Propagation Networks for Model-Based Control Under Partial Observation

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Abstract—There has been an increasing interest in learning dynamics simulators for model-based control. Compared with off-the-shelf physics engines, a learnable simulator can quickly adapt to unseen objects, scenes, and tasks. However, existing models such as Mujoco [1] and Bullet [1] use approximate contact interactions, which make contact-rich tasks hard to solve with these physics engines. For example, approximate, analytical differentiable rigid body equations of motion have been deployed for tool manipulation and generalizable to novel, partially observable scenes and tasks.

I. INTRODUCTION

Physics engines are critical for planning and control in robotics. To plan for a task, a robot may use a physics engine to simulate the effects of different actions on the environment and then select a sequence of actions to reach a desired goal configuration. The utility of the resulting action sequence depends on the accuracy of the physics engine’s predictions; so a high-fidelity physics engine plays an important role in robot planning. Most physics engines used in robotics (such as Mujoco [1] and Bullet [1]) use approximate contact models, and recent studies [2], [3], [4] have demonstrated discrepancies between their predictions and real-world data. These mismatches make contact-rich tasks hard to solve with these physics engines.

Recently, researchers have started building general-purpose neural physics simulators, aiming to approximate complex physical interactions with neural networks [5], [6]. They have succeeded to model the dynamics of both rigid-bodies and deformable objects (e.g., strings). More recent work has used interaction networks for discrete and continuous control [7], [8], [9], [10].

Interaction networks, however, have two major limitations. First, interaction nets only consider pairwise interactions between objects, restricting its use in real-world scenarios, where simultaneous multi-body interactions often occur. Typical examples include Newton’s cradle (Fig. 1a) or string manipulation (Fig. 1b). Second, they need to observe the full states of the environment; however, many real-world control tasks involve dealing with partial observable states.

Experiments demonstrate that PropNet consistently outperforms existing differentiable physics simulators in various forms [11], [12]. For example, approximate, analytical differentiable rigid body simulators [12], [13] have been deployed for tool manipulation and tool-use planning [14].
Among them, two notable efforts on learning differentiable simulators include interaction networks [5] and neural physics engines [6]. These methods restrict themselves to pairwise interactions for generalizability. However, this limits their ability to handle simultaneous, multi-body interactions. In this paper, we tackle this problem by learning to propagate the signals according to the interaction graph. Gilmer et al. [15] have recently explored message passing networks, but with a focus on quantum chemistry.

B. Model-Predictive Control with a Learned Simulator

Recent work on model-predictive control with deep networks [16], [17], [18], [19], [20] often learns an abstract-state transition function, instead of an explicit account of the environment [21], [22]. Subsequently, they use the learned model or value function to guide the training of the policy network. Instead, PropNet learns a general physics simulator that takes raw object observations (e.g., positions, velocities) as input. We then integrate it into classic trajectory optimization algorithms for control.

There have been a few papers that exploit the power of interaction networks for planning and control. Many of them use interaction networks to imagine—rolling out approximate predictions—to facilitate training a policy network [7], [8], [9]. In contrast, we use propagation networks as a learned dynamics simulator and directly optimize trajectories for continuous control. By separating model learning and control, our model generalizes better to novel scenarios. Recently, Sanchez-Gonzalez et al. [10] also explored applying interaction networks for control. Compared with them, our propagation networks can handle simultaneous multi-body interactions and deal with partially observable scenarios.

III. LEARNING THE DYNAMICS

A. Preliminaries

We assume that the interactions within a physical system can be represented as a directed graph, $G = (O, R)$, where vertices $O$ represent the objects, and edges $R$ correspond to relations (Fig. 3). Graph $G$ can be represented as

$$O = \{o_i\}_{i=1}^{\mid O\mid} \quad R = \{r_k\}_{k=1}^{\mid |R|\}$$ (1)

Specifically, $o_i = \langle x_i, a_i^o, p_i \rangle$, where $x_i = \langle q_i, \dot{q}_i \rangle$ is the state of object $i$, containing its position $q_i$ and velocity $\dot{q}_i$. $a_i^o$ denote its attributes (e.g., mass, radius), and $p_i$ is the external force on object $i$. For relations, we have

$$r_k = \langle u_k, v_k, a_k^r \rangle, \quad 1 \leq u_k, v_k \leq |O|, \quad (2)$$

where $u_k$ is the receiver, $v_k$ is the sender, and $a_k^r$ is the type and attributes of relation $k$ (e.g., collision, spring connection).

Our goal is to build a learnable physical engine to capture the underlying physical interactions using function approximators. We can then use it to infer the system dynamics and predict the future from the observed interaction graph $G$:

$$G_{t+1} = \phi(G_t),\quad (3)$$

where $G_t$ denotes the scene states at time $t$ and $\phi$ is a learnable dynamics model.

Below we review our baseline model Interaction Networks (IN) [5]. IN is a general-purpose, learnable physics engine, performing object- and relation-centric reasoning about physics. IN defines an object function $f_O$ and a relation function $f_R$ to model objects and their relations in a compositional way. The future state at time $t + 1$ is predicted as

$$e_{k,t} = f_R(o_{u_k,t}, o_{v_k,t}, a_k^r), \quad k = 1 \ldots |R|,$$

$$\dot{o}_{i,t+1} = f_O(o_{i,t}, \sum_{k \in N_i} e_{k,t}), \quad i = 1 \ldots |O|,$$ (4)

where $o_{i,t} = \langle x_{i,t}, a_i^o, p_i \rangle$ denotes object $i$ at time $t$, $u_k$ and $v_k$ are the receiver and sender of relation $r_k$, and $N_i$ denotes the relations where object $i$ is the receiver.

B. Propagation Networks

IN defines a flexible and efficient model for explicit reasoning of objects and their relations in a complex system. It can handle a variable number of objects and relations and has shown good performance in domains like n-body systems, bouncing balls, and falling strings. However, one fundamental limitation of IN is that at every time step $t$, it only considers local information in the graph $G$ and cannot handle instantaneous propagation of forces, such as the Newton’s cradle shown in Fig. 2, where ball A’s impact produces a compression wave that propagates through the balls immediately [23]. As force propagation is a common

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**Fig. 2: Newton’s Cradle.** (a) shows the initial states of a Newton’s cradle, based on which both the Interaction Networks and Propagation Networks try to predict future states; (b-i) The Interaction Networks can only propagate the force along a single relation at a time step, thus results in a false prediction (c-i); (b-ii) Our proposed method can propagate the force correctly which leads to the correct prediction (c-ii); (d) A downstream task that to achieve a specific goal using the learned model; (e-i) Model-based control methods fail to produce the correct control using Interaction Networks while (e-ii) our model can give the desired control signal.
phenomenon in rigid-body dynamics, this shortcoming has limited IN’s practical applicability.

To address the above issues, we propose Propagation Networks (PropNet) to handle the instantaneous propagation of forces efficiently. Our method is inspired by message passing, a classic algorithm in graphical models.

1) Effect propagation: Effect propagation requires multi-step message passing along the directed edges in the graph \( G \). Forces ejected from ball A (Fig. 2) should be propagated through the connected balls to ball B within a single time step. Force propagation is hard to analyze analytically for complex scenes. Therefore, we let PropNet learn to decide whether an effect should be propagated further or withheld.

At time \( t \), we denote the propagating effect from relation \( k \) at propagation step \( l \) as \( e^l_{k,t} \), and the propagating effect from object \( i \) as \( h^l_{i,t} \). Here, we have \( 1 \leq l \leq L \), where \( L \) is the maximum propagation steps within each step of the simulation. Propagation can be described as

\[
\text{Step 0: } h^0_{i,t} = 0, \quad i = 1 \ldots |O|,
\]

\[
\text{Step } l = 1, \ldots, L: \quad e^l_{k,t} = f^l_O(o_{uk,t}, o_{vk,t}, e^{l-1}_{uk,t}, e^{l-1}_{vk,t}), \quad k = 1, \ldots, |R|,
\]

\[
h^l_{i,t} = f^l_R(o_{it}, \sum_{k \in N_i} e^l_k), \quad i = 1 \ldots |O|,
\]

\[
\text{Output: } \hat{o}_{i,t+1} = h^L_{i,t}, \quad i = 1 \ldots |O|,
\]

where \( f^l_O \) denotes the object propagator at propagation step \( l \) and \( f^l_R \) denotes the relation propagator. Depending on the complexity of the task, the network weights can be shared among propagators at different propagation steps.

We name this model Vanilla PropNet. Experimental results show that the selection of \( L \) is task-specific, and usually a small \( L \) (e.g., \( L = 3 \)) can achieve a good trade-off between the performance and efficiency.

2) Object- and relation-encoding with residual connections: We notice that Vanilla PropNet is not efficient for fast online control. As information such as states \( o_{i,t} \) and attributes \( a^t_k \) are fixed at a specific time step, they can be shared without re-computation between each sequential propagation step. Hence, inspired by the ideas on fast RNNs training [24], [25], we propose to encode the shared information beforehand and reuse them along the propagation steps. We denote the encoders for objects as \( f^l_O \) and the encoder for relations as \( f^l_R \). Then,

\[
e^l_{i,t} = f^l_O(o_{i,t}), \quad e^l_k = f^l_R(o_{uk,t}, o_{vk,t}, a^t_k).
\]

In practice, we add residual links [26] between adjacent propagation steps that connect \( h^l_{i,t} \) and \( h^{l+1}_{i,t} \). This helps address the gradient vanishing/exploding problem and provides access to the historical effects. The update rules become

\[
e^l_k = f^l_R(v^l_k, h^l_{uk,t}, h^{l-1}_{vk,t}),
\]

\[
h^l_{i,t} = f^l_R(o_{it}, \sum_{k \in N_i} e^l_k, h^{l-1}_{i,t}),
\]

where propagators \( f^l_O \) and \( f^l_R \) now take a new sets of inputs, which is different from Vanilla PropNet.

Based on the assumption that the effects between propagation steps can be represented as simple transformations (e.g., identity-mapping in the Newton’s cradle), we can use small networks as function approximators for the propagators \( f^l_O \) and \( f^l_R \) for better efficiency. We name this updated model Propagation Networks (PropNets).

C. Partially Observable Scenarios

For many real-world situations, however, it is often hard or impossible to estimate the full state of the environments. We extend Eqn. 3 using PropNets to handle such partially observable cases by operating on a latent dynamics model:

\[
\tau(G_{t+1}) = \phi(\tau(G_t)),
\]

where \( \tau \) is an encoding function that maps the current state to a latent representation. Depending on the actual scenarios, both \( \phi \) and \( \tau \) can be realized as PropNets. Note that in fully observable environments, \( \tau \) reduces to an identity mapping.

To train such a latent dynamics model, we seek to minimize the loss function:

\[
\mathcal{L}_\text{forward} = \|\tau(G_{t+1}) - \phi(\tau(G_t))\|.
\]

Using this loss alone leads to trivial solutions such as \( \phi(x) = \tau(x) \) for any valid \( x \). We tackle this based on an intuitive idea: an ideal encoding function \( \tau \) should reserve information about the scene state. Hence, we introduce a decoding function \( \psi \) to ensure a nontrivial \( \tau \) by minimizing an additional auto-encoder reconstruction loss [27]:

\[
\mathcal{L}_\text{encode} = \| G - \psi(\tau(G)) \|.
\]

IV. CONTROL USING LEARNED DYNAMICS

Compared to model-free approaches, model-based methods offer many advantages, such as generalization and sample efficiency, as it can approximate the policy gradient or value estimation without exhausted trials and errors.

However, an accurate model of the environment is often hard to specify and brings significant computational costs for even a single-step forward simulation. It would be desirable to learn to approximate the underlying dynamics from data.

A learned dynamics model is naturally differentiable. Given the model and a desired goal, we can perform forward simulation, optimizing the control inputs by minimizing a loss between the simulated results and the goal. The model can
also estimate the uncertain attributes online by minimizing
the difference between the predicted future and the reality.
Alg. 1 outlines our control algorithm, which provides a good
testbed for evaluating the modeling of the dynamics.

\textbf{a) Model predictive control using shooting methods:}
Let \( G_g \) be our goal and \( u_{1:T} \) be the control inputs (decision
variables), where \( T \) is the time horizon. These task-specific
control inputs are part of the dynamics graph. Typical
choices include observable objects’ initial velocity/position
and external forces/attributes on objects/relations. We denote
the graph encoding as \( G^r = \tau(G) \), and the resulting trajectory
after applying the control inputs as \( G = \{G^r_i\}_{i=1:T} \). The task
here is to determine the control inputs by minimizing the gap
between the actual outcome and the specified goal \( L_g(G, G_g) \).

Our propagation networks can do forward simulation by
taking the dynamics graph at time \( t \) as input, and produce
the graph at next time step, \( \hat{G}^r_{t+1} = \phi(G^r_t) \). Let’s denote
the forward simulation from time step \( t \) as \( G^r = \{G^r_i\}_{i=t+1:T} \)
and the history until time \( t \) as \( G = \{G^r_i\}_{i=1:t} \). We can back-
propagate from the loss \( L_g(G, G_g) \) and use stochastic
gradient descent (SGD) to update the control inputs. This is
known as the shooting method in trajectory optimization [28].

If the time horizon \( T \) is too long, the learned model might
deviate from the ground truth due to accumulated prediction
errors. Hence, we use Model-Predictive Control (MPC) [29]
to stabilize the trajectory by doing forward simulation at
every time step as a way to compensate the simulation error.

\textbf{b) Online adaptation:} In many situations, without
actually interacting with the objects, inherent attributes such
as masses, friction, and damping are not directly observable.
PropNet can estimate these attributes online (denoted as \( A \))
with SGD updates by minimizing the difference between
the predicted future states and the actual future states
\( L_a(G^r_t, G^o_t) \).

\section{Experiments}

In this section, we proceed to evaluate our model’s
performance on both simulation and control in three scenarios:
Newton’s Cradle, String Manipulation, and Box Pushing. We
also test how it generalizes and learns to adapt online.

\textbf{A. Physics Simulation}

We aim to predict the future states of physical systems. We
first describe the network used across tasks and then present
the setup of each task as well as the experimental results.

\textbf{a) Model architecture.} For the IN baseline, we use the
same network as described in [5]. For Vanilla PropNet, we
adopt similar network structure where the relation propagator
\( f^r(t) (1 \leq t \leq L) \) is an MLP with four 150-dim hidden layers
and the object propagator \( f^o(t) (1 \leq t \leq L - 1) \) has one 100-
dim hidden layer. Both output a 100-dim propagation vector.
For fully observable scenarios, \( f^o(t) \) has one 100-dim hidden
layer and outputs a 2-dim vector representing the velocity at
the next time step. For partially observable cases, \( f^o(t) \) outputs
one 100-dim vector as the latent representation.

For PropNet, we use an MLP with three 150-dim hidden
layers as the relation encoder \( f^r_{enc} \) and one 100-dim hidden
layer MLP as the object encoder \( f^o_{enc} \). Light-weight neural
networks are used for the propagators \( f^r \) and \( f^o \), both of
which only contain one 100-dim hidden layer.

\textbf{b) Newton’s cradle.} A typical Newton’s cradle consists
of a series of identically sized rigid balls suspended from
a frame. When one ball at the end is lifted and released, it
strikes the stationary balls. Forces will transmit through the
stationary balls and push the last ball upward immediately.
In our setup, we assume full-state observation and the graph
\( G \) of \( n \) balls has \( 2n \) objects representing the balls and the
corresponding fixed pinpoints above the balls, as can be seen
in Fig. 2a, where \( n = 5 \). There will be \( 2n \) directed relations
describing the rigid connections between the fixed points and
the balls. Collisions between adjacent balls introduce another
\( 2(n - 1) \) relations.

We generated 2,000 rollouts over 1,000 time steps, of which
85% of the rollouts are randomly chosen as the training set,
while the rest are held as the validation set. The model was
trained for 2,000 epochs with a mini-batch of 32. We use the
Adam optimizer [30] with an initial learning rate of 0.001. We
downscaled the learning rate by 0.8 each time the validation
error stops decreasing for over 20 epochs.

Fig. 2a-c show some qualitative results, where we compare
IN and PropNet. IN can not propagate the forces properly:
the rightmost ball starts to swing up before the first collision
happens. Quantitative results also show that our method
significantly outperforms IN in tracking object positions. For
1,000 forward steps, IN results in an MSE of 336.46, whereas
PropNet achieves an MSE of 7.85.

\begin{algorithm}
\begin{algorithmic}
\caption{Control on Learned Dynamics at Time Step $t$}
\Input{Learned forward dynamics model $\phi$
\hspace{2em}predicted dynamics graph encoding $G^r_t$
\hspace{2em}current dynamics graph encoding $G^o_t$
\hspace{2em}goal $G_g$, current estimation of the attributes $A$
\hspace{2em}current control inputs $u_{t:T}$
\hspace{2em}states history $\{G^r_i\}_{i=1:t}$
\hspace{2em}forward simulation time $N$ and time horizon $T$}
\Output{Controls $u_{t:T}$; predicted next time step $\hat{G}^r_{t+1}$}
\State Update $A$ by descending with the gradients $\nabla_A L_a(G^r_t, G^o_t)$
\For{$i = 1, \ldots, N$}
\State Forward simulation using the current graph encoding $G^r_{t+1} = \phi(G^r_t)$
\State Make a buffer for storing the simulation results $G_i \leftarrow G_i \cup G^r_{t+1}$
\For{$t = t + 1, \ldots, T - 1$}
\State Forward simulation $G^r_{t+1} = \phi(G^r_t)$; $G_i \leftarrow G_i \cup G^r_{t+1}$
\EndFor
\State Update $\hat{u}_{t:T}$ by descending with the gradients $\nabla_{\hat{u}_{t:T}} L_g(G, G_g)$
\EndFor
\State Return $\hat{u}_{t:T}$ and $\hat{G}^r_{t+1} = \phi(G^r_t)$
\end{algorithmic}
\end{algorithm}
Two circular obstacles are placed at random positions near the center and the rest of the string is free to move. As can be seen from the figures, although in this case, the length of the underlying force propagation is fewer than Newton’s Cradle’s, our proposed method can still track the ground truth much more accurately and outperform IN with a large margin.

d) Box pushing.: In this case, we are pushing a pile of boxes forward (Fig. 4c). We place a camera at the top of the scene, and only red boxes are observable. More challengingly, the observable boxes are not tracked. Therefore, the visibility of a specific box might change over time. The vertices in the graph are then defined as the state of the observable boxes and edges are defined as directional relations connecting every pair of observable boxes. Specifically, if there are \( n \) observable boxes, \( n(n-1) \) edges are automatically generated. We augment the encoding function \( \tau \) by averaging the object-centric outputs before feeding to \( \phi \). The dynamics function \( \phi \) then takes both the scene representation and the action (i.e., position and velocity of the pusher) as input to perform an implicit forward simulation. As it is hard to explicitly evaluate a latent dynamics model, we evaluate the downstream control tasks instead.

e) Ablation studies.: We also provide ablation studies on how the number of propagation steps \( L \) influences the final performance. Empirically, a larger \( L \) can model a longer propagation path. They are however harder to train and more likely to overfit the training set, often leading to poor generalization. Fig. 5a and 5b show the ablation studies regarding the choice of \( L \). PropNet achieves a good accuracy at \( L = 3 \), which also has a good speed/accuracy trade-off. Vanilla PropNet achieves its best accuracy at \( L = 2 \) but generalizes less well as \( L \) increases further. This shows the benefits of using the shared encoding and residual connections as described in Section III-B.2.
We first assume the attributes of the physics graph is known ("Normal”), where the value of the attributes are unknown ("Bias”), where algorithms actively estimate these attributes online ("Adapt”), and where strings of varied length between 10 to 20 when the model is only trained on strings of length 15 ("Genealize"). DRL has the same performance for “Bias” and “Adapt” as it is model-free; it requires a fixed length input, and thus cannot generalize to strings of a different length. (b) For box pushing, propagation networks again outperforms the other methods.

B. Control

We now evaluate the applicability of the learned model on control tasks. We first describe the three tasks: Newton’s Cradle, String Manipulation, and Box Pushing, which include both open-loop and feedback continuous control tasks, as well as fully and partially observable environments. We evaluate the performance against various baselines and test its ability on generalization and online adaptation.

a) Newton’s cradle.: In this scenario, we assume full-state observation and a control task would be to determine the initial angle of the left-most ball, so as to let the right-most ball achieve a specific height, which can be solved with an accurate forward simulation model.

This is an open-loop control task where we only have control over the initial condition. We thus use a simplified version of Alg. 1. Given the initial physics graph and a learned dynamics model, we iteratively do forward simulation and update the control inputs by minimizing the loss function $L_\theta(g, G)$. In this specific task, the loss $L_\theta$ is the $L_2$ distance between the target height of the right-most ball and the highest height that has been achieved in $G$.

We initialize the swing up angle as 45° and then optimize the angle with a learning rate of 0.1 for 50 iterations using Adam optimizer. We compare our model with IN. Qualitative results are shown in Fig. 2e. Quantitatively, PropNet’s output angle has an MSE of 3.08 from the ground truth initial angle, while the MSE for interaction nets is 296.66.

b) String Manipulation.: Here we define the task as to move the string to a target configuration, where the only controls are the top two masses at the moving end of the string (Fig. 4b). The controller tries to match the target configuration by “swinging” the string, which requires to leverage the dynamics of the string. The loss $L_\theta$ here is the $L_2$ distance between the resulting configuration and the goal configuration.

We first assume the attributes of the physics graph is known (e.g., mass, friction, damping) and compare the performance between Proportional-Derivative controller (PD) [31], Model-free Deep Reinforcement Learning (Actor-Critic method optimized with PPO [32] - DRL), as well as Interaction Networks (IN) and Propagation Networks (PropNet) with Alg. 1. Fig. 6 shows quantitative results, where bars marked as “Normal” are the results in this task (a hand-tuned PD controller has an MSE of 2.50). PropNet outperforms the competing baselines. Fig. 4b shows a qualitative sample. Compared with the PD controller, our method leverages the dynamics and manages to match the target, instead of naively matching the free end of the string.

We then consider situations where some of the attributes are unknown and can only be guessed before actually interacting with the objects. We randomly add noise of 15% of the original scale to the attributes as the initial guesses. The “Bias” bars in Fig. 6 show that models trained with ground-truth attributes will encounter performance drop when the supplied attributes are not accurate. However, model-based methods can do online adaptation using the actual output from the environment as feedback to correct the attribute estimation. By updating the estimated attributes over the first 20 steps of the time horizon with standard SGD, we can improve the manipulation performance so as to catch up with the situations where attributes are accurate (bars marked as “Adapt” in Fig. 6).

We further test whether our model generalizes to new scenarios, where the length of the string is varied between 10 to 20. As can be seen in Fig. 6, our proposed method can still achieve a good performance, even though the original PropNet is only trained in situations with a fixed length 15 (PD has an MSE of 2.72 for generalization).

c) Box Pushing: In this case, we aim to push a pile of boxes to a target configuration within a predefined time horizon (Fig. 4c). We assume partial observation where a camera is placed at the top of the scene, and we can only observe the states of the boxes marked in red. The model trained with partial observation is compared with two baselines: DRL and IN. The loss function $L_\theta$ used for MPC is the $L_2$ distance between the resulting scene encoding and the target scene encoding.

We evaluate the performance by the Chamfer Distance [33] between the observable boxes at the end of the episode and the target configurations. The negative of the distance is used as the reward for DRL. Fig. 4c and Fig. 6b show qualitative and quantitative results, respectively. Our method outperforms the baselines due to its explicit modeling of the dynamics and its ability to handle multi-object interactions.

VI. Conclusion

We have presented propagation networks (PropNet), a general learnable physics engine that outperforms the previous state-of-the-art with a large margin. We have also demonstrated PropNet’s applicability in model-based control under both fully and partially observable environments. With propagation steps, PropNet can propagate the effects along relations and model the dynamics of a long-range interactions within a single time step. We have also proposed to improve PropNet’s efficiency by adding residual connections and shared encoding.
REFERENCES


