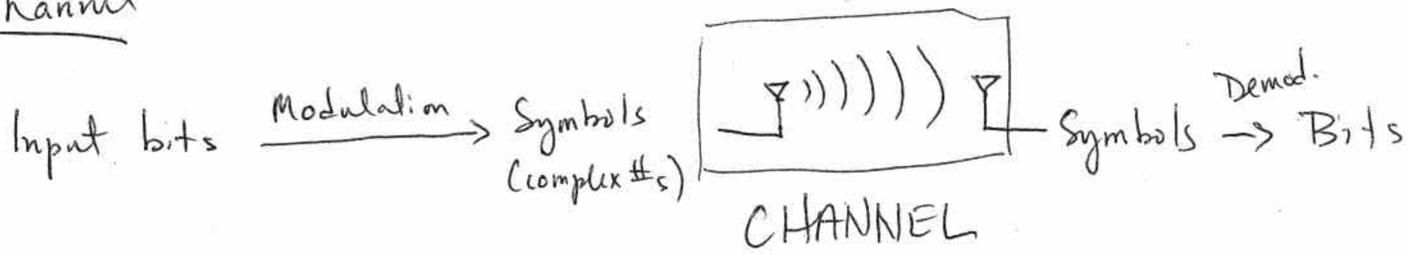
6.888 Fall 2013, Lecture 2 (9/12) (scribe notes)

Topics Covered in this lecture:

① Concept of channel
② Narrowband vs. Wideband
③ OFDM
  - Packet detection
  - Cyclic Prefix

Channel



The channel will transform a transmitted symbol:

- Attenuation
- Rotation
- Noise (assume Gaussian)

Consider a symbol x(t). After going through the channel, it is received as y(t). Then,

$$y(t) = \underset{\substack{\uparrow \\ \text{channel} \\ (\text{complex } \#)}}{h} \cdot x(t) + \underset{\substack{\uparrow \\ \text{noise}}}{n(t)}$$

Multiplying by h expresses attenuation & rotation.

- We assume h remains constant throughout the lifetime of a single packet (set of symbols).
  At walking speed, h may change every few seconds.

- So, by estimating h at the receiver, we can recover the original symbol. Let $\hat{h}$ be the estimate. Then,

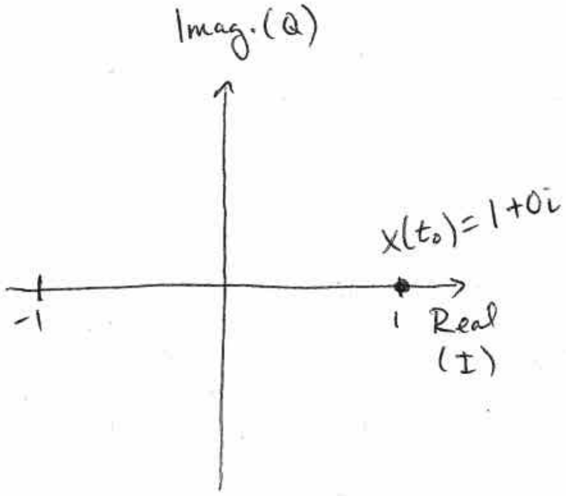
$$\hat{x}(t) = \frac{y(t)}{\hat{h}} \quad \text{(ignoring noise)}$$

The channel is estimated by using a <u>preamble</u>, which is a known sequence of symbols.

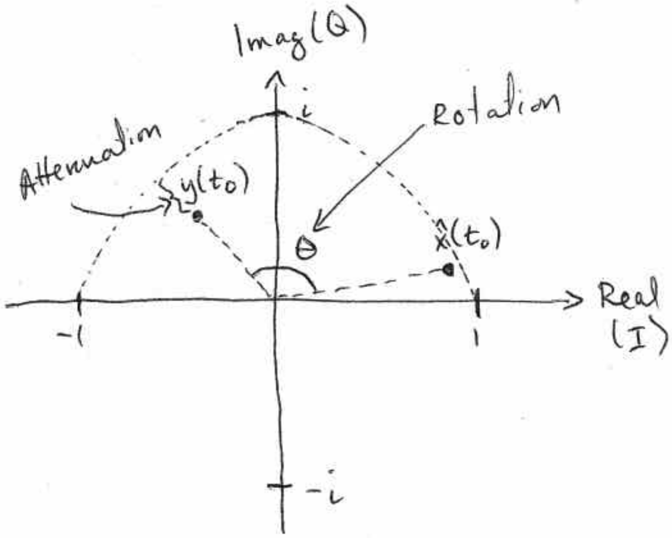
e.g. suppose $x(0)$ is known. Then, $\hat{h} = \frac{y(0)}{x(0)}$.

Improve the estimate by averaging over several symbols.

<u>Ex</u>: BPSK, which maps "0" bit -> 1, "1" bit -> -1.



<u>At Tx</u>



<u>At Rx</u>

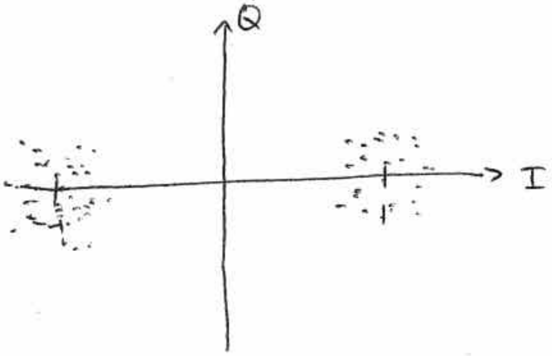
The receiver has (from previously), for example,

$$\hat{h} = \frac{2}{3} e^{j\theta}$$

Attenuation factor (pointing at $\frac{2}{3}$)   rotation by θ (pointing at $e^{j\theta}$)

Then, $\hat{x}(t_0) = \frac{y(t_0)}{\hat{h}}$. But, notice it is not exactly at $x(t_0) = 1+0i$. This is due to the noise.

Thus, over many symbols, the receiver sees a "cloud" around each modulation point:



So, in order to demodulate, simply pick the closest modulation point to the received symbol $\hat{x}(t)$.

BPSK encodes one bit per symbol, as each bit is mapped to one of the two possible modulation points.

Ex 4 QAM

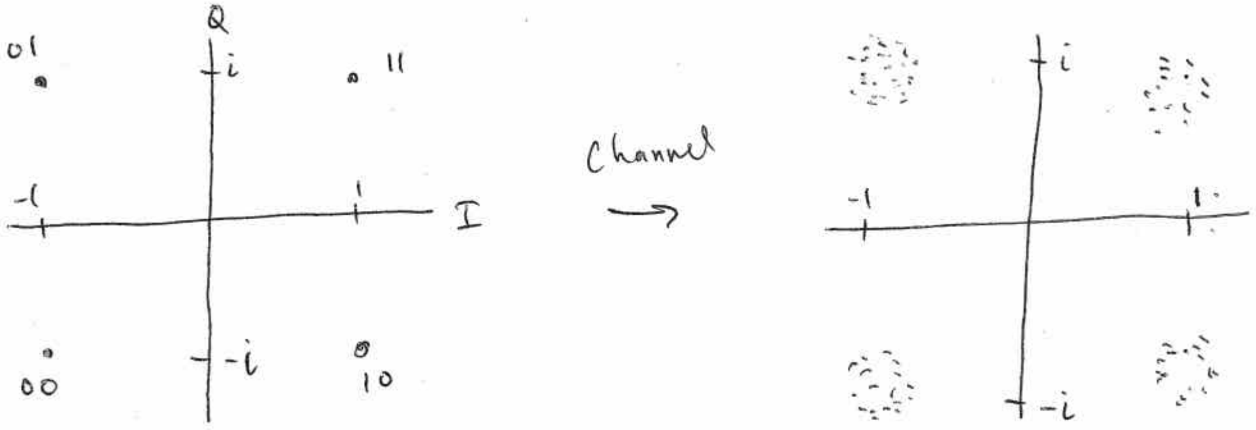
Encodes 2 bits per symbol:

$00 \to -1 - i$

$01 \to -1 + i$

$10 \to 1 - i$

$11 \to 1 + i$



At Tx At Rx

Note that noise (i.e. the cloud effect) limits the achievable rate. With too much noise, symbols become indistinguishable.

By increasing power (i.e. increase magnitude of symbol vector) we can space out the modulation points more, allowing greater noise tolerance.

Vector magnitude varies with power:

$$|v| = \sqrt{P}$$

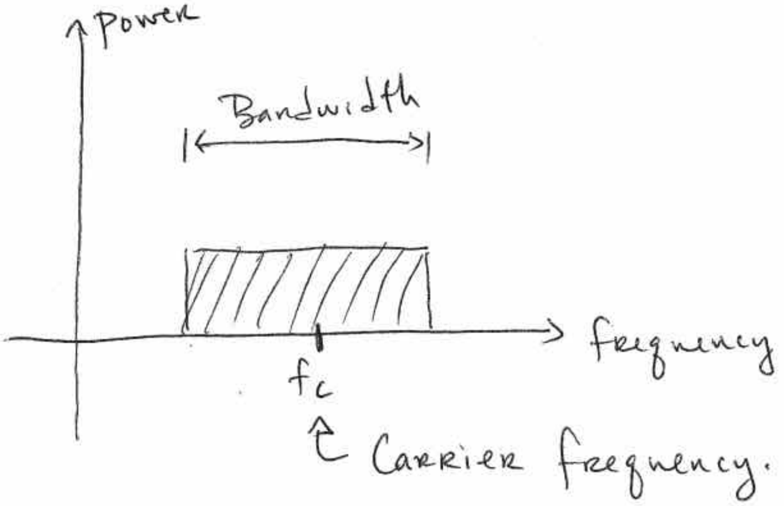
Now we can define the signal-to-noise ratio (SNR):

$$SNR = \frac{\text{signal power @ Rx}}{\text{Noise power}}$$

Because different receivers experience different SNR, the transmitter must choose a modulation scheme that each receiver can decode.

Narrowband vs. Wideband

Definition of bandwidth:



Typical 802.11 (wifi) has $f_c = 2.4$ GHz with 20 MHz bandwidth.

<u>Narrowband</u>: ≤ 1 MHz

<u>Wideband</u>: $\gg 1$ MHz

$\Rightarrow$ 20 MHz "wide ish"

(As an aside, maximum frequency is a practical limit: very high frequency oscillator hardware is very expensive to manufacture. 60-90 GHz is feasible today, but was not 5-10 years ago.)

To illustrate the difference between NB & WB, it's helpful to consider the channel models.

Consider a transmitter & receiver in free space placed at a distance d apart.

$$Tx \xrightarrow[d]{} Rx$$

Then,

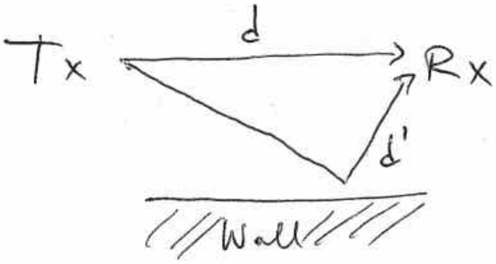
$$h = \boxed{\frac{1}{d}}\, e^{j\boxed{2\pi\, d/\lambda}}$$

Attenuation: inverse w/ d.

Change in phase. λ = wavelength = c/f
You rotate 2π every wavelength, so multiply by d to get total rotation.

This model ignores noise (among other complicating factors).

Now consider the same setup but with a reflecting surface (like a wall).



(total length of 2nd path is d').

This is called <u>multipath</u>. The new channel at the receiver is:

$$h = h_1 + h_2 = \frac{1}{d} e^{j2\pi d/\lambda} + \frac{1}{d_1} e^{j2\pi d_1/\lambda}$$

Which we can approximate by:

$$\approx \frac{1}{d}\left(e^{j2\pi d/\lambda}\right)\left[1 + e^{j2\pi \Delta d/\lambda}\right]$$

Then, consider the channel behavior at two wavelengths λ_1 and λ_2. Suppose λ_1 is set to a value such that $\frac{\Delta d}{\lambda_1} = \frac{1}{2}$. Then,

$$h = \frac{1}{d} e^{j2\pi d/\lambda_1}\left(1 + e^{j2\pi \cdot \frac{1}{2}}\right)$$

$$= \frac{1}{d} e^{j2\pi d/\lambda_1}\left(1 + -1\right) \qquad \{\text{Euler's formula}\}$$

$$= 0$$

In other words, the frequency $f_1 = c/\lambda_1$ will <u>not</u> arrive at the receiver.

Now suppose λ_2 is chosen such that $\Delta d / \lambda_2 = 1$. Then,

$$h = \frac{1}{d} e^{j 2\pi d/\lambda_2} \left(1 + e^{j 2\pi \cdot 1} \right)$$

$$= \frac{1}{d} e^{j 2\pi d/\lambda_2} \left(1 + 1 \right)$$

$$= 2 \cdot \frac{1}{d} e^{j 2\pi d/\lambda_2}$$

Meaning frequency $f_2 = c/\lambda_2$ will arrive at the channel <u>maximum</u> at the receiver.

The different response of the channel to different frequencies means that for wideband the different channels must be treated independently.

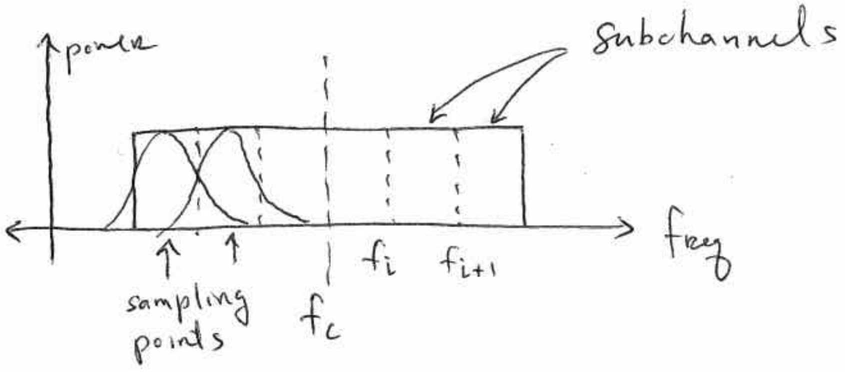
A wideband channel + multipath ⇒ <u>frequency selective</u>, i.e. different frequencies experience different attenuation and rotation.

OFDM

Orthogonal frequency division multiplexing.

Main idea: increase throughput by multiplexing a single wide channel into multiple orthogonal subchannels.

Orthogonality ⇒ adjacent subchannels do not nterfere.



Note the contribution of neighboring subchannels is 0 at the sampling point.

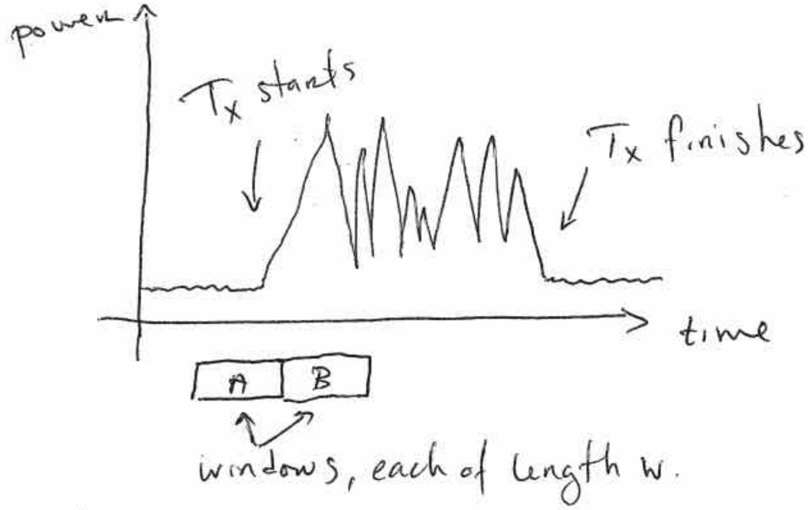
Data is modulated in the frequency domain in the array of subchannels, and then translated into the time domain (via IFFT) for transmission

Packet Detection

How does the receiver know when a packet (also called a frame) has begun, in order to start demodulating?

Simplest solution: sliding window algorithm.

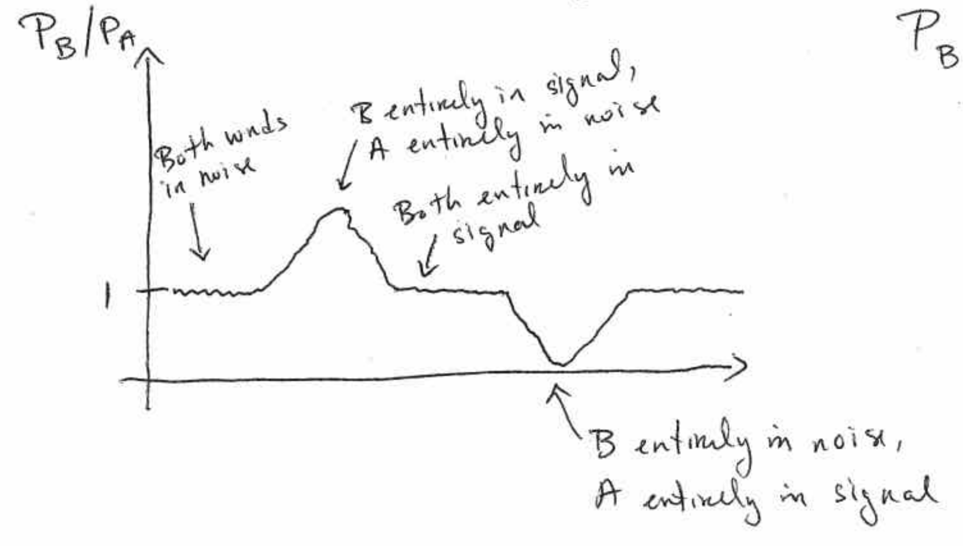
Maintain two windows in time, monitoring the spectrum. Use the average power ratio between the two windows to identify packet start/end.



Let $P_A = \sum_{t=1}^{w} |y(t)|^2$

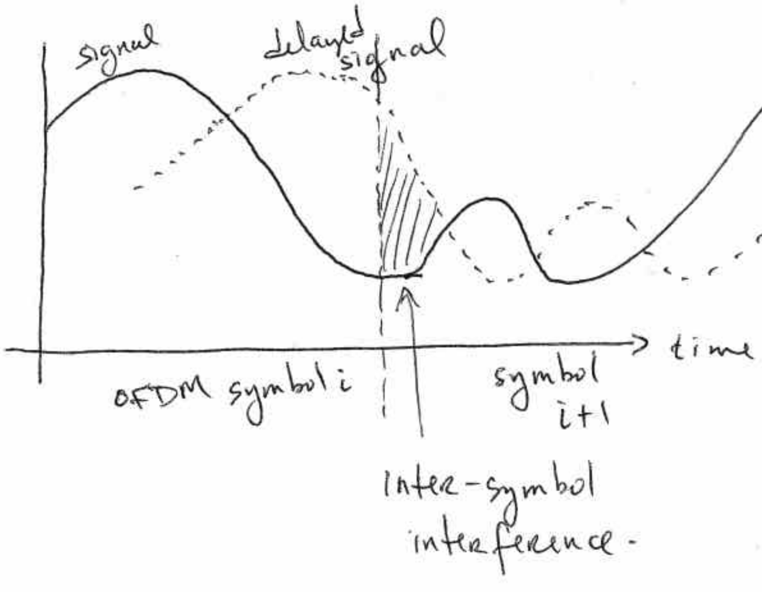
be the power in window A.

$$P_B = \sum_{t=w+1}^{2w} |y(t)|^2$$

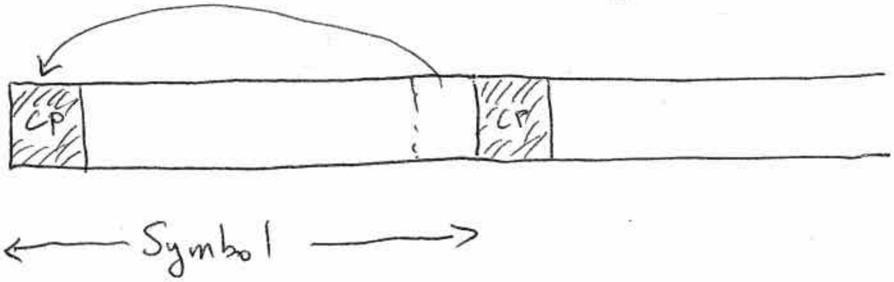


OFDM Cyclic Prefix

Addresses Inter-Symbol Interference (ISI), referring to interference between adjacent OFDM symbols.
(one OFDM symbol = N time samples (complex #s)).



Solution: add a cyclic prefix: put some samples from the end of the OFDM symbol on the beginning.



Due to the cyclic nature of the FFT, now a small shift in starting point does not matter.