Diagnosis as Semiring-based Constraint Optimization

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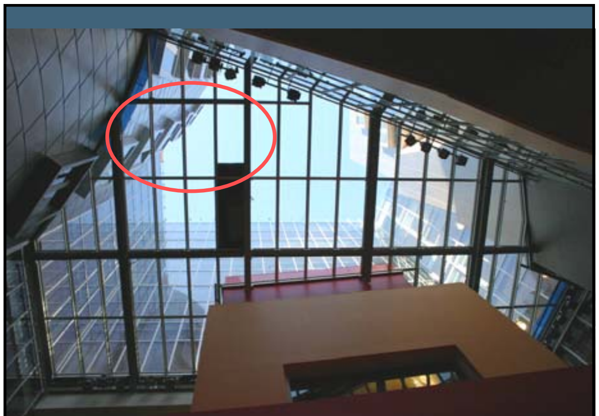
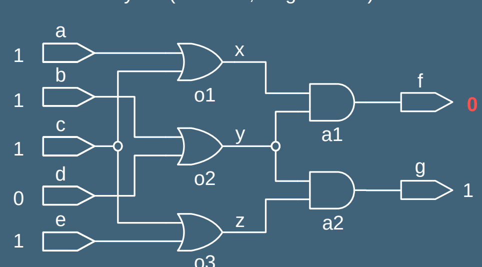


Overview

- Diagnosis traditionally viewed as logical reasoning
  - (de Kleer and Williams 87), (Reiter 87), …
- But more naturally viewed as constraint optimization
  - Minimal set of faulty components, most likely fault, …
- Framework that unifies qualitative and quantitative notions of diagnosis using semiring-based CSP
  - Choose appropriate semiring and construct constraints
- Diagnosis algorithms based on optimization methods
  - Dynamic programming with focus on leading solutions

Diagnostic Example

- Boolean Polycell (Williams, Ragno 2003)



Classical Formulation of Diagnosis

- Component Models (CSP)
  - Domains $D = D_1, \ldots, D_n$
  - Variables $X = x_1, x_2, \ldots, x_n$
  - Constraints $F = f_1, f_2, \ldots, f_m$
  - Constraints are functions $var(f_j) \rightarrow \{\perp, \top\}$
  - Solution is assignment to $Z \subseteq X$ satisfying constraints
- Preference Model
  - Cover faults minimally (Subset-Minimal Diagnosis)
  - Fewest faults (Cardinality-Minimal Diagnosis)
  - Most likely faults (Probabilistic Diagnosis)

Example: Component Models

- Constraints F = { f_{O1}, f_{O2}, f_{O3}, f_{A1}, f_{A2} }

O1	a	c	x	
G	1	1	1	T
B	1	1	0	T
B	1	1	1	T

O2	b	d	y	
G	1	1	1	T
B	1	1	0	T
B	1	1	1	T

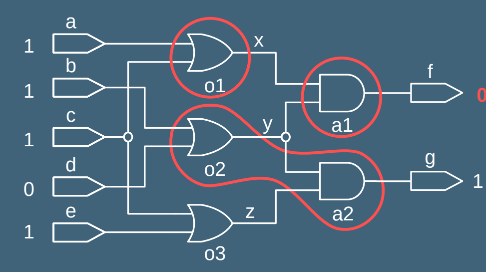
A1	x	y	f	
G	0	0	0	T
G	0	1	0	T
G	1	0	0	T
B	0	0	0	T
B	0	1	0	T
B	1	0	0	T
B	1	1	0	T

O3	c	e	z	
G	1	0	1	T
B	1	0	0	T
B	1	0	1	T

A2	y	z	g	
G	1	1	1	T
B	0	0	1	T
B	0	1	1	T
B	1	0	1	T
B	1	1	1	T

Example: Subset-Minimal Diagnosis

- Cover faults: o1=B {o1}, a1=B {a1}, o2=a2=B {o2,a2}



Diagnosis as Optimization

- Component Models (CSP)
  - Domains D = { D_1 , …, D_n }
  - Variables X = { x_1, x_2, …, x_n }
  - Constraints F = { f_1, f_2, …, f_m }
  - Constraints are functions var(f_i) $\rightarrow$ { $\perp$,$\top$ }
  - Solution is assignment to Z $\subseteq$ X satisfying constraints
- Preference Model
  - **Set of preferences A**
  - **Objective function U: Z $\rightarrow$ A**
  - **Partial order $\leq_A$ on A forming lattice**
    - **Complete: Each I $\subseteq$ A has lub, glb**
    - **Distributive: glb(a,lub(b,c)) = lub(glb(a,b),glb(a,c))**
  - **Solution maximizes U**

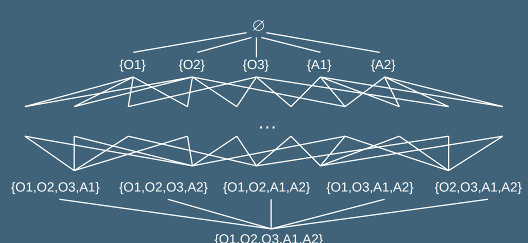
Example: Subset-Minimal Diagnosis

- Objective Function U: Z $\rightarrow$ 2^Z

O1	O2	O3	A1	A2	
G	G	G	G	G	∅
G	G	G	G	B	{A2}
G	G	G	B	G	{A1}
G	G	G	B	B	{A1,A2}
G	G	B	G	G	{O3}
G	G	B	G	B	{O3,A1}
…					…

Example: Subset-Minimal Diagnosis

- Partial order $\leq_A$ defined by set inclusion $\subseteq$



Soft Constraints

- Include Preferences in CSP
  - Domains D = D_1 , …, D_n
  - Variables X = x_1, x_2, …, x_n
  - Variables of interest Z $\subseteq$ X
  - Set of preferences A
  - Constraints F = f_1, f_2, …, f_m
  - Constraints are functions **var(f_i) $\rightarrow$ A**
- **How to combine preferences?**
  - **Generalize constraint combination (⊗)**
- **How to compare preferences?**
  - **Generalize constraint projection (⇓)**

Semiring-based CSPs (Bistarelli 95)

- Operator $\times$ to combine a,b $\in$ A (defines $\otimes$)
- Operator $+$ to compare a,b $\in$ A (defines $\Downarrow$)
  - a $\leq_A$ b iff a + b = b (b "better" than a)
- (A, +, ×, 0, 1) forms a c-semiring
  - + is commutative, associative, a + 0 = a
  - × is associative, a × 0 = 0
  - × distributes over +
  - + is idempotent
  - × is commutative
  - a + 1 = 1

Diagnosis as Semiring-based CSP

Component Models

Hard Constraints f_j: var(f_j) $\rightarrow$ { $\perp$,$\top$ }

Preference Model

Objective Function U: Z $\rightarrow$ A

Partial Order $\leq_A$

Separation

Soft Constraints f_j: var(f_j) $\rightarrow$ A

C-semiring

Semiring-based CSP (SCSP)

Construct Semiring

- Let 0 = $\perp$ = lub(A), 1 = $\top$ = glb(A), + = lub, × = glb

A2	y	z	g	
G	1	1	1	∅
B	0	0	1	∅
B	0	1	1	∅
B	1	0	1	∅
B	1	1	1	∅

Component Model

A1	O2	O3	A1	A2	
G	G	G	G	G	∅
G	G	G	G	B	{A2}
G	G	G	B	G	{A1}
G	G	G	B	B	{A1,A2}
G	G	B	G	G	{O3}
…					…

Preference Model

Separate Objective Function

- Faults are independent: U(t) = u_1(t) × u_2(t) × … × u_k(t)

O1	O2	O3	A1	A2	
G	G	G	G	G	∅
G	G	G	G	B	{A2}
G	G	G	B	G	{A1}
G	G	G	B	B	{A1,A2}
…					…

Preference Model

O1	
G	∅
B	{O1}

O2	
G	∅
B	{O2}

…

A2	
G	∅
B	{A2}

Construct Soft Constraints

- Apply each u_i to constraint f_j with var(u_i) $\subseteq$ var(f_j)

A2	y	z	g	
G	1	1	1	∅
B	0	0	1	∅
B	0	1	1	∅
B	1	0	1	∅
B	1	1	1	∅

Component Model

$\otimes$

A2	
G	∅
B	{A2}

Preference Model Part

=

A2	y	z	g	
G	1	1	1	∅
B	0	0	1	{A2}
B	0	1	1	{A2}
B	1	0	1	{A2}
B	1	1	1	{A2}

Soft Constraint

Notions of Diagnosis as SCSP

- Subset-Minimal Diagnosis
  - S_s = (2^Z, ∩, ∪, Z, ∅)
- Cardinality-Minimal Diagnosis
  - S_c =(N_0∪ ∞,min,+,∞,0)
- Probabilistic Diagnosis
  - S_p= ([0,1], max, ·, 0,1)

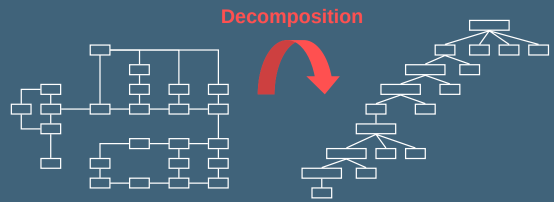
O1	a	c	x	
G	1	1	1	∅
B	1	1	0	{O1}
B	1	1	1	{O1}

O1	a	c	x	
G	1	1	1	0
B	1	1	0	1
B	1	1	1	1

O1	a	c	x	
G	1	1	1	.99
B	1	1	0	.01
B	1	1	1	.01

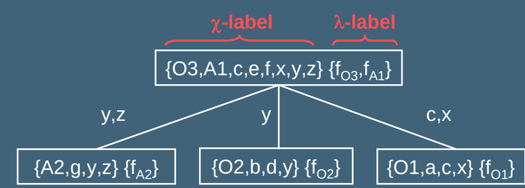
Diagnosis using Optimization Methods

- Construction of soft constraints doesn't affect network
- Decompose SCSP into equivalent acyclic instance
  - Combine constraints responsible for cyclicity



Tree Decomposition

- Tree T = (V,E) with labeling functions χ, λ such that:
  - For each $f_j \in F$, there is exactly one $v \in V$ such that $f_j \in \lambda(v)$; For this v, $vars(f_j) \subseteq \chi(v)$ ("covering")
  - For each $x_i \in X$, the set $\{v \in V \mid x_i \in \chi(v)\}$ induces a connected subtree of T ("connectedness")

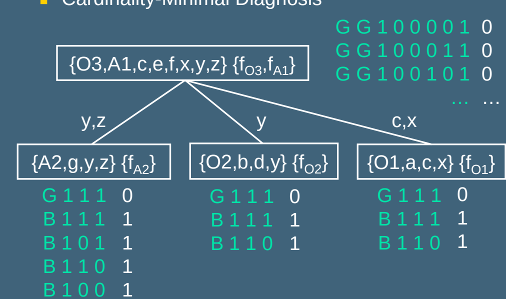


Tree-based Diagnosis Algorithms

- Solve tree-structured SCSP instance in two phases
  - Bottom-up dynamic programming phase
  - Top-down solution enumeration phase
- Focus on leading solutions using SCSP properties
  - Early pruning due to extensiveness

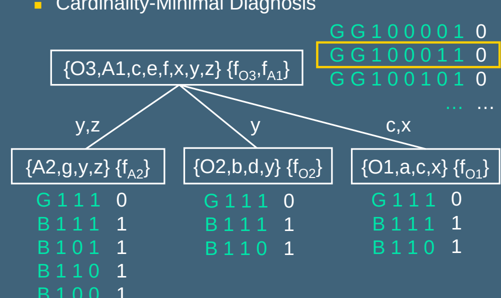
Bottom-Up Dynamic Programming

- Cardinality-Minimal Diagnosis



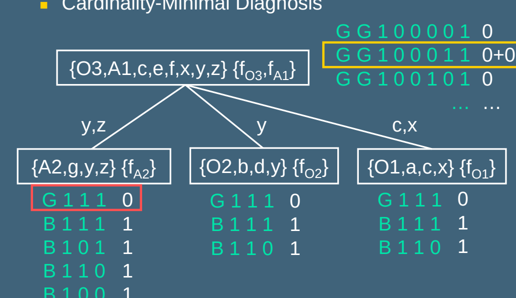
Bottom-Up Dynamic Programming

- Cardinality-Minimal Diagnosis



Bottom-Up Dynamic Programming

- Cardinality-Minimal Diagnosis



Bottom-Up Dynamic Programming

- Cardinality-Minimal Diagnosis

G G 1 0 0 0 0 1	0
G G 1 0 0 0 1 1	0+0+0
G G 1 0 0 1 0 1	0
…	…

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

y,z y c,x

{A2,g,y,z} $\{f_{A2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 0 1	1
B 1 1 0	1
B 1 0 0	1

{O2,b,d,y} $\{f_{O2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

{O1,a,c,x} $\{f_{O1}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

Bottom-Up Dynamic Programming

- Cardinality-Minimal Diagnosis

G G 1 0 0 0 0 1	0
G G 1 0 0 0 1 1	0+0+0+1
G G 1 0 0 1 0 1	0
…	…

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

y,z y c,x

{A2,g,y,z} $\{f_{A2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 0 1	1
B 1 1 0	1
B 1 0 0	1

{O2,b,d,y} $\{f_{O2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

{O1,a,c,x} $\{f_{O1}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

Bottom-Up Dynamic Programming

- Cardinality-Minimal Diagnosis

G G 1 0 0 0 0 1	3
G G 1 0 0 0 1 1	1
G G 1 0 0 1 0 1	2
…	…

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

y,z y c,x

{A2,g,y,z} $\{f_{A2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 0 1	1
B 1 1 0	1
B 1 0 0	1

{O2,b,d,y} $\{f_{O2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

{O1,a,c,x} $\{f_{O1}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

Example

G G 1 0 0 0 0 1	3	B G 1 0 0 1 0 0	3
G G 1 0 0 0 1 1	1	B G 1 0 0 1 0 1	3
G G 1 0 0 1 0 1	2	B G 1 0 0 0 1 1	4
G B 1 0 0 1 1 1	1	B G 1 0 0 1 0 1	4
G B 1 0 0 0 1 1	2	B B 1 0 0 1 1 1	2
G B 1 0 0 1 0 1	3	B B 1 0 0 0 1 1	3
G B 1 0 0 0 0 1	4	B B 1 0 0 1 1 0	3
B G 1 0 0 0 1 1	2	B B 1 0 0 0 1 0	4
B G 1 0 0 0 1 0	3	B B 1 0 0 1 0 1	4
		B B 1 0 0 1 0 0	4
		B B 1 0 0 0 0 1	5
		B B 1 0 0 0 0 0	5

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

y,z y c,x

Focussing on Leading Diagnoses

- Interested in a few best diagnoses, not all diagnoses
- Extensiveness property of c-semirings: $a \times b \leq_A a$
  - Allows cutting off solutions worse than threshold

Focussing on Leading Diagnoses

- Threshold: 2 (Double faults)

~~G G 1 0 0 0 0 1~~	~~3~~
G G 1 0 0 0 1 1	1
G G 1 0 0 1 0 1	2
…	…

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

y,z y c,x

{A2,g,y,z} $\{f_{A2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 0 1	1
B 1 1 0	1
B 1 0 0	1

{O2,b,d,y} $\{f_{O2}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

{O1,a,c,x} $\{f_{O1}\}$

G 1 1 1	0
B 1 1 1	1
B 1 1 0	1

Example

~~GG100001 3~~
GG100011 1
GG100101 2
GB100111 1
GB100011 2
~~GB100101 3~~
~~GB100001 4~~
BG100011 2
~~BG100010 3~~

~~BG100100 3~~
~~BG100101 3~~
~~BG100011 4~~
~~BG100101 4~~
BB100111 2
~~BB100011 3~~
~~BB100110 3~~
~~BB100010 4~~
~~BB100101 4~~
~~BB100100 4~~
~~BB100001 5~~
~~BB100000 5~~

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

y,z   y   c,x

Top-Down Solution Extraction

- Cardinality-Minimal Diagnosis

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

GG100011 1
GG100101 2
… …

y,z   y   c,x

{A2,g,y,z} $\{f_{A2}\}$
G111 0
B111 1
B101 1
B110 1
B100 1

{O2,b,d,y} $\{f_{O2}\}$
G111 0
B111 1
B110 1

{O1,a,c,x} $\{f_{O1}\}$
G111 0
B111 1
B110 1

Top-Down Solution Extraction

- O3=G,A1=G,c=1,x=0,y=1,z=1

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

GG100011 1
GG100101 2
… …

y,z   y   c,x

{A2,g,y,z} $\{f_{A2}\}$
G111 0
B111 1
B101 1
B110 1
B100 1

{O2,b,d,y} $\{f_{O2}\}$
G111 0
B111 1
B110 1

{O1,a,c,x} $\{f_{O1}\}$
G111 0
B111 1
B110 1

Top-Down Solution Extraction

- O3=G,A1=G,A2=G,c=1,x=0,y=1

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

GG100011 1
GG100101 2
… …

y,z   y   c,x

{A2,g,y,z} $\{f_{A2}\}$
G111 0
B111 1
B101 1
B110 1
B100 1

{O2,b,d,y} $\{f_{O2}\}$
G111 0
B111 1
B110 1

{O1,a,c,x} $\{f_{O1}\}$
G111 0
B111 1
B110 1

Top-Down Solution Extraction

- O2=G,O3=G,A1=G,A2=G,c=1,x=0

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

GG100011 1
GG100101 2
… …

y,z   y   c,x

{A2,g,y,z} $\{f_{A2}\}$
G111 0
B111 1
B101 1
B110 1
B100 1

{O2,b,d,y} $\{f_{O2}\}$
G111 0
B111 1
B110 1

{O1,a,c,x} $\{f_{O1}\}$
G111 0
B111 1
B110 1

Top-Down Solution Extraction

- O1=B,O2=G,O3=G,A1=G,A2=G

{O3,A1,c,e,f,x,y,z} $\{f_{O3},f_{A1}\}$

GG100011 1
GG100101 2
… …

y,z   y   c,x

{A2,g,y,z} $\{f_{A2}\}$
G111 0
B111 1
B101 1
B110 1
B100 1

{O2,b,d,y} $\{f_{O2}\}$
G111 0
B111 1
B110 1

{O1,a,c,x} $\{f_{O1}\}$
G111 0
B111 1
B110 1

SAB and TREE*

- SAB (Fattah Dechter 95)
  - Cardinality-Minimal Diagnosis
  - No threshold
- TREE* (Stumptner Wotawa 01)
  - Cardinality-Minimal Diagnosis
  - Combines bottom-up and top-down phases
  - Threshold
- Both are special instances of our framework

Conclusion

- Shift from logic view to optimization view of diagnosis
- Unifying framework for qualitative and quantitative diagnosis using semiring-based CSPs
- Solution methods based on decomposition and dynamic programming