Omega_{N_{a0}})^{-1} N_i^T (y_{a0} - \alpha - \hat{g}_{a0}).
\end{align*}
\]

We note that \( l \)-th row of \( \hat{h}_l \) in (16) is a linear combination of the elements of \( \theta_{a0} \) in the form of (10), and that \( l \)-th row of \( \hat{g}_a \) in (17) is a linear combination of the elements of \( \theta_{a0} \) in the form of (7). Therefore (16) and (17) together consists a set of \( \mathcal{K} \) equations with \( \mathcal{K} \) unknowns, where

\[
\mathcal{K} = \sum_{i \in r(I)} K_i + \sum_{a \in r(A)} K_a.
\]

Multiplying both sides of (16) and (17) by \( N_i \) and \( N_a \), respectively, from the left, we have

\[
\begin{align*}
    \hat{g}_i &= N_i \hat{h}_i \\
    &= N_i (N_i^T N_i + \lambda_g \Omega_{N_i})^{-1} N_i^T (y_i - \alpha - \hat{h}_i) \\
    &= S_i (y_i - \alpha - \hat{h}_i) \quad \text{for all } i \in r(I) \quad (18) \\
    \hat{h}_a &= N_a \hat{h}_a \\
    &= N_a (N_a^T N_a + \lambda_h \Omega_{N_a})^{-1} N_a^T (y_a - \alpha - \hat{g}_a) \\
    &= S_a (y_a - \alpha - \hat{g}_a) \quad \text{for all } a \in r(A). \quad (19)
\end{align*}
\]

Each of \( S_l = N_l (N_l^T N_l + \lambda_g \Omega_{N_l})^{-1} N_l^T \) and \( S_a = N_a (N_a^T N_a + \lambda_h \Omega_{N_a})^{-1} N_a^T \) in (18) and (19) is called a smoother matrix for a cubic smoothing spline. We can obtain the solutions \( \hat{g}_i \) for all \( i \in r(I) \) and \( \hat{h}_a \) for all \( a \in r(A) \) using the backfitting algorithm, an iterative method depicted in Fig. 2, in which these smoother matrices are applied as smoothing operators in turn until convergence. This backfitting algorithm is equivalent to a block-wise Gauss-Seidel method [13] to solve the linear system of (16) and (17).

In our current implementation, we have adopted the arguments in [7] and have used more computationally manageable \((K + 4)\) B-spline basis functions, replacing \( K \) basis functions of natural cubic spline (Appendix A). It is also suggested that if the number of knots is very large, it is not necessary to use all the knots and some thinning strategy will save in computations with negligible effect on the fit [7]. In our current implementation, therefore, we just adopt one every ten time points for the use as knots.

### 3. Experiments and Results

We have recently been developing a speech synthesizer for Japanese based on our finite-state transducer-based framework [14], [15], and have created a preliminary version for a weather forecast domain [16]. We have therefore attempted to evaluate the use of our \( F_0 \) modeling technique for Japanese as well. In our current implementation, we made
The number of moras in the accentual phrase and
n
Therefore, the variable
A
position of the nucleus of accent (often called
accent type
sumed to be identified by the number of moras in it and the
intonational phrase. An accentual phrase component is as-
ponent of
F
an simplifying assumption that an intonational phrase com-
ponent of
F_0
is identified by its mora length. The predictor
variable
f
represents the number of moras (or morae) in the
intonational phrase. An accentual phrase component is as-
sumed to be identified by the number of moras in it and the
position of the nucleus of accent (often called
accent type
). Therefore, the variable
A
represents a pair
(m, n)
, where
m
is the number of moras in the accentual phrase and
n
means that the nucleus is associated with the
n
-th mora.

We have implemented the algorithm mentioned above in
Matlab [17], and estimated component functions
G_i
's and
h_n
's in the log frequency domain using a corpus of Japanese
utterances read by a female speaker. \( \lambda_g \) and \( \lambda_h \) are both set to be 1.0.

The corpus consisted mostly (around 90%) of general
sentences taken from news, novels and other types of gen-
eral texts. The rest consists of weather (approx. 7%) and
stock market report (approx. 3%). The speaker was in-
structed to read in a fairly neutral manner (but not so neutral
as to be completely unexpressive), in other words to give a
delivery typical of that of a newscaster.

In the transcription of this corpus, the intonational
phrase boundaries were defined simply in terms of pauses
the speaker made, and assigned in the labels by human tran-
scriptionists, who checked and marked the phrase bound-
aries for all of the recorded material. It was attempted to
check carefully the location of pauses during recordings
so that they did not occur in unnatural spots, although the
speaker was not given specific instructions about where to
pause in the text and where not to.

The corpus comprised 7,282 utterances, which in turn
consist of 16,181 intonational phrases and 44,717 accen-
tual phrases. The number of distinct types of intonational
phrases (or distinct mora lengths) was 49, and there were
130 unique accentual phrase types. Utterances in the cor-
opus are annotated with accentual phrase and intonational
phrase boundary information as well as phone labels. \( F_0 \)
values were extracted from the corpus every 10 ms using the
Snack Sound Toolkit, a public domain toolkit devel-
oped at KTH [18], [19]. These \( F_0 \) data were used as is, and
no particular postprocessing such as elimination of “micro-
prosody” was performed. The mean and the standard devia-
tion of the corpus \( F_0 \) were 207 Hz and 48.2 Hz, respectively.

Before the estimation, the original pitch samples were
normalized to have the same number of samples per mora
by uniformly interpolating or decimating each accentual
phrase. The data instances for which no pitch was extracted
for more than half of the mora interval at the beginning or
end of all the instances of an accentual phrase type were dis-
carded before estimation, which resulted in accentual phrase
types available for training reduced to 116, i.e. 89% of the
number of distinct types before discarding. As a side effect,
the number of unique intonational phrase types was reduced
to 46, which is 94% of the number before discarding.

The backfitting iteration (Fig. 2, (2)) converged well
when sixth loop was over. As a result, estimates for 46
distinct intonational phrases, and 116 types of accentual
phrases were obtained.

Figure 3 shows examples of extracted intonational and
accentual phrase components. Figure 4 illustrates an exa-
ple of the estimated \( F_0 \) contour plotted with the actual \( F_0 \)
data in the training corpus. As an objective evaluation, we
measured the goodness of fit in terms of root mean square er-
er (RMSE) and correlation coefficient (Corr) in the voiced
portions of the data, which are often used in the evaluation
of \( F_0 \) modeling [5], [20]. On the training data, RMSE was
28.9 Hz, and the Corr was 0.806. Measured on 85 intona-
tional phrases set aside from the training data, RMSE and
Corr were 29.8 Hz, and 0.777, respectively (Table 1). Mea-
sured in the log frequency domain to the base 2, the RMSE
for training and test set were 0.195 (octave) and 0.203 (oc-
tave), respectively.

Although it can be difficult to compare performance
across different speech corpora and languages, we believe
these results are quite promising. For example, state-of-the-
art results of 33–34 Hz RMSE, and 0.6–0.72 Corr have been
reported on a female-speaker English radio news corpus [5],
[20] with the standard deviation reported as e.g. 53 Hz in
[20].

![Figure 3 Examples of intonational phrase components and accentual phrase components estimated with the proposed method. (a) Intonational phrase components with the length of 8 through 12 moras. (b) 3-, 4- and 5-mora accentual phrase components with all distinct accent nucleus positions.](image-url)
Table 1  Experimental results for the additive $F_0$ model. “RMSE” stands for the root mean square error and “Corr” for the correlation coefficient.

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>Corr</th>
</tr>
</thead>
<tbody>
<tr>
<td>training</td>
<td>28.9</td>
<td>0.806</td>
</tr>
<tr>
<td>test</td>
<td>29.8</td>
<td>0.777</td>
</tr>
</tbody>
</table>

4. Conclusions and Future Work

In this paper, we have proposed a novel two-layer approach to $F_0$ modeling that uses a statistical learning technique for nonparametric regression called Additive Models. We confirmed by experiment that intonational and accentual phrase components that shows a quite regular patterns can be successfully estimated from a large Japanese speech corpus with the proposed method.

The fundamental frequency predicted by the model can be used as the reference for deriving a substitution (target) cost for unit selection in a corpus-based speech synthesizer. It may also be used in part of a post-processor to modify the waveform units to have pitch contour closer to the target.

Although current paper has only examined a two-layer modeling with the proposed additive framework, there is no theoretical limitation to the number of layers. It is expected that we may be able to add more layers as far as additivity of the component effects holds and those components are linearly independent from each other.

We plan to incorporate the $F_0$ measures predicted by the model, as one of the target measures to evaluate the goodness of the corpus units, into our next generation speech synthesis system we are currently developing. We also plan to apply this framework to $F_0$ modeling for English speech synthesis.

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References

Suppose we have real numbers $\xi_1, \ldots, \xi_K$ on some interval $[a, b]$, satisfying $a < \xi_1 < \xi_2 < \ldots < \xi_K < b$. A function $g$ defined on $[a, b]$ is a cubic spline if two conditions are satisfied. First, on each of the intervals $(a, \xi_1), (\xi_1, \xi_2), \ldots, (\xi_K, b)$, $g$ is a cubic polynomial. Second, the polynomial pieces fit together at the points $\xi_i$ in such a way that $g$ itself and its first and second derivatives are continuous at each $\xi_i$, hence on the whole of $[a, b]$. The points $\xi_i$ are called knots.

A natural cubic spline has additional constraints, namely that its second and third derivatives are zero at $a$ and $b$. These constraints incur that the function is linear beyond the two boundary knots, $\xi_1$ and $\xi_K$. It is known that a natural cubic spline with $K$ knots can be represented in the form of a linear combination of $K$ basis functions

$$g(x) = \sum_{j=1}^{K} \theta_j N_j(x), \quad (A.1)$$

where each of the basis functions $N_j$ is a some polynomial with an order up to three. See, for example, [7] (pp.120–122) for the detailed discussion.

**Appendix B: A Property of Natural Cubic Spline Interpolant (Green and Silverman [21])**

Suppose that $N \geq 2$ and that $g$ is the natural cubic spline interpolant to the pairs $(x_i, z_i)$ ($i = 1, \ldots, N$) with $a < x_1 < \ldots < x_N < b$. This is a natural cubic spline with knots at $x_i$ ($i = 1, \ldots, N$). Let $\bar{g}$ be any other twice continuously differentiable function on $[a, b]$ that also interpolates the $N$ pairs, i.e. $\bar{g}(x_i) = z_i$ for $i = 1, \ldots, N$. Then, it holds that

$$\int_a^b \bar{g}''(x)^2 \, dx \geq \int_a^b g''(x)^2 \, dx,$$

with equality only if $\bar{g}$ and $g$ are identical.

**Proof** Let $h(x) = \bar{g}(x) - g(x)$. Since $\bar{g}$ and $g$ both interpolate the pairs $(x_i, y_i)$, $h$ is zero at all $x_i$ ($i = 1, \ldots, N$).

Using the boundary conditions that $g''$ is zero at $a$ and $b$, integration by parts yields

$$\int_a^b g''(x)h''(x) \, dx = \int_a^b g''(x)h'(x) \, dx - \int_a^b g''(x)h'(x) \, dx$$

$$= -\int_a^b [g''(x)h'(x) - g''(x)h'(x)] \, dx - \int_a^b g''(x)h'(x) \, dx$$

$$= -\sum_{j=1}^{N-1} \int_{x_j}^{x_{j+1}} g''(x)h'(x) \, dx$$

$$= -\sum_{j=1}^{N-1} g''(x_j^+) \int_{x_j}^{x_{j+1}} h'(x) \, dx$$

$$= -\sum_{j=1}^{N-1} g''(x_j^+) (h(x_{j+1}) - h(x_j)) = 0. \quad (A.2)$$

We have used the fact that $g''$ is zero on the intervals $(a, x_1)$ and $(x_N, b)$ and is constant on each of the intervals $(x_j, x_{j+1})$ with the value $g''(x_j^+)$. Using (A.2), it follows that

$$\int_a^b \bar{g}''(x)^2 \, dx = \int_a^b g''(x)^2 \, dx + 2 \int_a^b g''(x)h''(x) \, dx + \int_a^b h''(x)^2 \, dx$$

$$\geq \int_a^b g''(x)^2 \, dx,$$

and equality will hold only if $\int_a^b h''(x)^2 \, dx$ is zero, so that $h$ is linear on $[a, b]$. But since $h$ is zero at $x_1, \ldots, x_N$, and since $N \geq 2$, this can only happen if $h$ is identically zero, which means that $g$ and $\bar{g}$ are the same function. \hfill \Box