Massachusetts Institute of Technology

# DNNF-based Belief State Estimation

Paul Elliott
Brian Williams

Model-based Embedded & Robotic Systems

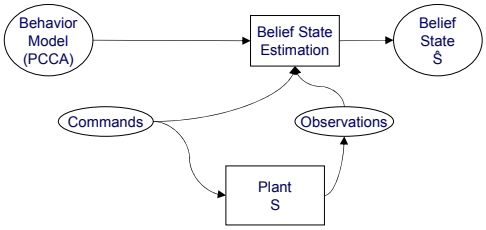
CSAIL

## Motivation

- Complex embedded systems

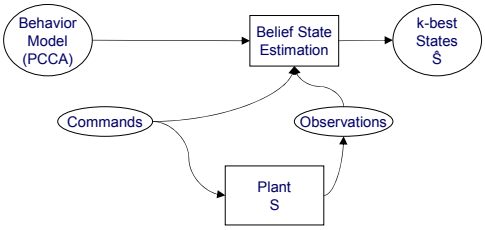


## Belief State Estimation



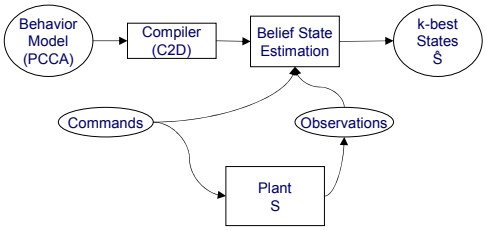



## Approximate Belief State Estimation





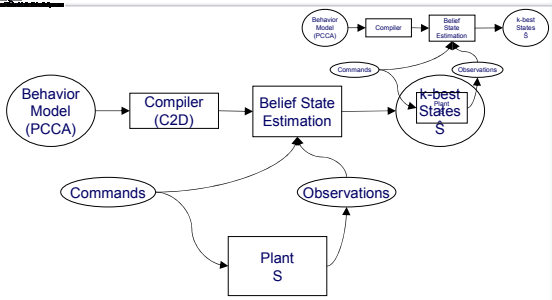

## Compiled Approx Belief State Estimation




Darwiche C2D 2005 v2.20

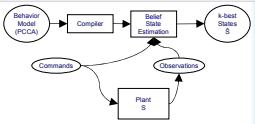


## Compiled Approx Belief State Estimation



## Prior Work

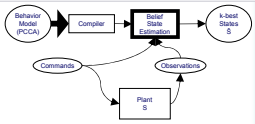
- MEXEC (Barrett '05)
  - Trajectories
    - Max-Prod
  - sd-DNNF
- BFBSU (Martin '05)
  - Approximate Belief State
    - Max-Sum-Prod
  - Optimal constraint-solver
- Dynamic Bayesian Networks
  - Max-Prod
  - Sum-Prod
  - Distribution on variables





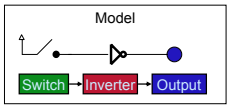

## Contributions

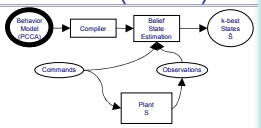
- New encoding of estimation for compilation
- New use of sd-DNNF
- New k-best algorithm for sd-DNNF
- New bounded believe state estimation algorithm





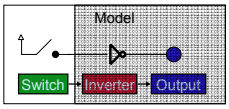

## Model: Probabilistic Concurrent Constraint Automata (PCCA)

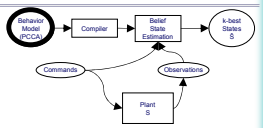


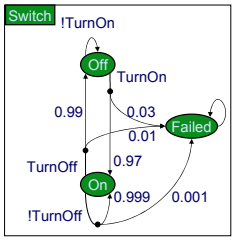






## PCCA: Switch



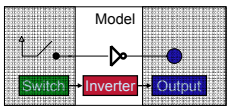



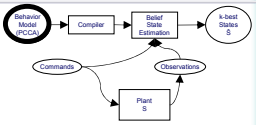

Constraints

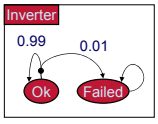
- Off : $S_{out}$ = Low
- On : $S_{out}$ = High
- Failed : $S_{out} \in$ {Low, High}



## PCCA: Inverter



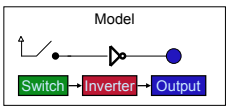

Constraints

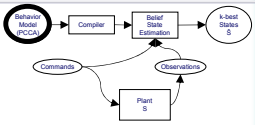
| | $I_{in}$ | $I_{out}$ |
|---|---|---|
| Ok : | High | Low |
| | Low | High |

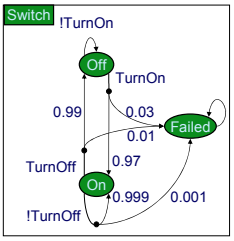
Failed : $I_{in} \in$ {Low, High}

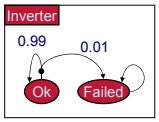


## PCCA Model



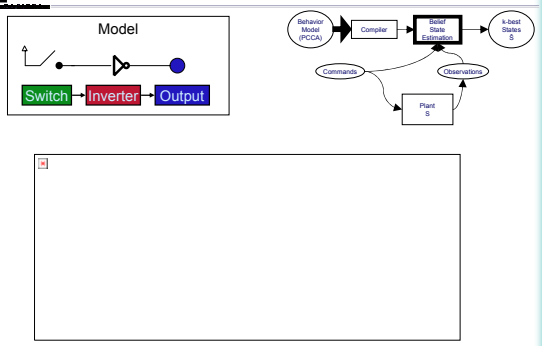




Constraints

- On ∧ Ok ➔ Low
- Off ∧ Ok ➔ High

## Estimation Function

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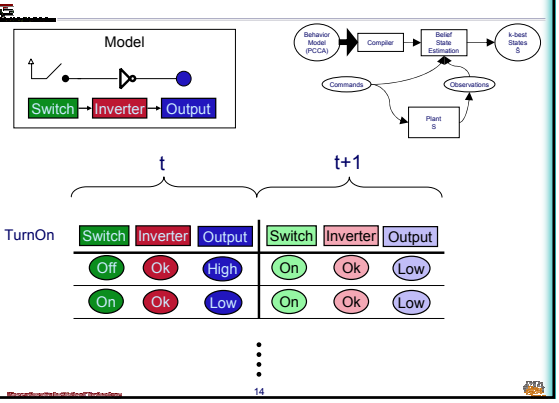
## PCCA as a Relation

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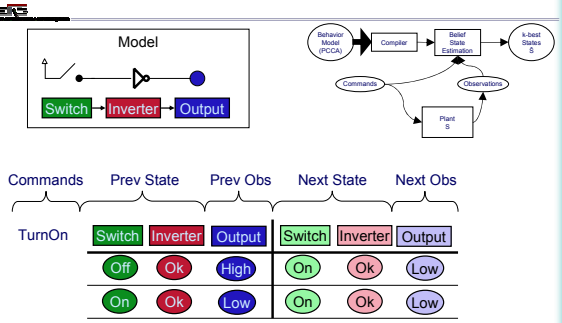
## PCCA as a Probabilistic Relation

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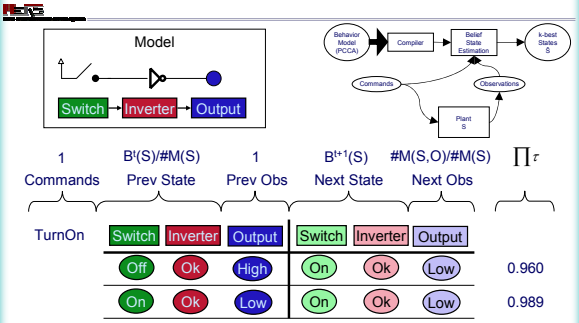
## PCCA as a Probabilistic Relation

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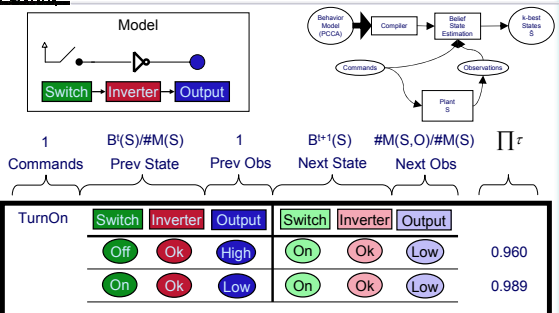
## sd-DNNF: Relation Encoding

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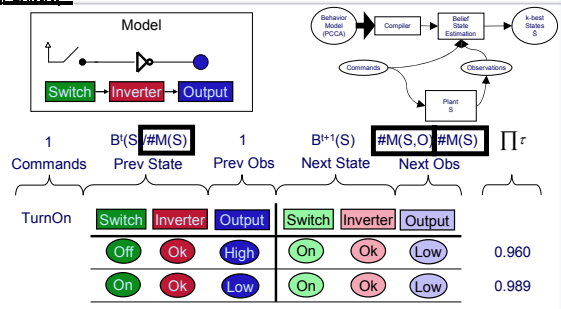
## sd-DNNF: Model Counting

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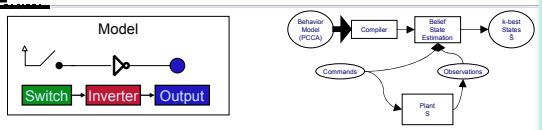
## sd-DNNF: Probabilities

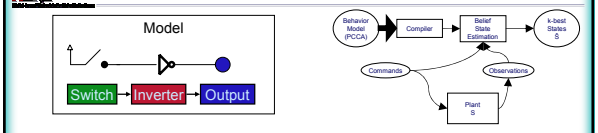



| 1 | $B^t(S)/\#M(S)$ | 1 | $B^{t+1}(S)$ | $\#M(S,O)/\#M(S)$ | $\prod \tau$ |
|---|---|---|---|---|---|
| Commands | Prev State | Prev Obs | Next State | Next Obs | |
| TurnOn | Switch Inverter Output | | Switch Inverter Output | | |
| | Off Ok High | | On Ok Low | | 0.960 |
| | On Ok Low | | On Ok Low | | 0.989 |

⋮



## sd-DNNF: States

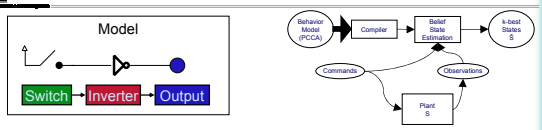



| 1 | $B^t(S)/\#M(S)$ | 1 | $B^{t+1}(S)$ | $\#M(S,O)/\#M(S)$ | $\prod \tau$ |
|---|---|---|---|---|---|
| Commands | Prev State | Prev Obs | Next State | Next Obs | |
| TurnOn | Switch Inverter Output | | Switch Inverter Output | | |
| | Off Ok High | | On Ok Low | | 0.960 |
| | On Ok Low | | On Ok Low | | 0.989 |

⋮



## Estimation

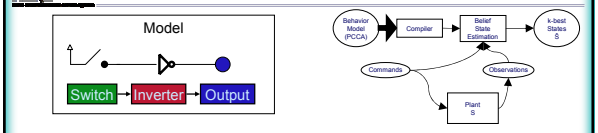



| | | | | | | |
|---|---|---|---|---|---|---|
| TurnOn | Switch | Inverter | Output | Switch | Inverter | Output |
| $B^{t+1}(S)$ | Off | Ok | High | On | Ok | Low 0.960 |
| On Ok | On | Ok | Low | On | Ok | Low 0.989 |

⋮



## Estimation: k-best Approx



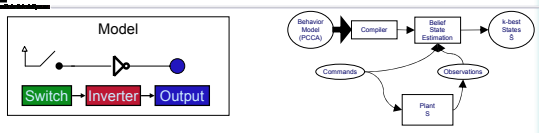

$B^t(S)/\#M(S)$ $\#M(S,O)/\#M(S)$ $\prod \tau$

| | | | | | | |
|---|---|---|---|---|---|---|
| TurnOn | Switch | Inverter | Output | Switch | Inverter | Output |
| $\underset{k}{\text{argmax}}\ B^{t+1}(S)$ | Off | Ok | High | On | Ok | Low 0.960 |
| On Ok | On | Ok | Low | On | Ok | Low 0.989 |

⋮



## Estimation: k-best Approx




$B^t(S)/\#M(S)$ $\#M(S,O)/\#M(S)$ $\prod \tau$

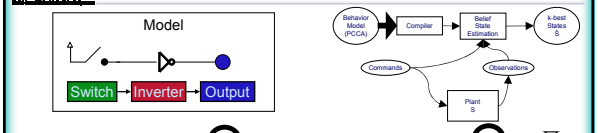
| | | | | | | |
|---|---|---|---|---|---|---|
| TurnOn | Switch | Inverter | Output | Switch | Inverter | Output |
| $\underset{k}{\text{argmax}}\ B^{t+1}(S)$ | Off | Ok | High | On | Ok | Low 0.960 |
| On Ok | On | Ok | Low | On | Ok | Low 0.989 |

Max-Sum-Product-Sum

⋮



## Estimation: k-best Approx (Division)



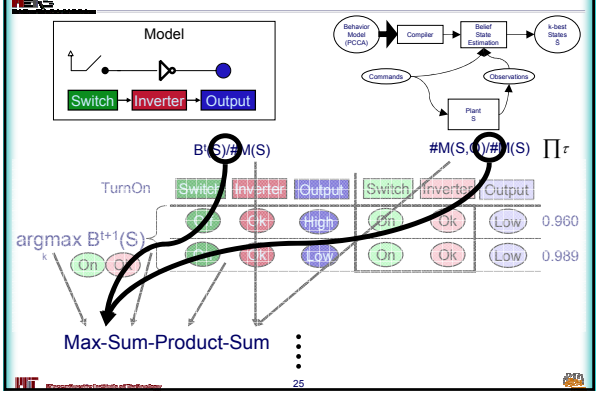

$B^t(S)/\#M(S)$ $\#M(S,O)/\#M(S)$ $\prod \tau$

| | | | | | | |
|---|---|---|---|---|---|---|
| TurnOn | Switch | Inverter | Output | Switch | Inverter | Output |
| $\underset{k}{\text{argmax}}\ B^{t+1}(S)$ | Off | Ok | High | On | Ok | Low 0.960 |
| On Ok | On | Ok | Low | On | Ok | Low 0.989 |

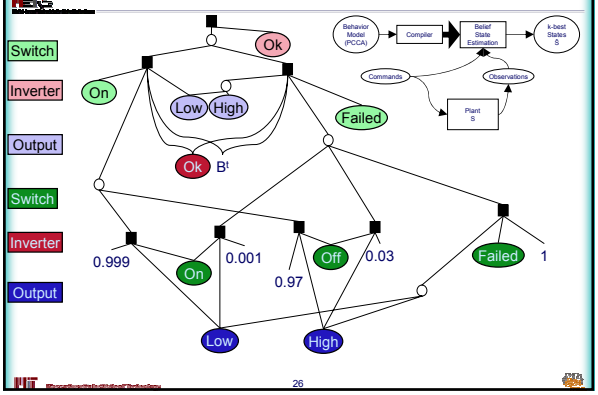
Max-Sum-Product-Sum

⋮

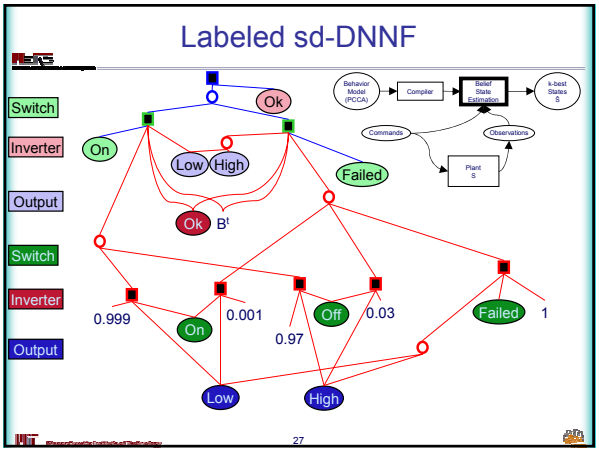
## Estimation: k-best Approx (Division)




## C2D 2005 v2.20: Compilation

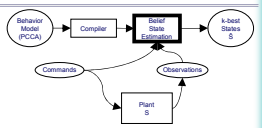



## Labeled sd-DNNF

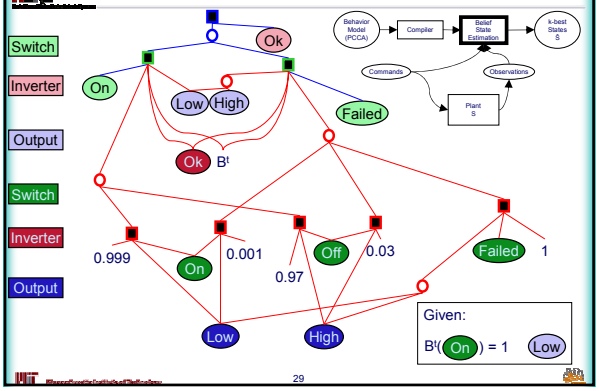



## Estimation Algorithm

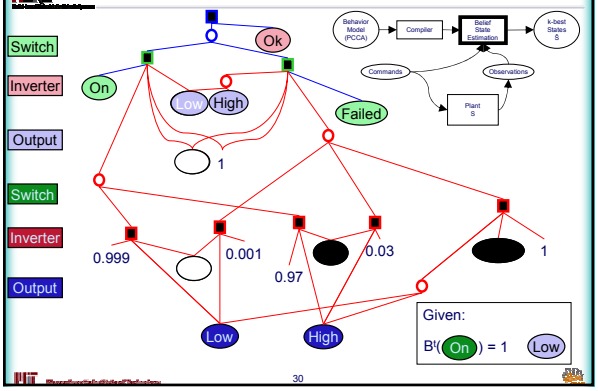
1. Assign known values and probabilities
2. First Pass: Compute all summations
3. Merge summation results
4. Second Pass: Extract the k-best solutions
   1. Upward part: Compute the probabilities and paths of the k-best estimates
   2. Downward part: Follow the paths to extract the k-best estimates

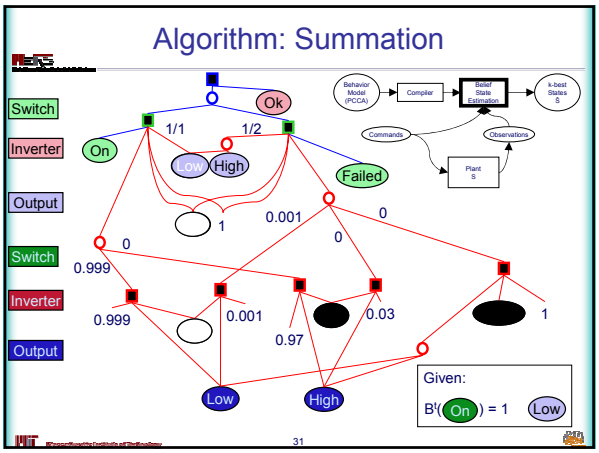


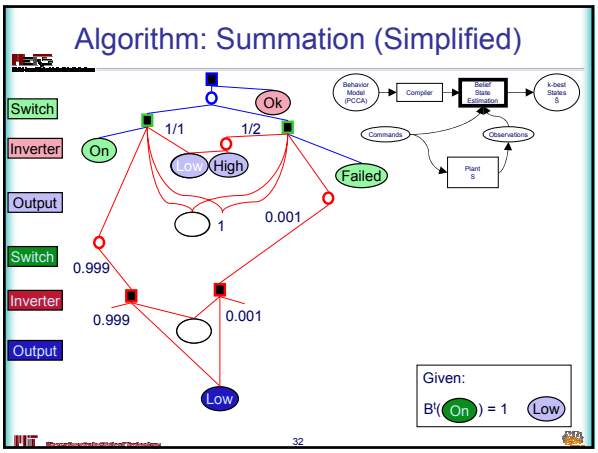

## Algorithm: Initial Assignment

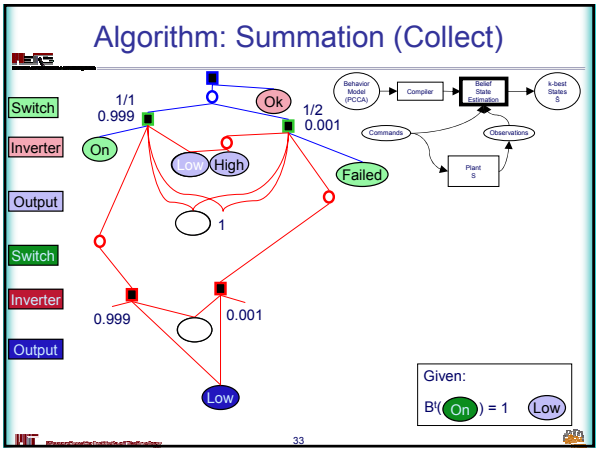


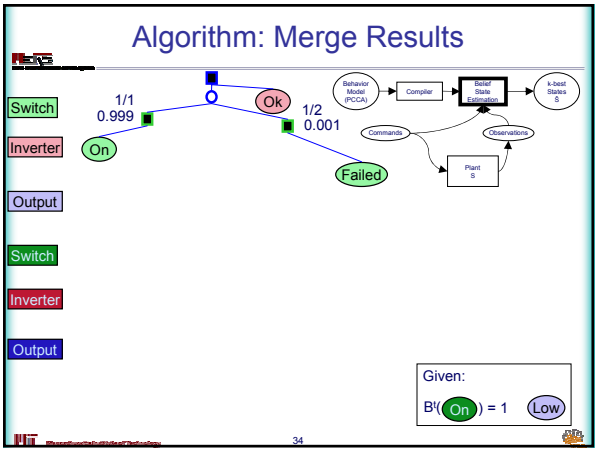

## Algorithm: Initial Assignment

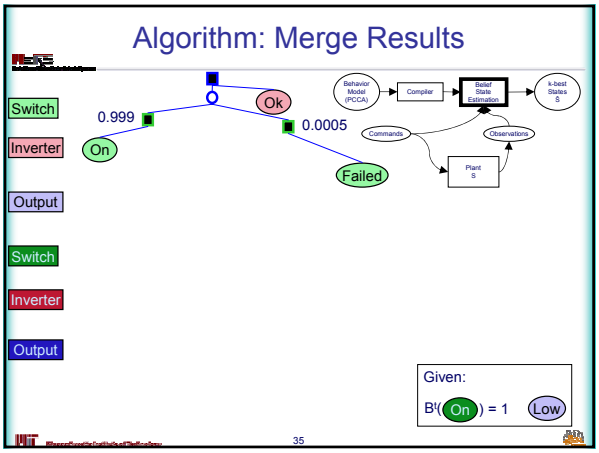


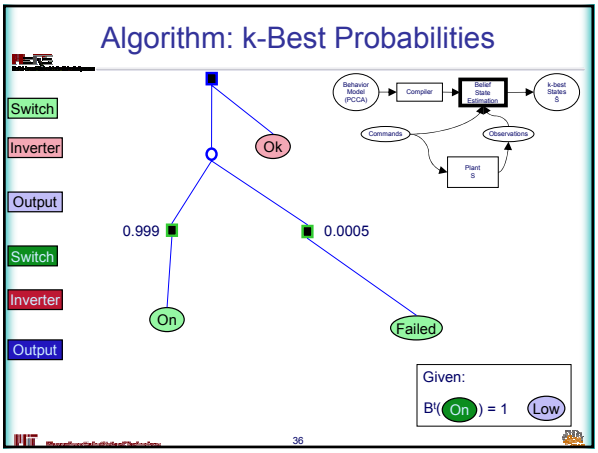

Algorithm: Summation
Switch
Inverter
Output
Switch
Inverter
Output
Ok
On
Low
High
Failed
1/1
1/2
1
0.001
0
0
0
0.999
0.999
0.001
0.97
0.03
1
Low
High
Given:
B^t( On ) = 1 Low
31


Algorithm: Summation (Simplified)
Switch
Inverter
Output
Switch
Inverter
Output
Ok
On
Low
High
Failed
1/1
1/2
1
0.001
0.999
0.999
0.001
Low
Given:
B^t( On ) = 1 Low
32

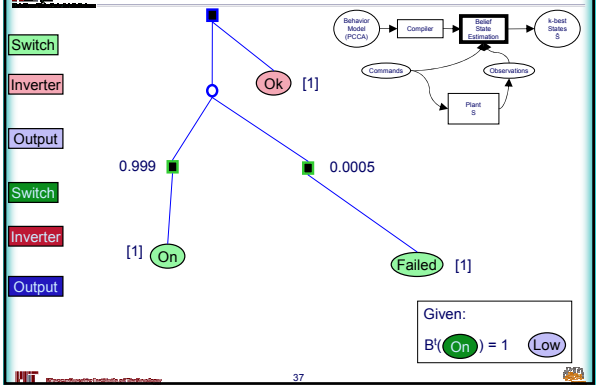

Algorithm: Summation (Collect)
Switch
Inverter
Output
Switch
Inverter
Output
1/1
0.999
Ok
1/2
0.001
On
Low
High
Failed
1
0.999
0.001
Low
Given:
B^t( On ) = 1 Low
33


Algorithm: Merge Results
Switch
Inverter
Output
Switch
Inverter
Output
1/1
0.999
Ok
1/2
0.001
On
Failed
Given:
B^t( On ) = 1 Low
34

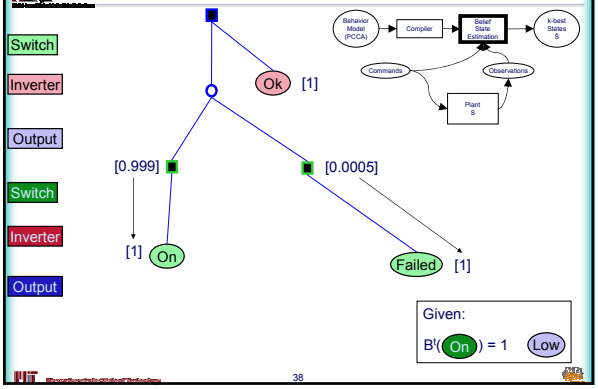

Algorithm: Merge Results
Switch
Inverter
Output
Switch
Inverter
Output
0.999
Ok
0.0005
On
Failed
Given:
B^t( On ) = 1 Low
35


Algorithm: k-Best Probabilities
Switch
Inverter
Output
Switch
Inverter
Output
Ok
0.999
0.0005
On
Failed
Given:
B^t( On ) = 1 Low
36

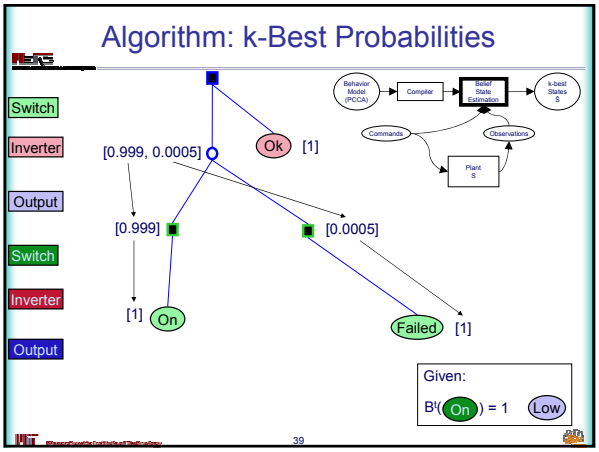
## Algorithm: k-Best Probabilities



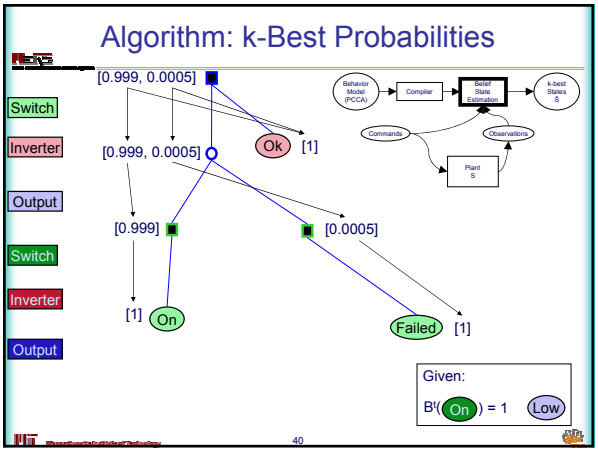

## Algorithm: k-Best Probabilities



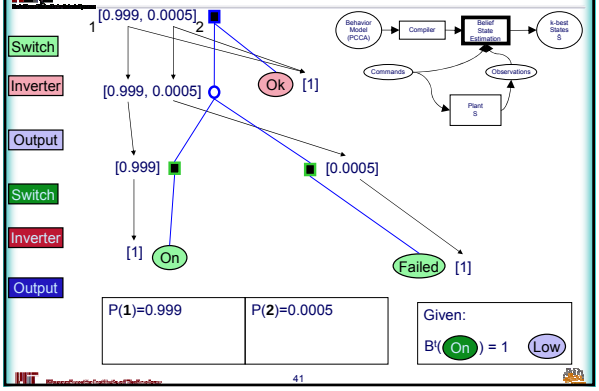

## Algorithm: k-Best Probabilities



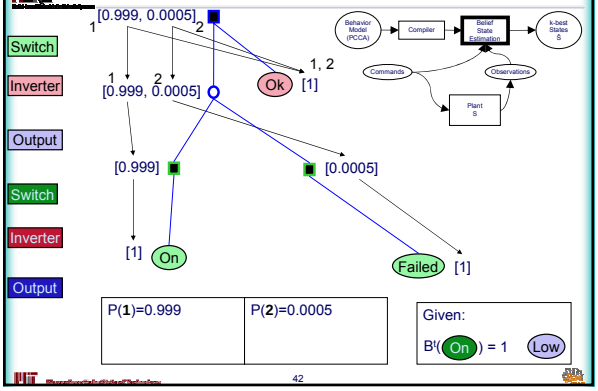

## Algorithm: k-Best Probabilities



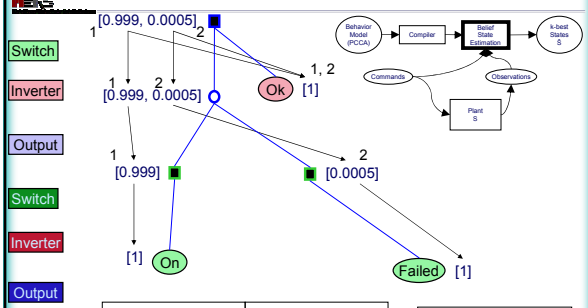

## Algorithm: k-Best Solutions




## Algorithm: k-Best Solutions



## Algorithm: k-Best Solutions



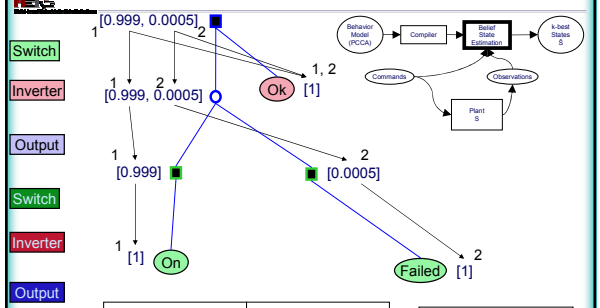

| P(**1**)=0.999 | P(**2**)=0.0005 |
|---|---|

Given:

$B^t$( On ) = 1 Low



## Algorithm: k-Best Solutions



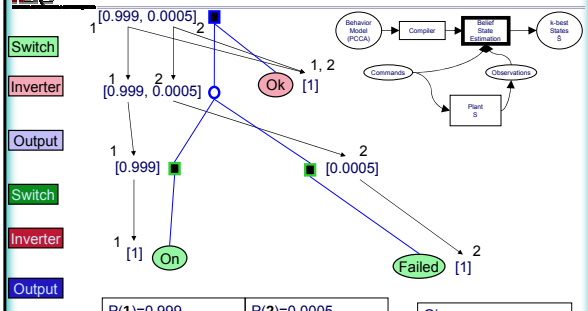

| P(**1**)=0.999 | P(**2**)=0.0005 |
|---|---|

Given:

$B^t$( On ) = 1 Low



## Algorithm: k-Best Solutions




| P(**1**)=0.999 | P(**2**)=0.0005 |
|---|---|
| On Ok | Failed Ok |

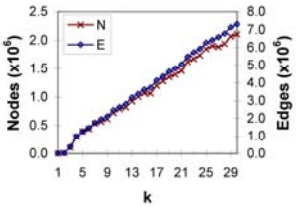
Given:

$B^t$( On ) = 1 Low



## Results

- EDL Model
  - 42 Variables
  - 10 State Variables
  - 4.4 Values/Variable
- Model Size (k > 3)
  - $(71 \pm 2.5) \times 10^3 \times k$ nodes
  - $(244 \pm 4.7) \times 10^3 \times k$ edges
- Model Size (k < 3)
  - $(4.4 \pm 0.37) \times 10^3 \times k$ nodes
  - $(14.8 \pm 0.34) \times 10^3 \times k$ edges




- Algorithm Complexity
  - $O(k^2 e_n)$ space
  - $O(k^3 e_n)$ time

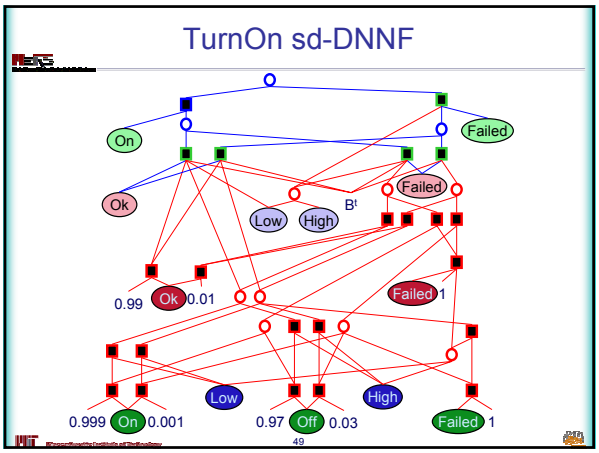


## Conclusion

- New Estimation Algorithm
  - Linear in the size of the sd-DNNF, cubic time with k
  - Estimates k belief states
- New Algorithm for computing Max-Sum-Product on an sd-DNNF
  - Operates in two phases
    1. Computes Sum-Product
    2. Computes Max-Product
- An encoding of PCCA estimation as a probabilistic relation



## Questions?


TurnOn sd-DNNF
On
Failed
Ok
Low
High
B'
Failed
0.99
Ok
0.01
Failed
1
Low
High
0.999
On
0.001
0.97
Off
0.03
Failed
1
49