The Sparse FFT: From Theory to Practice

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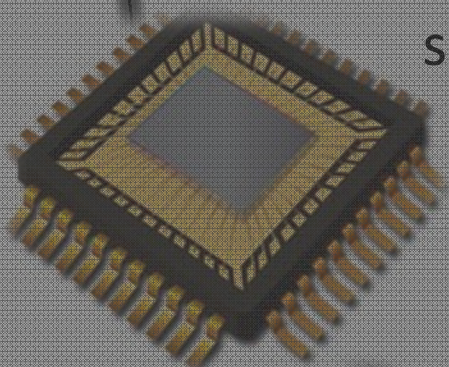
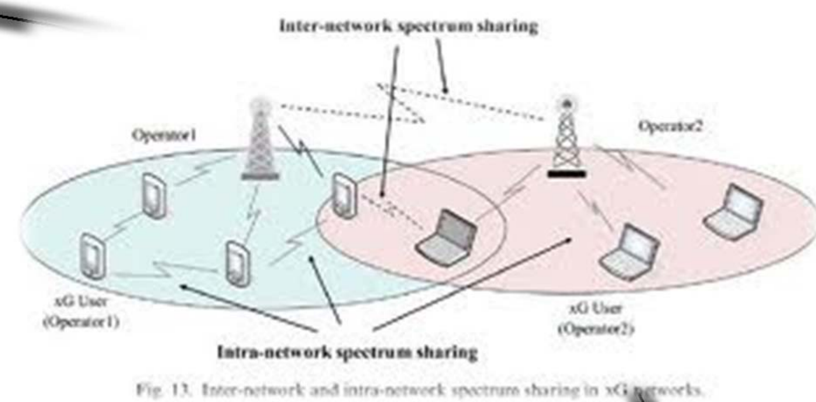
Ongoing sFFT Projects (Beyond Theory)

GPS



sFFT Chip

Spectrum Sharing



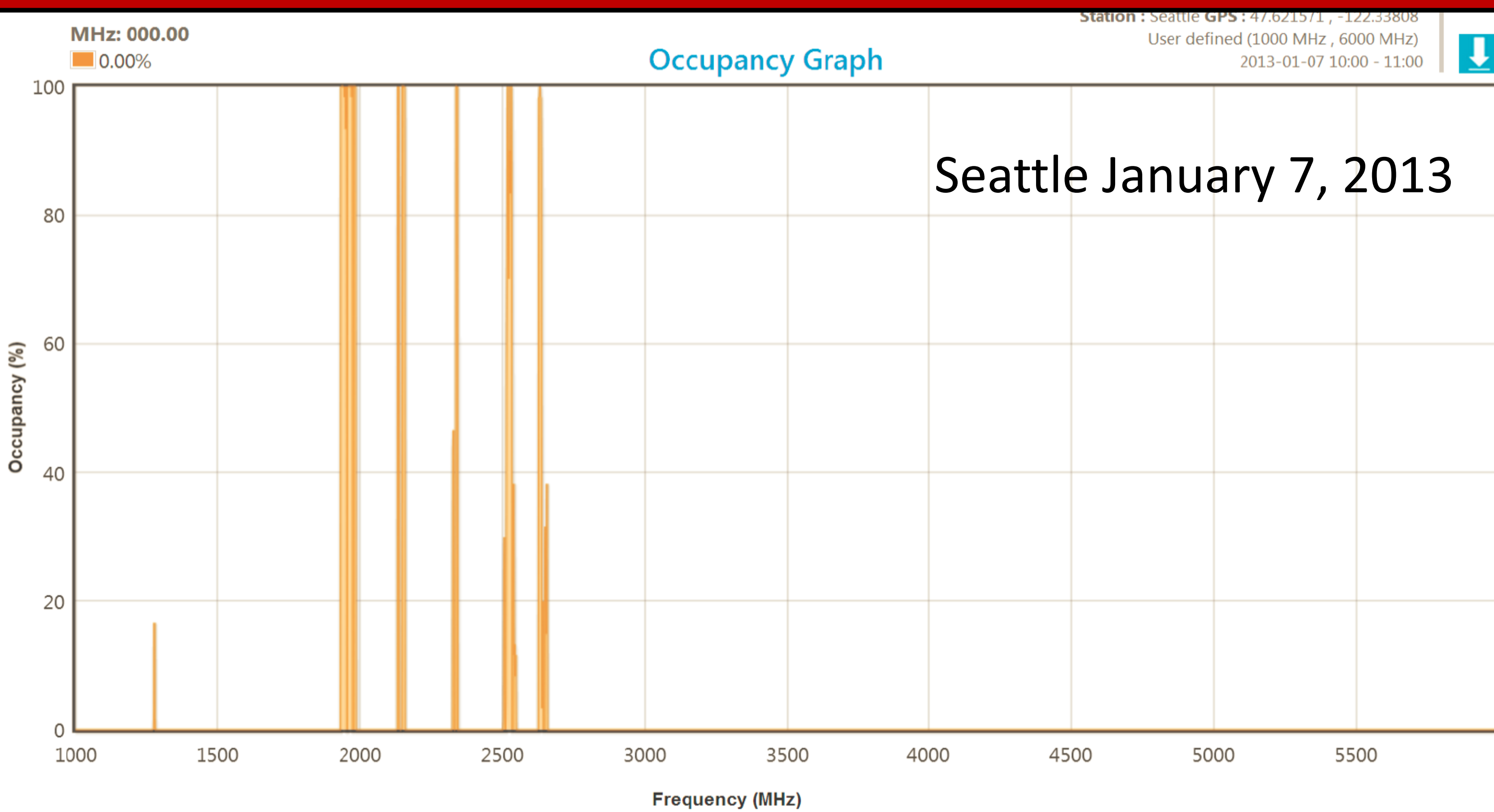
Medical Imaging

Light Field Photography

Spectrum Sharing

Sense to find unused bands; Use them!

How do you capture GHz of spectrum?



Challenges in Sparse GHz Acquisition

- GHz sampling is expensive and high-power

Tens of MHz ADC
< a dollar
Low-power

A Few GHz ADC
Hundreds of dollars
10x more power

- Compressive sensing using GHz analog mixing is expensive, and requires heavy computation

Recap of sFFT

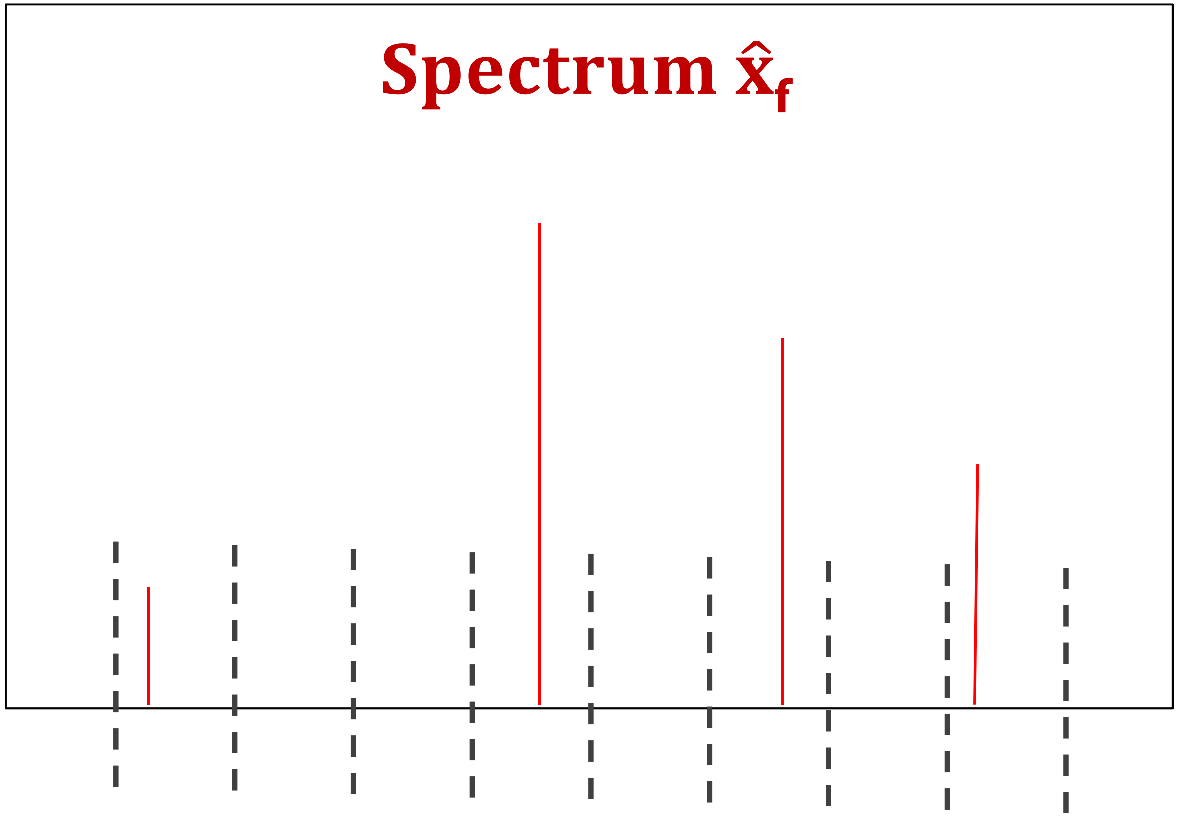
1- Bucketize

Hash the spectrum into a few buckets

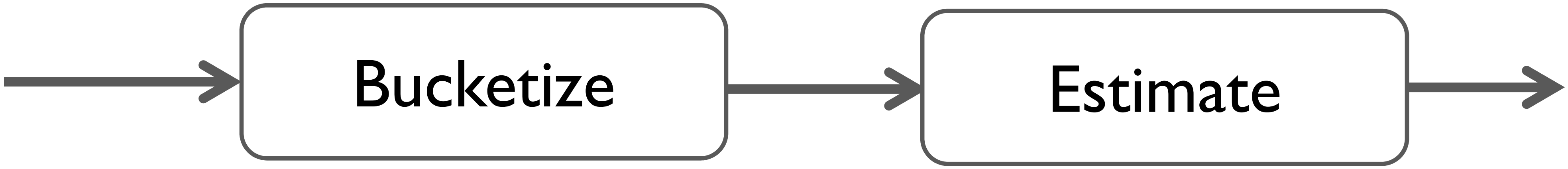
→ Can ignore empty bucket

2- Estimate

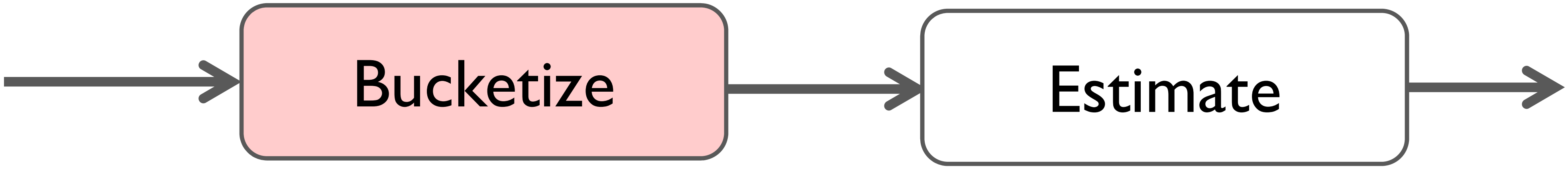
Estimate the large coefficient in each non-empty bucket



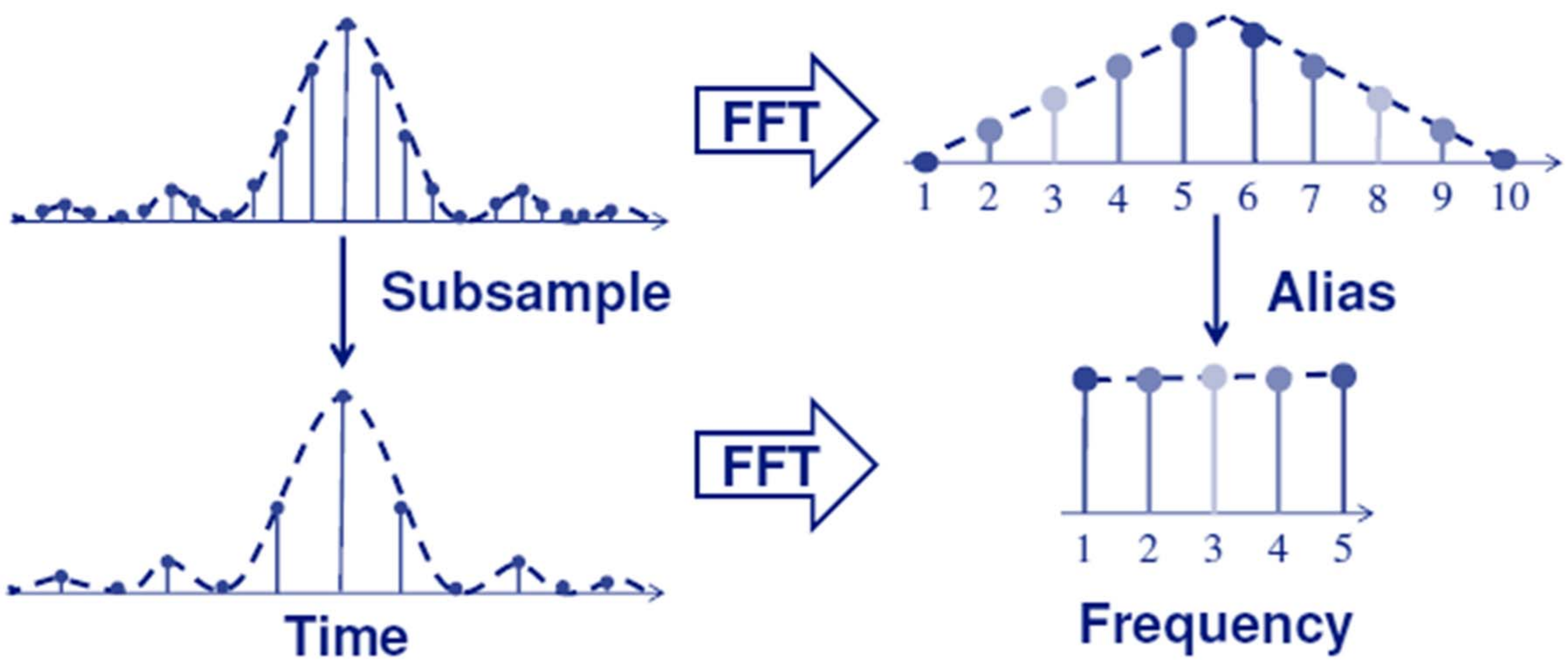
Spectrum Sensing & Decoding with sFFT



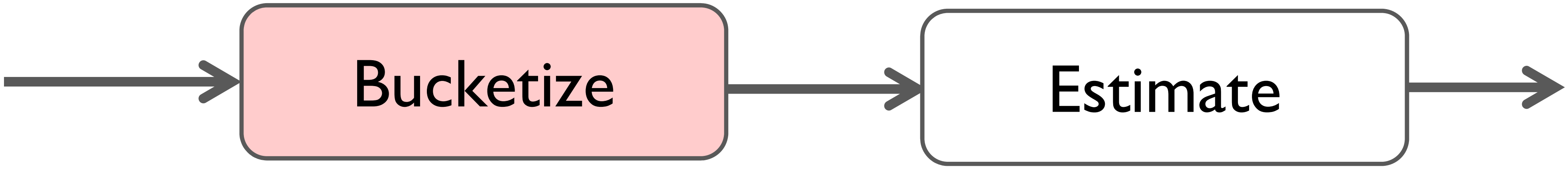
Spectrum Sensing & Decoding with sFFT



Sub-sampling time ➔ Aliasing the frequencies



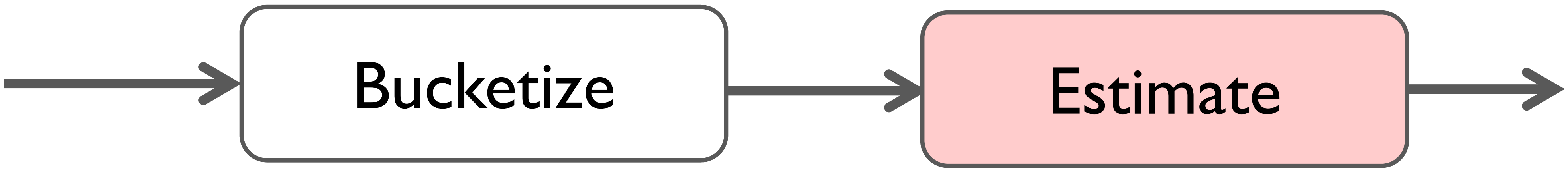
Spectrum Sensing & Decoding with sFFT



- Hash freqs. using multiple co-prime aliasing filters
  - Same frequencies don't collide in two filters
- Identify isolated freq. in one filter and subtract them from the other; and iterate …

Low-speed ADCs, which are cheap and low-power

Spectrum Sensing & Decoding with sFFT



Estimate frequency by repeating the bucketization with a time shift ΔT

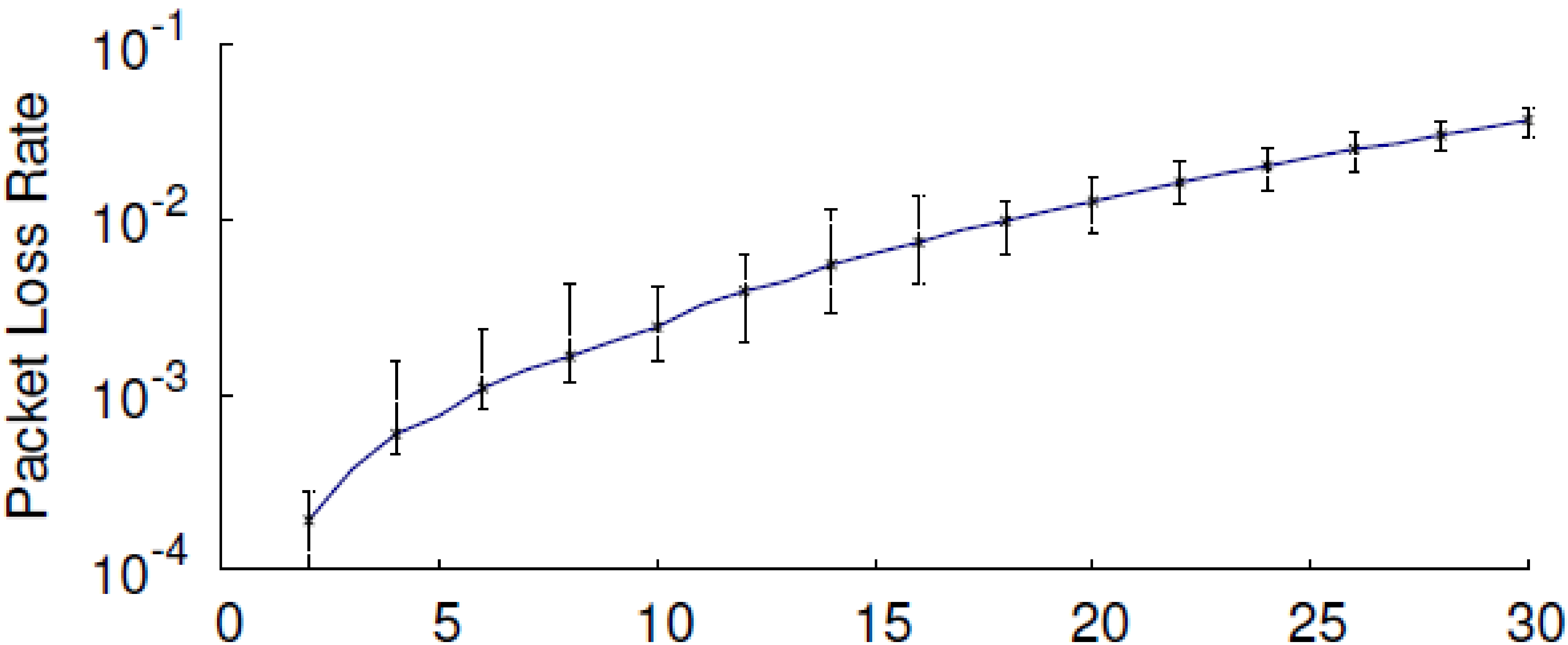
$$\Delta Phase = 2\pi f \, \Delta T$$

Low-Power GHz Receiver

- Built a 0.9 GHz receiver using three 50 MHz software radios

- First off-the-shelf receiver that captures a sparse signal larger than its own digital bandwidth

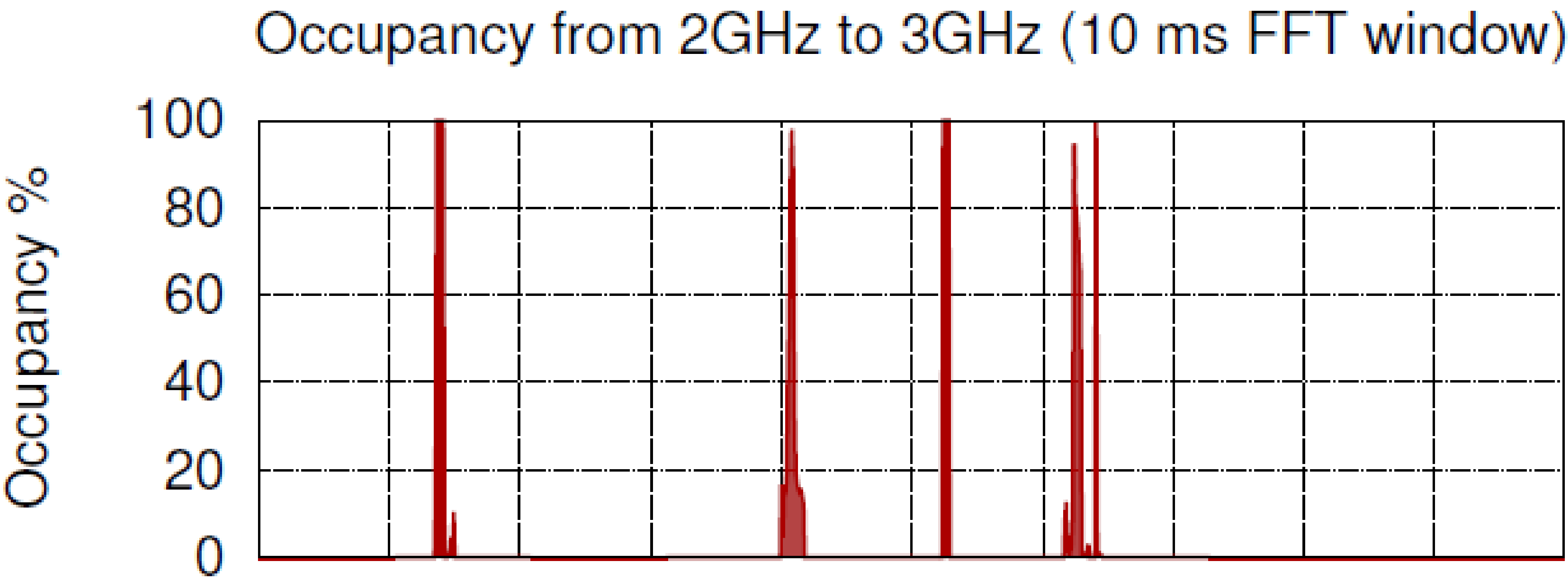
Concurrent Senders Hopping in 0.9 GHz



Number of MHz Senders Randomly Hopping gin in 0.9 GHz

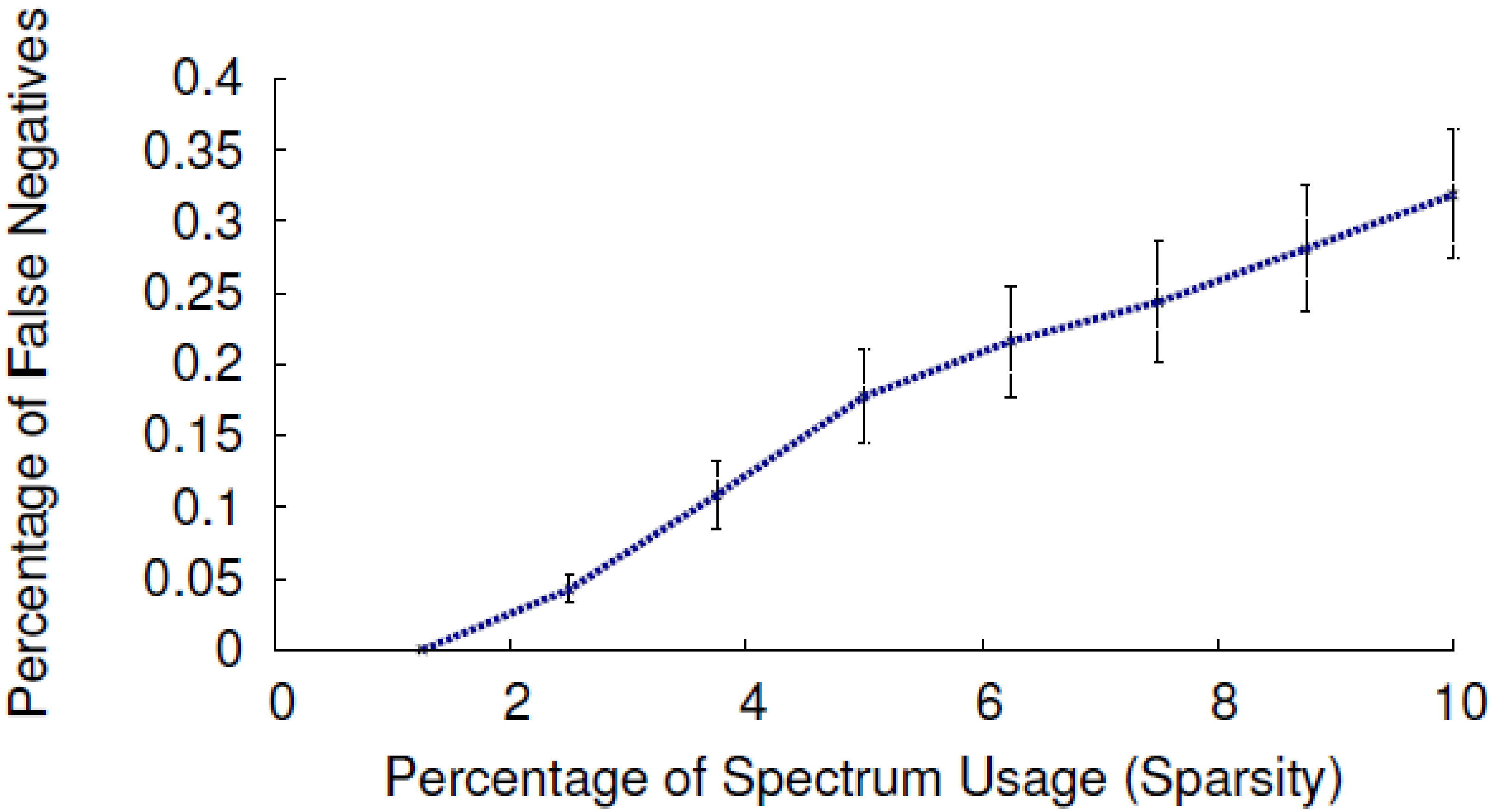
Realtime GHz Spectrum Sensing

Cambridge, MA January 2013



sFFT enables a GHz low-power receiver using only a few MHz ADCs

Probability of Declaring a Used Frequency as Unused



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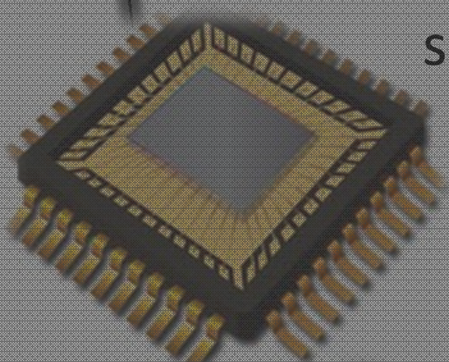
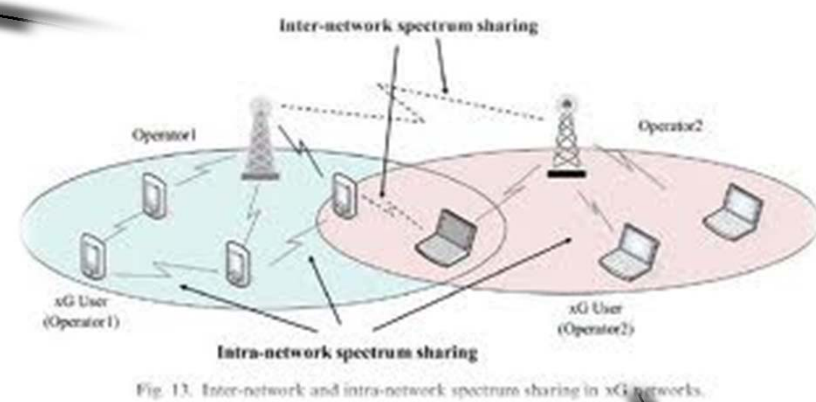
GPS



sFFT Chip

Spectrum Sharing



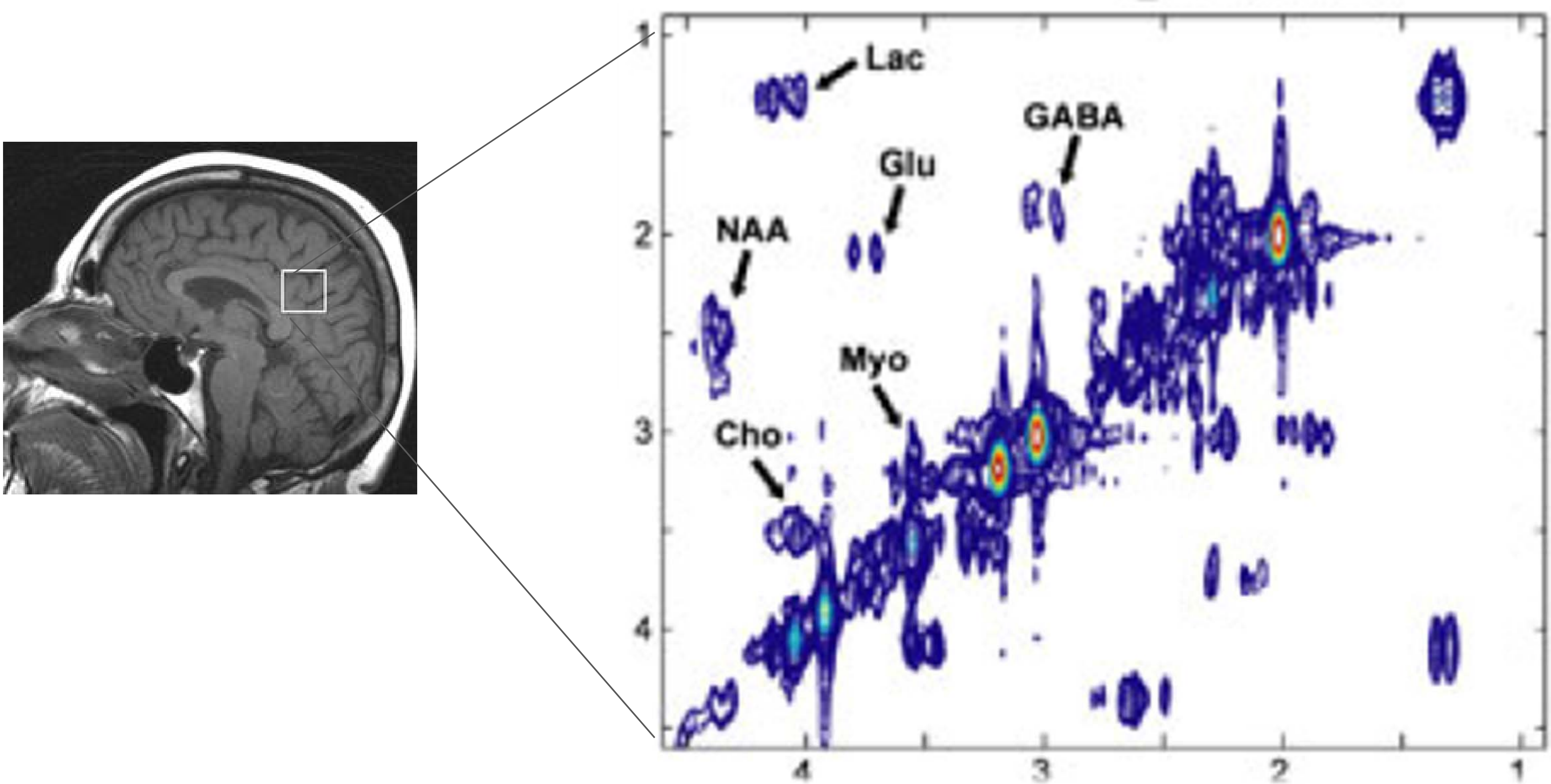
Medical Imaging

Light Field Photography

Magnetic Resonance Spectroscopy

Analyses the chemical making of a brain voxel

➔Disease Bio-markers

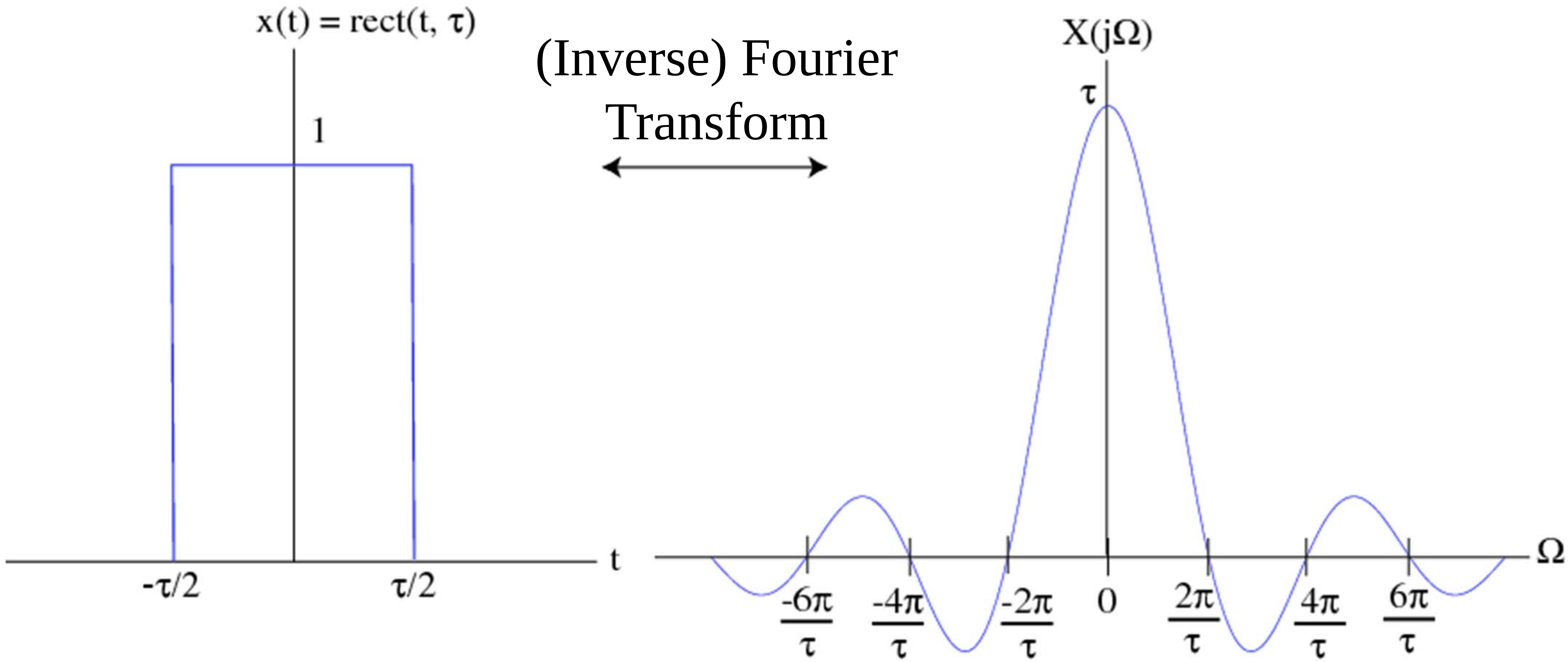


Challenges

- Long acquisition time
  - patient is in the machine for 40min to hours
- Artifacts due to acquisition window

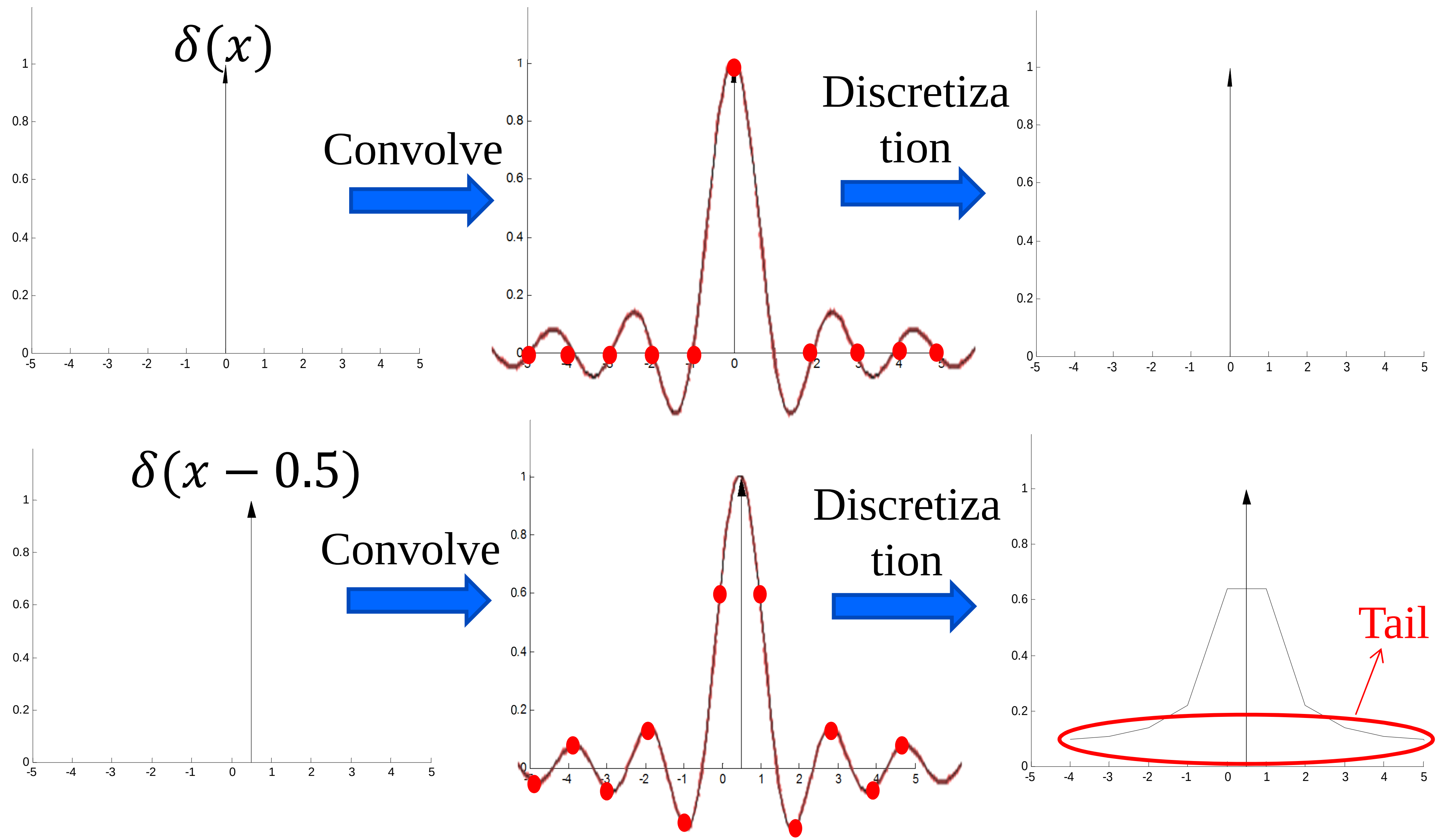
Windowing Artifacts

- Fourier transform of a window is a sinc



Acquisition Window ➔ Convolution with a sinc

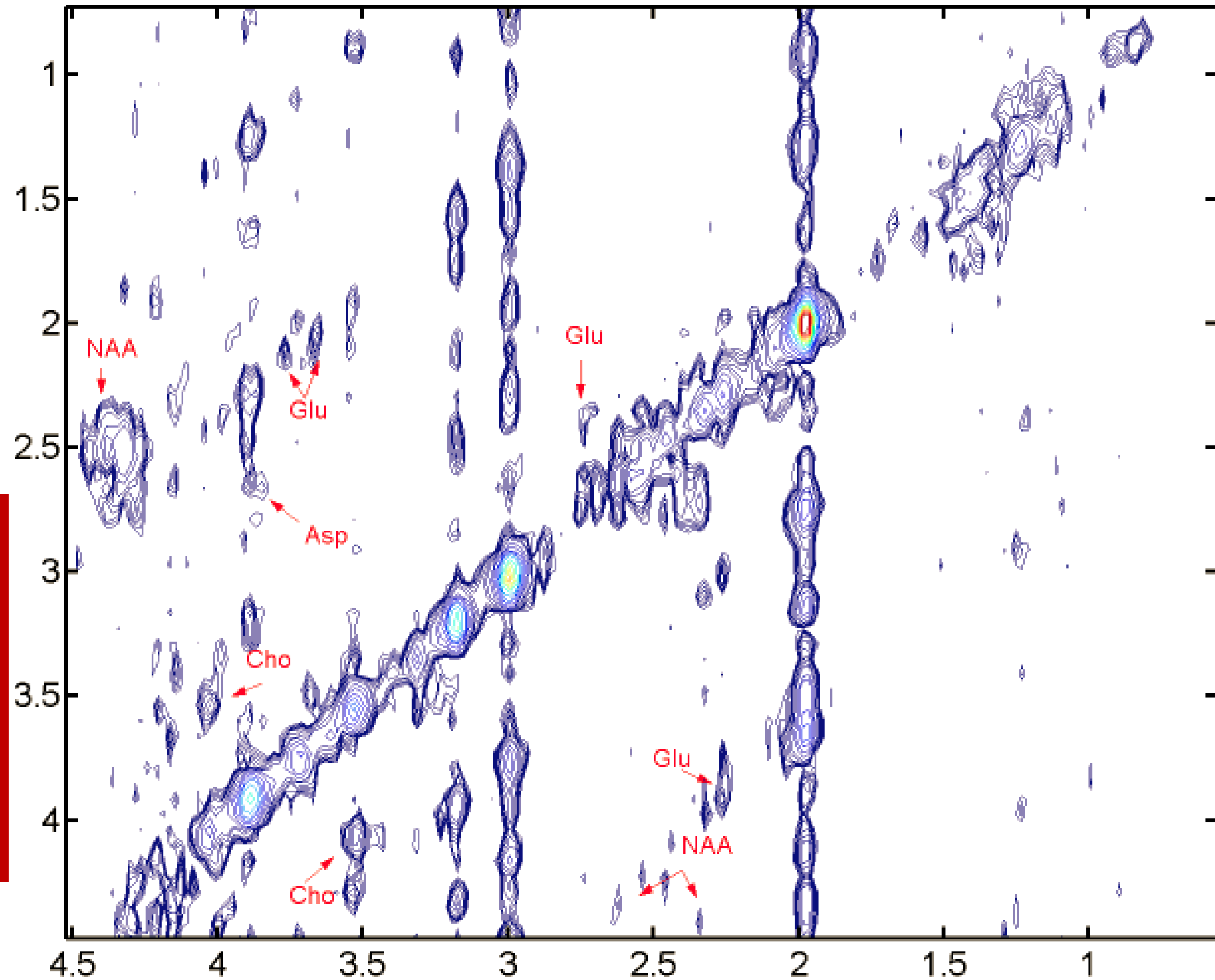
Windowing Artifacts



Challenges with In-Vivo Brain MRS

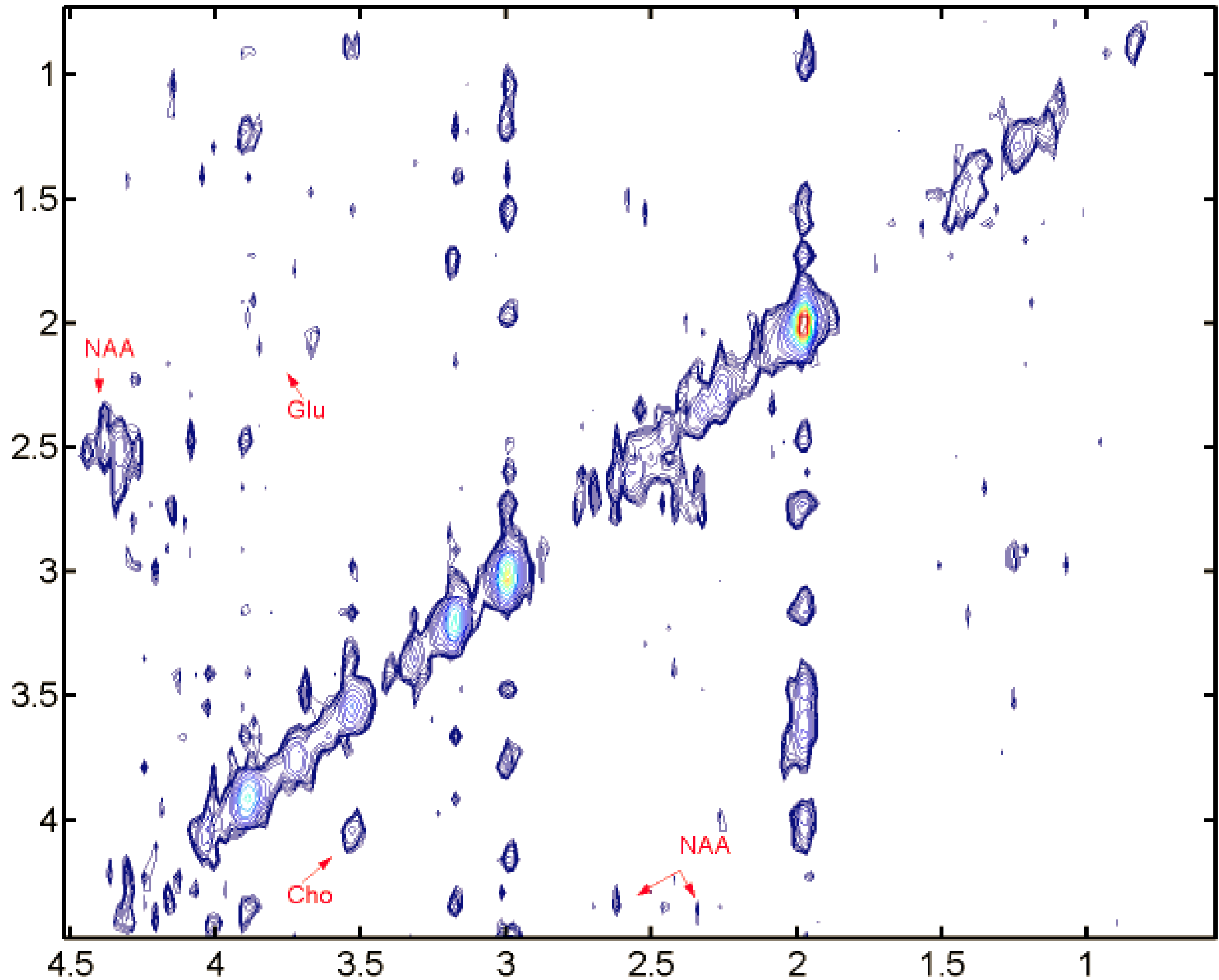
1) clutter due to sinc tail
2) hours in machine

Can sparse recovery help?



Compressive Sensing + 30% data

Lost some Biomarkers



Non-Integer Sparse FFT

- Problem and Model
  - Sparse in the continuous case
  - The railings are because of non-integer frequencies
- Algorithm
  - Use original sparse FFT to estimate integer frequencies
  - Use gradient descent algorithm to find the non-integer frequencies to minimize the residue of our estimation over the samples

Challenges with In-Vivo Brain MRS

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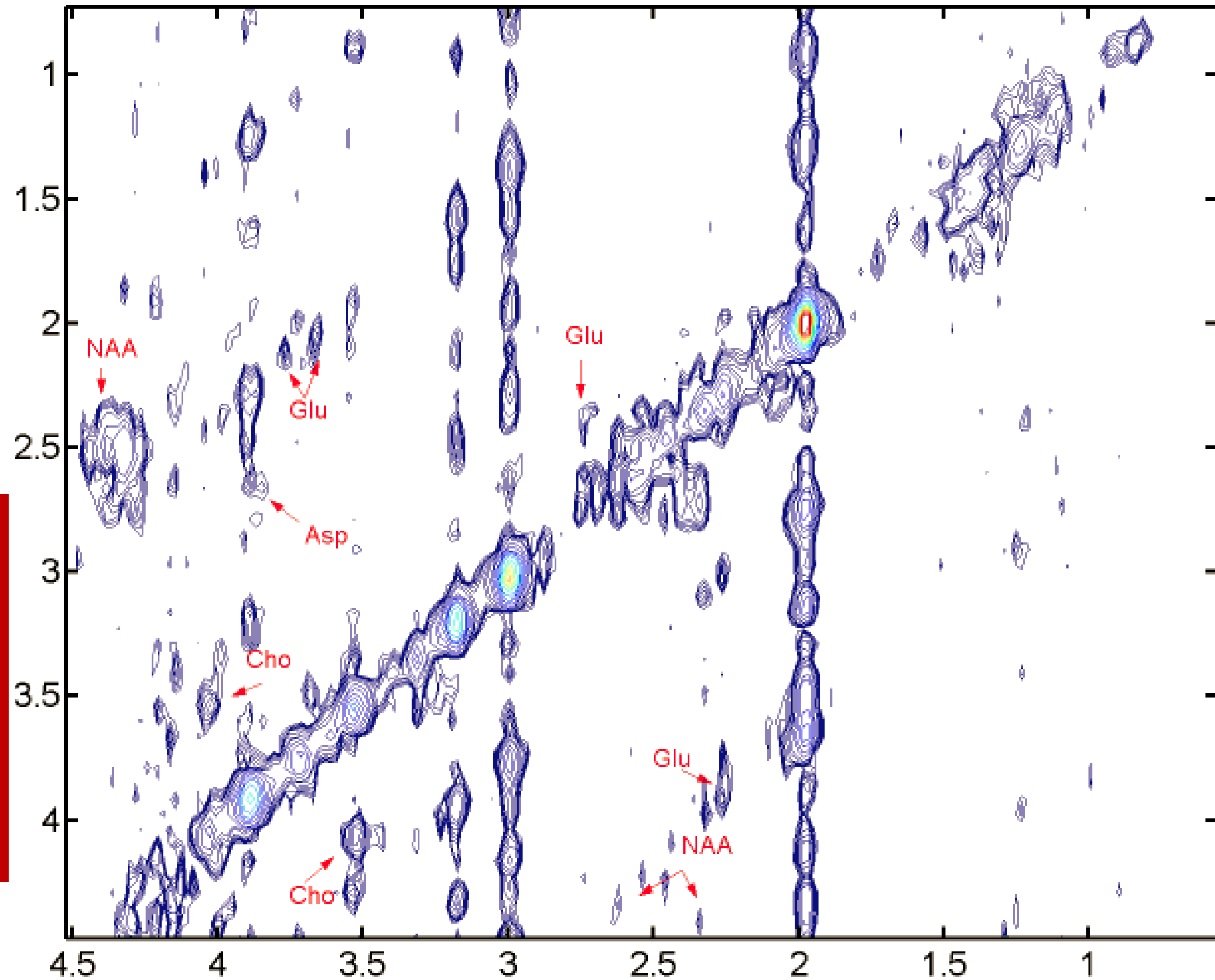
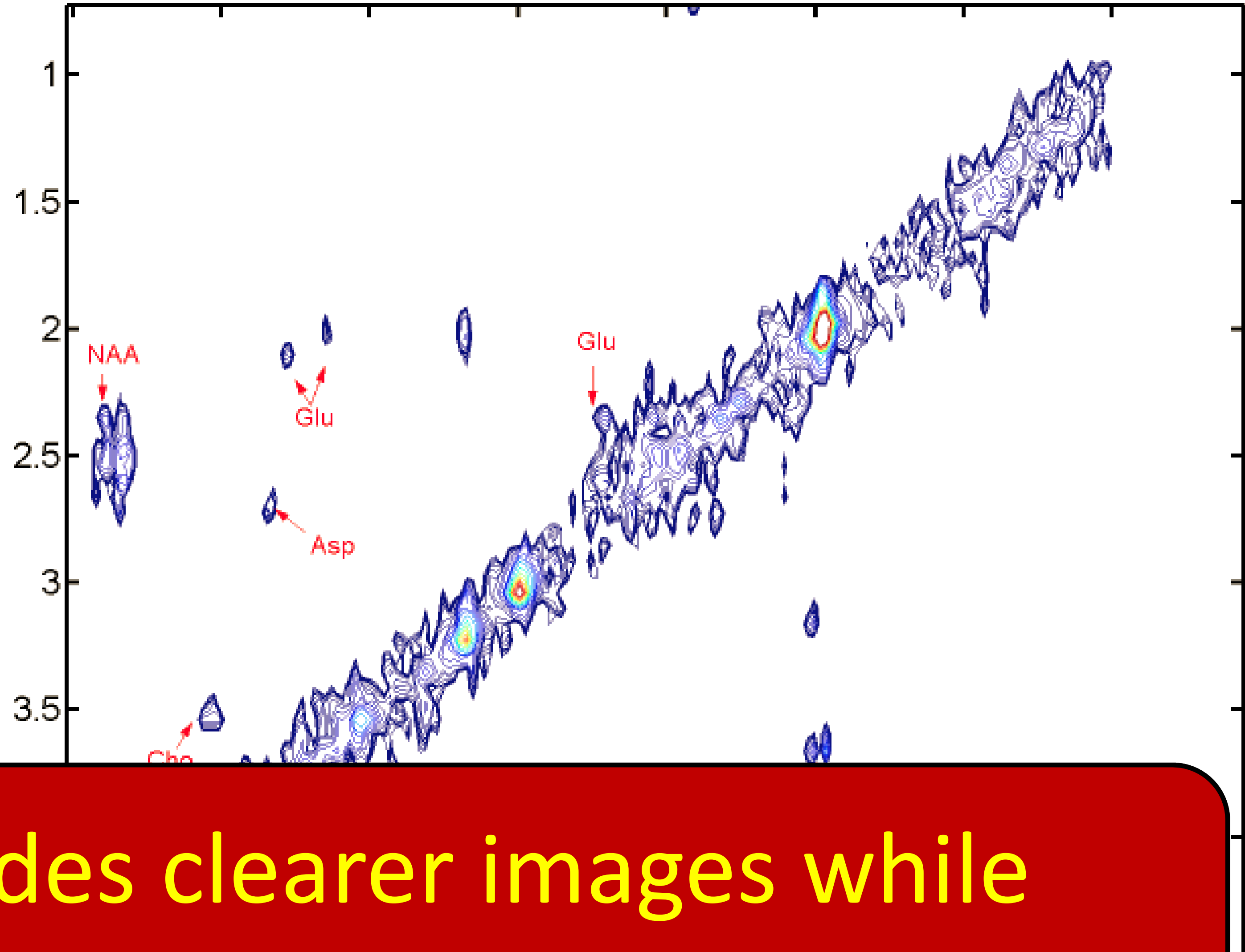


Can sparse recovery help?

Sparse FFT + 30% of data

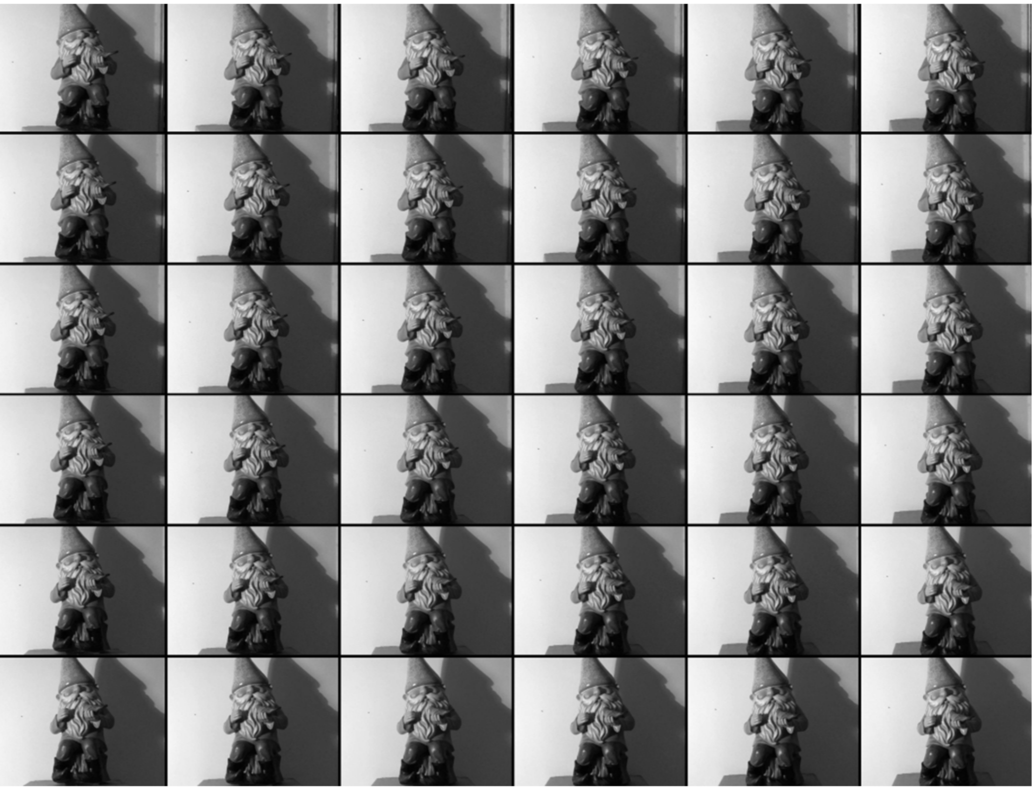
Removed Clutter without losing Biomarkers



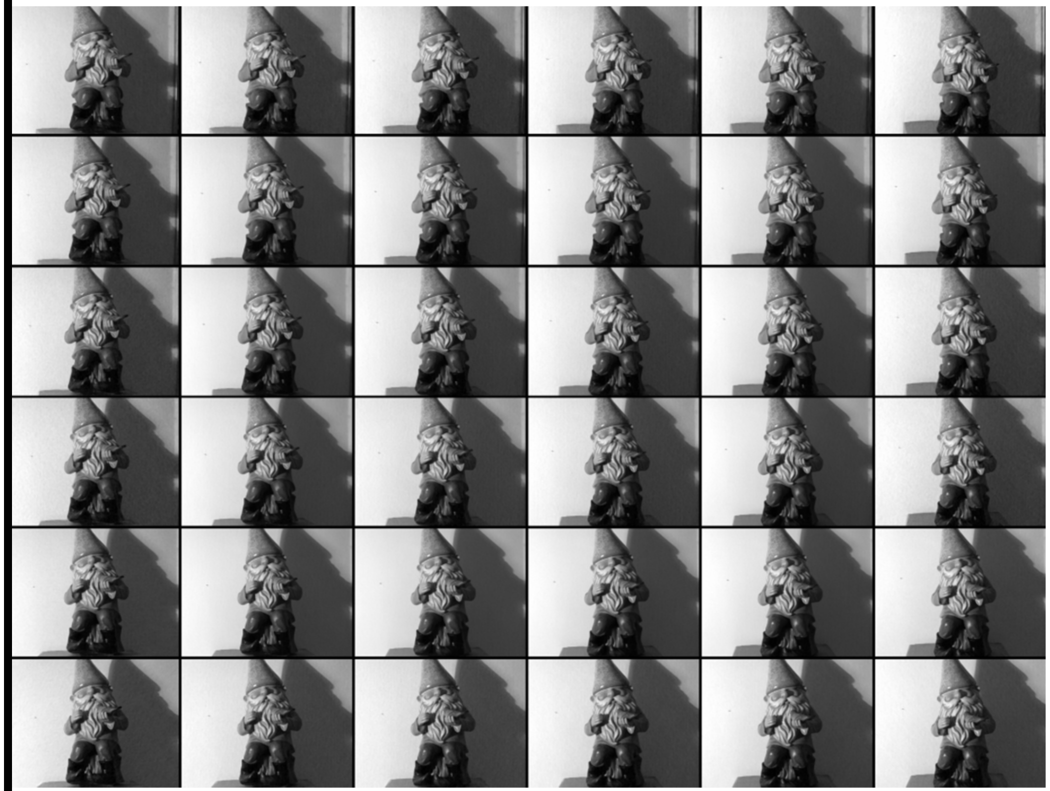
sFFT provides clearer images while reducing the acquisition time by 3x

Light-Field Photography

- Generate depth and perspective using images from a 2D camera array
- Images are correlated →4D frequencies are sparse
- <u>Goal</u>: Same performance but with fewer images



Original



Reconstructed with 11% of data

Conclusion

- Many applications are sparse in the frequency domain and hence can benefit from sFFT

- We showed that sFFT enables GHz low-power spectrum sensing and decoding, and improves MRS medical imaging and 4D light-filed capture

- We just scratched the surface and expect more applications soon