Stability of Passive Dynamic Walking on Uneven Terrain

Katie Byl and Russ Tedrake
Massachusetts Institute of Technology
email: katiebyl@mit.edu, russt@mit.edu, web: http://groups.csail.mit.edu/locomotion/

INTRODUCTION

We present a stability analysis for passive compass gait walkers on uneven (rough) downhill terrain. Although deterministic definitions of stability do not apply, for sufficiently low levels of noise the resulting dynamics are governed by a stochastic convergence to a "metastable" (long-living) limit cycle. For unbounded noise models (e.g. Gaussian), the walker will eventually exit (or “escape”) from this metastable cycle with a probability of one as time goes to infinity, entering an absorbing state (falling or standing still). Experimental walking machines are subject to similar random disturbances; statistics of the associated stochastic process, such as the mean first passage time (MFPT) to a fallen state, may be the correct way to quantify walking stability.

METHODS

The data presented here come from Monte Carlo simulations of the equations of motion for a 2D compass gait (CG) as it walked down an uneven slope. Each simulation run began with a particular double-stance initial condition, and histograms of the number of successful steps taken were recorded and used to estimate the MFPT as a function of the initial state. The CG has a mass, m_h, at its hip and a mass, m, at distance a measured from the toe along each unit-length leg (L=1); ground collisions were modeled as inelastic. The ground slope between the stance and swing leg at each collision had a Gaussian distribution, with a mean slope of 4 degrees. We compare two walkers, listed in Table 1.

RESULTS AND DISCUSSION

Figure 1 shows a slice of the deterministic basin of attraction (top) and a plot showing MFPT as a function of initial conditions (the “stochastic basin of attraction”) on terrain with a std of 0.5° for walker #1. Along the ridge of the stochastic basin, the MFPT is about 20 steps. Qualitatively, the stochastic basin resembles a low-pass filtered version of the deterministic basin; it identifies how "safe" states are relative to each other. The deterministic basin for walker #2 (not shown) is wider than that of #1, and this walker is also not as sensitive to noise in ground slope. For a std of 1°, the MFPT for walker #2 is about 150 steps, compared to about 6 steps for walker #1.

For a std of 0.5 deg, the failure or “leakage” rate (the inverse of the MFPT) from the metastable basin for walker #2 is empirically so slow that Monte Carlo estimation seems impractical. We are developing algorithms to efficiently estimate the entire FPT distribution for these systems by discretizing the state-space and writing the stochastic step-to-step return map as a Markov chain. Iterating the transition matrix of the Markov chain can produce the entire first passage time distribution, and the *mean* FPT can be calculated directly (no iterations) from the transition matrix.

Table 1: CG geometry and MFPTs on uneven terrain.

	m_h/m	a (m)	MFPT: std .5°	MFPT: std 1.0°
walker #1	**2.0**	**.6**	**20**	**6**
walker #2	**0.3**	**.7**	**>>100,000**	**150**

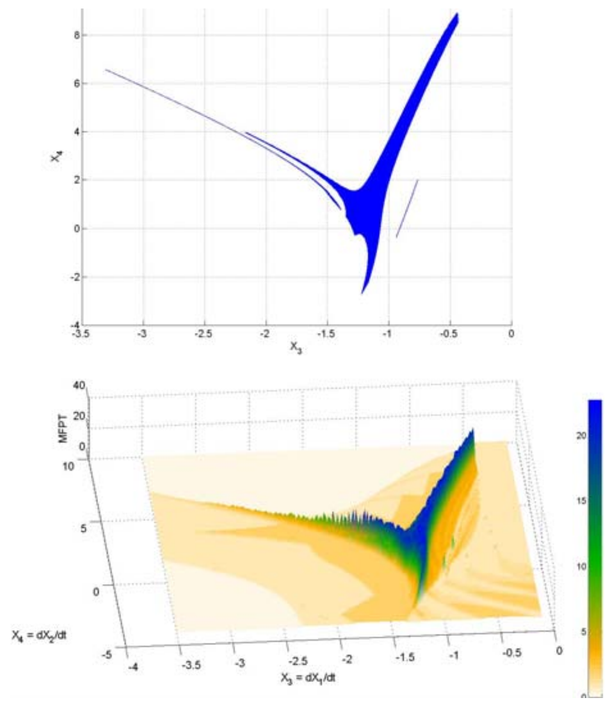

Figure 1: Deterministic basin of attraction (top) and a plot of MFPT (bottom) for walker #1. x-y axes are the stance-leg and inter-leg angular velocities (rad/s), X_3 and X_4, for a post-collision initial condition on a 4° slope with an inter-leg angle of approximately 33.4°.

CONCLUSIONS

The concept of metastability can be used in modeling a variety of stochastic noise sources (uneven terrain, elasticity of ground collision, disturbance forces and torques, etc.). The MFPT provides a way of comparing the relative stability of different mechanical passive designs, and it can also be used in optimizing actively-controlled walkers based on passive dynamic principles. In our own research group, for instance, this metric has particular significance as a goal for optimization in reinforcement learning.