Generalized Conflict Learning for Hybrid Discrete/Linear Optimization

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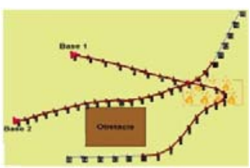
Outline

Motivation

The continuous model-based execution problem:

- A dynamic system (a *plant*)
- A desired evolution of the plant state over time (a *temporally flexible state plan*)
- A consistent control sequence

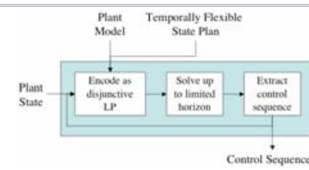


Fire fighting example [Léauté-AAAI-05]

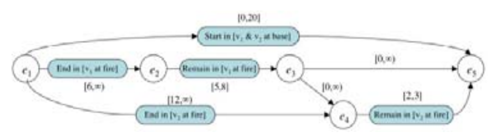
- Two UAVs
- A fire to extinguish
- Avoid obstacles
- Drop water
- Assess the damage

Motivation

Receding horizon continuous planner:



The state plan of the fire fighting example:



Encodings

In [Léauté-AAAI-05] the plant and the state plan within each limited horizon are encoded in disjunctive linear programs (DLPs) [Balas-ADM-79].

- State plan encoding example

$$\bigwedge_{t=0\ldots N_t} \left\{ \begin{array}{c} T(e_S) \le T_0 + t \cdot \Delta T \\ \wedge \quad T(e_E) \ge T_0 + t \cdot \Delta T \end{array} \right\} \Rightarrow \mathbf{s}_t \in D_v$$

- Plant model encoding example

$$\bigwedge_{t=1\ldots N_t} \bigvee_{i=1\ldots n_{P_R}} \mathbf{a}_i^T \mathbf{s}_t \ge b_i$$

A fast algorithm is needed to solve the DLPs for each horizon.

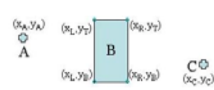
⇨ Generalized Conflict-Directed Branch and Bound

Disjunctive Linear Programs

Definition:

$$\text{Minimize } f(x)$$
$$\text{Subject to } \bigwedge_i (\underbrace{\bigvee_j C_{ij}(x) \le 0)}_{\text{clause}}$$

Example:



$$\text{Minimize } f(x)$$
$$\text{Subject to } g(x) \le 0$$
$$x_t \le x_L \vee x_t \ge x_R \vee y_t \le y_B \vee y_t \ge y_T, \ \forall t = 1, \ldots, n$$

Other Formulations

DLP:

$$\begin{aligned}&\textit{Minimize } f(x)\\&\textit{Subject to } \; g(x) \le 0\\&\qquad x_t \le x_L \vee x_t \ge x_R \vee y_t \le y_B \vee y_t \ge y_T,\\&\qquad \forall t = 1,\dots,n\end{aligned}$$

Binary Integer Program (BIP):

$$\begin{aligned}&\textit{Minimize } f(x)\\&\textit{Subject to } \; g(x) \le 0\\&\qquad x_t - x_L \le M(1 - b_{t1})\\&\qquad x_t - x_R \ge M(b_{t2} - 1)\\&\qquad y_t - y_B \le M(1 - b_{t3})\\&\qquad y_t - y_T \ge M(b_{t4} - 1)\\&\qquad \sum_{j=1,\dots,4} b_{tj} \ge 1\\&\qquad b_{tj} \in \{0,1\},\ \forall j = 1,\dots,4\\&\qquad \forall t = 1,\dots,n\end{aligned}$$

- LCNF [Wolfman-IJCAI-99]
  "Trigger" linear inequalities with propositional variables
- Mixed Logical Linear Programs (MLLPs) [Hooker-JDAM-99]
  - Generalization from LCNF
  - optimization
  - variables over finite domain
  - logic forms other than CNF

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Generalized Conflict-Directed Branch and Bound

- Building upon Branch and Bound (B&B)
- Two key features:
  - Generalized Conflict Learning
    - infeasibility
    - sub-optimality
  - Forward Conflict-Directed Search
    - constituent kernel
    - kernel
    - DLP candidate

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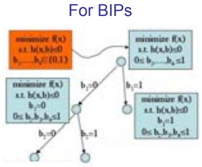
Branch and Bound

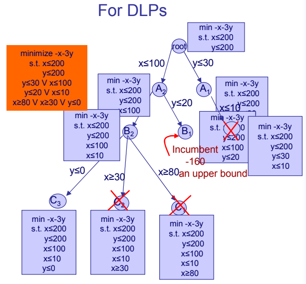
- Discrete + Continuous
- "Branch"
- "Bound"

For BIPs



p' is a relaxed LP of an optimization problem p, if the feasible region of p' contains the feasible region of p, and they have the same objective function.
⟶ a lower bound

For DLPs



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Generalized Conflict Learning

- Infeasibility conflict

$$\begin{aligned}&\textit{Minimize } -x-3y\\&\textit{Subject to } \; x \le 200\\&\qquad y \le 200\\&\qquad x \le 100 \vee y \le 50\\&\qquad y \le 0 \vee x \ge 80 \vee x \ge 30\\&\qquad x \le 10 \vee y \le 5 \vee y \le 4\end{aligned}$$

An infeasibility conflict, since the constraints are not satisfiable for any value of x.

A *minimal* infeasibility conflict

- Sub-optimality conflict

$$\begin{aligned}&\textit{Minimize } -x-3y\\&\textit{Subject to } \; x \le 200\\&\qquad y \le 200\\&\qquad x \le 100 \vee y \le 50\\&\qquad y \le 0 \vee x \ge 80 \vee x \ge 30\\&\qquad x \le 10 \vee y \le 5 \vee y \le 4\end{aligned}$$

A sub-optimality conflict, since the best solution that satisfies it has value -100 > f(x*).

A *minimal* sub-optimality conflict

Incumbent: x*(200,0)

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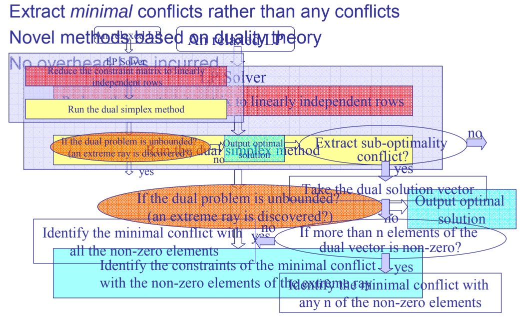
Minimal Conflict Extraction

- Extract *minimal* conflicts rather than any conflicts
- Novel methods based on duality theory
- No overhead is incurred



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Forward Conflict-Directed Search

- *Backward* conflict-directed methods use conflicts to select backtrack points and as a cache to prune nodes without testing consistency.
  - dependency-directed backtracking [Stallman77]
  - conflict-directed backjumping [Prosser93]
  - dynamic backtracking [Ginsberg93]
  - LPSAT [Wolfman99].
- *Forward* conflict-directed search guides the forward step of search away from regions of the state space that are ruled out by known conflicts [Williams - CD-A* - JDAM05].
- Our experimental results on a range of cooperative vehicle plan execution problems show that forward conflict-directed search significantly outperforms backtrack search with conflicts.

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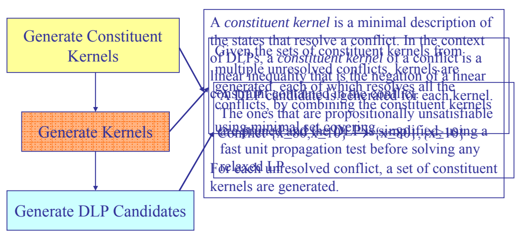
Forward Conflict-Directed Search



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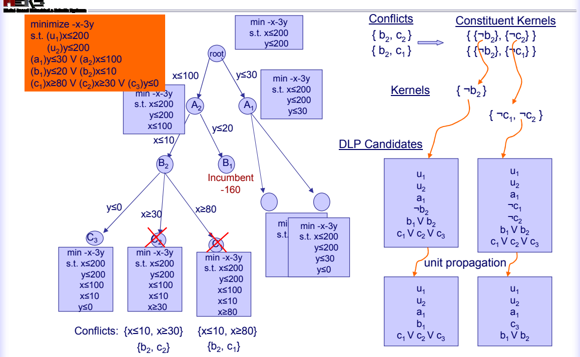
Example



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Best-First Search (BFS) v.s. Depth-First Search (DFS)

- BFS is more efficient than DFS in time.
- BFS can take dramatically more memory space than DFS.
- With conflict learning and forward conflict-directed search, BFS takes similar memory space to DFS.
- The concept of sub-optimality is rooted in maintaining an incumbent. Hence, it can be applied to DFS but not to BFS.

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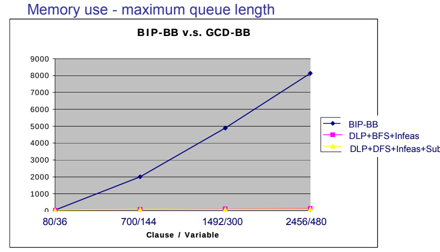
Empirical Evaluation

Test problems: Model-based temporal plan execution for cooperative vehicles [Léauté-AAAI-05].
Comparisons: GCD-BB v.s. BIP-BB
algorithmic variants of GCD-BB
Measures: Computation time - number of LPs & average LP size
Memory use - maximum queue length



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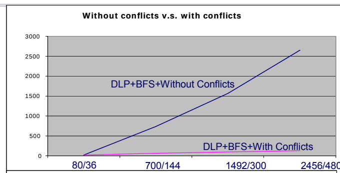
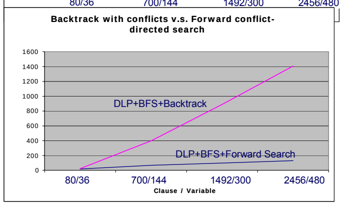
Empirical Evaluation





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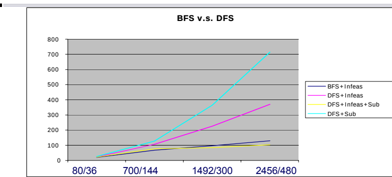

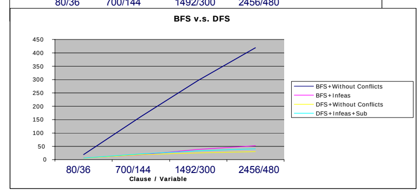

Conclusion

Generalized Conflict-Directed Branch and Bound (GCD-BB)
- branch and bound for DLPs
- generalized conflict learning
- forward conflict-directed search

An order of magnitude speed-up over BIP-BB.

Future Work

- Run GCD-BB on a range of well-known benchmark problems, and compare its actual runtime against that of BIP-BB.
- Study empirically the reason why sub-optimality conflicts do not speed up search as much as infeasibility conflicts.
- Apply GCD-BB to a more general form of HDLOPs than DLPs.
- Extend conflict learning to non-linear programming.