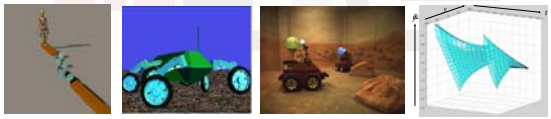
Massachusetts Institute of Technology

Exploiting Spatial and Temporal Flexibility for Plan Execution of Hybrid, Under-actuated Systems

Andreas Hofmann and Brian Williams



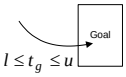
Model-based Embedded & Robotic Systems

CSAIL

Temporal Plan Execution for Continuous Systems



- Plan temporal and state constraints
- Plant dynamics and actuation limits



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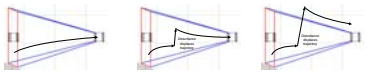
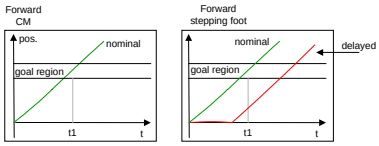


Synchronization Example: Trip Recovery



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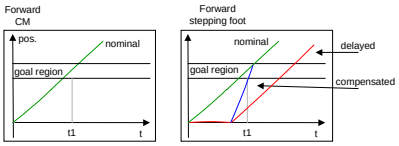

Problem Statement

Muybridge

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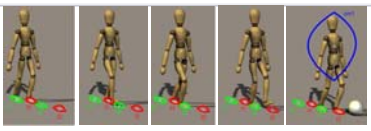
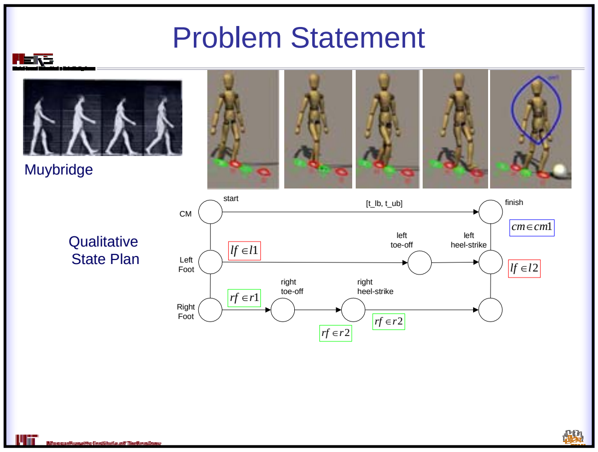
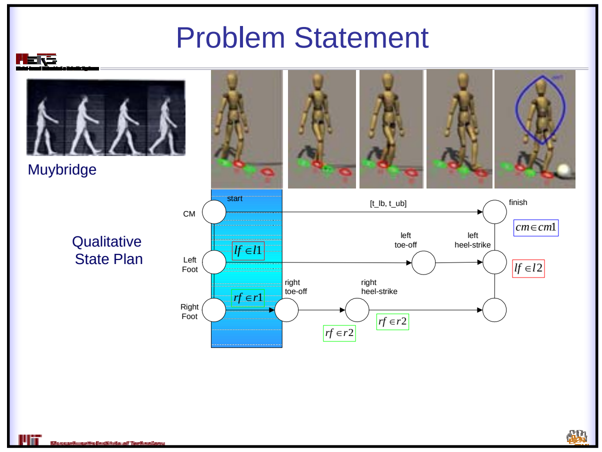



Problem Statement
Muybridge
Qualitative State Plan
start
[t_lb, t_ub]
finish
CM
Left Foot
Right Foot
left toe-off
left heel-strike
right toe-off
right heel-strike
lf ∈ l1
rf ∈ r1
cm ∈ cm1
lf ∈ l2
rf ∈ r2
rf ∈ r2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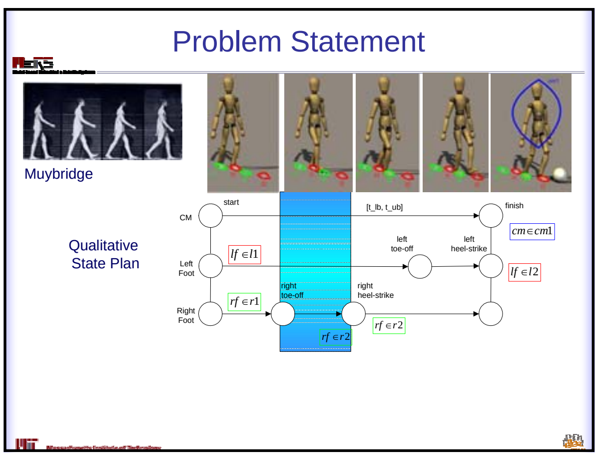
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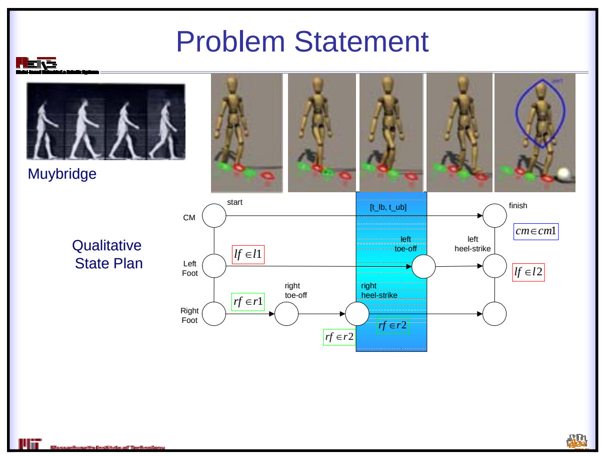
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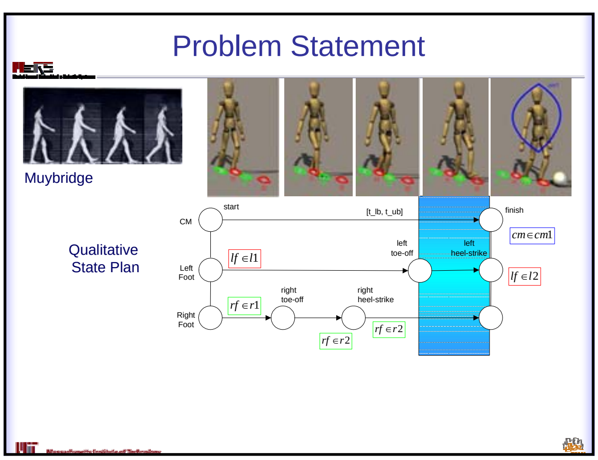
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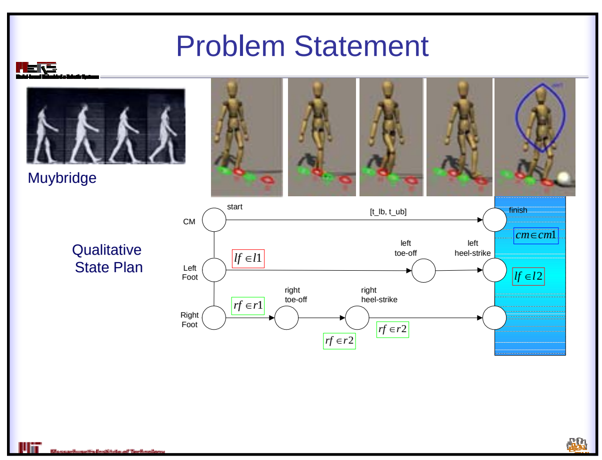
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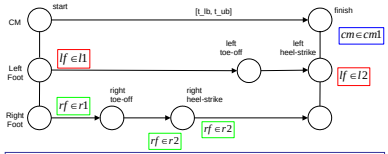
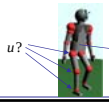


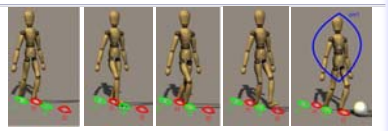
Plant



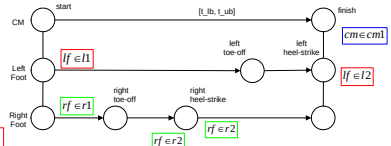
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u})$$
$$\mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{0}$$

Problem Statement

Muybridge

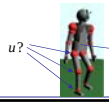


Qualitative State Plan



Compute u such that resulting state trajectory satisfies plan
-state constraints
-temporal constraints

Plant



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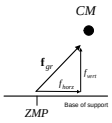
Problem Statement

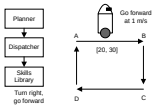
- Achieve state-space and temporal goals specified in plan
- Achieve robustness by exploiting plan flexibility
- Detect plan failure as early as possible

- Challenges
  - High dimensionality
  - Actuation limits
  - Interaction of limits from plan with limits of plant



Key Innovations

Temporal plan executive



Key Innovations

Temporal plan executive

Robust to temporal disturbances

Ignores continuous dynamics

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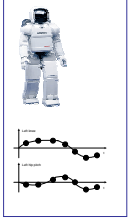
Temporal plan executive

Robust to temporal disturbances

Ignores continuous dynamics

High impedance tracking

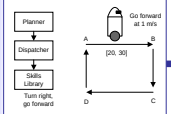


Takes continuous dynamics into account

Not flexible to disturbances

Key Innovations

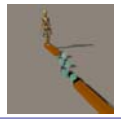
Temporal plan executive



Model-based executive

Takes continuous dynamics into account

Uses plan flexibility to handle state and temporal disturbances



High impedance tracking

Robust to temporal disturbances

Ignores continuous dynamics

Takes continuous dynamics into account

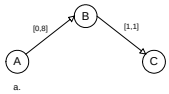
Not flexible to disturbances

Roadmap

- Introduction
  - Problem statement, innovations
- Background and approach
  - Temporal plan execution systems
  - Robot trajectory tracking systems
  - Flow tubes
- Architecture and implementation
- Results
- Summary

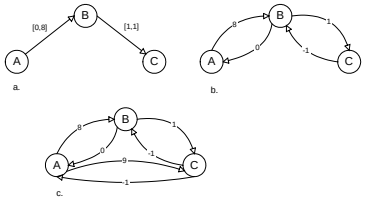
Temporal Plan Execution Systems

- Activity plan consists of events and activities.
- Activities have temporal constraints.



Temporal plan execution systems

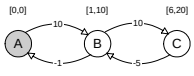
- Activity plan consists of events and activities.
- Activities have temporal constraints.
- Activity plan compiled into distance, dispatchable graph [Muscettola, 1998].



Temporal plan dispatcher

1. Initialize execution windows.

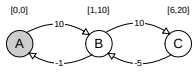
a. T=0



Temporal plan dispatcher

1. Initialize execution windows.
2. Schedule next event.
   - Set event execution time to valid time in window
3. Wait until event time.

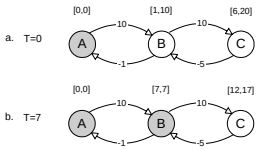
a. T=0



Ex. Event time for B = 7

Temporal plan dispatcher

1. Initialize execution windows.
2. Schedule next event.
   - Set event execution time to valid time in window
3. Wait until event time.
4. Update execution windows.



Temporal plan dispatcher

1. Initialize execution windows.
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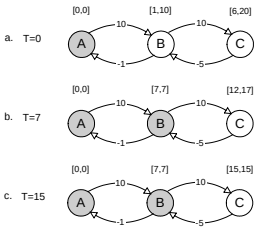
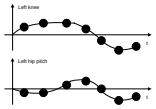
   - Set event execution time to valid time in window
3. Wait until event time.
4. Update execution windows.
5. If no more events, then done, else, go to 2.



High impedance ref. trajectory tracking

Hirai, et al., 1998



Takes continuous dynamics into account

Not flexible to disturbances

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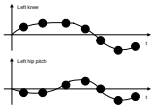

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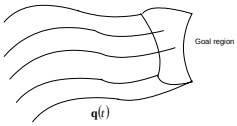


Flow tube: all trajectories that satisfy plan goal

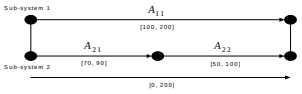
- fully exploits plan flexibility
- always know if state is feasible

[Bradley and Zhao, 1993]

[Frazzoli, 2000]

Extend temporal plan execution

- Extend to hybrid systems through use of flow tubes
- Begin with plan specifying temporal constraints for activities



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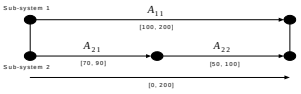
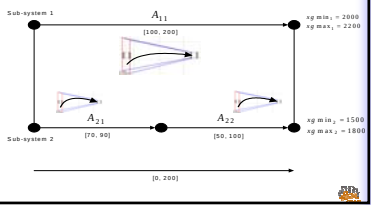
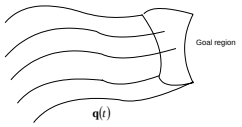


- Add continuous state constraints (CM, foot placement)
- Compute flow tubes

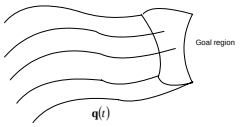


Flow tube computation



Flow tubes computed by reachability analysis
[Bradley and Zhao, 1993]
[Bemporad, et al., 2002]

Flow tube computation



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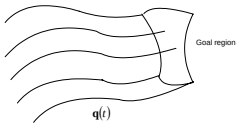
Problem: highly nonlinear system, $\mathbf{q} \in \Re^{18}$

Reachability analysis only works for small, linear systems!

Flow tube computation



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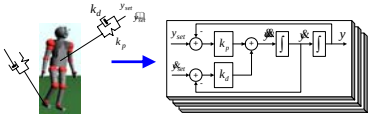
Problem: highly nonlinear system, $\mathbf{q} \in \Re^{18}$

Reachability analysis only works for small, linear systems!

Solution: linearize and decouple plant into set of smaller linear systems.

[Hofmann, et al., 2004]
[Khatib, et al., 2004



Abstracted Plant

- Use of abstracted plant presents new challenges.
  - Decoupled sub-systems must be synchronized
  - Example: Fwd. movement of CM and swing foot

Key idea: goal region arrival time can be adjusted by adjusting parameters

- Range of arrival times is controllable, subject to initial state, actuation limits
- Controllable temporal range of an activity: a key concept in temporal plan execution systems

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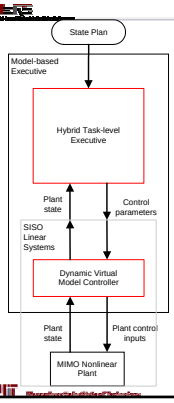
Model-based Executive



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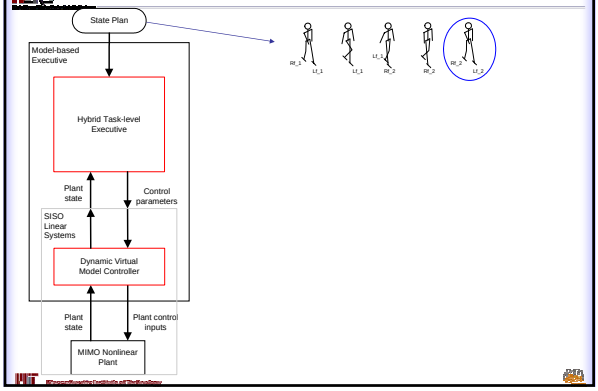

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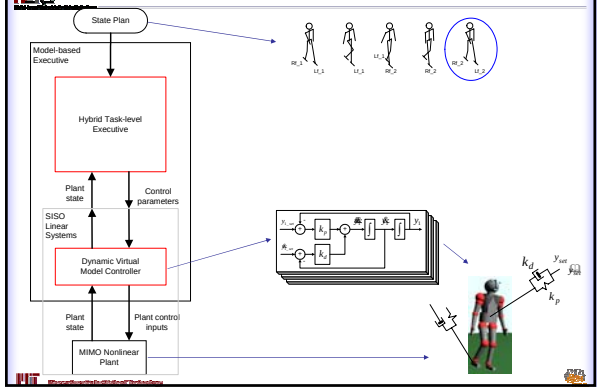

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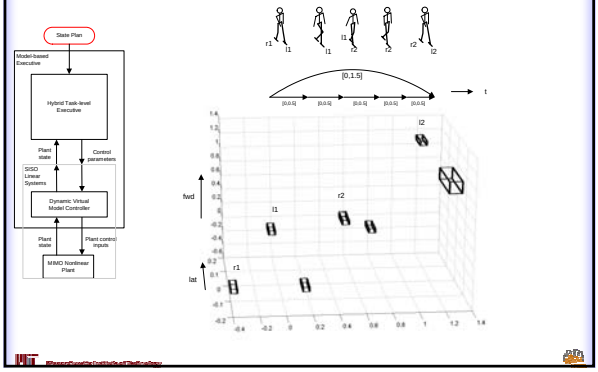

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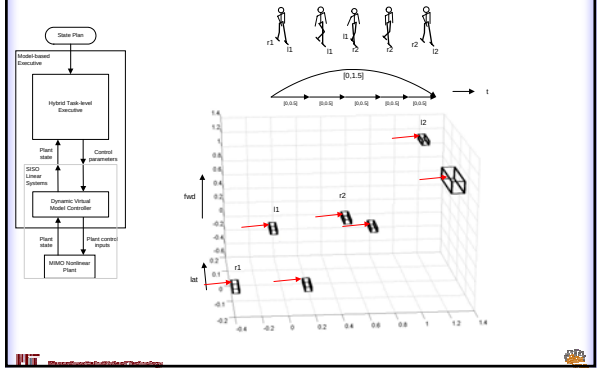

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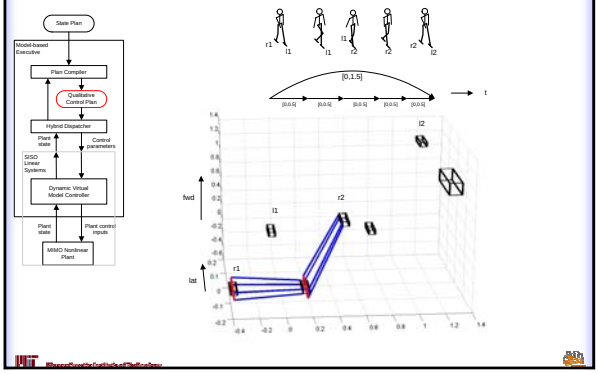
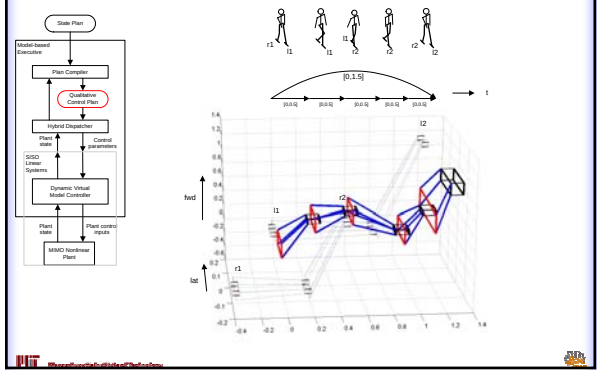
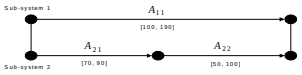


Model-based Executive



Plan Compilation

1. Compute dispatchable graph [Muscettola, 1998]
   - This gives tightest duration bounds for all activities



Plan Compilation

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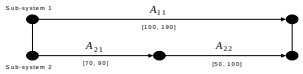
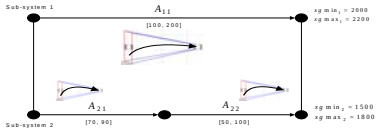
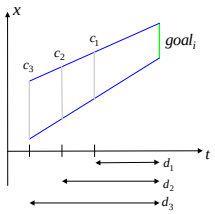


2. For each activity, compute flow tubes, based on duration bounds
   - Using reachability analysis with input and state constraints



Flow Tube Computation

- Flow tube cross section
  - Set of states, c, from which goal can be reached in duration d $c = ft(goal_i, d)$
- Flow tube formed by set of cross sections

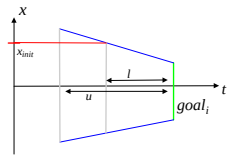


If $x_{init} \in c_1$, then d_1 is a possible duration

If $x_{init} \in c_2$, then d_2 is a possible duration

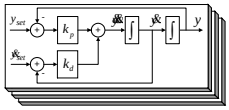
If $x_{init} \in c_3$, then d_3 is a possible duration

Temporal controllability of an initial state



- Initial state is in flow tube over duration range $[l, u]$
  - Initial state is an element of all cross sections in this range
- For any duration in this range
  - Control input can be adjusted so that state is in goal region after this duration

Flow Tube Computation



Actuation limits

$$y_{set\_min} \le y_{set} \le y_{set\_max}$$
$$\dot{y}_{set\_min} \le \dot{y}_{set} \le \dot{y}_{set\_max}$$
$$k_{p\_min} \le k_p \le k_{p\_max}$$
$$k_{d\_min} \le k_d \le k_{d\_max}$$

- Plant dynamics $\ddot{y} = k_p(y_{set} - y) + k_d(\dot{y}_{set} - \dot{y})$

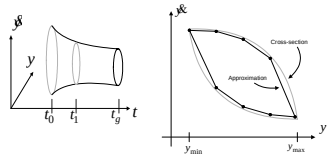
$$y = e^{\alpha t}(K_1 \cos \beta t + iK_2 \sin \beta t) + u/c \qquad K_1 = y(0) - u/c,\ K_2 = (i/\beta)(\alpha K_1 - \dot{y}(0))$$

$$\dot{y} = e^{\alpha t}(\beta(-K_1 \sin \beta t + iK_2 \cos \beta t) + \alpha(K_1 \cos \beta t + iK_2 \sin \beta t)) \qquad \alpha = -k_d/2,\ \beta = \left(-i\sqrt{k_d^2 - 4k_p}\right)/2,\ u = k_p y_{set} + k_d \dot{y}_{set}$$

$$y = f_1(y(0), \dot{y}(0), y_{set}, \dot{y}_{set})$$
$$\dot{y} = f_2(y(0), \dot{y}(0), y_{set}, \dot{y}_{set})$$

Linear for fixed t, Kp, Kd

Flow Tube Approximation

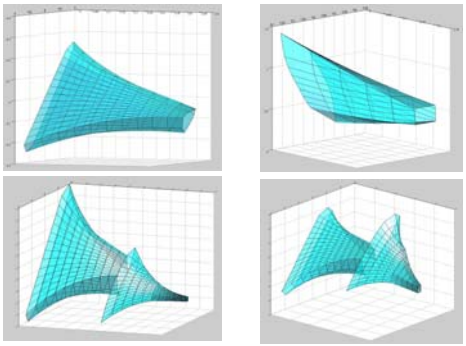


LP Formulation

$$y = f_1(y(0), \dot{y}(0), y_{set}, \dot{y}_{set}, d)$$
$$\dot{y} = f_2(y(0), \dot{y}(0), y_{set}, \dot{y}_{set}, d)$$
$$y_{set\_min} \le y_{set} \le y_{set\_max}$$
$$\dot{y}_{set\_min} \le \dot{y}_{set} \le \dot{y}_{set\_max}$$
$$\langle y(d), \dot{y}(d) \rangle \in R_{goal}$$

- Approximate cross section with polyhedron
- Find y_min, y_max
- Discretize interval [y_min, y_max]
- For each position, find min and max velocity

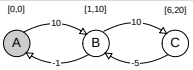
Flow Tube Approximations



Plan Execution – Temporal plan dispatcher

1. Update execution windows

a. T=0



2. Schedule next event
   - to a time consistent with window

T(B) = 7

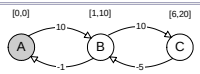
3. Wait for event

4. If no more events, done,
   else, go to 1

Plan Execution – Hybrid dispatcher

1. Update execution windows

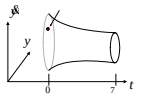
a. T=0

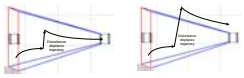


2. Schedule next event
   - to a time consistent with window

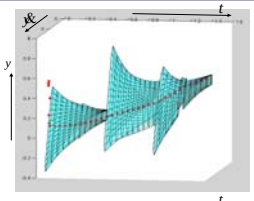
T(B) = 7

3. Find a flow tube consistent with
   - Current state
   - Activity duration implied by event
4. Wait for event
   - Monitor progress to goal
   - If out of flow tube, plan has failed
   - If in flow tube, adjust control parameters, if necessary
   - If in goal, done with activity
5. If no more events, done,
   else, go to 1

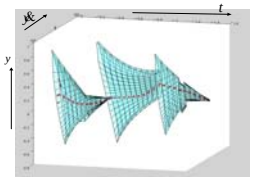




Results

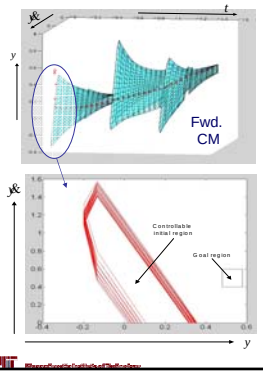


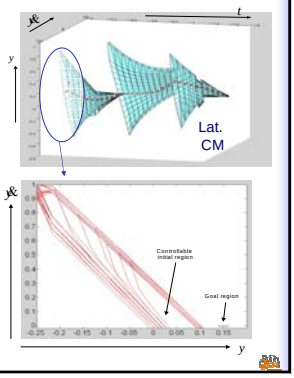
Fwd. CM



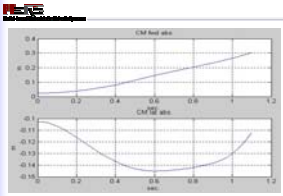
Lat. CM

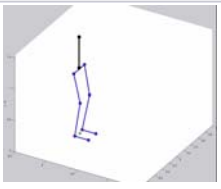
Results



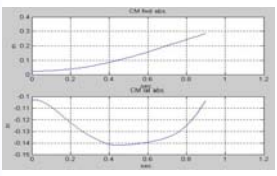


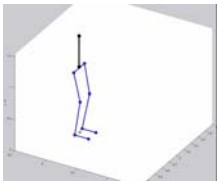
Results





Slow

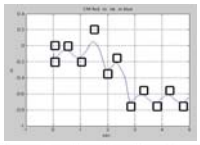


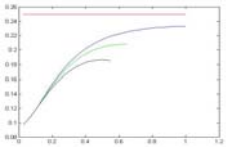


Fast

Done!

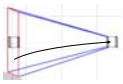
Results



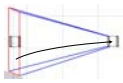


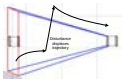
Lateral CM with push disturbance
-Blue – 40 N
-Green – 35 N
-Black – 25 N

Discussion



Discussion





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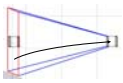

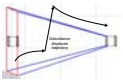

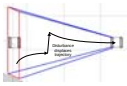

Notes

- In discussion, consider showing video from soccer
  - One where player kicks a ball
  - One where player gives up on chasing a ball
  - This is a key point – emphasize that using plan flexibility to respond to disturbances, but also, limits are defined. Thus, knowing when to quit is important

Summary

- Requirements for walking task execution different from those of periodic walking

Summary

- Requirements for walking task execution different from those of periodic walking
  - Observe state-space, temporal constraints