

Hybrid Diagnosis with Unknown Behavioral Modes

Michael W. Hofbaur¹ & Brian C. Williams²

¹⁾ Department of Automatic Control,
Graz University of Technology, Austria

²⁾ Artificial Intelligence & Space Systems Laboratories
MIT, USA

Motivation / Aim

- Hybrid mode estimation / diagnosis of a highly complex artifact that exhibits both, continuous and discrete behaviors
- model-based: does the model capture every possible situation?
- unknown environment
- how can we cope with unmodeled situations?



⇒ show how *structural analysis and decomposition techniques* can enable “*unknown mode detection*” for hybrid estimation

Overview

- Concurrent Probabilistic Hybrid Automata
 - Hybrid Estimation
 - Unknown Mode
 - Decomposition - *intuitively*
 - Decomposition - *algorithmically*
 - Example
 - Discussion & Conclusion
-

Probabilistic Hybrid Automata

Probabilistic Hybrid Automata $\langle \mathbf{x}, \mathbf{w}, F, T, X_d, U_d, T_s \rangle$

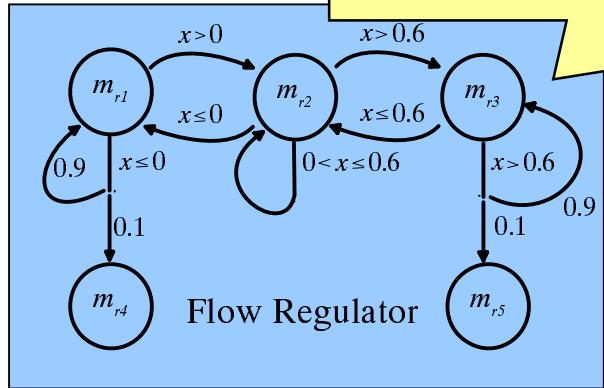
$\mathbf{x} \dots \{x_d\} \cup \mathbf{x}_c \dots \dots \dots \quad x_d$ mode (discrete state) with domain X_d
 \mathbf{x}_c continuous state with domain \mathbb{R}^n

$\mathbf{w} \dots \mathbf{u}_d \cup \mathbf{u}_c \cup \mathbf{y}_c \dots \dots \dots \quad \mathbf{u}_d$ discrete command with domain U_d
 \mathbf{u}_c continuous command with domain \mathbb{R}^{m_i}

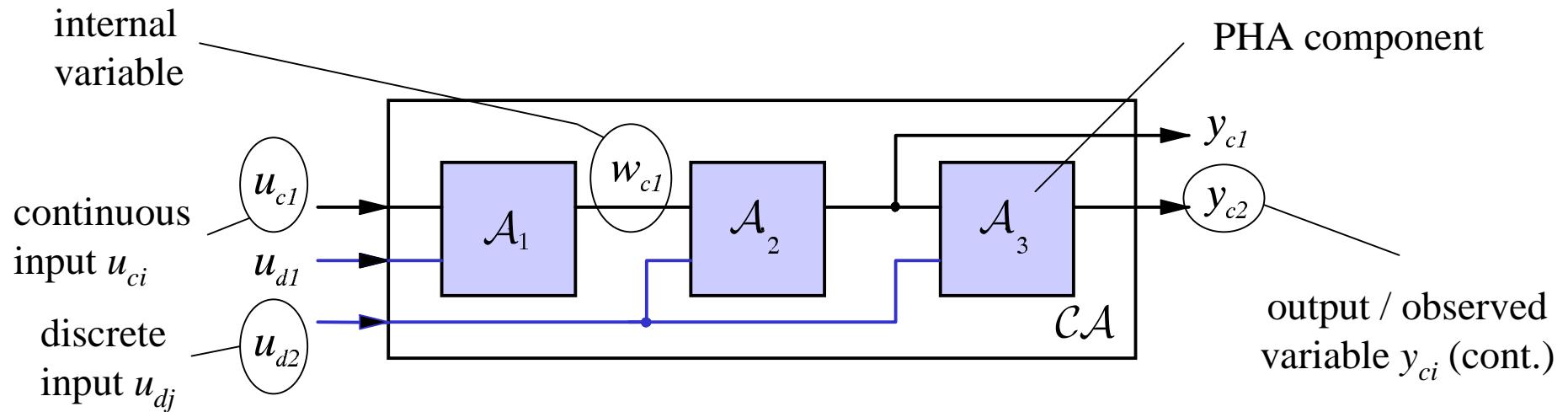
\mathbf{y}_c continuous output with domain \mathbb{R}^{m_o}

$F \dots \dots \dots \dots \dots \quad$ discrete-time dynamics for each mode
(sampling-period T_s)

$T \dots \dots \dots \dots \dots \quad$ guarded probabilistic transitions between modes



concurrent Probabilistic Hybrid Automata



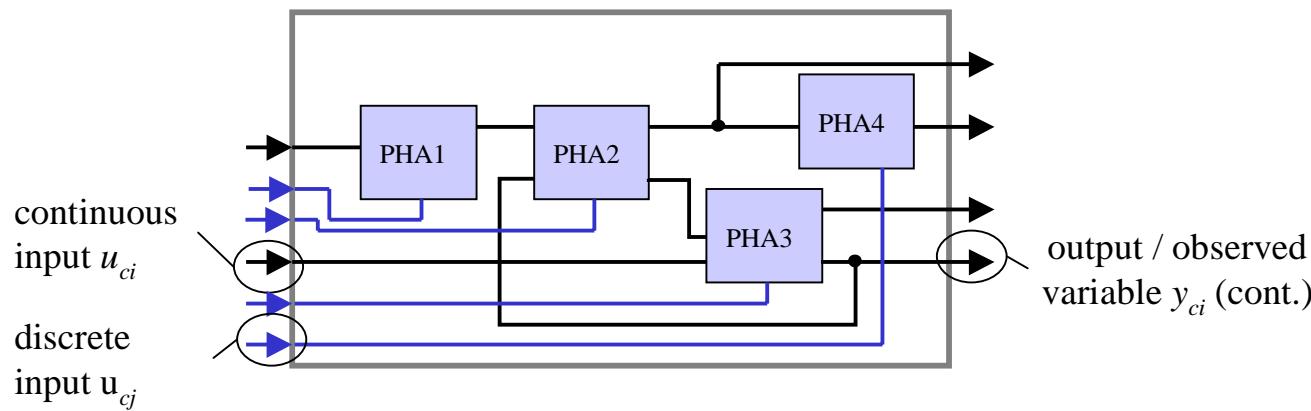
- *observed variables:* internal variable + additive Gaussian noise
- *independent variables:* $\mathbf{u}_c, \mathbf{u}_d$, noise
- *dependent variables:* $\mathbf{x}_c, \mathbf{x}_d$, internal variables

$$\mathbf{x}_{c,(k)} = f_{(k)}(\mathbf{x}_{c,(k-1)}, \mathbf{u}_{c,(k-1)}) + \mathbf{v}_{s,(k-1)}$$

$$\mathbf{y}_{(k)} = g_{(k)}(\mathbf{x}_{c,(k)}, \mathbf{u}_{c,(k)}) + \mathbf{v}_{o,(k)}$$

Hybrid Mode / State Estimation

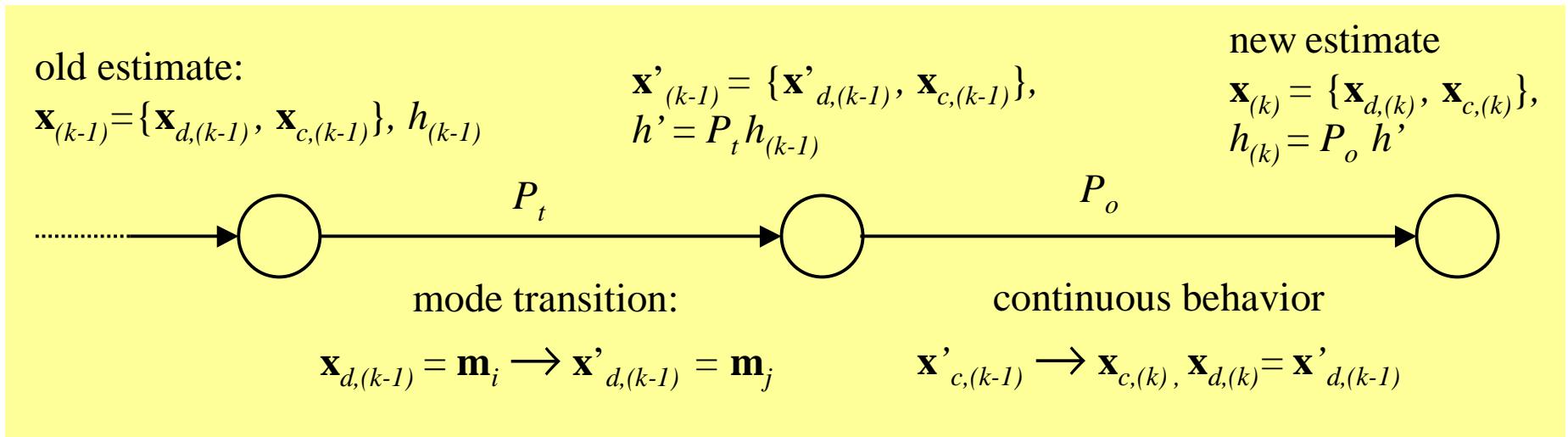
Task Overview:



Hybrid Estimation Problem: Given a cPHA model for a system, a sequence of observations and the history of the control inputs generate *the leading set of most likely states at time-step k*

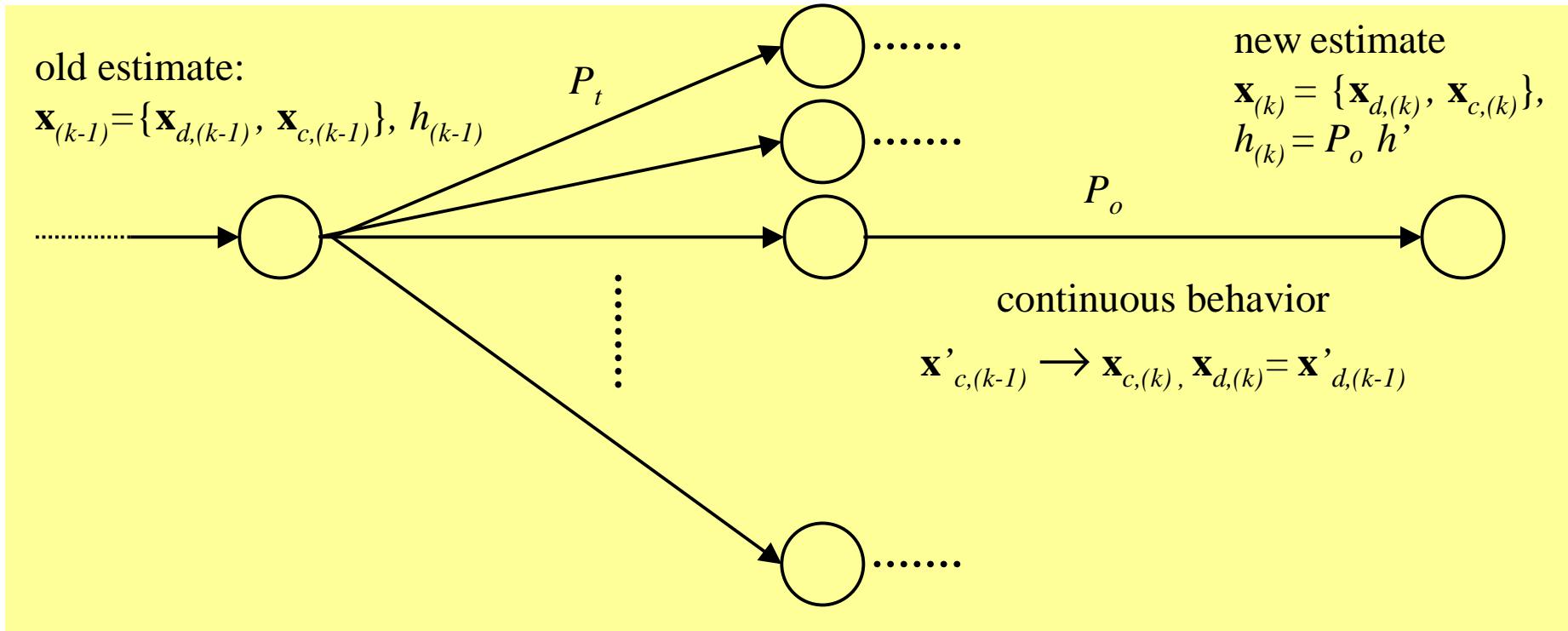
hybrid Mode Estimation

At each time step k , we evaluate for each trajectory:



hybrid Mode Estimation

At each time step k , we evaluate for each trajectory:



Filter Calculation

Hypothesis: mode $\mathbf{m}_j = \{m_{11}, m_{21}, m_{31}\}$

1) retrieve cPHA equations for \mathbf{m}_j

$$F_1(m_{11}) = \{u_{c1} = 5 w_{c1}\}$$

$$F_2(m_{21}) = \{x_{c1,(k)} = 0.8 x_{c1,(k-1)} + w_{c1,(k-1)}, y_{c1} = x_{c1}\}$$

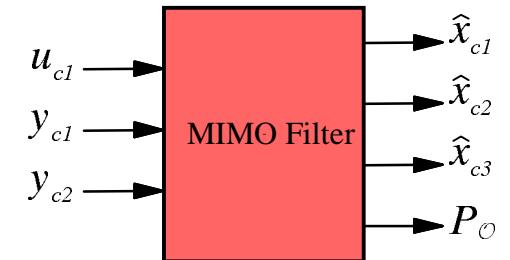
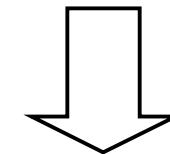
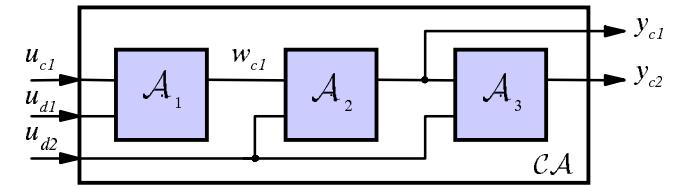
$$\begin{aligned} F_3(m_{31}) &= \{x_{c2,(k)} = x_{c3,(k-1)} + y_{c1,(k-1)}, \\ &\quad x_{c3,(k)} = 0.4 x_{c2,(k-1)} + 0.5 u_{c1,(k-1)}, \\ &\quad y_{c2} = 2 x_{c2} + x_{c3}\} \end{aligned}$$

2) solve equations

independent vars: \mathbf{u}_c , observed vars: \mathbf{y}_c

$$\begin{aligned} \mathbf{x}_{c,(k)} &= \mathbf{f}_{(k)}(\mathbf{x}_{c,(k-1)}, \mathbf{u}_{c,(k-1)}) + \mathbf{v}_{s,(k-1)} \\ \mathbf{y}_{c,(k)} &= \mathbf{g}_{(k)}(\mathbf{x}_{c,(k)}, \mathbf{u}_{c,(k)}) + \mathbf{v}_{o,(k)} \end{aligned}$$

3) calculate extended Kalman filter and evaluate it



Unknown Mode

Hypothesis: mode $\mathbf{m}_j = \{\text{?}, m_{21}, m_{31}\}$

- 1) retrieve cPHA equations for \mathbf{m}_j

$$F_1(\text{?}) = \{ \}$$

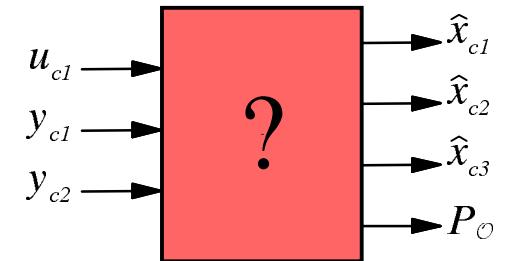
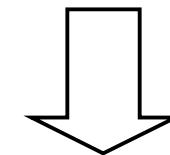
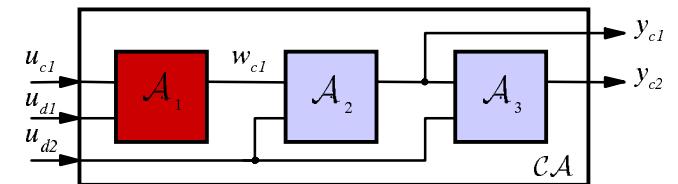
$$F_2(m_{21}) = \{x_{c1,(k)} = 0.8 x_{c1,(k-1)} + w_{c1,(k-1)}, y_{c1} = x_{c1}\}$$

$$\begin{aligned} F_3(m_{31}) &= \{x_{c2,(k)} = x_{c3,(k-1)} + y_{c1,(k-1)}, \\ &\quad x_{c3,(k)} = 0.4 x_{c2,(k-1)} + 0.5 u_{c1,(k-1)}, \\ &\quad y_{c2} = 2 x_{c2} + x_{c3}\} \end{aligned}$$

- 2) solve equations

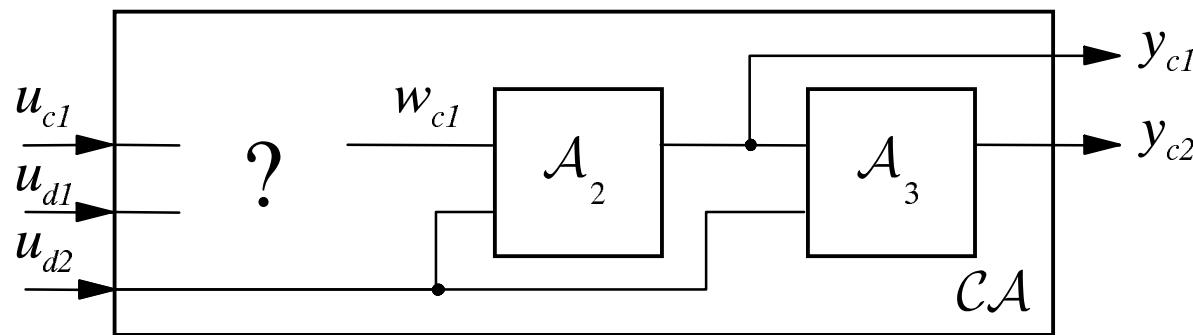
FAILS - additional independent variable: w_{c1}

**we cannot calculate the extended Kalman Filter
that is necessary for hybrid estimation**



Unknown Mode

What about partial estimation?

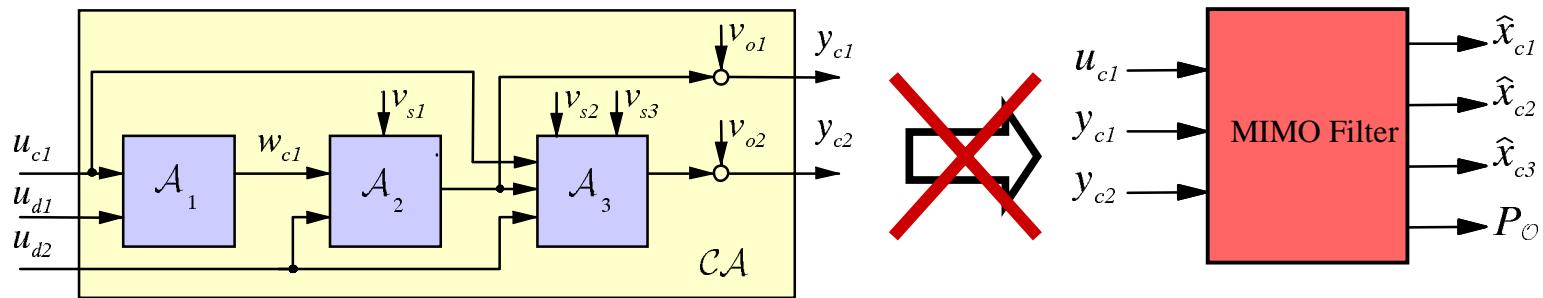


y_{c1} ... noisy measurement of outputsignal of PHA \mathcal{A}_2



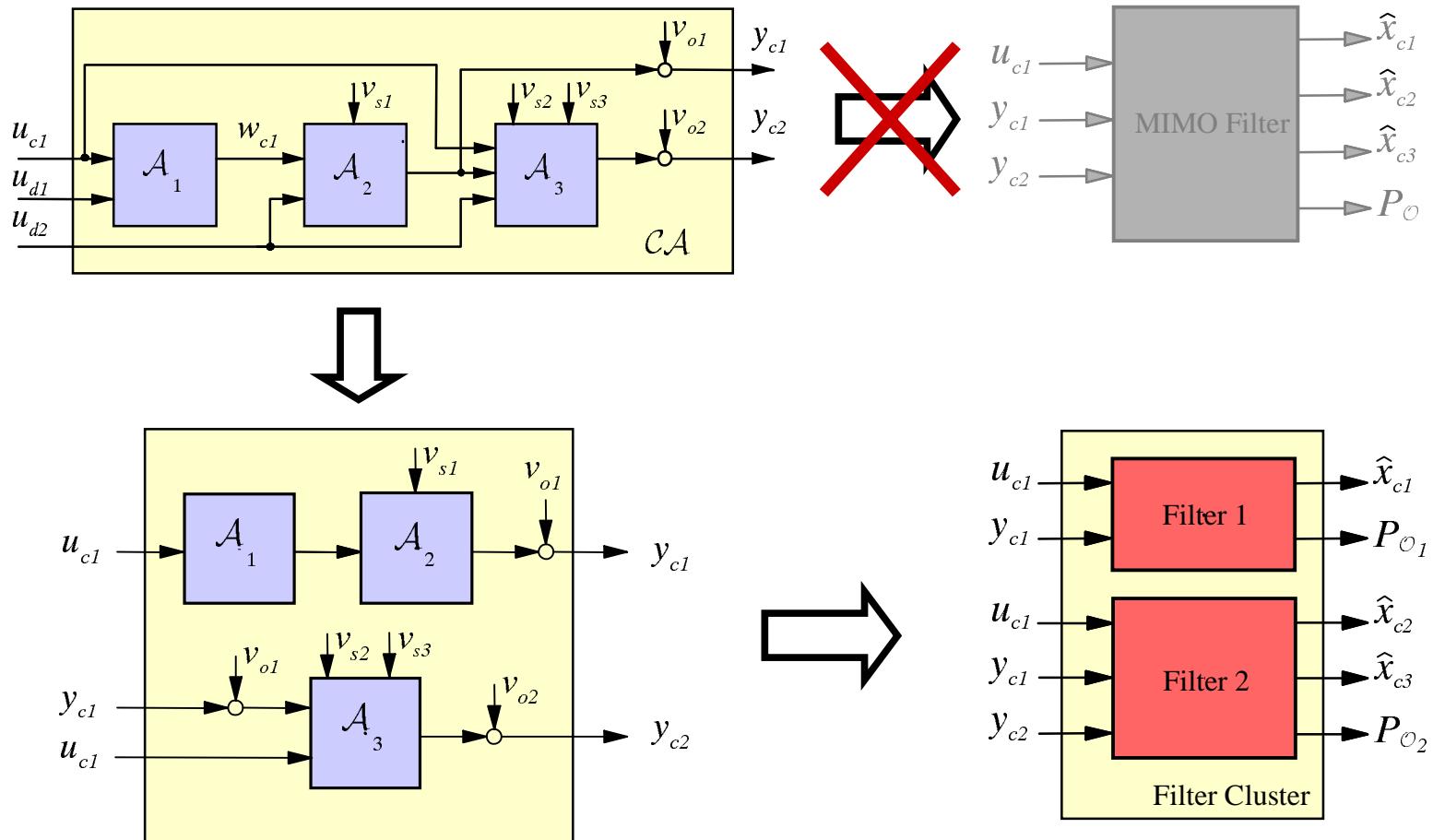
PHA \mathcal{A}_3 fully specified \rightarrow partial estimation possible

Intuitive Decomposition

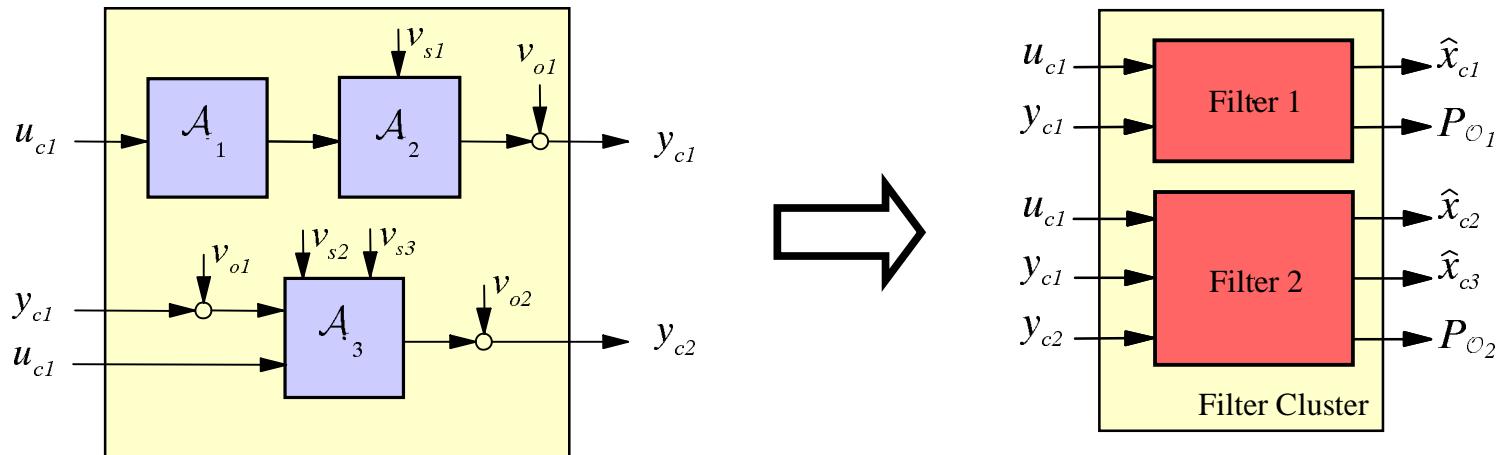


alternative: calculate *clustered* Filter

Intuitive Decomposition



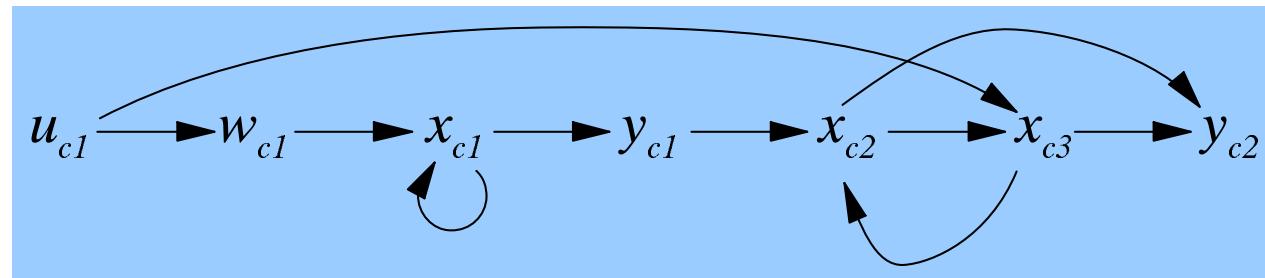
Implications of Decomposition



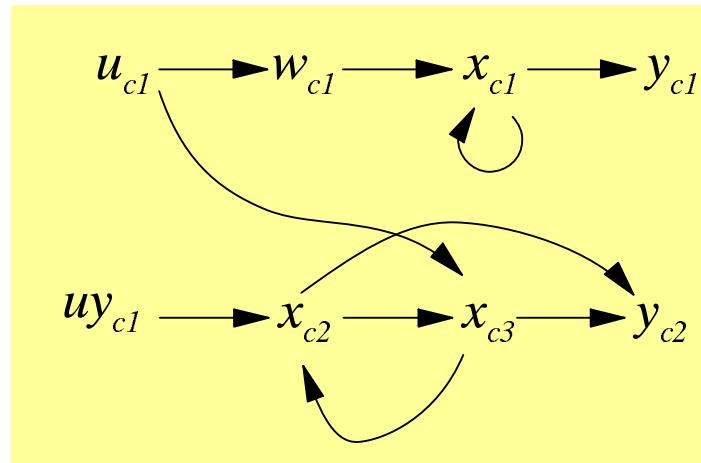
- Factorization of P_O : $P_O = \prod P_{Oj}$
eg. unknown mode in A_1 : $P_O \leq P_{O2}$
- Additional (virtual) noise at inputs (e.g. v_{o2} acting upon y_{c2})
- Reduced computational complexity of the filter cluster
[Kalman filter $O(n^3) <$ filter cluster $O(n_1^3 + n_2^3)$]

Algorithmic Decomposition

- 1) generate the *causal graph* from the equations
[bipartite matching based algorithm, Nayak-95]



- 2) Remapping:
insert *virtual inputs*

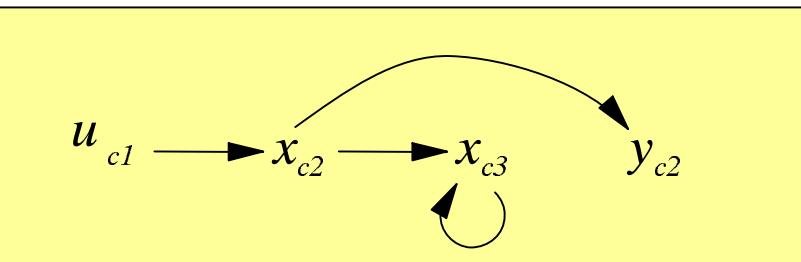


Algorithmic Decomposition

continuous estimation is limited to the *observable part* of the system - observable with respect to the measurements \mathbf{y}_{ci} and the known input values \mathbf{u}_{ci}

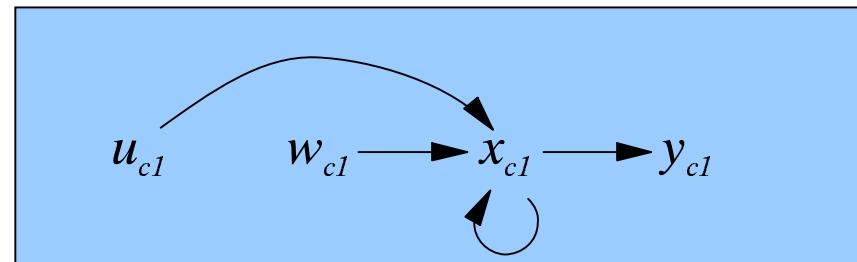
structurally observable (SO) variable:

Either directly observed, or there exists at least one path in the causal graph that connects the variable to an output variable



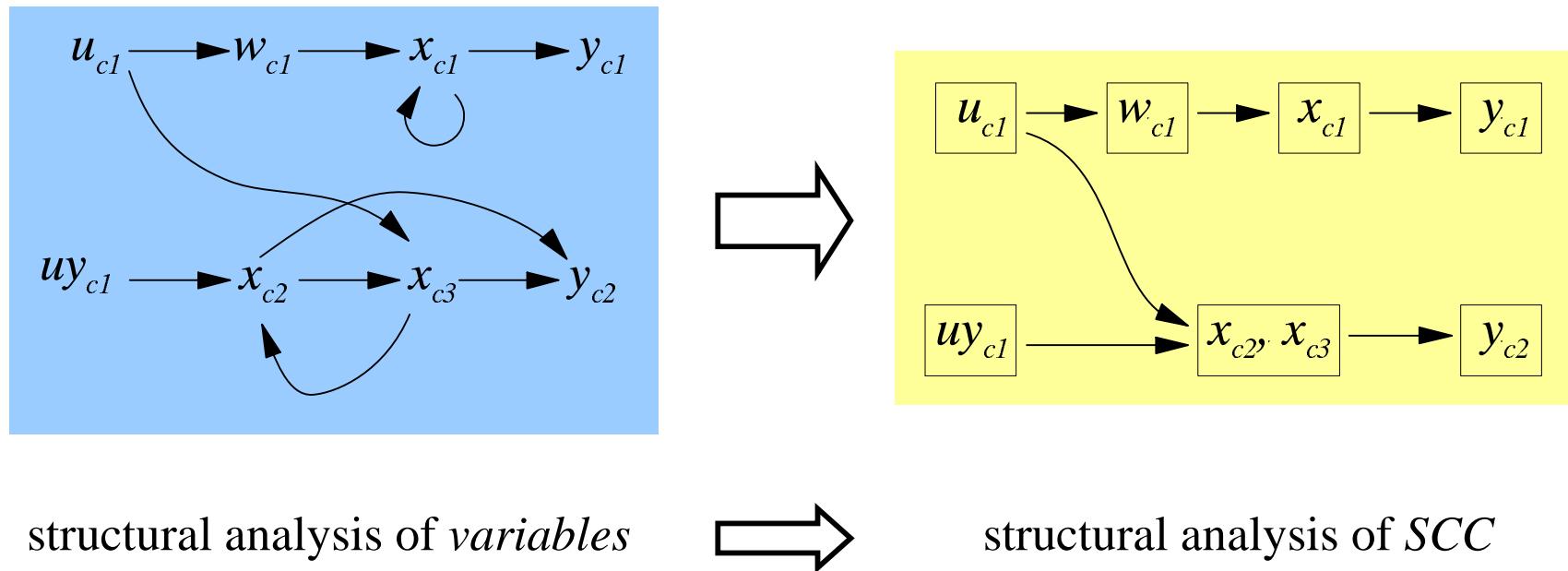
structurally determined (SD) variable:

Either an input variable of the automaton, or there does not exist a path in the causal graph that links the variable with an undetermined exogenous variable



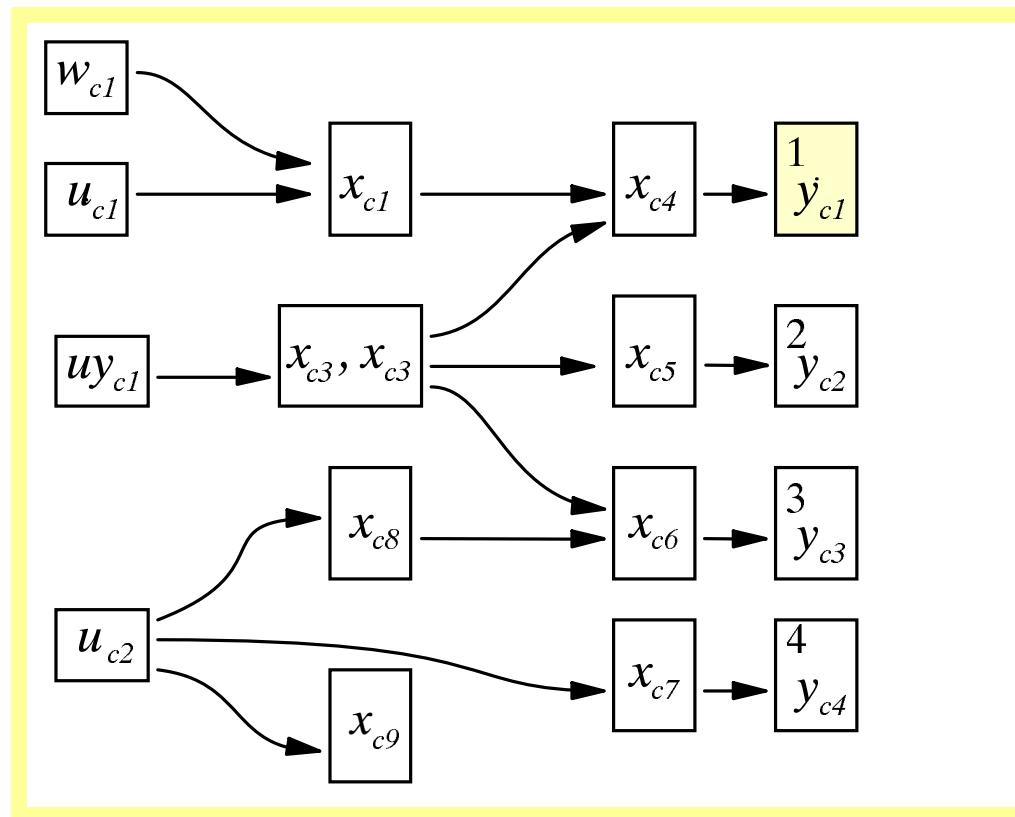
Algorithmic Decomposition

prior structural analysis: eliminate loops by calculating the
strongly connected components (SCC) [Aho-83]



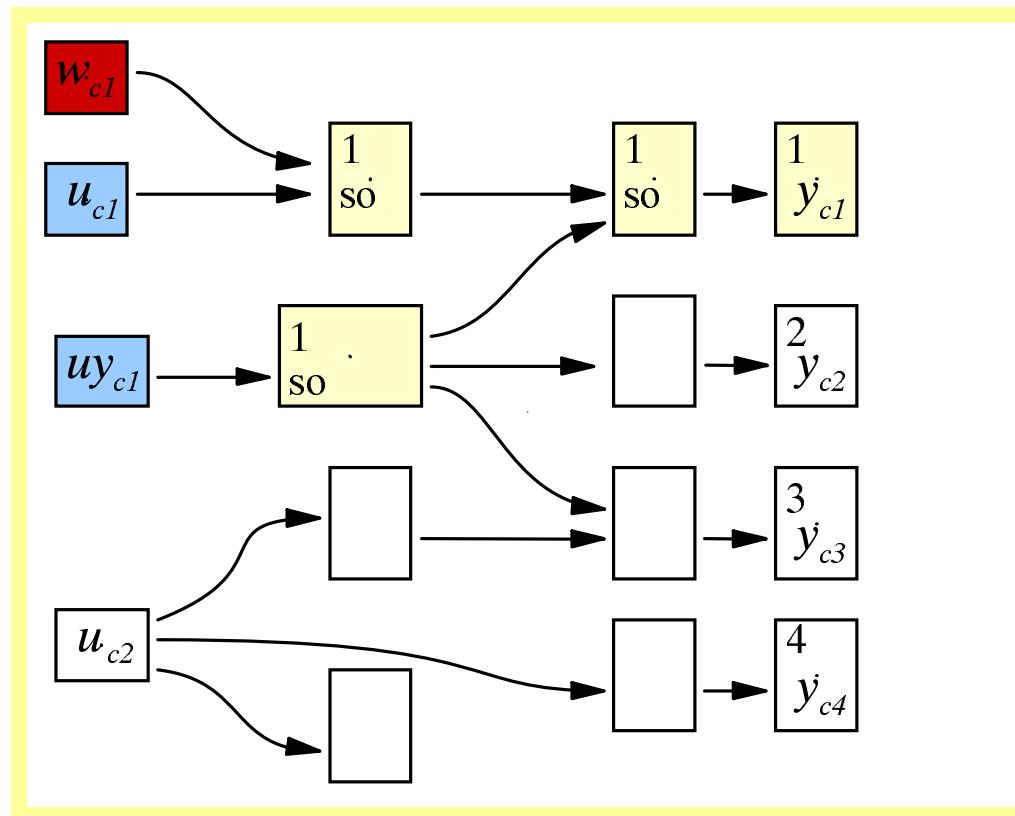
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



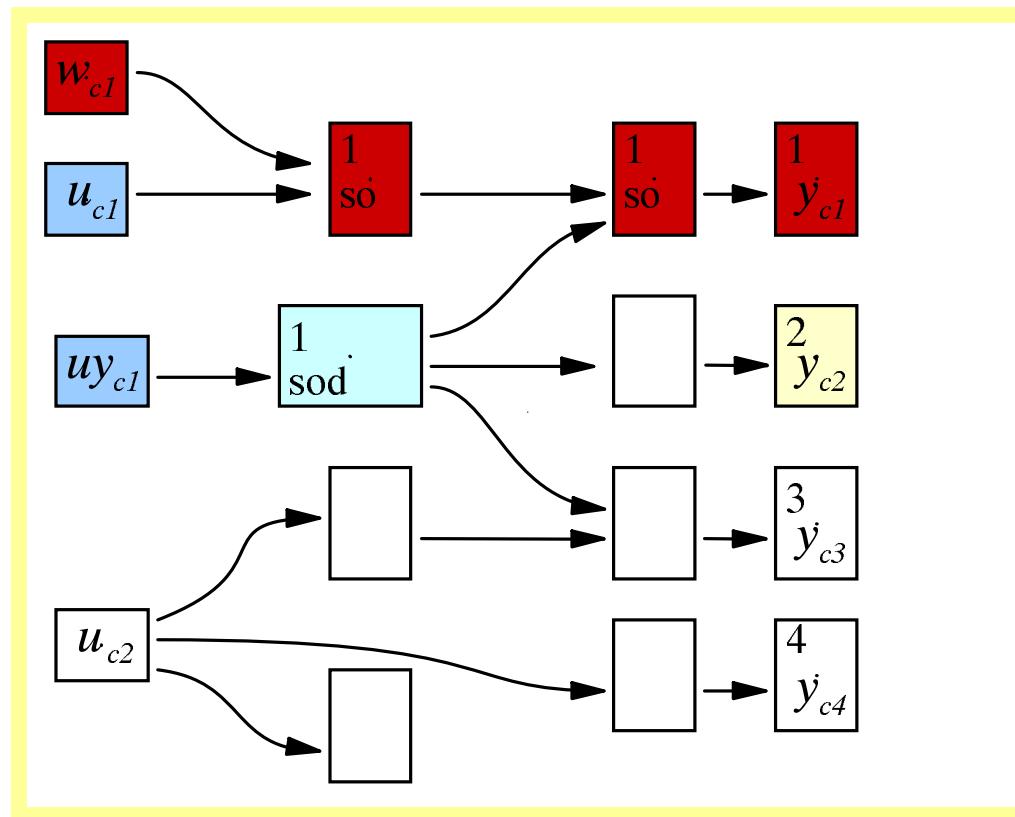
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



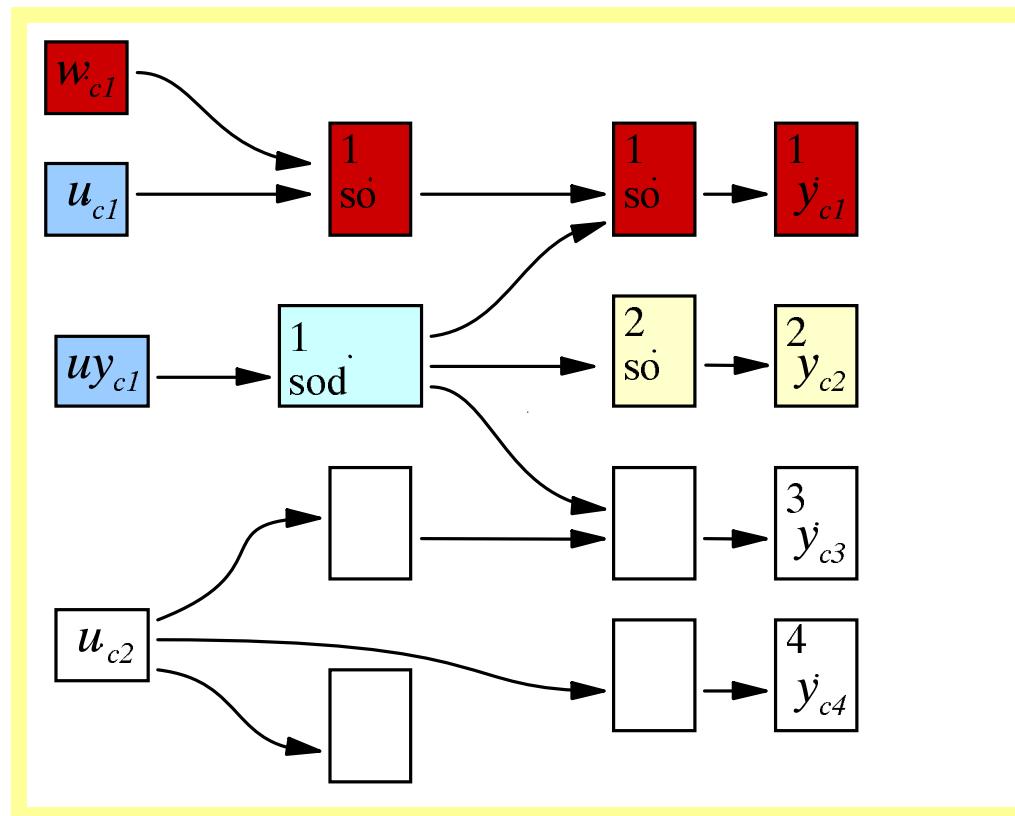
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



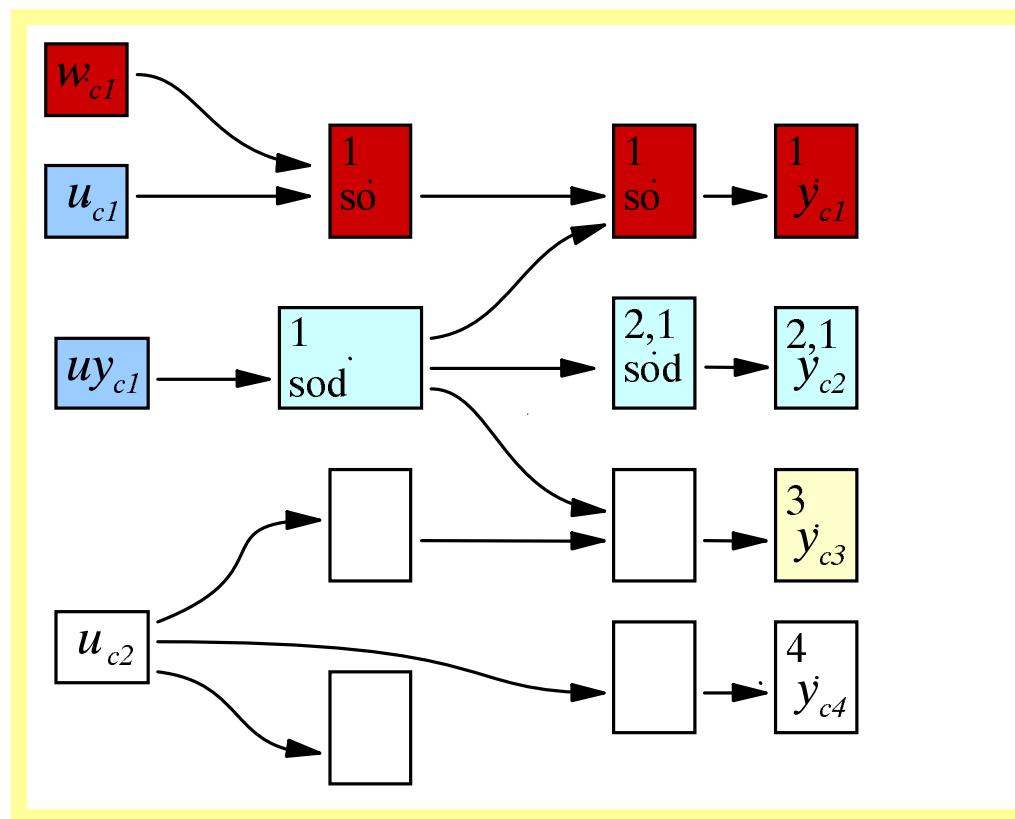
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



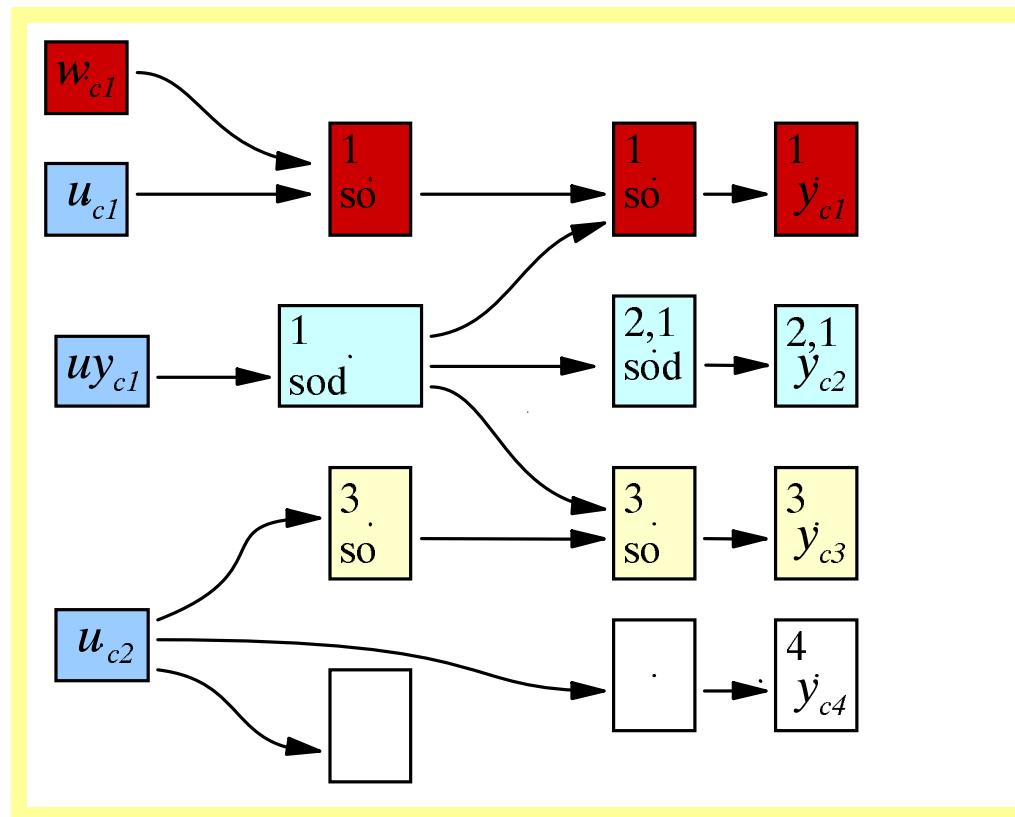
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



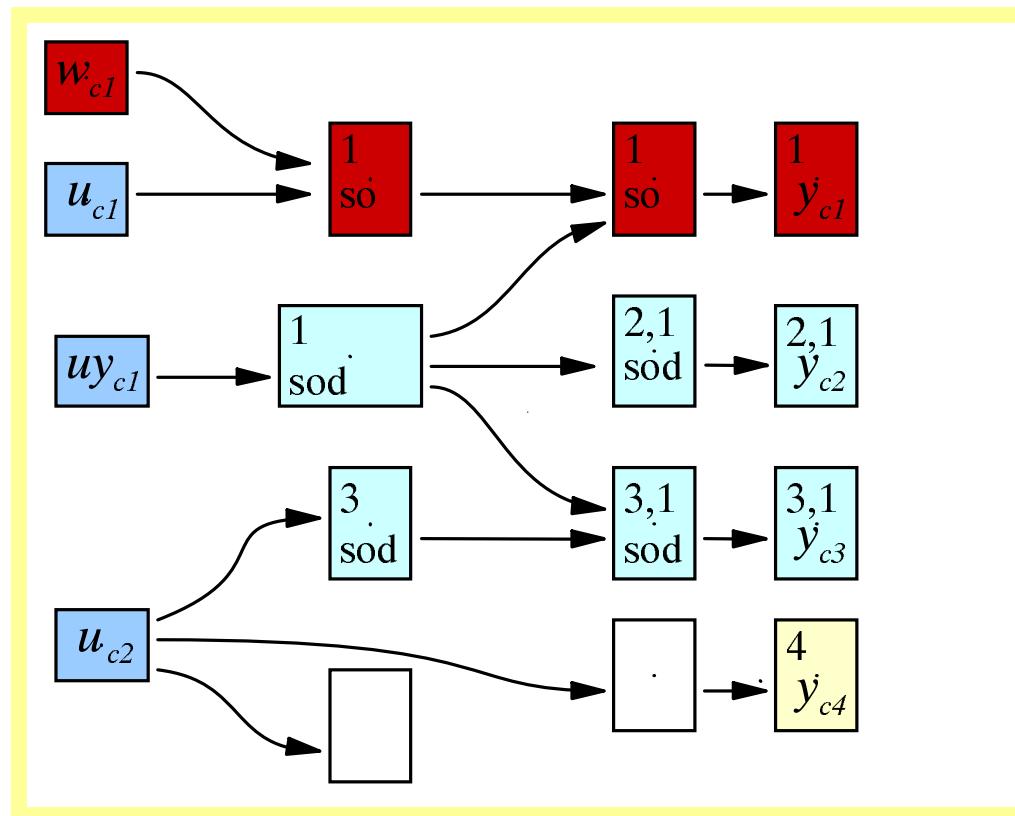
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



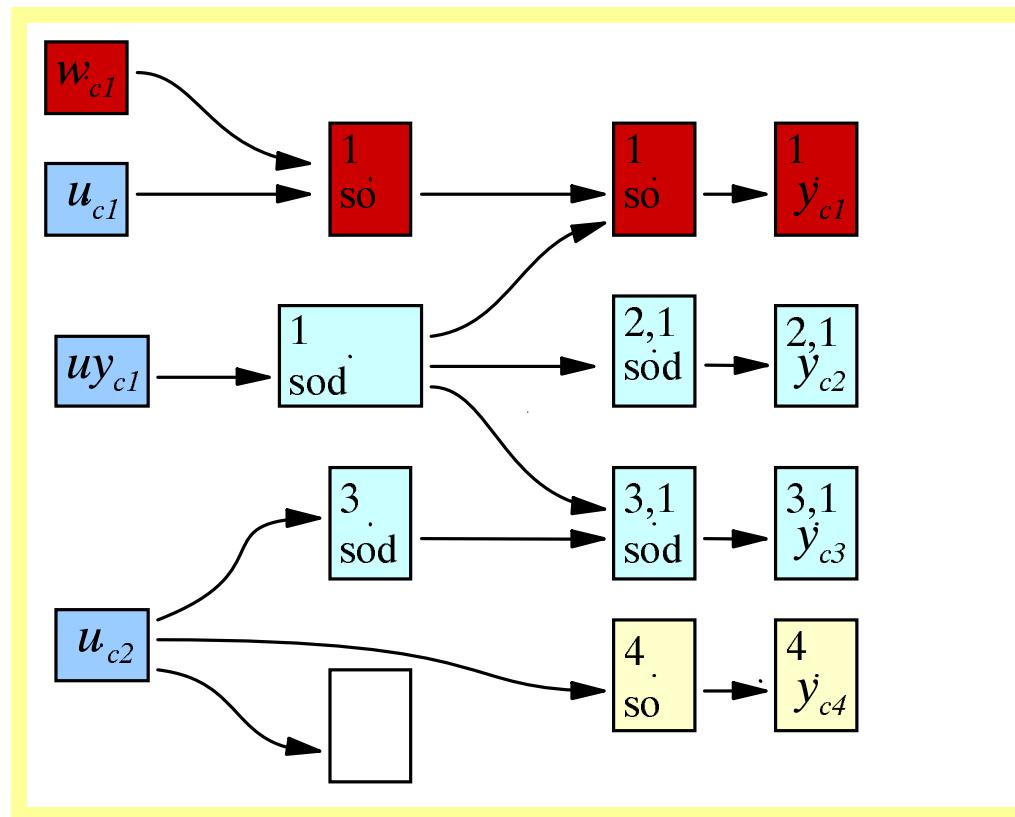
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



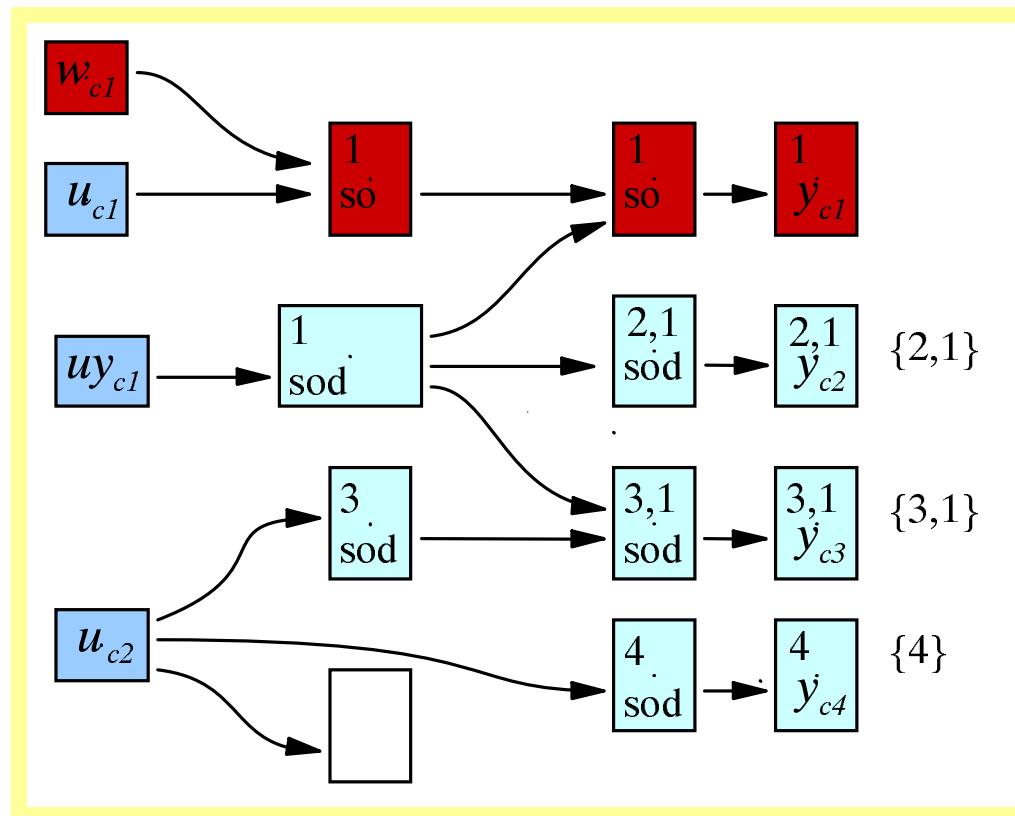
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



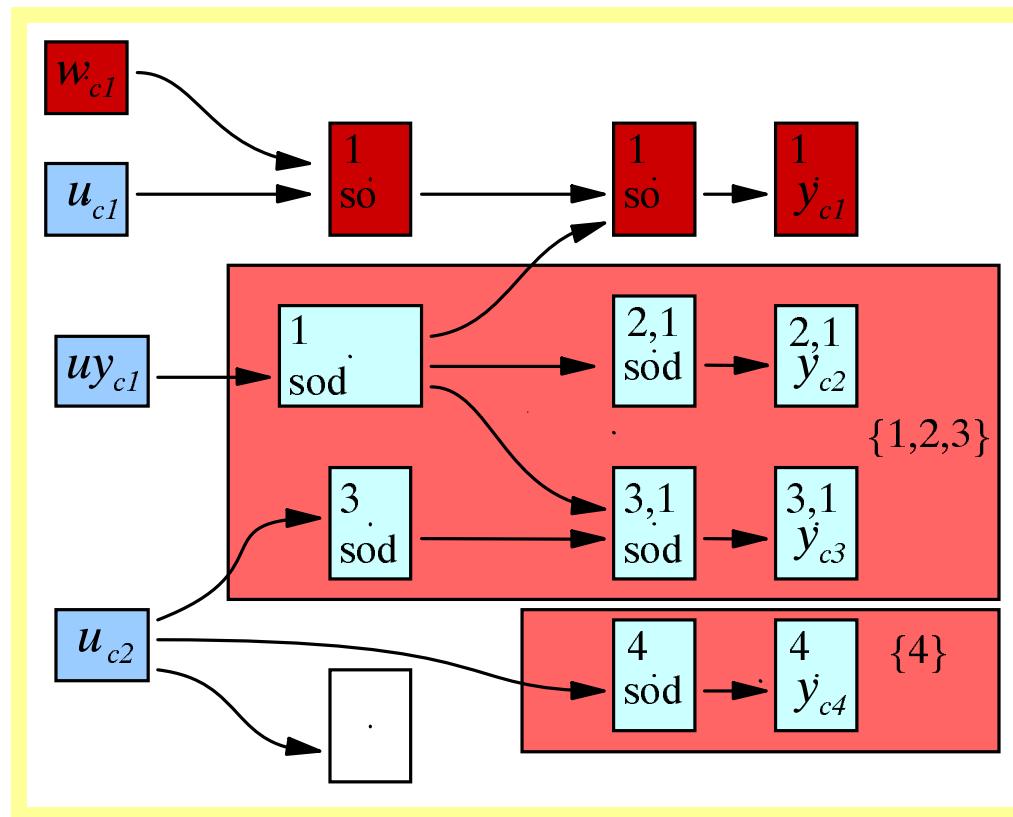
Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning

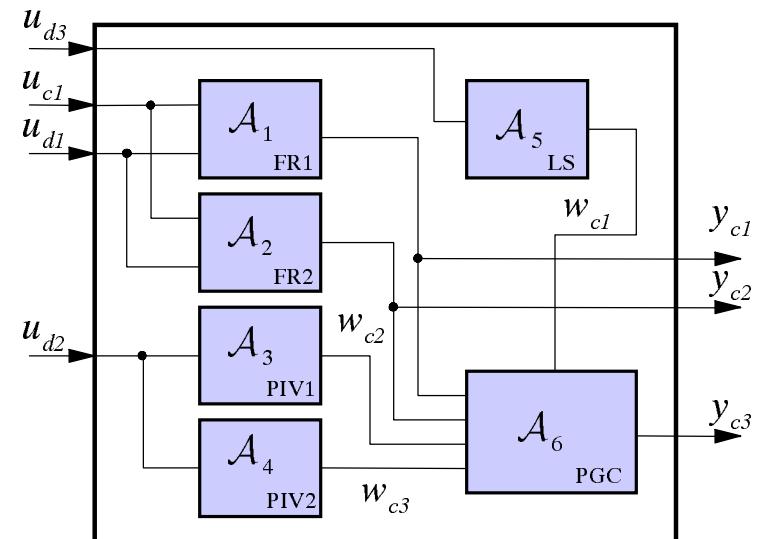
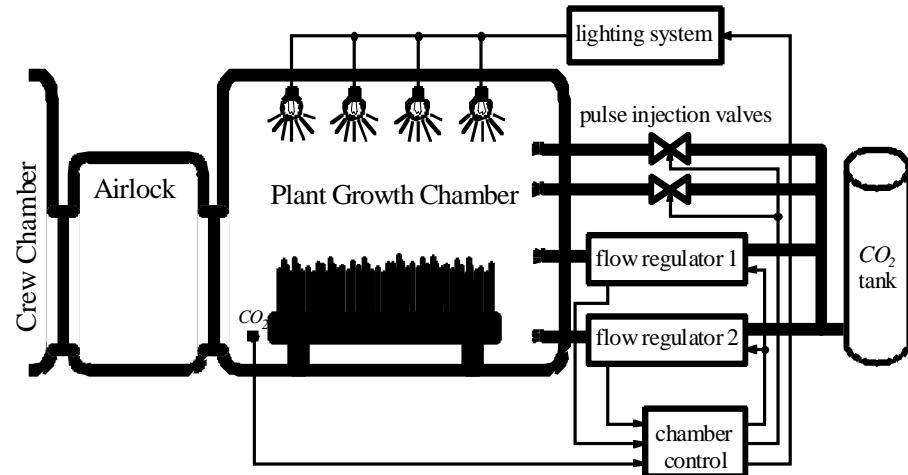
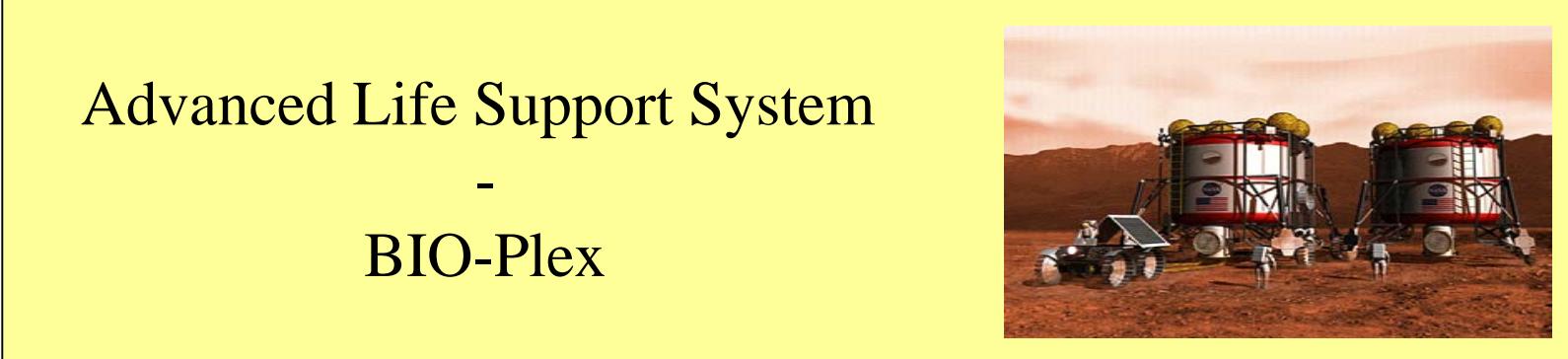


Algorithmic Decomposition

Observability analysis algorithm and labeling/partitioning



Example - BIO Plex



Example

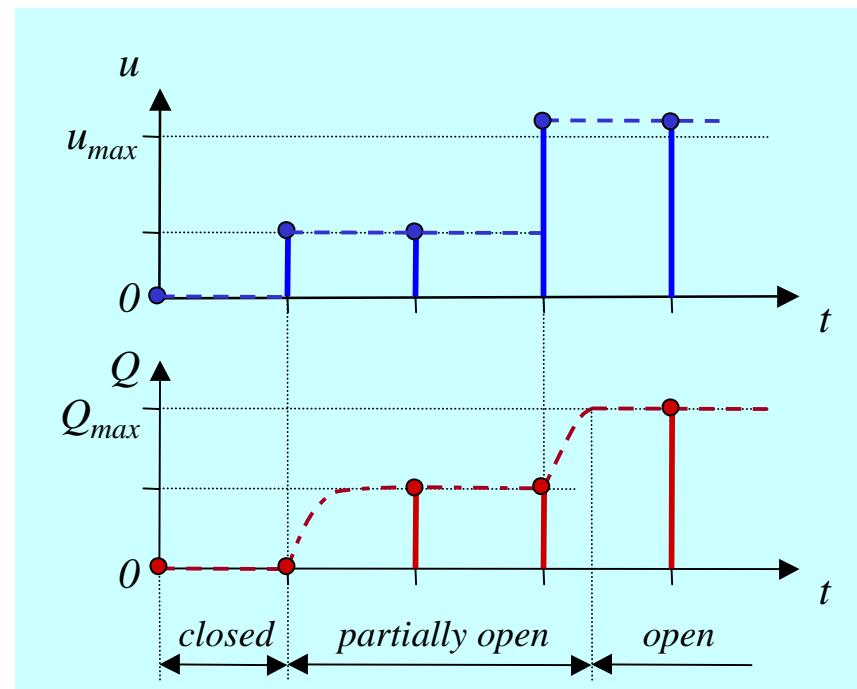
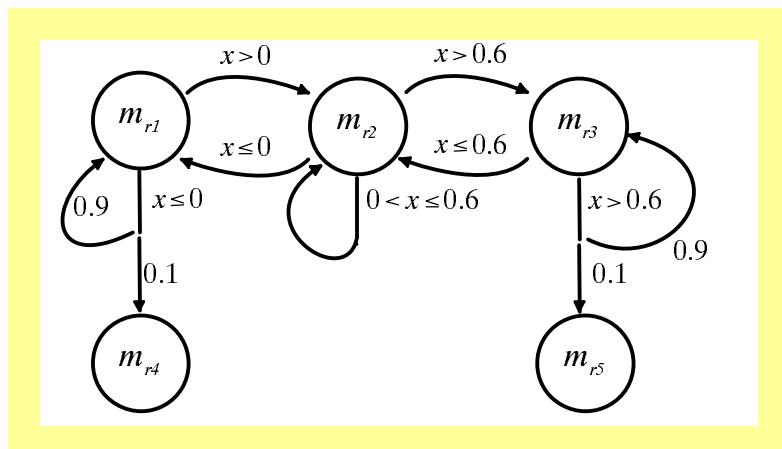
PHA example: Flow Regulator

operational modes:

closed m_{r1} , partially open m_{r2} , open m_{r3}

fault modes:

stuck closed m_{r4} , stuck open m_{r5}



Example

PHA example: Flow Regulator

operational modes:

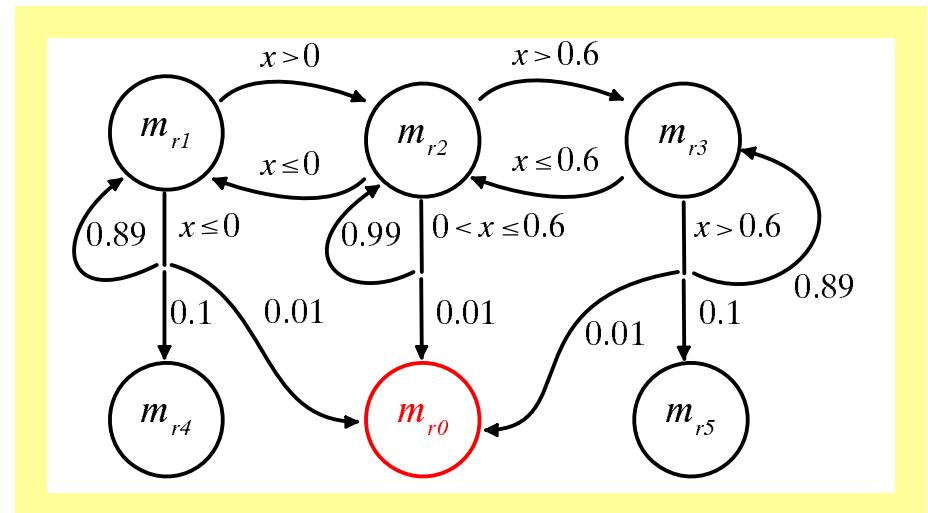
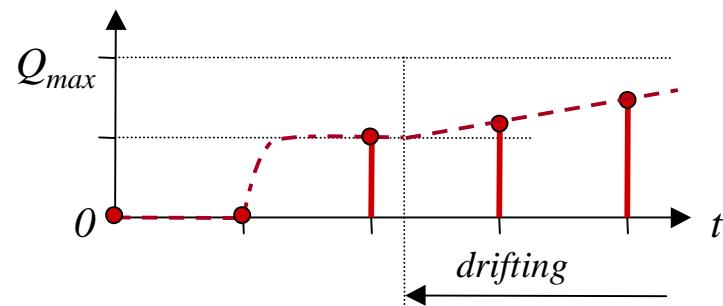
closed m_{r1} , partially open m_{r2} , open m_{r3}

fault modes:

stuck closed m_{r4} , stuck open m_{r5}

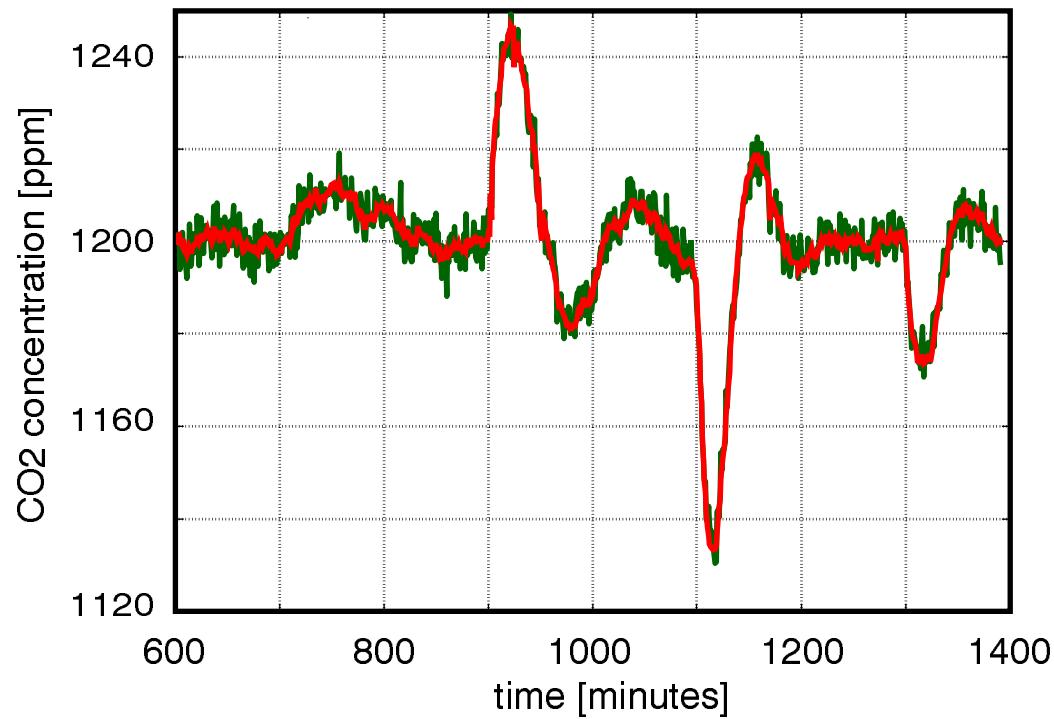
unknown mode m_{r0}

example: DRIFT FAULT



Example

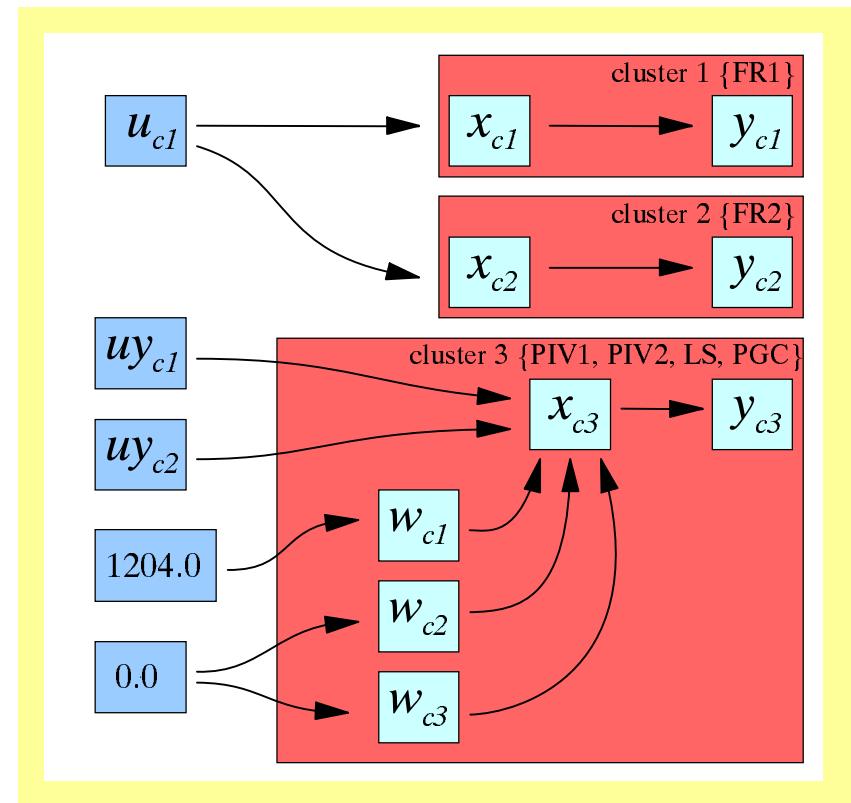
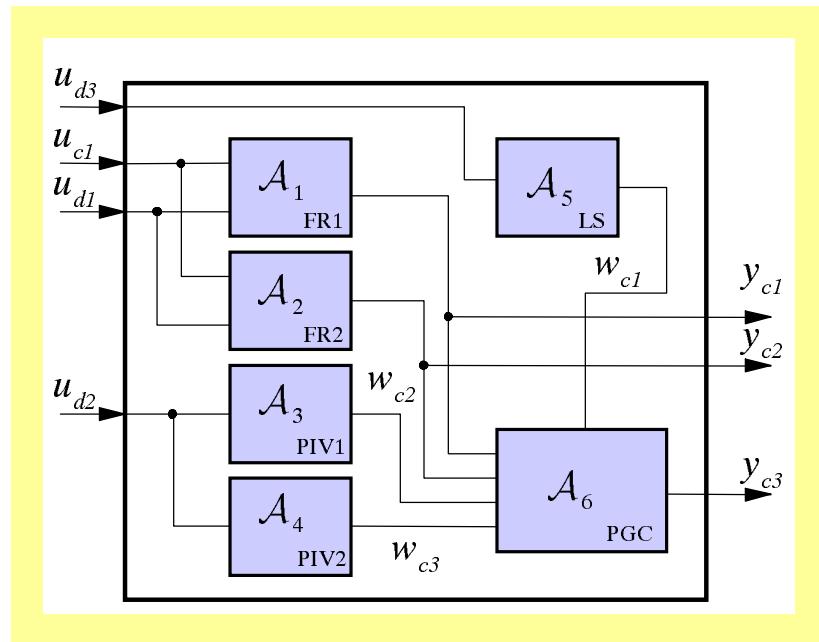
Hybrid estimation with a cPHA model of BIO-Plex and
Simulation data obtained from NASA's simulator



t = 700: flow regulator drift
t = 900: partial lighting blackout
t = 1100: flow regulator repair
t = 1300: light repair

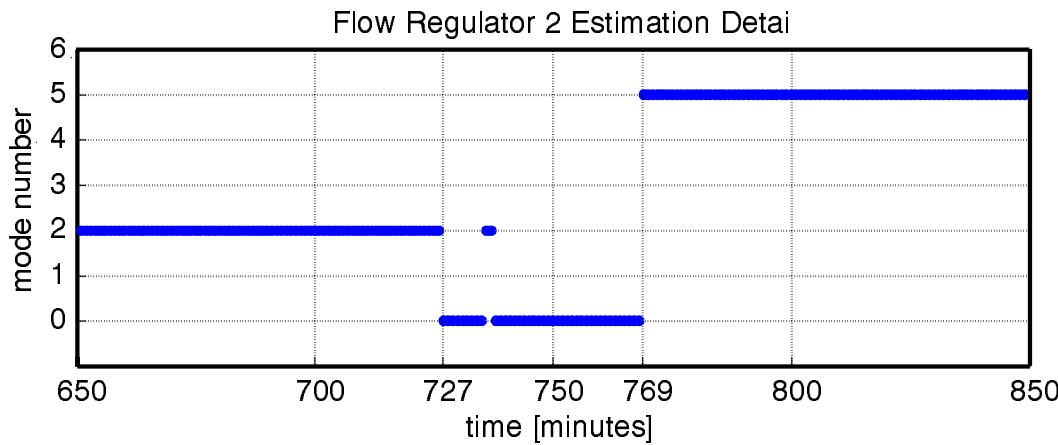
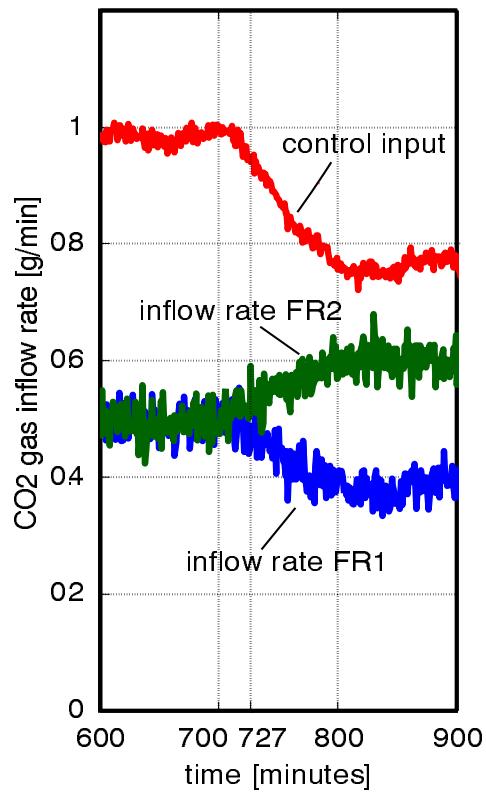
Example

cPHA model and decomposition



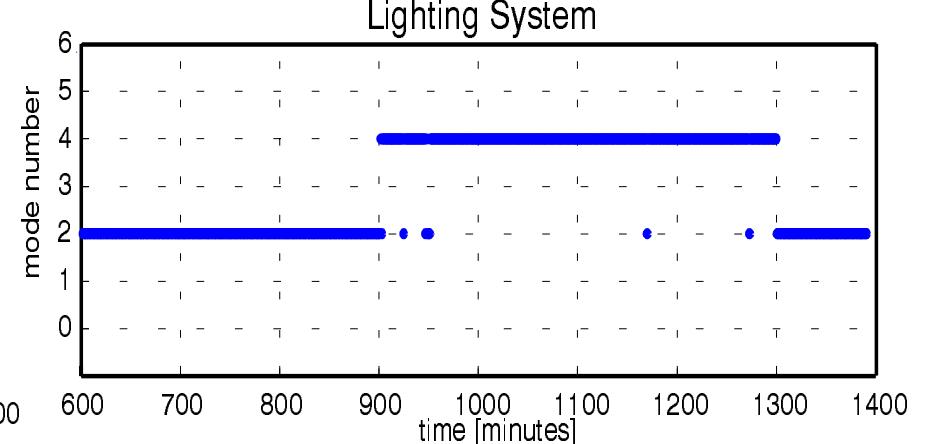
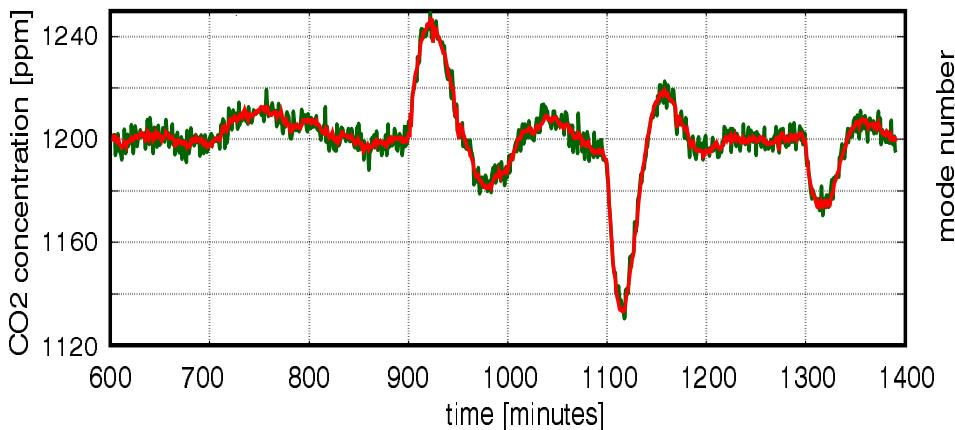
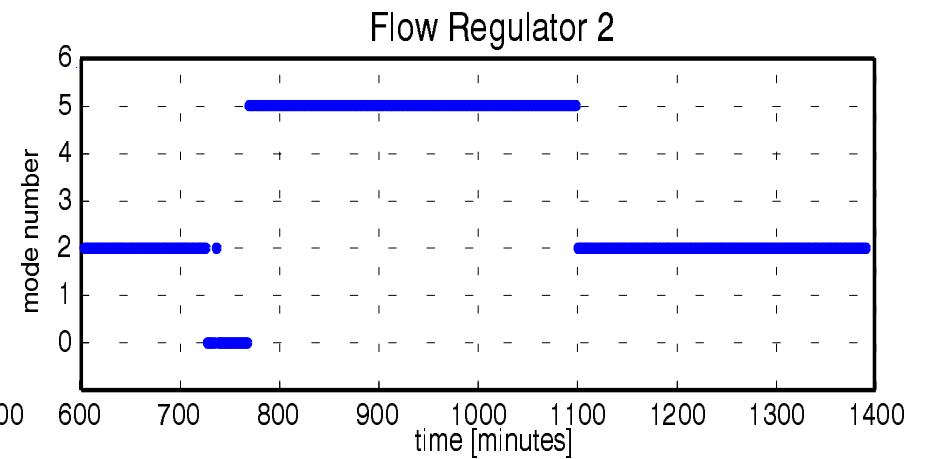
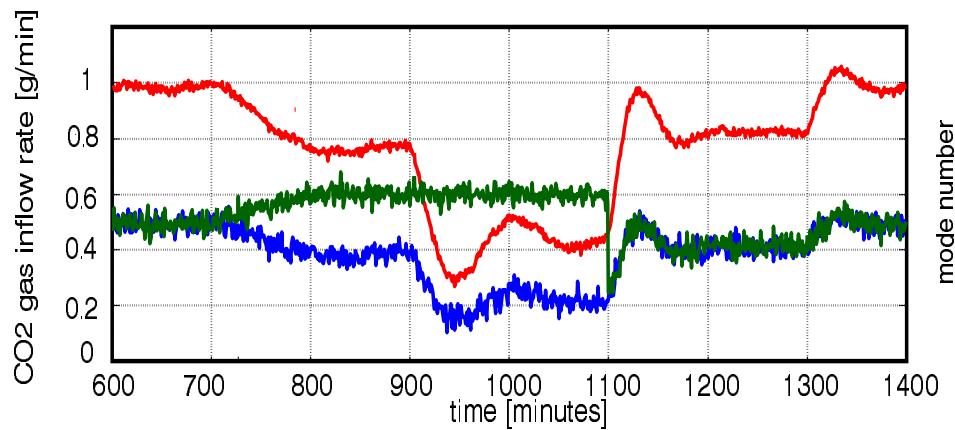
Example

1. Flow regulator drift detection detail



- $t = 700$: Flow regulator 2 drifts
- $t = 727$: *unknown mode* classification
- $t = 769$: hME prefers *stuck open* as symptom explanation
- $t = 800$: FR2 becomes fully open

Example



Discussion / Conclusion

Summary

- *unknown mode* for hybrid estimation
- structural analysis and decomposition
- example

Current & Future Research

- decomposition → *conflicts*
- *conflict directed search* to improve hybrid estimation
- hybrid mode estimation & reconfiguration
- *fault tolerant control - autonomous automation*

optional slides

concurrent Probabilistic Hybrid Automata

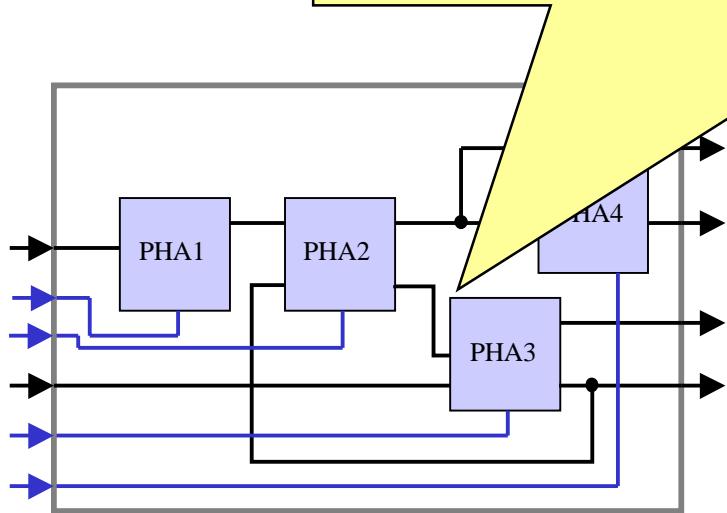
Concurrent Probabilistic Hybrid Automata $\langle A, \mathbf{u}, \mathbf{y}_c, \mathbf{v}_s, \mathbf{v}_o, N_x, N_y \rangle$

A set of PHAs

$\mathbf{u} \dots \mathbf{u}_d \cup \mathbf{u}_c \dots$ continuous and discrete command variables

\mathbf{y}_c observed continuous variables

$\mathbf{v}_s, \mathbf{v}_o$ state disturbances and sensor noise inputs
characterized by N_x, N_y



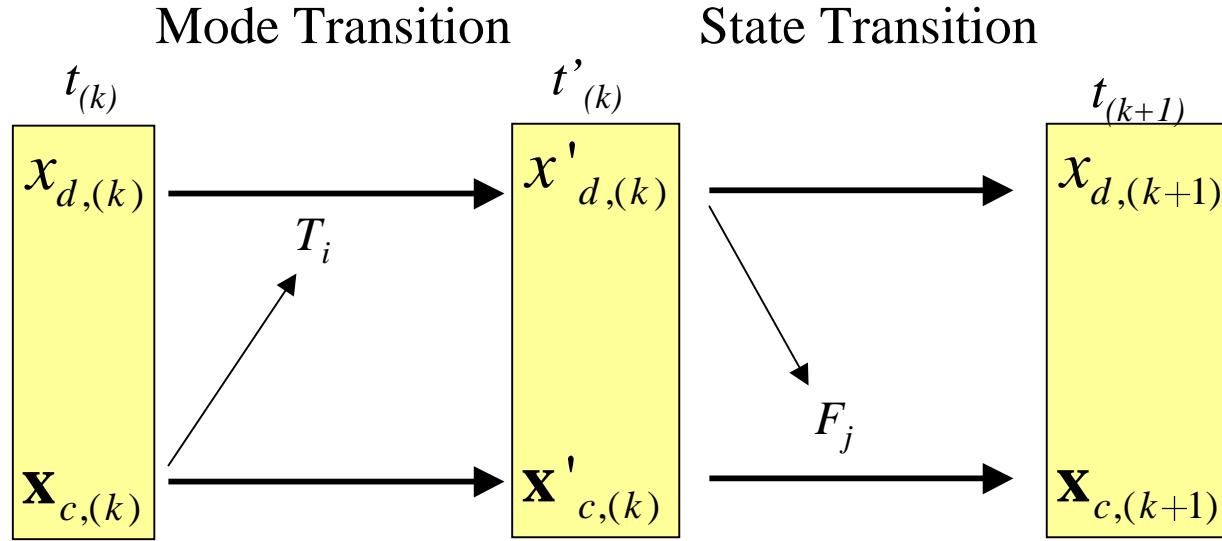
$$\mathbf{x}_c = \mathbf{x}_{c1} \cup \mathbf{x}_{c2} \cup \dots \cup \mathbf{x}_{cl}$$

$$\mathbf{x}_d = \{x_{d1}, x_{d2}, \dots, x_{dl}\}$$

$$\mathbf{x}_{c,(k)} = f(\mathbf{x}_{c,(k-1)}, \mathbf{u}_{c,(k-1)}, \mathbf{x}'_{d,(k-1)}) + \mathbf{v}_{s,(k-1)}$$

$$\mathbf{y}_{(k)} = g(\mathbf{x}_{c,(k)}, \mathbf{u}_{c,(k)}, \mathbf{x}_{d,(k)}) + \mathbf{v}_{o,(k)}$$

Mode / State Transition



Mode transition: time proceeds only infinitesimally $t'_{(k)} = t_{(k)} + \varepsilon$ so that the evolution of the continuous state $\mathbf{x}_{c,(k)} \rightarrow \mathbf{x}'_{c,(k)}$ can be neglected: $\mathbf{x}'_{c,(k)} = \mathbf{x}_{c,(k)}$

State transition: no transition is triggered ($x'_{d,(k)} = x_{d,(k+1)}$) and time proceeds for one sampling period: $t_{(k+1)} = t_{(k)} + T_s$. The evolution of the continuous state $\mathbf{x}'_{c,(k)} \rightarrow \mathbf{x}_{c,(k+1)}$ is captured by the discrete-time dynamic model that holds for $x'_{d,(k)}$.

Observation Probability P_o

We compare the sensor signal $\mathbf{y}_{c(k)}$ with its estimation for mode \mathbf{m}_j using an extended Kalman filter.

operation performed by an (*extended*) *Kalman filter*:

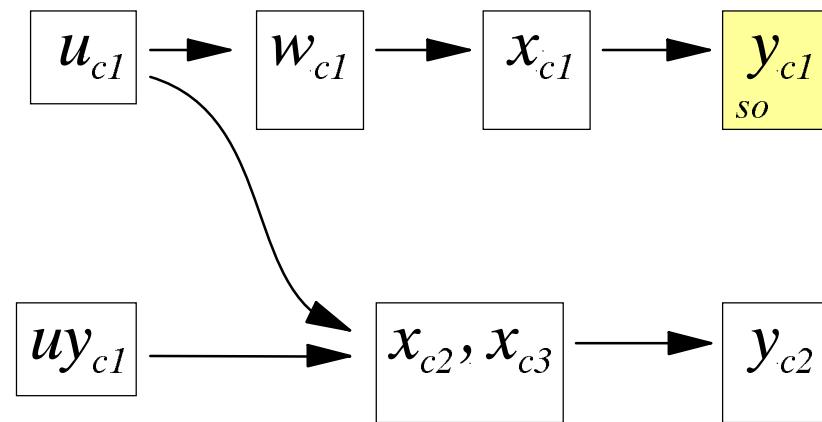
- state prediction: $\mathbf{x}_{c,(k-1)}, \mathbf{P}_{(k-1)}, \mathbf{u}_{c,(k-1)} \rightarrow \mathbf{x}'_{c,(k)}, \mathbf{P}'_{(k)}$
- residual calculation: $\mathbf{x}'_{c,(k)}, \mathbf{P}'_{(k)}, \mathbf{y}_{c(k)} \rightarrow \mathbf{r}_{(k)}, \mathbf{S}_{(k)}, P_o$
- Kalman filter gain calculation: $\mathbf{P}'_{(k)}, \mathbf{S}_{(k)} \rightarrow \mathbf{k}_{(k)}$
- state estimate refinement: $\mathbf{x}'_{c,(k)}, \mathbf{P}'_{(k)}, \mathbf{k}_{(k)}, \mathbf{r}_{(k)} \rightarrow \mathbf{x}_{c,(k)}, \mathbf{P}_{(k)}$

$$P_o = e^{-\mathbf{r}^T \mathbf{S}^{-1} \mathbf{r}}$$

→ *one extended Kalman filter for each hypothesis*

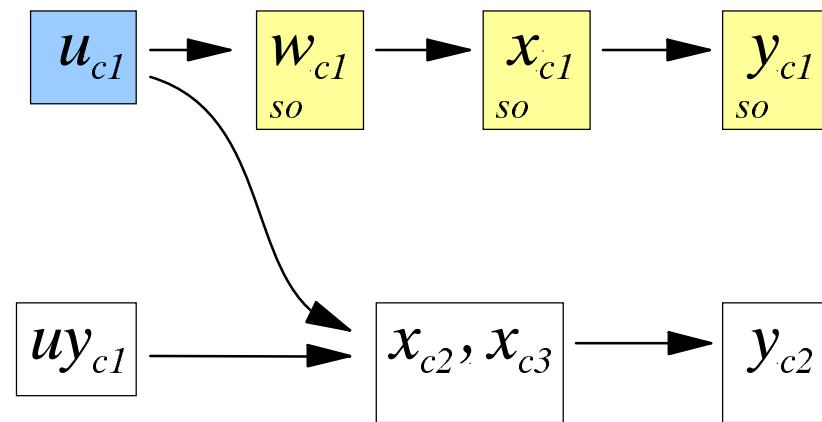
Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning



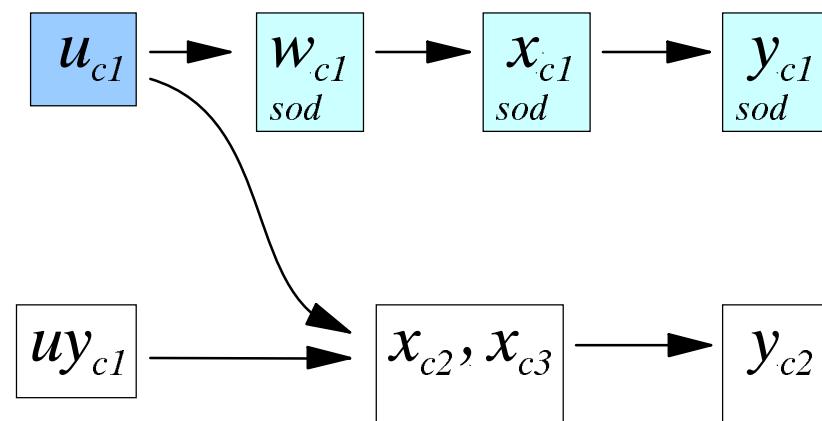
Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning



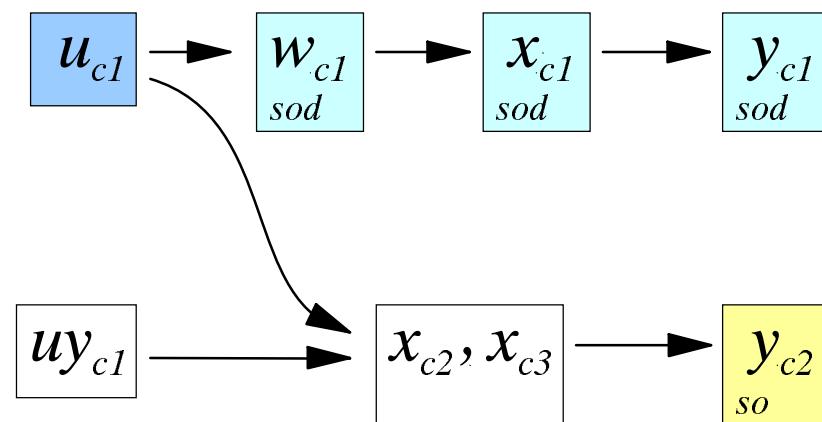
Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning



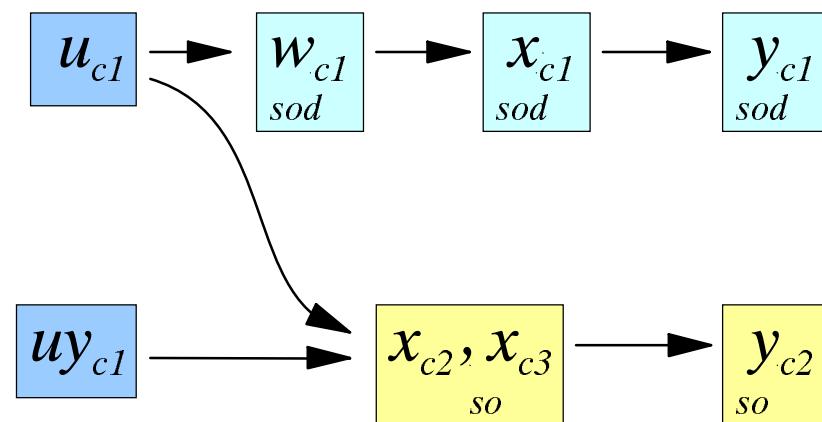
Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning



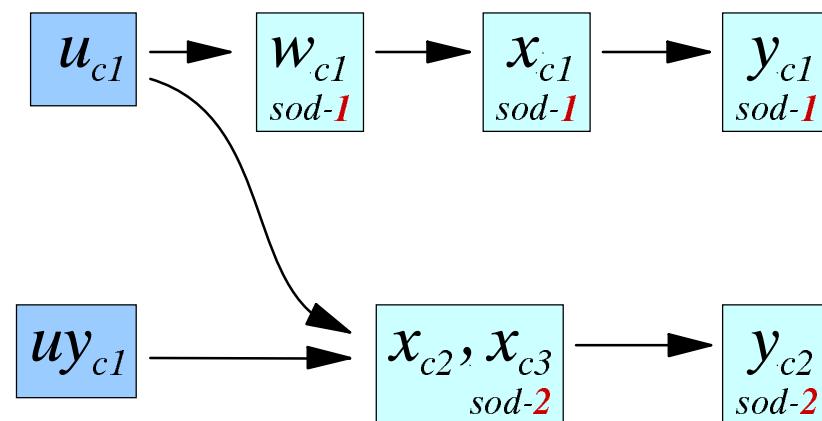
Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning



Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning



Algorithmic Decomposition

- Observability analysis algorithm and labeling/partitioning

